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# The spatial ultimatum game

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In the ultimatum game, two players are asked to split a certain sum of money. The proposer has to make an offer. If the responder accepts the offer, the money will be shared accordingly. If the responder rejects the offer, both players receive nothing. The rational solution is for the proposer to offer the smallest possible share, and for the responder to accept it. Human players, in contrast, usually prefer fair splits. In this paper, we use evolutionary game theory to analyse the ultimatum game. We first show that in a non-spatial setting, natural selection chooses the unfair, rational solution. In a spatial setting, however, much fairer outcomes evolve.

**Keywords:** evolution; fairness; rationality; game theory; spatial dynamics

## 1. INTRODUCTION

Since its introduction by Güth *et al.* (1982), the ultimatum game has fascinated game theorists and experimental economists. The rules can be stated in a couple of lines. Two players are offered a gift, provided they manage to share it. One of the players—the proposer—suggests how to split the offer, the other player—the responder—can either agree or else reject the deal. In each case the decision is final.

A rational responder bent on maximizing his utility should accept even the smallest positive offer, because the alternative is getting nothing. A rational proposer who believes that his opponent is rational should therefore claim almost the entire sum. But it was found in a large number of experiments, in many countries and for varied stakes, that this is not how humans play the game. Most proposers offer a fair share—in fact, some 60–80% of proposers offer fractions between 0.4 and 0.5, and only 3% offer less than 0.2. They are well advised to do this—indeed, some 50% of responders reject any split offering them less than one-third of the sum (for surveys see Thaler 1988; Güth & Tietze 1990; Roth *et al.* 1991; Bolton & Zwick 1995; Roth 1995).

There are many explanations for this uneconomical emphasis on a fair division; this seems to reflect the psychological fact that humans use a utility function which does not simply correspond to the expected pay-off, but is also a decreasing function of the difference between the pay-off values of the two players engaged in the game (Kirchsteiger 1994; Bethwaite & Tompkinson 1996; Fehr & Schmidt 1999). In particular, the rejection of a low offer by the responder can be seen as a kind of punishment inflicted by the responder on the proposer (who loses much more than the responder, in that case). Many theoretical and experimental investigations (see Boyd & Richerson (1992), for instance, or Fehr & Gächter (1999)) have stressed the important role of punishment for inhibiting selfish individuals.

A frequently used explanation for the human behaviour in the ultimatum game is that the players do not realize that they interact only once. They are expecting repeated interactions, even if the experimenter makes it clear that there will be no repetition. The game can be repeated in different ways. A fixed proposer and responder may simply play a number of times and sum their pay-offs from each game. This may lead to an incentive for the responder to reject low offers to obtain more in subsequent rounds. Alternatively, players may play a repeated ultimatum game in which they take turns in the role of proposer and have to divide a single sum. The game can continue until they reach agreement. This is equivalent to haggling over a price. If it is assumed that the sum is discounted from round to round ('the shrinking pie'), one obtains a very convincing model of bargaining (Rubinstein 1982; Binmore 1992).

The tacit expectation of a further round is certainly as plausible to explain fairness in the ultimatum game, as to explain cooperation in the prisoner's dilemma. However, it has been shown that a spatial population structure changes the outcome of the one-round prisoner's dilemma to a considerable extent (Nowak & May 1992; Nowak *et al.* 1994). In this paper, we investigate similar effects of a spatial structure on the ultimatum game. Again, neighbourhood relations and structured interactions affect the evolution of the strategies—in particular, fair shares become a likely outcome.

This approach is related to Huck & Öchssler (1999), where it is shown—in a non-spatial context—that ultimatum games played in small groups can lead to positive offers. Small groups localize the competition. For reproductive success, the possibility that the co-player gets a relative advantage might become more important than the size of the player's own pay-off. This effect, however, can only lead to offers which are less than the reciprocal of the group size. In our spatial set-up, we get considerably higher offers.

