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A gravitational diffusion model without dark matter

(galactic rotation curves/clusters and superclusters/zero dark matter/initial gravitational collapse/galaxy formation)

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ABSTRACT In this model, without dark matter, the flat rotation curves of galaxies and the mass-to-light ratios of clusters of galaxies are described quantitatively. The hypothesis is that the agent of gravitational force is propagated as if it were scattered with a mean free path of ≈5 kiloparsecs. As a result, the force between moderately distant masses, separated by more than the mean free path, diminishes as the inverse first power of the distance, following diffusion equations, and describes the flat rotation curves of galaxies. The force between masses separated by <1 kiloparsec diminishes as the inverse square of distance. The excess gravitational force (ratio of $1/r{:}1/r^2$) increases with the scale of structures from galaxies to clusters of galaxies. However, there is reduced force at great distances because of the ≈12 billion years that has been available for diffusion to occur. This model with a mean free path of ≈5 kiloparsecs predicts a maximum excess force of a few hundredfold for objects the size of galactic clusters a few megaparsecs in size. With only a single free parameter, the predicted curve for excess gravitational force vs. size of structures fits reasonably well with observations from those for dwarf galaxies through galactic clusters. Under the diffusion model, no matter is proposed in addition to the observed baryons plus radiation and thus the proposed density of the universe is only a few percent of that required for closure.

The concept of diffusion of gravity arose from recognition that flat rotation curves of galaxies would result from the equations of diffusion (1). The model that has developed includes: (*i*) an unspecified agent responsible for the force of gravity, probably traveling at the speed of light over small distances; (*ii*) for distances more than a few kiloparsecs (kpc), the agent propagates following the diffusion equations; (*iii*) the effective mean free path is ≈5 kpc, apparently independent of the local matter density; and (*iv*) the process that causes the propagation according to diffusion equations is probably not scattering of the direction of travel of gravitational elements but something more subtle, involving distortion of the metric. A process for which the continued propagation is proportional to the concentration of elements in local regions follows the diffusion equations. The phrases "gravitational elements" or the "agents of gravity" used in this article are shorthand for an unknown underlying process that amounts to propagation of the curvature of the metric.

In the diffusion model, the retardation of the agent of gravity increases the gravitational force (compared with inverse square) at distances from a few kpc to many megaparsecs (Mpc), owing to the effective higher concentration of the agent. This increase is described as "excess gravitational force." It explains the observed differences in the mass-to-light ratio for structures of various sizes. There is no known reason to propose that the mean free path or diffusion constant varies over space or time or that the propagation is affected by the presence of matter or radiation. Questions about the nature of gravitation and the mechanism of its propagation are bypassed. A scattering process is not favored because of the problem of preservation of the vector of attractive force through scattering events. This article reports the agreement of astronomical observations of excess gravitational force with the quantitative predictions of the diffusion concept.

Quantitative Description of Diffusion

The standard solution for diffusion in three dimensions from a point source can be transformed to amount per spherical shell (*Ps*):

$$Ps = C\ r\ \mathrm{erfc}[r/2\ (D\ t)^{0.5}], \qquad [1]$$

where C is a constant; r is radius; t is time; and D is a diffusion constant. This equation is accurate for distances much greater than the mean free path but does not apply for small distances. To obtain an equation suitable for small distances, Monte Carlo calculations were made for elements traveling at constant speed (c) that are scattered in a totally random direction after traveling an average distance p. The following equation matched the results quite well:

$$Ps = (1 + br/p)\mathrm{erfc}[r/a\ (pct)^{0.5}], \qquad [2]$$

where *Ps* again is the number per spherical shell. For the best fit, $b = 3.1$ and $a = 1.1$. With this definition of the mean free path, the diffusion coefficient is approximated by $p\ c$ (over a small factor) and thus the gravitational force (F) in this model becomes

$$F = Gmm\ r^{-2}\ (1 + br/p)\mathrm{erfc}[r/a\ (pct)^{0.5}], \qquad [3]$$

where r is distance; a and b are constants; p is mean free path; c is speed of travel of a gravitational element; and t is the time since the start. This equation is graphed in Fig. 1. The process equivalent to scattering might be represented as many small deflections instead of the large deflections used in this Monte Carlo model, but tests show that this does not affect to a great degree the approximation, although it could affect the constant b, which is uncertain by perhaps 50%.

