

## CICATRIZATION OF WOUNDS.

### X. A GENERAL EQUATION FOR THE LAW OF CICATRIZATION OF SURFACE WOUNDS.

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It has been shown<sup>1</sup> by the extrapolation formula

$$S_n = S_{n-1} [1 - i(t + \sqrt{t + nt})]$$

that the normal progress of cicatrization of surface wounds follows a definite curve. The fact that many biological and chemical phenomena are expressed by exponential formulas suggested the comparison, if possible, of the curve for the cicatrization of wounds with other curves expressing biological phenomena. It is well known that the exponential function plays an important part in natural phenomena. It expresses the general law called by Lord Kelvin "the compound interest law," and by Mellor, the "ubiquitous law."

I had already studied a formula of the form

$$y = \frac{K}{x - a} \quad (\text{hyperbola})$$

which was suggested to me by Professor Houssay, by means of which he expresses the phenomenon of regression of certain organs in animals, under special conditions; but this proved to be unsuccessful.

On the basis that during the short time  $dt$  the cicatrized area  $ds$  remains proportional to the total area, we can write

$$(1) \quad - ds = KSdt$$

<sup>1</sup> du Noüy, P. L., *J. Exp. Med.*, 1916, xxiv, 451, 461; 1917, xxv, 721.

by integration in respect to time,

$$(2) \quad T = - \int_{S_0}^S \frac{ds}{KS}$$

or

$$T = - \frac{1}{K} \int_{S_0}^S \frac{ds}{S}$$

hence

$$T = \frac{1}{K} \text{Log}_e \frac{S_0}{S}$$

which is similar to the equation of Slater,<sup>2</sup>

$$T = \frac{1}{K} \text{Log}_e \frac{N+n}{N}$$

and finally,

$$(3) \quad KT = \text{Log}_e \frac{S_0}{S}$$

that is,

$$S = S_0 e^{-KT}$$

We can then compute the values of the coefficient  $K$  for the different values of  $T$ .  $K$  increases regularly. Therefore, the curve obtained from the equation

$$S = S_0 e^{-KT}$$

does not correspond to the facts, and gives for every value of  $T$  a certain value of  $S$  which deviates more and more from that calculated according to formula (1) (extrapolation form). We were then obliged to introduce a new coefficient, stating the problem in the following way: Is it better to attempt to find this new coefficient by giving to  $T$  its real value and by studying the variations of  $K$ , or is it more advisable to study the variations of the exponent if  $K$  remains

<sup>2</sup> Slater, A., *Biochem. J.*, 1912-13, vii, 197.

constant; that is, the variations of a certain coefficient  $\alpha$  as in the exponent

$$(4) \quad -K(T + \alpha)$$

The study of a large number of cases showed that by trying to find the correction of the coefficient  $K$ , I encountered a practical difficulty from the fact that since this coefficient is small in respect to  $T$ , the smallest numerical variations such as those arising from calculation errors with 2 or 3 decimal numbers were of sufficient importance to destroy the concordance of the curves. On the contrary, in the second case, fairly important variations in a certain coefficient  $K_2$ , the connection of which with  $\alpha$  can be expressed as

$$(5) \quad \alpha = \frac{T^2}{K_2},$$

interfered very little with the accuracy of the calculation.

Text-fig. 1 shows the variations of the coefficient  $K$  in function of time. The angular coefficient of the lines seems to vary proportionally with the index of cicatrization, as

$$\text{index } i = \frac{S_o - S}{S_o(t + \sqrt{i})}$$

It is by no means certain that these lines are straight lines mathematically (see the straight dotted line in Text-fig. 1), but the observations are limited by time and it is difficult to determine this point. In this chart the value of  $K$  is given by equation (3) from which the following formula is obtained:

$$(6) \quad K = \frac{\text{Log } S_o - \text{Log } S}{T}$$

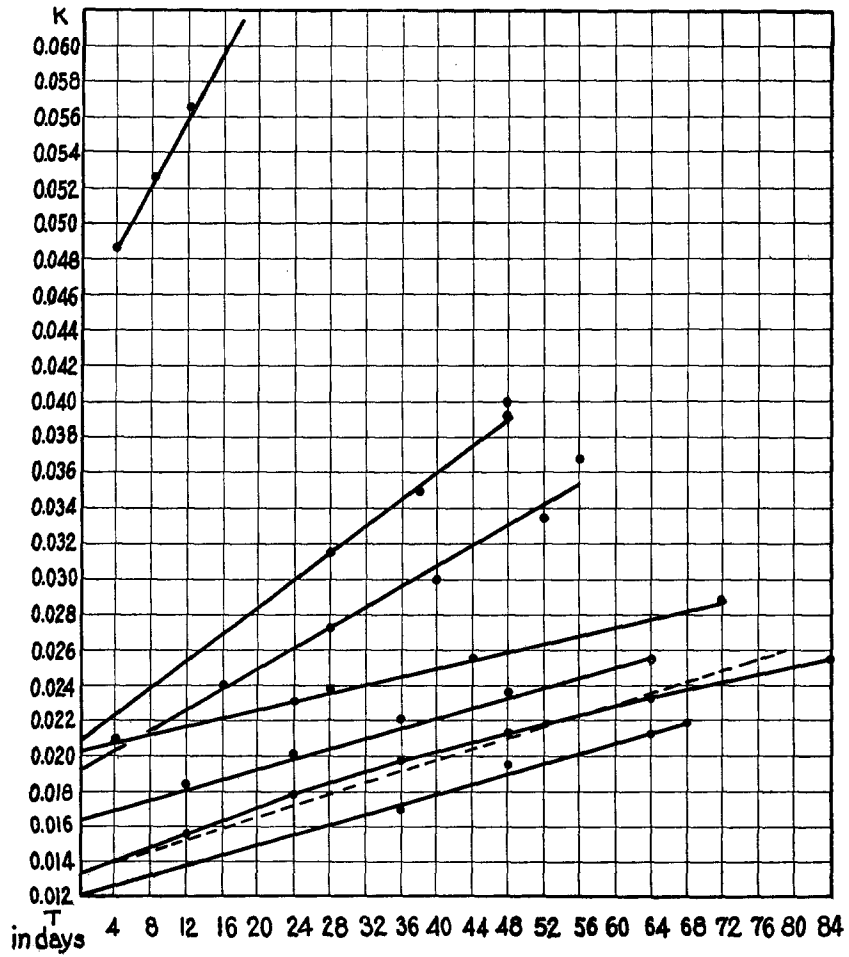
Text-fig. 2, on the contrary, shows the variations of the coefficient  $\alpha$  previously determined, the value of which is

$$(7) \quad \alpha = \frac{\text{Log } S_o - \text{Log } S}{K} - T$$

By plotting in ordinates the values of  $\alpha$  which represent the difference between the curve resulting from equation (3) and that resulting

from equation (1), we obtain a curve which expresses the law of these differences. It is a branch of parabola and the equation is

$$y^2 = 2 px$$



TEXT-FIG. 1. Variations of the coefficient *K*, in function of time.

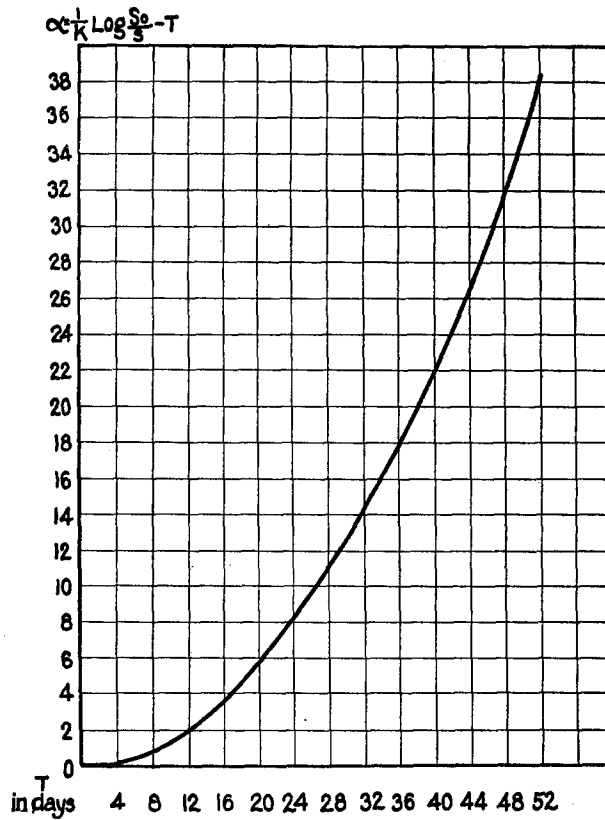
That is, by replacing the letters by those we have adopted, *viz.*,  $y = T$ , and  $x = \alpha$ ,