In a biological context, the ultimatum game could perhaps describe two individuals trying to divide in advance the reward of a task that they can only perform jointly, such as cooperative hunting or forming an alliance against another group member. If one individual is dominant, then it is possible that that individual would determine the split and that the other individual would simply have to take it or leave it. The ultimatum game could also reflect dilemmas of food sharing: if the players do not

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Table 1. *Evolution in the non-spatial ultimatum game leads to a population of near 'rational' players*

(The table shows the mean offer and acceptance level and also the standard deviations (spread within the population) in a population of 100 individuals, for simulations with various mutation errors, $\varepsilon$. Initially all individuals have randomly distributed offers and acceptance rates. Everyone plays everyone else and the number of offspring of a given individual is proportional to his total pay-off. The values given are averages over time, sampled at $10^4$ generation intervals between $10^5$ generations and $10^6$ generations. The outcome approaches rational behaviour as $\varepsilon$ becomes infinitesimally small. For larger $\varepsilon$, the heterogeneity of the population favours non-zero acceptance rates which in turn favour non-zero offers.)

| $\varepsilon$ | $\bar{p}$ | $\bar{q}$ |
|---|---|---|
| 0.001 | 0.064 ± 0.002 | 0.049 ± 0.002 |
| 0.002 | 0.077 ± 0.004 | 0.053 ± 0.005 |
| 0.01 | 0.109 ± 0.016 | 0.051 ± 0.019 |
| 0.02 | 0.145 ± 0.027 | 0.057 ± 0.031 |
| 0.1 | 0.269 ± 0.089 | 0.104 ± 0.080 |
| 0.2 | 0.327 ± 0.142 | 0.141 ± 0.120 |

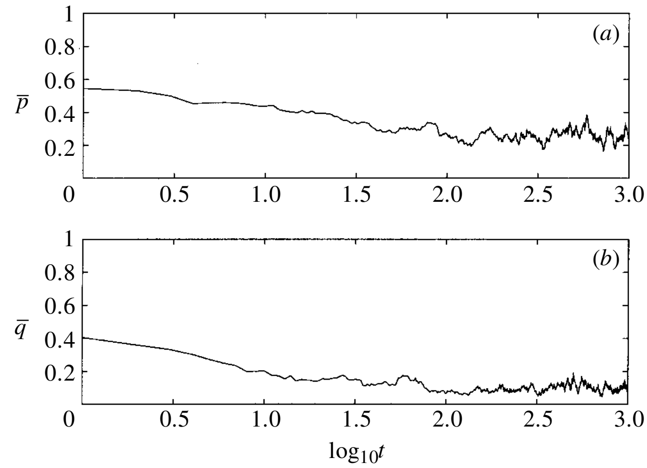


Figure 1. This figure shows the time evolution of the average offer (*a*) and acceptance levels (*b*) in simulations of the non-spatial ultimatum game. Initially the 100 individuals in the population have random offers and acceptance levels. Everyone plays everyone else (both as proposer and responder) and the number of offspring of a given individual is proportional to his total pay-off. The mutation error, $\varepsilon$, is 0.1. The time-scale is logarithmic to illustrate the long term (albeit noisy) convergence.

agree on a split, there is a possibility that someone will take away the food or the food will disappear (for example, in cases of stream feeding of fish; M. Milinski, personal communication). Hence, while the experimental situation of an isolated, anonymous ultimatum game is somewhat artificial, it is very likely that situations similar to it have shaped the fairness instinct of animals and humans for millions of years.

In addition, experiments on the ultimatum game shed a striking light on our mental equipment for social and economic life. Why do fairness considerations matter more, to many of us, than rational utility maximization?

## 2. RANDOM ENCOUNTERS

We normalize the sum which is to be divided by the two players to be equal to unity, and consider strategies given by two parameters $p$ and $q$ in the unit interval. The parameter $p$ denotes the amount offered to the other player if one is in the role of the proposer, while $q$ denotes the minimum acceptance level (or aspiration level) when one is the responder.

