An Analysis of Some Data on the Linkage Between Xg and Colorblindness in Man

J. H. RENWICK¹ AND JANE SCHULZE²

¹Department of Genetics, University of Glasgow ²Division of Medical Genetics, Department of Medicine, Johns Hopkins University School of Medicine, Baltimore

INTRODUCTION

JACKSON, SYMON AND MANN (1964) in the preceding paper have presented extensive pedigree data bearing on the linkage between the Xg blood group locus and the *deutan* and *protan* sites on the X chromosome. The lod scores (log odds) are here calculated from 26 pedigrees in that study. A further eight small pedigrees were totally uninformative. An IBM 7090 computer was used (Renwick and Schulze, 1961), the program treating each pedigree as a single unit and considering all possible genotypes for persons of uncertain or unknown genotype. In these calculations, the usual frequencies for Americans of European origin were used for the Xg^a , deutan, and protan alleles (0.64, 0.06, 0.02), although it appears possible, from evidence in the pedigrees themselves, that the frequency of protan defects in this inbred group might be somewhat higher than such figures would predict. For example, there are two men with a protan defect and one with a deutan defect among 36 unrelated husbands tested for color vision and lacking tested grandsons via daughters. (Any such grandson might, by being colorblind, have caused the pedigree to be selected for study. Hence the maternal grandfather in these pedigrees is rather more likely to be colorblind than the average man in the same community and has been excluded from the count.)

RECOMBINATION FRACTION BETWEEN Xg and the DEUTAN-PROTAN REGION OF THE X

Table 1 gives the lod scores (Morton, 1955; Smith, 1959) for various values of the recombination fraction, θ , between Xg and the *deutan-protan* region (on the approximation that the *deutan* and *protan* sites are sufficiently close together that they have very similar true recombination fractions, θ , with Xg). In this analysis, no allowance has been made for possible misclassification of phenotype or for any undetected illegitimacies. There was no selection at all at one locus (the Xg locus), and therefore no ascertainment correction is needed (Morton, 1955). Figure 1 (antilods plotted against θ) shows the relative likelihood of obtaining these families for various values of θ , the peak being at $\theta = 0.42$ (0.42, 0.40 for Xg : *deutan* and Xg : *protan* separately).

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Fig. 1. Relative likelihoods (antilods) plotted against the recombination fraction, θ , between Xg and the *deutan-protan* region. Prior probabilities are not included here.

PROCEDURE FOR ESTIMATING LIMITS

To obtain an estimate and limits of the map interval, after incorporating prior probabilities, a graphical method has been used (see Fig. 2), in which the final relative probabilities are plotted against the map interval, w (as suggested for a slightly different method by Dr. J. H. Edwards, personal communication) using a suitable transformation for θ to w. The formula, $4w = \tanh^{-1} 2\theta + \tan^{-1} 2\theta$, is the best of the simpler transformations so far available for mouse data (Carter and Falconer, 1951) and has been used here. The usual formula, $2w = \tanh^{-1} 2\theta$, of Kosambi (1944) was found by Carter (1954) to give a rather poor fit, i.e. mouse map intervals are not closely additive if the Kosambi formula is used. Table VIIi of Fisher and Yates (1963), $z = \frac{1}{2} \tanh^{-1} r$, is invaluable for both transformation procedures. See Bailey (1961) for an excellent review of mapping transformations.

The prior probabilities of map intervals are assumed to be proportional to L-w (Morton, 1955), where L is the genetical (map) length of the chromosome. Our estimate of L for the human X chromosome is two morgans. (One morgan, M, = 100 centimorgans, cM, or 100 map units. The map interval, in morgans, if small, is equal to the recombination fraction expressed as a decimal.)

This estimate is based on approximately 30 M for the total autosomal map length in males, as deduced from chiasma counts (Ford and Hamerton, 1956). In mammals (and in other orders), recombination fractions and, therefore, total genetical lengths in the XX sex are generally greater than in the opposite sex (Haldane, 1922). Human linkage data show that man is probably no