(8) 
$$\alpha = \frac{T^2}{2p}$$

The significance of the coefficient  $K_2$  in equation (5) appears now clearly, and the equation may be written

$$(9) \quad S_T = S_0 e^{-K \left( T + \frac{T^2}{2p} \right)}$$

which is the general equation of the law.



TEXT-FIG. 2. Variations of the coefficient  $\alpha$  in function of time in the formula  $S_T = S_0 e^{-K(T + \alpha)}$

Before we begin a thorough study of the coefficients it may be interesting to compare, for example, two series of figures representing the ordinates, *viz.* the areas of wounds in square centimeters, of two

cicatrization curves obtained, the first (figures of the upper row) by means of the last exponential equation (9), the second (figures of the lower row) by means of the former extrapolation formula (1). It is obvious that the concordance is almost perfect and that the differences are beyond the errors of experimentation (Table I).

These two examples suffice to show that the proposed equation fulfills the required conditions; in all the cases the coincidence is equally satisfactory. Slight differences, however, sometimes may be observed at the beginning of the curve (for  $T = 4, 8, 12$  days), but since the exponential equation has been mathematically studied in a different manner from the first formula, and since, on the other hand, these differences may be affected by errors of measure of the area of wounds, it cannot be concluded that the equation previously proposed is more accurate than the new one.

*Study of the Coefficients  $K$  and  $2p$ .*

As the coefficient  $K$  can be determined within 4 days, that is from two points on the curve, 4 days apart, and as the contraction, especially for the large wounds, plays the principal part at the beginning of cicatrization, this coefficient characterizes the contraction, and during the first days the relative rate of repair, with reference to the total area of the wound. But it has been stated<sup>1</sup> that this rate is itself a function of the age of the man, within certain limits. Hence the coefficient  $K$  must logically be proportional to the index of cicatrization  $i$  which plays the same part in formula (1). The calculation of a number of curves shows that this is so.

The velocity of repair is originally determined by the area of the wound. We have assumed that at the beginning of the phenomenon it remained proportional to the area for a very short time. We proceeded from this assumption to state the differential equation

$$- ds = KSdt$$

If the velocity remained proportional to the area, this would explain the increasing delay due to the reduction of the area of the wound. On account of this delay, the phenomenon is expressed by a logarithmic curve and not by a straight line, for, at a certain moment  $T$ , the area which is not yet cicatrized is  $TM = S$  (Text-fig. 3).

TABLE I.

Area calculated with.	Time in days.																			
	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
Patient 263.																				
Equation (9).....	61.8	50.8	41.4	33.4	26.8	21.2	16.7	13.1	10.2	7.7	6.0	4.5	3.4	2.5	1.8	1.4	1.0	0.7	0.5	0.4
“(1).....	61.8	51.0	41.6	33.6	26.9	21.3	16.7	13.1	10.2	7.7	6.0	4.5	3.4	2.5	1.8	1.4	1.0	0.7	0.5	0.4
Patient 706.																				
Equation (9).....	27.4	21.8	17.1	13.4	9.9	7.2	5.1	3.6	2.4	1.6	1.1	0.69	0.43							
“(1).....	27.4	22.0	17.1	13.0	9.6	7.2	5.1	3.6	2.4	1.6	1.1	0.7	0.46							