Let us suppose that in an interaction between a player using strategy $S_1 = (p_1, q_1)$ and a player using strategy $S_2 = (p_2, q_2)$, each player can be in the role of proposer with equal probability. The expected value of the pay-off for the $S_1$ player against the $S_2$ player, $E(S_1, S_2)$, is given (up to the factor $\frac{1}{2}$, which we henceforth omit) by

$$E(S_1, S_2) = \begin{cases} 1 - p_1 + p_2 & \text{if } p_1 \geqslant q_2 \text{ and } p_2 \geqslant q_1 \\ 1 - p_1 & \text{if } p_1 \geqslant q_2 \text{ and } p_2 < q_1 \\ p_2 & \text{if } p_1 < q_2 \text{ and } p_2 \geqslant q_1 \\ 0 & \text{if } p_1 < q_2 \text{ and } p_2 < q_1. \end{cases} \quad (1)$$

Consider the following evolutionary dynamics. There is a population of $N$ players. In each round (generation), every player interacts with every other player. The pay-offs are added up. For the next generation, players leave offspring in numbers proportional to their fitness. Offspring inherit (or learn) their parent's strategy subject to some small mutation: their $p$- and $q$-values are randomly chosen within an interval of size $\varepsilon$ centred around their parent's $p$- and $q$-values.

In simulations of a population of $N = 100$ players, we calculated the time averages of the mean $p$- and $q$-values, $\bar{p}$ and $\bar{q}$ respectively, and the average values (over time) of the standard deviations of $p$ and $q$ within the population. These are shown in table 1, for various values of $\varepsilon$. For very small $\varepsilon$, the $\bar{p}$- and $\bar{q}$-values tend to zero. Thus accurate reproduction (or imitation) of strategies favours the rational outcome. For larger values of $\varepsilon$, however, we observe significant positive offers and aspiration levels. For example, $\varepsilon = 0.1$ leads to $\bar{p} \approx 0.27$ and $\bar{q} \approx 0.10$. It should be noted that this effect is not simply a consequence of mutational noise, but involves selection at least on the value of $\bar{p}$.

In § 2(a), we show that heterogeneity in the population favours non-zero offers. In this context, we also refer to the paper by Gale *et al.* (1995) where the important role of errors is analysed in the context of mini-games (games with only two levels of offer and demand).

Figure 1 shows the time evolution of $\bar{p}$ and $\bar{q}$ in a simulation of a population of 100 individuals, who initially have randomly assigned $p$- and $q$-values. The mutation error, $\varepsilon$, is 0.1. For small mutation errors, evolution leads to a population of near rational players.

### (a) *What is the best response to a random strategy?*

We have seen in § 2 that a small error in imitating a parent's strategy can lead to a diversity of strategies within the population. This leads to a pressure to offer a non-zero amount. Perhaps it is smart to have a strategy which, instead of being the optimal response to another rational player, fares well against a population in which there is some diversity. As an illustration that these are not the same in the ultimatum game, we consider the best