There are three important domains as follows. If $r \ll p$, the inverse square term dominates and gravitational force is inverse square. If r is greater than p but less than a few Mpc, there is a $1/r$ relationship, which applies for galaxies and for small clusters. Finally, if r is many Mpc, then as a result of the slowness of diffusion, the gravitational force only partially reaches the distant regions and the decay of the erfc function dominates. Thus, under the diffusion model, the present gravitational force reaches a maximum ratio to inverse square

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Abbreviations: kpc, kiloparsec; Mpc, megaparsec; MOND, modification of Newtonian gravity.

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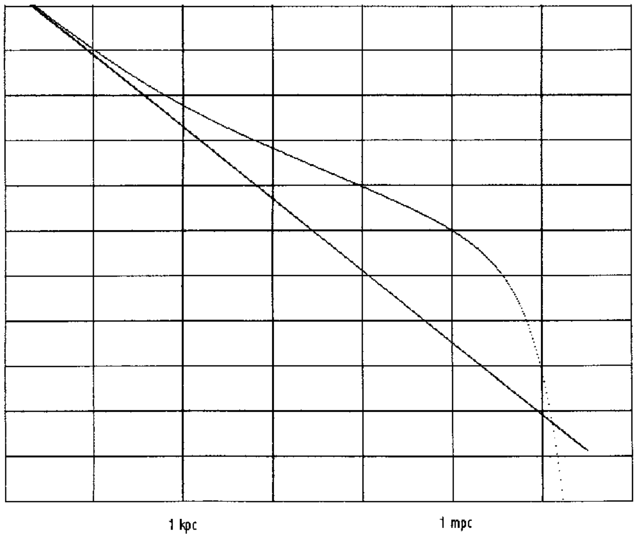

FIG. 1. Log–log graph of Eq. **3** showing the effect of diffusion on the force of attraction compared with the inverse square law. Eq. **3**: $r^{-2}(1 + br/p)\mathrm{erfc}[r/a(pct)^{0.5}]$ is the upper curve using the values $a = 1.1$, $b = 3.1$, $p = 5$ kpc and r is shown in Mpc. Numerically, it becomes $r^{-2}(1 + 600\mathrm{r})\mathrm{erfc}(r/3)$ because $(pct)^{0.5}$ is 7.5 E24 cm at the present time, assuming diffusion has been occurring for 1.2 E10 years. Below this curve is a line for the inverse square law. The index lines are spaced at factors of 10 with the vertical scale half of the horizontal to accommodate the 1.0 E11 range. Note that, at the maximum, the diffusion curve lies more than two orders of magnitude above the inverse square line.

for major clusters of galaxies and becomes very small at large distances.

Eq. **3** is used in Fig. 2 to describe the excess force external to galaxies and clusters. For large distances, it decays in a way that does not represent what is expected for larger objects. Because astronomical observations of excess force depend on the velocities of objects in apparent orbit around clusters, calculation was made for the excess force on a test mass immediately external to a large distributed mass. For this purpose, numerical integration was carried out over an extended object such as the Great Wall (2) with the contribution of each point calculated according to Eq **3**. The result is the upper curve in Fig. 2. This curve closely approximates Eq. **3** up to several Mpc and then remains high because major regions of the distributed mass are not far from the adjacent test mass and an excess attractive force is caused by this matter.

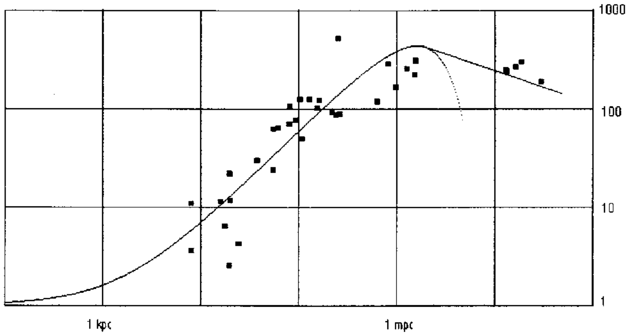

FIG. 2. Comparison of the observed excess gravitational force for galaxies and clusters of galaxies with the prediction of the gravitational diffusion model. The lower curve is Eq. **3**, using the numerical values described in the legend of Fig. 1, without the $1/r^2$ factor so that it directly represents the excess gravitational force. The upper curve on the right is based on calculations of the excess force on a test mass immediately external to a large distributed mass for a diffusion case with a mean free path of 5 kpc, as described in the text. The principal part of the data has been extracted from figure 2 from the study by Bahcall *et al.* (3). The lower points are the median values for Sa, Sb, and Sc galaxies from figure 5 from the study of Rubin (4). Reference is to the original papers for the identification of the individual objects, which include spiral galaxies, elliptical galaxies, clusters, and superclusters.