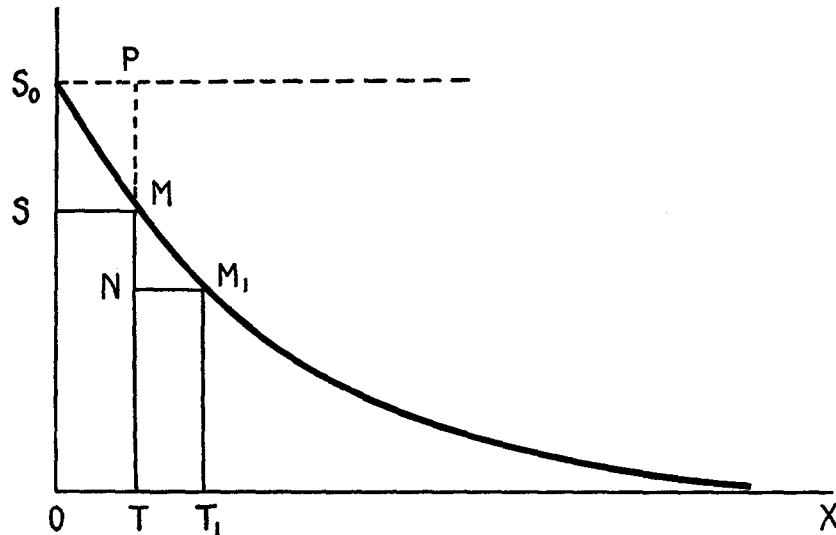
The surface which is already cicatrized is represented by

$$MP = S_0 - S$$

The law of the curve, if logarithmic, is that the decrease  $MN$  of the ordinate  $S$ , when passing from the time  $T$  to the time  $T_1$ , is proportional to the length  $S$  of the ordinate; that is, the area cicatrized during the time  $T_1 - T$  is proportional to the area which is not yet cicatrized. This is what we have written in mathematical symbols, for infinitesimal values

$$- ds = KSdt$$

for  $ds$  corresponds to  $MN$  and  $dt$  to  $T_1 - T$ .



TEXT-FIG. 3. Logarithmic curve.

But this hypothesis, true at the beginning of the phenomenon, under certain conditions, grows rapidly erroneous, since we have stated that the curve resulting from this equation deviates more and more from the experimental facts. Hence the diminution of the area is not the only factor which governs the real curve. A careful study of the latter and a comparison with the plain logarithmic curve shows that to the *decreasing* acceleration a uniformly *increasing* acceleration is opposed, which at every moment counteracts the effect of the delay due to the decrease of the area.



But if the hypothesis is justified at a certain moment, the simple equation which proceeds from it

$$S_T = S_0 e^{-KT}$$

must represent the phenomenon at the beginning and must express the part played by the first factor, the contraction, which intervenes alone at this moment, as long as the second disturbing factor does not enter into action, or its part is small with reference to that of the first one.

We can verify the correctness of this statement by drawing the curve representing the contraction of a wound; this can be done by measuring the total area of the new scar tissue, no longer merely the area of granulations. For we know that the decrease of this area measures solely the contraction.<sup>3</sup> Then the contraction curve obtained in this way should logically, within certain limits, comply with the law expressed by equation (3). Text-fig. 4 illustrates this fact, and our first hypothesis was therefore justifiable.

It is easily seen that the phenomenon follows the law until, owing to the decrease of the wound area, a more important part of the work of reparation in respect to the area of the granular surface is carried out by the second factor. Then, the decrease of the area being much greater than indicated by equation (3), the contraction, which depends obviously on the area not yet cicatrized, slackens gradually, until it ceases entirely. These observations would show plainly, if we were not already aware of it, that the second factor is the epithelization, and it is then understood that its action is represented in equation (9) by the quotient  $\frac{T^2}{2p}$ , which expresses that its efficiency, feeble at the beginning, increases slowly at first, then more rapidly, according to a parabolic law.