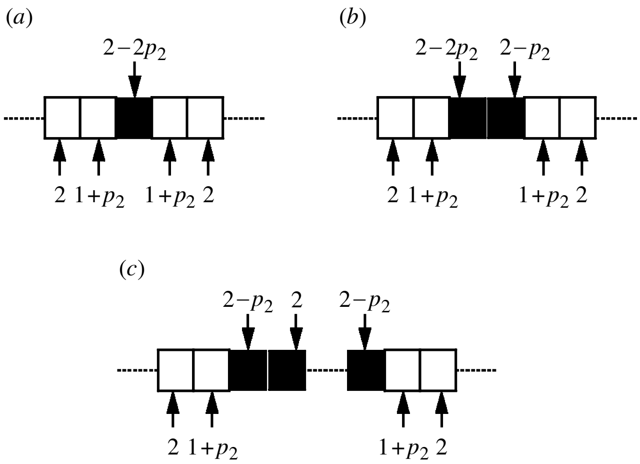


Figure 2. When do mutant clusters spread in the one-dimensional spatial ultimatum game? The majority of the population play strategy $S_1 = (p_1, q_1)$. A cluster of mutants forms which play strategy $S_2 = (p_2, q_2)$, where $p_2 \geqslant q_2 \geqslant p_1 \geqslant q_1$. (*a*) A single player playing the mutant strategy, $S_2$, is shown in black, in a ring of players playing the resident strategy, $S_1$, in white. The pay-off obtained by each player is indicated by the arrows. The mutant is likely to propagate if $p_2 \leqslant 0.39\ldots$. (*b*) An adjacent pair of mutant players is likely to spread if $p_2 \leqslant 0.43\ldots$. (*c*) A cluster containing at least three players is likely to grow if $p_2 \leqslant 0.5$.

response to a population the strategies of which are randomly distributed. We note that the rational solution does not maximize the expected pay-off in a population of players whose $(p, q)$-strategies are randomly distributed in the unit square. This can be shown as follows.

If we assume that the $p$- and $q$-values are uniformly distributed between zero and unity, we see immediately that the strategy $S = (\frac{1}{2}, 0)$ does best. The average pay-off, $P$, of a strategy $S = (p, q)$ against opponents in the square is given by

$$\begin{aligned} P &= \int_0^1 \int_0^1 (1-p) I[p \geqslant q'] + p' I[p' \geqslant q] \mathrm{d}q' \mathrm{d}p' \qquad (2) \\ &= (1-p) \int_0^p \mathrm{d}q' + \int_q^1 p' \mathrm{d}p' \\ &= p(1-p) + \tfrac{1}{2}(1-q^2), \end{aligned}$$

where $I[\ldots]$ is the indicator function, taking value unity if its argument is true and zero if it is false. The average pay-off, $P$, is thus maximized by $(p, q) = (\frac{1}{2}, 0)$.

Thus, the best response to the rational strategy is $(0, 0)$ itself, and the best response to a random strategy is $(\frac{1}{2}, 0)$.

## 3. THE ONE-DIMENSIONAL SPATIAL ULTIMATUM GAME

So far we have assumed that players meet randomly. If there is any social structure—due, for instance, simply to a spatial arrangement—the outcome can be very different. Because encounters are no longer randomized, the success of a strategy $S_2$ invading a population using strategy $S_1$ depends not only on the pay-offs $E(S_1, S_1)$ and $E(S_2, S_1)$, i.e. on encounters with the vast majority, but also on $E(S_1, S_2)$ and $E(S_2, S_2)$. The invading minority can affect locally the resident's pay-off. Beating one's neighbours is now all important.

Table 2. *Fairness emerges in the spatial ultimatum game*

(The table shows the average offer and acceptance level in and the standard deviations for simulations with mutation error $\varepsilon = 0.001$. As in table 1, the values given are averages over time, sampled at $10^4$ generation intervals between $10^5$ generations and $10^6$ generations. $N$ is the number of individuals in the population, which is arranged one-dimensionally with periodic boundary conditions (a ring). The neighbourhood size is the number of individuals with whom a given individual plays the game. Each individual competes to keep his position in the ring in the following generation with the same individuals with whom he plays. Initially all individuals have random offer and acceptance levels.)

| $N$ | neighbourhood size | $\bar{p}$ | $\bar{q}$ |
|---|---|---|---|
| 100 | 2 | $0.453 \pm 0.007$ | $0.432 \pm 0.008$ |
| | 6 | $0.326 \pm 0.004$ | $0.306 \pm 0.004$ |
| | 10 | $0.243 \pm 0.003$ | $0.226 \pm 0.003$ |
| 500 | 2 | $0.468 \pm 0.016$ | $0.446 \pm 0.019$ |
| | 6 | $0.455 \pm 0.009$ | $0.438 \pm 0.013$ |
| | 10 | $0.394 \pm 0.006$ | $0.377 \pm 0.008$ |