TABLE 1. LOD	SCORES]	BEARING ON	THE LINK	LAGE RELA	HHSNOIT	BETWEE	IN THE X6	g Locus A	AND THE C	leutan-pr	otan Re	GION	
					Map i	nterval, w (in morgans)	, by Carter	and Falcone	r's formula			
			0	.10	.20	.26	.31	.37	.44	.55	69.	.95	2.0
							Recombine	ation fractio	n, <i>I</i> I				
Loci	reuigree	Sibship	0	.10	.20	.25	.30	.35	.40	.45	.482	.4975	5.
Xg : deutan	Reb	10	8 1	-1.151	597	429	300	197	115	050			0
1	Ove	133-134	+.062	+.050	+.038	+.032	+.026	+.019	+.013	+.007			0
	Lud	135-136	+.034	+.028	+.021	+.018	+.014	+.011	+.007	+.004			c
	loh	122-124	+.200	+.139	+.087	+.065	+.046	+.030	+.017	+.007			0
	Tho	129-130	057	036	022	016	011	008	004	002			0
	Rom	119	023	010	003	002	001	80	∭ ∭	0≋			0
	Sla	128	+.086	+.057	+.033	+.023	+.015	+.008	+.004	+.001			0
	Scw	6	-2.500	444	194	125	076	041	018	004			0
	Hai	144-148	+.063	$\pm.041$	+.024	+.017	+.011	+.006	+.003	+.001			0
	Dra	61	8	887	388	250	151	082	035	009			0
	Moc	1	8 1	891	418	288	193	122	068	028			0
	Dec	œ	8 1	855	364	223	124	057	016	+.003			0
	*Hom	3-7	8	-1.919	622	308	109	+.008	+.059	+.055			0
	Kyl	63	107	066	036	024	015	600'	004	001			0
Xg : protan	Har	132	107	066	036	024	015	-000	004	001			0
	Bus	12	+.455	+.321	+.185	+.121	+.065	+.021	006	014			0
	Nib	137-140	121	080	049	036	025	016	-000	004			0
	Swa	11	8 	-1.536	648	405	235	119	045	007			0
	T_{on}	149-152	005	001	+.002	+.003	+.004	+.004	+.003	+.002			0
	Kyl	14-17	8 	468	+.309	+.417	+.424	+.358	+.242	+.106			0
	Hau	141-143	+.021	+.002	008	010	011	010	008	004			0
	\mathbf{Reb}	50-52	+.336	+.251	+.169	+.130	+.095	+.063	+.037	+.015			0
	Kne	13	931	363	167	109	066	036	016	004			0
	Orr	18	819	342	159	104	064	035	015	004			0

COLORBLINDNESS AND Xg LINKAGE

1				
00000	0 - 0 0	0 1 0	0 1 0	
	+.007 1.016 1.05 1.067	+.005 † 1.012 1.063	+.002 1.005 1.055	
	+.060† 1.148 1.31 1.504	+.042† 1.102 1.444	+.019 1 1.044 1.368	
+.054 +.037 ≈0 001	+.148 1.406 1.45 2.039	+.096 1.247 1.808	+.053 1.130 1.638	
+.110 +.059 001 002 052	+.136 1.368 1.56 2.134	+.053 1.130 1.763	+.082 1.208 1.884	
+.163 +.053 001 005 129	132 .738 1.63 1.203	150 .708 1.154	+.018 1.042 1.698	
+.204 +.007 003 250	747 .179 1.69 .302	556 .278 .470	190 .646 1.092	
+.223 092 006 014 422	1.838 .014 1.74 .024	-1.230 .059 .103	608 .247 .430	
+.207 262 010 020 674	-3.602 .0002 1.80 .0004	-2.284 .005 .009	-1.318 .048 .086	
024 971 027 036 -1.629	-10.913 $\simeq 0$ 1.90 $\simeq 0$	-6.531 ≊0 ≊0	—4.381 ≊0 ≊0	
- 8 	0 0 0 0 0	8 o o 	800	js.
23-25 19-22 120-121 64 27-30	$\begin{pmatrix} y (L-w) \\ z \end{pmatrix}$, M		itional sibshi
Neu Dan Ste Dub	eut-prot) f_{*} 1) r probability probability ot) if $L =$	eutan)‡ probability 1) if $L = 2$	rotan) \ddagger probability) if $L = 2N$	lished add
Xg : deut-prot	Lods (Xg : d a Antilods (Fig b Relative prior $a \times b$ Final relative (Xg : $deut$ -pr	Lods ($Xg: d$ c Antilods $c \times b$ Final relative ($Xg: deutan$	Lods ($Xg: p$ d Antilods $d \times b$ Final relative ($Xg: protan$,	*Includes unpub

 β graphical interpolation on \log/θ curve. Including the relevant contribution from the deutan-protan families. Additions have been made since the analysis of these data by Davies *et al.* (1963).