Comparison with Larger Scale Observations of Excess Gravitational Force

Fig. 2 shows observations of excess gravitational force as a function of the linear size of structures including galaxies and clusters of galaxies. The curve rises slowly in the first decade because there is little excess gravitational force at distances less than the mean free path. Then it rises linearly at 45° for the next two decades because the excess gravitational force is proportional to the size of structures (ratio of $1/\mathrm{r}:1/\mathrm{r}^2$). The principal part of the data has been extracted from figure 2 of a study by Bahcall *et al.* (3). The lower points are the median values for Sa, Sb, and Sc galaxies from figure 5 of a study by Rubin (4). Refer to the original papers for the identification of the individual objects, which include spiral galaxies, elliptical galaxies, clusters, and superclusters. The upper curve in Fig. 2, described in the legend and in the previous paragraph, represents the force of attraction expected at the periphery of a massive distributed structure assuming a mean free path of 5 kpc under the diffusion model. The curve is in broad agreement with the excess gravitational forces observed. Thus, the diffusion model gives the correct increase in gravitational excess with size over a wide range of sizes and a lack of increase with size for objects above a Mpc. In the diffusion model, the reduced force at large distances is caused by the limited time ($\approx$12 billion years during which diffusion has been occurring) as described by Eq. **3**.

The implication is that very distant objects would at the present time have negligible gravitational attraction for each other. Thus, the major flows observed at present would have been established at earlier times when the various structures were under each other's gravitation influence before the expansion outran the diffusion process, which is slower on these large scales. This phenomenon is expected for structures whose current spacings are more than a few Mpc. Fig. 1 shows that, for this model, the force of attraction falls below that for inverse square where the spacing exceeds $\approx$10 Mpc and becomes very small for larger distances.

There is general agreement between the expectation from the diffusion model and the observed mass-to-light ratio or mass excess. However, the individual observations scatter around the curve by a factor of two or more. The deviation of the plotted values from the curve in Fig. 2 could be due to uncertainties in the characteristics of galaxies and clusters, specifically in the disk masses and the estimates of distances. Some uncertainty also arises from unknown peculiar velocities. Some of the variation in the excess gravitational force might be interpreted under the diffusion model as resulting from interaction between galaxies. It is possible that the diffusing clouds have not recovered fully from past tidal interactions and collisions or mergers. Many estimates are based on the dynamics of the clusters, but virial calculations are uncertain and require assumptions that are not independently verifiable. It would be a better corroboration, of course, if the curve passed through the points shown in Fig. 2, but the deviations shown in Fig. 2 do not represent strong conflict with the predictions of the diffusion model. In sum, there is general agreement between the diffusion model predictions and the observed excess gravitational forces, however, uncertainties are so great that it remains possible that they agree in detail.

The Rotation Curve Expected for NGC 2403 Under the Diffusion Model

Eq. **3** gives a nearly flat rotation curve at distances of 20 kpc from the center of a galaxy owing to the $1/r$ dependence of the gravitational force. However, it is worthwhile to ask whether the observed rotation curves are consistent with Eq. **3** at closer distances. Fig. 3 shows the observed rotation curve for NGC 2403 compared with a calculation of the rotation curve under the diffusion model based on estimates of the mass distribution from Albada and Sancisi (Figure 4 in ref. 5). Because the rotation curve of Albada and Sancisi (Figure 4 in ref. 5) does not match the observed rotation curve at the inner radii, mass was added near the center in a step that is essentially model-independent. The match in the inner regions makes it possible to compare the shape of the curve derived from the diffusion model with the observations, and the agreement appears excellent. The details of the calculation are mentioned in the legend of Fig. 3. The apparent asymptotic value of the flat part of the rotation curve is 132 km/sec. The mass of the galaxy directly calculated from this velocity under the diffusion model, as described in the next section, is 6.5 E9 solar masses, which agrees with a luminosity estimate of 7.9 E9 times the solar luminosity (6). This close agreement is probably fortuitous because it is a single value in a set that follows the Tully–Fisher relationship as described below.