Let us consider players located on a one-dimensional lattice, each interacting with his two nearest neighbours. Players also compete only with their nearest neighbours for offspring. We once again assume that offspring are apportioned probabilistically in proportion to the total scores of the parents. The probability that the offspring at a specific site belongs to a given member of the neighbourhood of that site is equal to that player's score divided by the total score of the three players in the neighbourhood. For a more general discussion of the formulation of spatial games, see Killingback & Doebeli (1998).

Consider the strategies $S_1$ and $S_2$ with $q_1 \leqslant p_1 < q_2 \leqslant p_2$. We know that in the non-spatial game there is a bistable equilibrium between these two strategies, because each strategy is the better response against itself. Hence a small proportion of $S_2$-strategists cannot invade a population of $S_1$-strategists. Suppose, however, that there is a single $S_2$-strategist ('mutant') isolated in a sea of $S_1$-strategists ('residents'). With the exception of the players at the frontiers, all have pay-off 2 (with one provided from each neighbour). At the frontier, the $S_2$-strategist will obtain $2(1-p_2)$, whereas its $S_1$-neighbours obtain $1 + p_2$ each (see figure 2*a*). Hence the probability of each of the neighbours of the mutant to become ousted by the mutant in the next generation is $2(1-p_2)/[2(1-p_2) + (1+p_2) + 2] = 2(1-p_2)/(5-p_2)$ and the probability of the mutant to become ousted by the resident-type in the next generation is $2(1+p_2)/[2(1+p_2) + 2(1-p_2)] = (1+p_2)/2$. Thus the expected number of offspring of the mutant exceeds unity if $2 \times 2(1-p_2)/(5-p_2) \geqslant (1+p_2)/2$, i.e. if $p < 0.26\ldots$. For an adjacent pair of $S_2$-players, the critical condition is $p_2 < 0.43\ldots$ (see figure 2*b*).

When there is a small cluster of at least three $S_2$-strategists (see figure 2*c*), all players, except those at the frontier, have pay-off 2. The $S_1$-player at each frontier obtains $1 + p_2$, the $S_2$-player next to it obtains $2 - p_2$. Hence the probability that a site at the boundary switches from resident to