RENWICK AND SCHULZE

413



FIG. 2. Relative probability densities plotted against the map interval, w, between Xg and the *deutan-protan* region using Carter and Falconer's transformation of θ to w. The prior distribution of w (dotted line) is assumed proportional to L - w where L, the genetical length of the X, is estimated as 2.0 morgans. The final distribution (continuous line) incorporates this prior distribution.

exception (Renwick, 1963; Cook, 1965; Renwick and Schulze, 1965) but more extensive data are available in the mouse, where the ratio is about 1.25:1(Green *et al.*, 1962; Carter, 1954). The cytological data of Slizynski (1960) suggest 1.15:1. The ratio 1.25:1 would imply a human female autosome map of 37.5 M. An estimate of about 2 M is then obtained for the X by assuming that physical lengths in mitotic metaphase are roughly proportional to genetical lengths, provided that the chromosomes appear to be equally densely coiled in mitosis. This condition is met reasonably well in male somatic cells, and in such cells the Ferguson-Smiths, Ellis, and Dickson (1962) find the X to be equal to 5.43% of the total length of the autosomal haploid set. The above condition is probably also met in the female for the autosomes and for the one X which is not late-replicating.

For the four *Drosophila melanogaster* chromosomes, the correlation coefficient is 0.97 between the *actual* genetical lengths, i.e. 0.66, 1.08, 1.06, 0.03 M (Warren and Novitski, 1962), and the genetical lengths which would be *expected*, 0.52, 1.05, 1.23, 0.03 M—if the same total genetical length were distributed among the chromosomes proportionally to their physical lengths measured from the (presumed interphase) salivary gland chromosomes (Bridges, 1942). The data of Elliott (1953) on two flowering plant species show correlations of more than 0.98 between the mitotic metaphase lengths of individual chromosomes and their deduced genetical lengths estimated from chiasma counts. For maize, the correlation between genetical lengths (Rhoades, 1962) and cytological lengths is only 0.8, whether the expectations are based on the mitotic diagrams of McClintock (1929) or on the meiotic prophase measurements of Longley (1939).

ESTIMATES OF MAP INTERVALS AND OF THEIR LIMITS

Estimates of map interval, derived with the aid of the Carter-Falconei transformation of θ to w, are given in Table 2a (see also Fig. 2) and those

derived with the aid of the Kosambi transformation in Table 2b. In both tables, the modal (maximum probability) and median estimates obtained graphically are accompanied by the appropriate 95% probability limits and by the corresponding θ values (recombination fractions). Values are given for the pooled Xg: deutan - protan data and for the Xg: deutan and Xg: protan intervals separately. The modal estimates of the three map intervals correspond to θ values of 0.42, 0.42, 0.40 on the Carter-Falconer scale, values which are almost identical to those given by the Kosambi scale or to the direct maximum likelihood estimates of θ from the raw lod scores (Table 1, Fig. 1). The prior assumptions have had a negligible effect on these estimates but they have allowed limits of the map intervals to be found. The data (on the Carter-Falconer scale) give a modal estimate of w = 0.48morgans for the Xg: deutan - protan interval and the narrowest limits which encompass 95% of the probability (i.e. 95% of the area under the graph shown in Fig. 2) are 0.31 M and 1.64 M ($\theta = 0.30$ and approximately 0.5).

The graphical determination of such limits is facilitated by the fact that these are also the limits which have the same relative probabilities (same ordinates) as each other and which together exclude 5% of the area under the curve, y = f(w). That this is so, even for an asymmetrical distribution (provided that it is unimodal and has no truncation or other discontinuity at either of the narrowest limits) can be shown by considering any two w values, w, and $w_1 + D$, which have an area, A (95% of total area under curve), between them. These are then the narrowest limits when w_1 is such that D is a minimum.

Suppose the lower limit is moved by δw and the upper limit is moved in the same direction by $\delta w + \delta D$, without change in the area, A, between the new limits. Then the changes in area at the two limits must be equal. Thus

$$\delta w \cdot f(w_1) = (\delta w + \delta D) \cdot f(w_1 + D) \text{ approximately.}$$

Hence
$$\delta D/\delta w = \frac{f(w_1) - f(w_1 + D)}{f(w_1 + D)}$$

At the limit, as $\delta D/\delta w$ tends to zero,

Hence

$$f(w_1) = f(w_1 + D)$$

i.e. the ordinates are equal. This is the condition for a minimal D and not for a maximal D, as is shown by the fact that $d^{*}D/dw^{*}$ is necessarily positive for the unimodal types of distribution specified above:

$$d^{s}D/dw^{s} = \frac{f(w_{1} + D)t_{1} - f(w_{1})t_{s}}{[f(w_{1} + D)]^{s}}$$

where t_1 and t_2 are the tangents to the curve at $w = w_1$ and $w = w_1 + D$ respectively. t_s is negative and the other items are positive, hence d^sD/dw^s is positive. (A closely related result is given by Neyman and Pearson, 1936.)