The Tully–Fisher Relationship and Galaxy Masses

Under the diffusion model, the mass of a galaxy is proportional to the square of the rotational velocity, assuming no major events have affected the rotation curve. Taking V_c rotation velocity in centimeters per second and V_k rotation velocity in kilometers per second, Eq. **3** gives

$$V_c^2 r^{-1} = Gmr^{-2}(1 + 3.1r/p)\mathrm{erfc}(r/a(pct)^{0.5}). \quad \textbf{[4]}$$

In the domain where $r > 10$ kpc and $r < 1$ Mpc, typical of galaxies, this equation reduces to

$$v_c^2 = 3.1Gm/\mathrm{p}. \quad \textbf{[5]}$$

Converting from cgs units to practical astronomical units, the mass in solar mass units (M) becomes

$$M = 3.73\ \mathrm{E5}\ V_k^2. \quad \textbf{[6]}$$

Thus, for the reference galaxy in the Tully–Fisher relationship from Peebles (7), p49, which has a flat rotation curve with 220 km/sec velocity, the expected mass is 1.8 E10 solar masses, which is an acceptable value for the baryonic mass of the reference galaxy. The Tully–Fisher relationship is as follows:

$$V_k = 220(L/Lref)^{0.22}, \quad \textbf{[7]}$$

where L is luminosity of the galaxy in the 2.2-μm band and $Lref$ is the luminosity for the reference galaxy. When Eqs. **6** and **7** are combined, there is an implication that the brightness of galaxies rises as the square root of their mass (actually the 0.44 power).

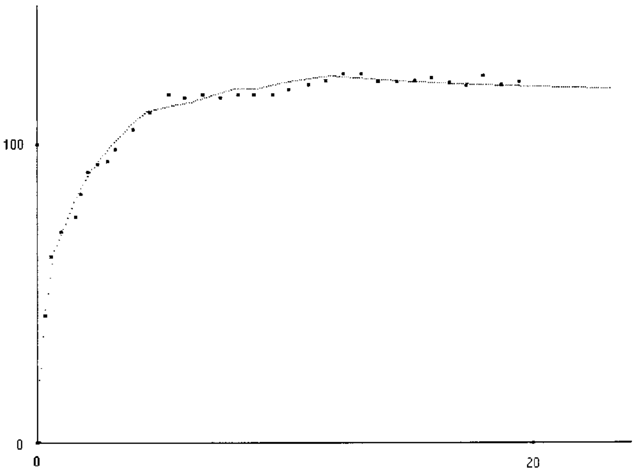

FIG. 3. The rotation curve of NGC 2403 derived from the diffusion model, km/sec vs. kpc. The squares are the observed velocities of rotation taken from the study by Albada and Sancisi (5). The fine dots are the calculation of the rotation curve expected from the diffusion model. The mass distribution for stars and gas was derived from the curve on figure 4 from the study of Albada and Sancisi (5). Mass was added near the center for agreement with the inner part of the rotation curve in a step that is essentially model independent. From this mass distribution (136 values) the rotation curve was calculated step-by-step: $V(R) = \mathrm{sum}\ (j = n$ to $136)[\mathrm{sqrt}\ (m(j)/R + m(j)/2)]$, for 260 values of the radius R, the sum being taken from each mass element or shell, outwards. This form is derived from Eq. **3** as $V^2/r = Gm\,r^{-2}$ $(1 + br/p)\mathrm{erfc}[r/a(pct)^{0.5}]$ where the erfc term is unity at these distances. The mean free path is taken to be 5 kpc and the constant $b = 3.1$. Figure 4 of Albada and Sancisi (5) shows a comparably good fit to the rotation curve, which is achieved by adding in an arbitrary curve for the mass of dark matter. Here, there is no such freedom and the parameters are the mean free path (5 kpc) and the mass of the galaxy NGC 2403, which, from the asymptotic value of $V = 132$ km/sec, is 6.5 E9 solar masses, which agrees with a brightness estimate (6).

Flat Rotation Curves and the "Conspiracy"

The flat rotation curves of spiral galaxies have been measured by optical and HI methods (e.g., see refs. 8–11) and form a remarkable body of evidence. In general, the Doppler shift is measured for both the receding and approaching sides of a galaxy, and the rotation velocity is known independently of the distance to the galaxy or its recession velocity. Thus, the effective gravitational field holding the peripheral stars or gas in orbit is directly calculable. A few examples are known in which the rotational velocity may drop 20 or 30% at the limit of measurement (12), and irregular rotation curves have been observed (see ref. 13). Nevertheless, many examples exhibit a flat rotation curve right to the limit of observation either by HI or optical means of detection, and it is this frequent regularity that will be discussed in the next three paragraphs; the irregular examples will be dealt with later.