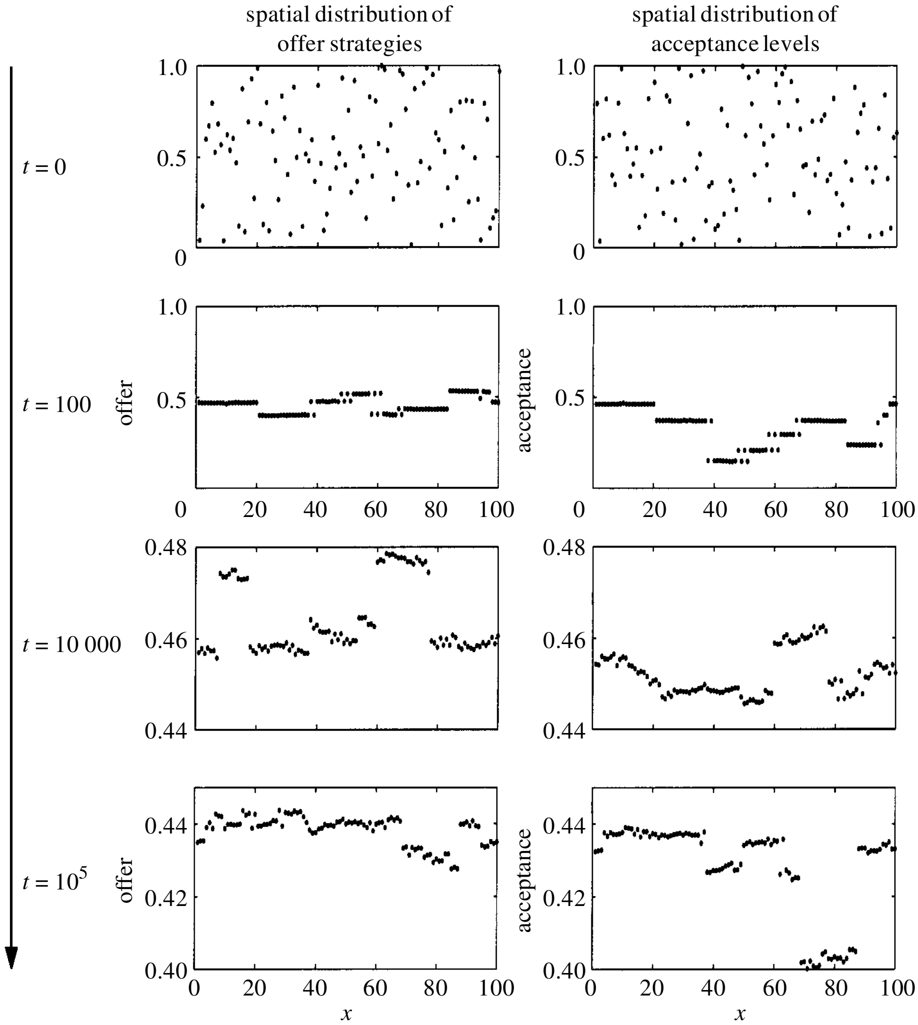


Figure 3. Temporal behaviour of the one-dimensional spatial ultimatum game. The ring contains 100 individuals who interact with their immediate neighbours. Initially, the offers and acceptance levels converge within clusters. A cluster with low offer and acceptance level spreads and dominates. Subsequently, clustering is established on a smaller scale. Finally, the whole ring of players evolves towards roughly fair strategies. The mutation error is $\varepsilon = 0.001$.

mutant in the next generation is $(2-p_2)/(2+2-p_2+1+p_2) = (2-p_2)/5$ and the probability for the opposite switch is likewise $(1+p_2)/5$. Thus clusters of the mutant strategy, $S_2$, tend to spread if $(2-p_2)/5 > (1+p_2)/5$, i.e. if $p_2 < \frac{1}{2}$.

We performed numerical simulations of the spatial ultimatum game with two total population sizes ($N = 100, 500$) and various neighbourhood sizes ($n = 2, 6, 10$). The neighbourhood size is the number of individuals who play the game with a given individual. The individual at a given site competes with its $n$ 'neighbours' to give rise to the offspring at that site. The resulting average offers and acceptance levels together with their standard deviations are shown in table 2. With increasing $N$ and decreasing $n$, evolution leads to players offering and demanding an almost fair split. Note that the results of these simulations were not significantly affected by whether each pair of neighbours played the game only once with the role of proposer assigned randomly or whether they played twice, each taking the role of proposer once. In the above analysis we have assumed the latter for simplicity.

Figure 3 shows the evolutionary dynamics of the system starting from a random initial condition. Initially, when the offers within these clusters are different from the acceptance levels, natural selection favours the domination of a cluster with offer and acceptance level close together. Of course, the offer always exceeds the acceptance level. Clustering then develops on a more microscopic scale and the system is slowly driven towards fairness.