The above statements are generally verified only if the first observations are made when the cicatrization has already begun, and little or no epithelization has yet appeared. The starting-points of both curves (contraction and cicatrization) are confounded, that is they have the same ordinate at the time 0, so that the surface of the wound itself and that of the cicatrix cannot be discerned from each

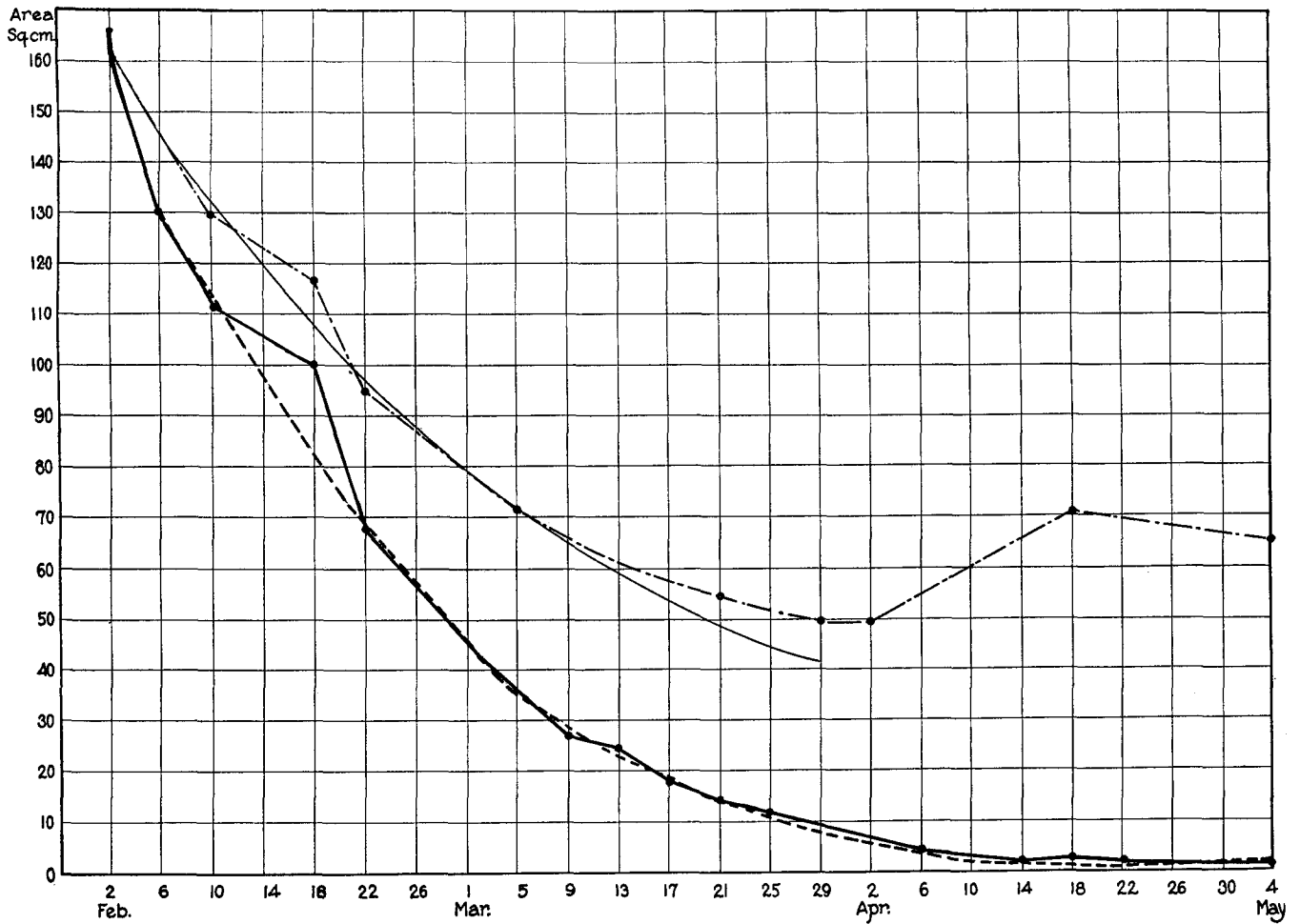
<sup>3</sup> Carrel, A., and Hartmann, A., *J. Exp. Med.*, 1916, xxiv, 429.