There has been discussion of the "conspiracy" that permits the visible and dark matter to just add up to give a flat rotation curve (see refs. 14 and 15). The striking part of the so-called conspiracy is the way the rising part of the rotation curves typically blend smoothly with the flat region. A direct explanation of this regularity is called for, and the diffusion model offers a simple one. As shown in Fig. 3, a smooth rise and flat rotation curve are direct predictions of the diffusion model because they are the result of the gravitational attraction of the baryonic matter that makes up the visible galaxies, following Eq. **3**. In the diffusion model, the flat part of the rotation curve results from the expectation that the gravitational field falls as $1/r$ in the size range of galaxies.

In contrast, for the usual dark matter model, there is no known underlying reason that the distribution of the dark matter is such as to yield a smooth rising rotation curve, which levels to the flat part of the curve. There are grounds for believing that the initial galaxy formation would lead to a $1/r^2$ distribution of matter, that is the isothermal pattern, but there are problems. It is not possible for the dark matter to follow the $1/r^2$ pattern in the interior region because that would yield a flat rotation to the very center because of the dark matter alone. To this result would be added the gravitational effect of the stars and gas giving a hump in the intermediate inner regions, a pattern that is observed rarely. In fact, in making models of individual galaxies, for example NGC 3198 (16) or NGC 1560 (17), the disk and halo are adjusted in relation to each other. There is a good deal of freedom in models ranging from "maximum disk" in which there is a large empty hole in the dark matter distribution to examples with more inner dark matter. In all cases, there is a shortage of dark matter corresponding to the baryonic matter. Some aspect of the interaction between the dark matter and the baryonic matter, during galaxy formation, leads to the smooth rise dominated in the inner region by the baryonic matter. However, no models of this process exist.

The process of galaxy formation is an unsolved problem, and as yet, there are no detailed models. Thus, without clear initial conditions, it is difficult to assess the dynamics of the galactic dark matter. N-body calculations (see ref. 18) with an appropriate initial mix of baryonic matter and the unknown form of dark matter were moderately successful, but the analysis was considered incomplete. Blumenthal *et al.* (18) assumed circular orbits, and by adjusting parameters, they could achieve flat rotation curves, although not tested over a range of radius as broad as many galaxies show, including the HI data. However, I am not aware of publications examining the long term gravitational stability of the $1/r^2$ distribution of dark matter under the Newtonian model, and there is as yet no way to deal with the interaction of the baryonic matter (stars and gas) and the unknown dark matter.

Deviation from Flat Rotation Curves

Quite a point has been made about deviations of some galaxies from flat rotation curves, specifically the decreased velocity in outer parts of the curves (12, 19). Such cases can be explained under the diffusion model by considering collisions and tidal interactions between galaxies. In this explanation, the excess gravitational force is considered to be caused by a "cloud" of the agent that carries gravitational force that always is diffusing freely, although more concentrated in some regions than others as a result of the time required for the diffusion process and the size of the regions involved. When tidal interactions have occurred between galaxies, some momentum could be transferred between stars, gas, and dust that would not be shared by the diffusing clouds, and therefore, asymmetries in the gravitational forces would result. For example, the cloud and galaxies could separate if the two galaxies merged because the galaxies would share their momentum and the clouds would remain independent and continue to diffuse. Then, new gravitational clouds would be built slowly by diffusion from the merged galaxy. The time required according to Eq. **3** is ≈100 million years to come within 5% of the steady value for one of the larger galaxies, at a distance of ≈20 kpc. The time is greater at a larger distance rising as the square of the distance, following diffusion rules. The peripheral fall in some rotation curves (12) could be due to incomplete recovery from past collisions.

Many cases have shown lack of symmetry between the measured rotation curves on the two sides of galaxies, and these cases generally have been excluded from further detailed examination. Opinions tend to favor the view that a significant fraction of observed galaxies has undergone tidal interaction, collisions, or mergers. In addition, very many galaxies are members of pairs or clusters of galaxies that are held by complex gravitational interactions. In the diffusion model, it is assumed that the gravitational elements will diffuse freely whereas the galaxies will follow other gravitationally determined paths. This process will result in various distortions of the rotation curves.