## 4. THE TWO-DIMENSIONAL SPATIAL ULTIMATUM GAME

We now consider players arranged on a two-dimensional square lattice. Each player interacts with his neighbours directly above, below, to the left and to the right. This is called a von Neumann neighbourhood. Unlike the one-dimensional case, there are are many configurations of mutant cluster that we could consider. However, it appears that the reproductive success of a $3 \times 3$ square mutant cluster is critical to the ability of the mutant to invade (Killingback *et al.* 1999). If we consider, once

Table 3. *In the two-dimensional spatial ultimatum game, we observe offers which are somewhere between fair* ($\frac{1}{2}$) *and rational* (0)

(The players are arranged on a square grid and play the game and compete for offspring with those players directly above, below, to the right and to the left of them (a von Neumann neighbourhood). The table shows the average offers and acceptance levels and standard deviations for $\varepsilon = 0.001$ on square grids of various size (fixed boundaries; players at the edges have fewer neighbours). The time averages and standard deviations are calculated as in tables 1 and 2.)

| grid size | $\bar{p}$ | $\bar{q}$ |
|---|---|---|
| 10 × 10 | 0.228 ± 0.006 | 0.205 ± 0.006 |
| 20 × 20 | 0.291 ± 0.007 | 0.273 ± 0.009 |
| 30 × 30 | 0.308 ± 0.008 | 0.287 ± 0.012 |
| 50 × 50 | 0.326 ± 0.009 | 0.300 ± 0.017 |
| 100 × 100 | 0.340 ± 0.012 | 0.312 ± 0.022 |
| 200 × 200 | 0.354 ± 0.014 | 0.326 ± 0.024 |
| 300 × 300 | 0.361 ± 0.016 | 0.334 ± 0.025 |
| 500 × 500 | 0.369 ± 0.019 | 0.342 ± 0.028 |

again, a resident strategy, $S_1$, and a mutant strategy, $S_2$, with $q_1 \leqslant p_1 < q_2 \leqslant p_2$, then we find that a $3 \times 3$ mutant cluster of the fairer type is likely to expand if and only if $p_2 < 0.342\ldots$ (see Appendix A).

The results of numerical simulations of the ultimatum game on square lattices of various sizes are shown in table 3. As the grid size becomes large, the average offer and acceptance rate in the population comes close to the value of $p$ for which the $3 \times 3$ mutant cluster spreads. Thus also in the two-dimensional situation, evolution leads to strategies which show some degree of fairness.

## 5. SUMMARY

A straightforward evolutionary approach to the ultimatum game in which all members of the population play each other with equal probabilities and their offspring are apportioned according to their total scores, predicts that the average offer and acceptance rate in the population will tend to some values near zero, provided that the mutation error is very small. Thus players ultimately display behaviour which is close to the 'rational' behaviour predicted by game theory and unlike observed human behaviour. Larger mutation errors lead to a heterogeneous population with consequently larger average offer and acceptance rate. For example, a large mutation error of 0.1 leads to an average offer of around 0.27. The aspiration levels, $q$, in simulations of the non-spatial ultimatum game are, however, considerably smaller than those found experimentally.

If players compete for offspring only with certain neighbours rather than with all of the population, then it is their score relative to those neighbours which is important. Thus there is more pressure not to allow an opponent to get away with an unfairly large share of the pie. In a one-dimensional geometry with nearest neighbour interactions, we find that the average offer and acceptance level approximate a fair split. When the players are arranged on a two-dimensional square lattice, we obtain offers around 0.35.

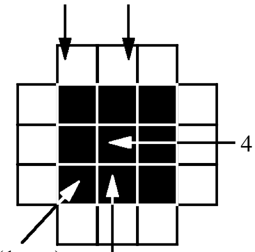


Figure A1. This figure shows (in black) a $3 \times 3$ cluster of players playing the mutant strategy, $S_2$, in a large square of players playing the host strategy, $S_1$, in white. The offers and acceptance rates of these strategies satisfy $p_2 \geqslant q_2 > p_1 \geqslant q_1$. The pay-off obtained by each player is indicated on the figure, with unlabelled squares having the same pay-offs as their counterparts under 90° rotation.