other, the edges of the wound being constituted by the old skin or a new and hardly visible epithelial border (Text-fig. 4). When epidermization has already begun, the ordinates at the time 0 are not coincident. If  $S_0$  is the ordinate of the area of granulations, in square centimeters, and  $S_1$  the area of the cicatrix, let us call  $A$  the difference  $S_0 - S_1$ .  $A$  represents the surface already covered by the new epithelium and the equation becomes

$$S_1 = A + S_0 e^{-\kappa T}$$

$A + S_0$  is what we have called the area of the cicatrix. But in this case it is often more difficult to verify formula (3) for the contraction, because the epithelization may have become important enough to disturb the simple phenomenon of contraction, the disturbing action being obviously the function of  $A$ . The difficult definition of the outline of the cicatrix is also a cause of error. This explains why it is difficult, except on experimental wounds, to find cases on which observation can be made accurately. However, Text-fig. 5 shows that this is possible. The measure of the cicatrix area is made by drawing on cellophane the common limit of the old skin and of the new epithelium, or scar tissue. It is essential to draw this outline on the skin itself, in order to prevent errors of interpretation and of drawing which are frequent, as this common limit often lacks sharpness. But if at the beginning it is tattooed (on animals) or drawn with a dermatographic pencil (on men), the measures become comparable and can be done with sufficient accuracy. Every time a drawing is taken, it is advisable to go over the outline again with the pencil where it shows a tendency to be obliterated.

As regards the term  $\frac{T^2}{2p}$ , what has already been said concerning its growing action in function of time must be taken merely from a mathematical standpoint and not as an assumption dealing with the mechanism of the phenomenon itself. The activities of the real factors are not known, and we can only measure one of the results of these activities, which may vary proportionally to the mathematical factors. Our knowledge does not go beyond that. For example, we know that  $\frac{T^2}{2p}$  increases slowly at first, then rapidly, and we assume



TEXT-FIG. 4. Patient 360. The dotted and broken line represents the contraction of the wound (for details of technique see Carrel and Hartmann<sup>3</sup>). The light line is the calculated curve, according to the formula

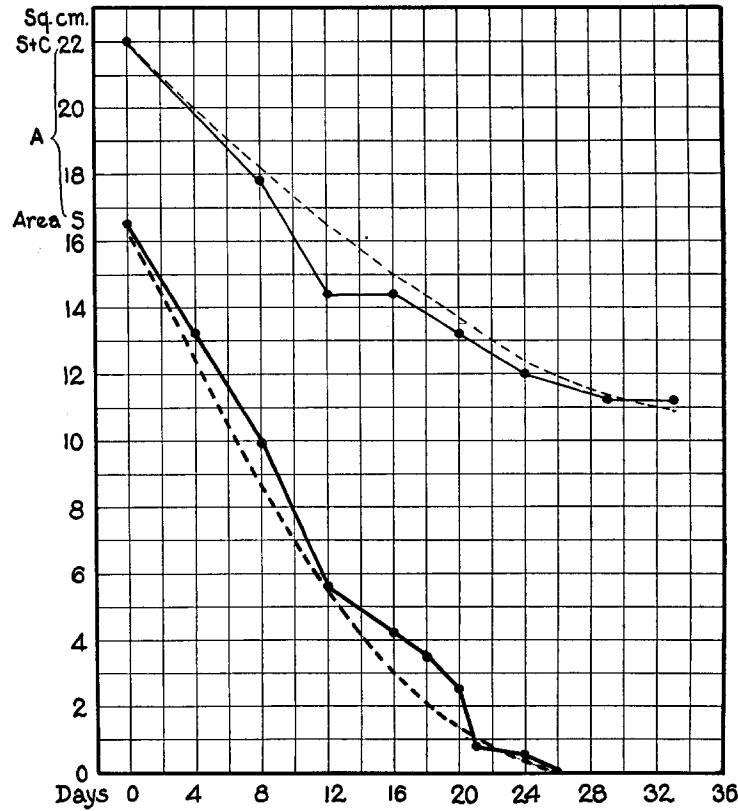
$$S_T = S_0 e^{-KT}$$

The heavy line represents the decrease of the area of the wound (curve of cicatrization), and the dotted line, the curve calculated according to the equation

$$S_T = S_0 e^{-K \left( T + \frac{T^2}{2f} \right)}$$

The decrease in the rate on February 18 is due to infection.