Comparison with MOND

Milgrom (20) has explored a modification of Newtonian gravity (MOND) that converts to a $1/r$ law at low gravitational fields. Although it is written in terms of acceleration, this model uses distance, and presumably, constants could be chosen so that the MOND formulation approximated Eq. **3** in the region of distances $\ll$1 Mpc, for a particular galaxy. All that is required to match is that the transition from $1/r^2$ to $1/r$ occurs at a similar distance. Thus, there is some similarity to MOND even though the basis is very different from that of the diffusion model. The constant required for matching this model with Milgrom's formulation would perhaps differ from galaxy to galaxy. Some detailed comparisons could be made to see which model gives the more acceptable fits to measurements. However, there are two major differences in the predictions of this model compared with MOND. First, there are many galaxies in which the velocities fall at larger distances or that show large asymmetries. As mentioned above, these cases can be attributed to a history of collisions under the diffusion model, and no explanation has been suggested for MOND. Second, as the scale of clusters of galaxies exceeds ≈1 Mpc, the observed mass excess does not continue to rise as exhibited in Fig. 2. Under MOND, the excess acceleration would continue to rise or the constant would have to be changed in an ad hoc fashion for each of the larger clusters. Of course, it is a major feature of this model that the limited time in which diffusion has occurred explains the lack of continuous increase in excess gravitational attraction for larger galactic clusters.

Implications for Gravitational Collapse During Early Expansion

Under the diffusion model, the excess gravitational force initiates regional gravitational collapse at early times during the expansion and the scale of the regions is established by the maximum distance that gravitational diffusion reaches as the collapse proceeds. Small scale N-body calculations have been made, but this is an essentially unexplored area. One possible assumption is that the mean free path early in the expansion is the same as at present, that is ≈5 kpc. If that is the case, early expansion is accelerated, compared with inverse square, for any degree of density variation, by the $1/r$ gravitational attraction in the appropriate range of distances. At much larger spacings, collapse fails to occur because of limited gravitational diffusion to such distances during the expansion. This result suggests an explanation for the size of the largest structures now observed (2).

General Implications

Because this model consists primarily of a proposal for a reduced effective rate of travel of the gravitational force, due to diffusion, it does not follow from present calculations based on the general theory of relativity. It is not necessarily inconsistent with the general theory because the diffusing gravitational elements might be interpreted as spatial curvature. There are other mechanisms besides scattering that mimic diffusion and lead to $1/r$ decay with distance from a source. This is the case for models in which local regions have to be charged or influenced for propagation to continue. To demonstrate this point, model calculations were done with ≈10,000 capacitors arrayed in a cube with resistors connecting all adjacent pairs and with peripheral elements grounded. Current was introduced at the center, and after steady–state was reached, the charge per capacitor fell with radius as $1/r$ in the inner half. Such a calculation carried out on a large scale with the boundary conditions set only by dynamics likely would give a result closely similar to the diffusion equations. A reasonable proposal is that the distortion of the metric induces distortion in adjacent regions, and it is the continued induction of distortion that is responsible for the propagation of the gravitational force. A deeper level of analysis would be required to obtain a quantitative understanding of the effective diffusion constant corresponding to a mean free path of ≈5 kpc.

It is not possible to predict the characteristics of a gravitational theory modified for propagation of gravity by diffusion or its analog. There is much at stake because of the scale of the intellectual investment and the subtle arguments in cosmology making use of the general theory. Binney and Tremaine (21) state in regard to the explanation of dark matter, "If a new theory of gravity is required, it will ultimately be accepted because of its beauty and unifying properties rather than because it eliminates the need for dark matter." That challenge may not be met at this time because beauty is a subtle concept. As for unification, it is a step to use a single, free parameter to explain the rotation curves of galaxies, the dependence of the gravitational excess of galaxies and galactic clusters on their dimensions and a maximum excess attractive force. A much bigger step toward unification would be made if it could be shown that this model explains the control of formation of large structures during the initial collapse and the main features of galaxy formation. Within the large structures that are expected to form under this model, there will exist exaggerated density fluctuations that may contribute to galaxy formation.

Under the diffusion model, the matter in the universe is best estimated as the baryons that are observable directly with methods from radio to x-rays. At present, that implies that the amount of matter is only a few percent of what would be required for closure under the general theory. The small amount of matter is consistent with calculation of element formation in the early period, and it is not likely that a change in the propagation of gravity will affect severely that calculation.

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