Clearly, just as with the prisoner's dilemma or the hawk–dove game (see Nowak & May 1992; Nowak *et al.* 1994; Killingback & Doebeli 1998), spatial population structure can have important effects on the evolutionary outcome of the ultimatum game. In another paper (Nowak *et al.* 2000), we show that some information on the co-player's past actions can lead to the prevalence of fair splits.

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## APPENDIX A. A $3 \times 3$ CLUSTER OF MUTANT PLAYERS SPREAD IN THE TWO-DIMENSIONAL SPATIAL GAME IF THE OFFER IS LESS THAN 0.342...

Experience shows that the fate of a random arrangement of two strategies on a grid depends essentially on whether a $3 \times 3$ cluster of one strategy can spread or not (Killingback *et al.* 1999). Thus let us assume that all members of the population play strategy $S_1 = (p_1, q_1)$, except for a $3 \times 3$ cluster of mutants who play the strategy $S_2 = (p_2, q_2)$, with $p_2 \geqslant q_2 \geqslant p_1 \geqslant q_1$ (the most interesting case). Let us consider a von Neumann neighbourhood: figure A1 shows the mutant players represented by black squares and the resident players represented by white squares. If the sum of the expected number of the nine mutants' offspring exceeds nine, then we expect the mutant cluster to spread. The expected number of offspring of a player is equal to the sum (over each site of the neighbourhood) of the probabilities that the player gives rise to offspring at that site. This probablity is determined by the player's total pay-off divided by the total pay-off of all players in the neighbourhood of that site. The figure shows the payoffs attained by players within the mutant cluster and just outside the boundary. The rest of the resident players further away from the mutant cluster all receive a total pay-off of four.

We can deduce that mutant players at a corner of the square have expected number of offspring given by

$$E[\text{no. offspring of mutant at corner}]$$
$$= (4 - 2p_2)\left[\frac{1}{4 - 2p_2 + 2(3 + p_2) + 2(4 - p_2)} + 2\frac{1}{4 - 2p_2 + 2(3 + p_2) + 2(4)} + 2\frac{1}{2(4 - 2p_2) + 4 - p_2 + 3 + p_2 + 4}\right]$$
$$= (4 - 2p_2)\left[\frac{1}{18 - 2p_2} + \frac{1}{9} + \frac{2}{19 - 4p_2}\right]. \quad \text{(A1)}$$

Mutants on the middle of the sides of the square have expected number of offspring given by

$$E[\text{no. offspring of mutant on side}]$$
$$= (4 - p_2)\left[\frac{1}{2(4 - 2p_2) + 4 - p_2 + 3 + p_2 + 4} + 2\frac{1}{4 - 2p_2 + 2(3 + p_2) + 2(4 - p_2)} + \frac{1}{3(3 + p_2) + 4 - p_2 + 4} + \frac{1}{4 + 4(4 - p_2)}\right]$$
$$= (4 - p_2)\left[\frac{1}{19 - 4p_2} + \frac{1}{9 - p_2} + \frac{1}{17 + 2p_2} + \frac{1}{20 - 4p_2}\right], \quad \text{(A2)}$$

and the mutant in the centre of the cluster has expected number of offspring given by

$$E[\text{no. offspring of mutant in centre}]$$
$$= 4\left[\frac{1}{4 + 4(4 - p_2)} + 4\frac{1}{2(4 - 2p_2) + 4 - p_2 + 3 + p_2 + 4}\right]$$
$$= 4\left[\frac{1}{20 - 4p_2} + \frac{4}{19 - 4p_2}\right]. \quad \text{(A3)}$$

Thus the expected number of mutant offspring is given by

$$E[\text{no. mutant offspring}]$$
$$= \frac{64 - 20p_2}{19 - 4p_2} + \frac{48 - 16p_2}{18 - 2p_2} + \frac{16 - 8p_2}{9} + \frac{16 - 4p_2}{17 + 2p_2} + 1. \quad \text{(A4)}$$

This value is greater than nine, and hence the cluster is likely to spread, if and only if $p_2 < 0.342$.

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