The narrowest limits are thus readily found by trial and error, using a planimeter, and have the reassuring property that the maximum probability value necessarily lies between them if the curve is unimodal. Similar noncentral limits for a parameter with a skewed distribution are more usually obtained, e.g. Morton (1956), by a symmetrizing transformation (with subsequent retranslation of the limits into original units) or by the ratio method of Haldane and Smith (1947), but the first is not possible here and the second

TABLE 2A. MAXIMUM PROBABILITY ESTIMATES AND MEDIAN ESTIMATES (IN MORGANS) OF MAP INTERVAL, w, between Xg and the *deutan-protan* Region, On the Assumption that the X Chromosome is Two Morgans Long

Corresponding values of the recombination fractions, θ , are also given. Carter and Falconer's transformation, $4w = \tanh^{-1}2\theta + \tan^{-1}2\theta$, has been used.

There a f		Fror	n Xg	Narrowest (asy accounting the pro	mmetrical) limits g for 95% of obability
estimate	Locus	w	θ	<i>w</i> .	θ
Maximum probability	deutan- protan	0.48	0.42	0.31 & 1.64	0.30 & ≅0.5
estimates	deutan	0.49	0.42	0.30 & 1.65	0.29 & $\cong 0.5$
	protan	0.44	0.40	0.25 & 1.64	$0.24 \& \cong 0.5$
				Conventional(sy accounting the pr	ymmetrical) limits g for 95% of obability
				w	θ
Median estimates	deutan- protan	0.77	0.49	0.36 & 1.73	0.34 & ≌0.5
	deutan	0.79	0.49	0.34 & 1.74	0.33 & $\simeq 0.5$
	protan	0.76	0.49	0.29 & 1.73	0.28 & $\cong 0.5$

TABLE 2B. On the Scale of the Kosambi Transformation, $2w = \tanh^{-1}2\theta$

		From Xg		Narrowest (asymmetrical) limits accounting for 95% of the probability	
Type of estimate	Locus	w	θ		θ
Maximum probability	deutan- protan	0.60	0.42	0.37 & 1.65	0.31 & ≅0.5
estimates	deutan	0.62	0.42	0.33 & 1.66	0.29 & $\simeq 0.5$
	protan	0.53	0.39	0.26 & 1.65	0.24 & $\simeq 0.5$
				Conventional(sy accounting the pr	ymmetrical) limits g for 95% of obability
				w	θ
Median estimates	deutan- protan	0.81	0.46	0.43 & 1.74	0.35 & ≅ 0.5
	deutan	0.83	0.47	0.39 & 1.75	0.33 & $\simeq 0.5$
	protan	0.79	0.46	0.32 & 1.74	0.28 & $\cong 0.5$

suffers, when applied to limited data, from an uneconomical safety factor. Central (i.e. conventional) 95% limits are also given in Table 2 together with the median estimate, the estimate to which they are clearly more appropriate.

TEST FOR ADDITIVITY OF MAP INTERVALS

The section of chromosome between Xg and *deutan* is spanned also by the two intervals Xg : G6PD (0.26 morgans by the above method) Adam *et al.*, 1963; Fraser *et al.*, 1964), and G6PD : deutan, 0.05 morgans (Porter, Schulze, and McKusick, 1962), G6PD being the symbol for the glucose 6-phosphate dehydrogenase locus. The combined length, 0.32 morgans, falls far short of the present estimate, 0.48 morgans, for the Xg : deutan interval but

RENWICK AND SCHULZE

the discrepancy is not surprising in view of the wide limits of all the estimates. Xg : G6PD has narrowest 95% limits of 0.13 M and 0.52 M, corresponding to $\theta = 0.13$ and 0.43. If the Kosambi formula is used throughout, the discrepancy from additivity is much larger, the estimate of the combined lengths remaining at 0.32 morgans and the Xg : deutan interval rising to 0.62 morgans.

When the publication of X-linkage data being collected by other workers is complete, it should be possible to construct a consistent map of the X chromosome and to discriminate further between the suitabilities of various mapping transformations for man.

SUMMARY

From the data of Jackson, Symon, and Mann (1964), linkage between the Xg blood group locus and the *deutan-protan* region of the X chromosome has been estimated via scores calculated by an IBM 7090 computer. The maximum probability estimate of the map interval is 0.48 morgans (48 map units) and corresponds to a recombination fraction, θ , of 0.42. The narrowest 95% limits of this estimate are wide and, on the basis of the mapping formula of Carter and Falconer (1951), may be estimated as 0.31 morgans and 1.64 morgans, if available knowledge of the probable genetical length of the X chromosome is used. A length of two morgans for the X chromosome has been chosen here, on the basis of indirect evidence. The data, although allowing estimates to be made of the map distance of Xg from the *deutan* and *protan* sites separately (0.49 M and 0.44 M) are inadequate to determine the ordering of *deutan* and *protan* with respect to Xg.

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