that this factor represents the epithelization. It must not be inferred that the latter remains proportional to  $\frac{T^2}{2p}$  and increases at first slowly, then rapidly. On the contrary, we know that epitheliza-



TEXT-FIG. 5. The upper curves are the contraction curves. The dotted curve is calculated according to the formula

$$S_T = S_0 e^{-KT}$$

The lower curves are the so called cicatrization curves expressing the decrease of the area of the granulations.

tion, or growth of cells, is likely to be much more active at the beginning of the cicatrization, according to the length of the epithelial edge, and then must decrease in absolute value. In proportion as the

wound decreases, the length of the epithelial edge diminishes, and at the same time the number which measures, in absolute value, the proliferation of cells. But, as it is likely that the number of cells produced by a unit of length is the same for each unit of time, and as, on the other hand, ordinarily the area decreases much faster than the perimeter (four times more rapidly for the square), it is clear that the production of cells by the edges seems to increase and that if it is expressed by units of covered area it increases really with reference to the area of the wound. What must therefore be understood by "the factor represents the epithelization," is that in the considered equation owing to the introduction of this factor, the relations existing between epithelization and the decrease of the area are satisfactorily expressed, and that it enables us to express the result of the phenomena in a way which is in accordance with the facts.

*End of the Phenomenon.*

Since in a logarithmic curve the diminution of the ordinate is always proportional to the ordinate, it never becomes zero. The curve, as well as that which had been established previously, is asymptotic to the axis of the time. But we have already stated, in a former paper, the moment at which cicatrization comes practically to an end.<sup>4</sup> This happens when our methods of measuring are unable to estimate the progress of the phenomenon. This moment is rapidly followed—in a few hours—by complete healing of the wound. The curve practically comes to an end, and experience has shown that it can be arbitrarily stopped, when the ordinate is inferior to 0.4 sq. cm. This means that, when the calculation comes to a figure smaller than 0.4 sq. cm., the corresponding abscissa, that is the time, indicates the date of complete cicatrization. Besides, this conforms to the facts in the majority of cases, as has been shown before, and the errors are small. In all natural phenomena, the law of which is expressed by an exponential equation, the same holds true.

*Numerical Value of the Coefficients. Relation of K, the Index of Cicatrization i, and the Parameter 2p.*

Calculation of fifteen cicatrization curves has shown principally three facts. The first, to which I referred above (page 334), is the pro-

<sup>4</sup> du Noüy, P. L., *J. Exp. Med.*, 1916, xxiv, 451; 1917, xxv, 721.

portional variations of  $K$  and of the index of cicatrization. Table II shows this plainly. The ratio  $\frac{i}{K}$  varies between 1.6 and 1.2 inversely to  $i$  and  $K$ . The second fact is the remarkable constancy of the factor  $2p$ , or parameter of the parabolæ expressing the acceleration due to the epithelization. The third fact is the relation which seems

TABLE II.  
*Comparative Numerical Results.*

No. of patient.	Area. <i>sq. cm.</i>	Index $i$ .	$K$	$2p$	$100 \frac{K}{i} = \delta$		$\frac{i}{K} = \beta$
						$\frac{1}{\beta'}$	
318	64.0	0.0200	0.0132	80	66		1.51
737	50.3	0.0200	0.0138	90	69	81.0	1.45
263	61.8	0.0205	0.0140	85	68	80.5	1.46
360	113.0	0.0210	0.0147	90	70	80.0	1.43
795	21.6	0.0255	0.0174	73	68.5	79.0	1.46
721	40.4	0.0285	0.0192	70	68	78.5	1.46
706	27.4	0.0325	0.0222	70	68.5	77.5	1.46
724	13.9	0.0346	0.0255	77	74	77.0	1.36
725	30.6	0.0375	0.0277	75	74	76.0	1.35
791	23.0	0.0400	0.0295	73	74	75.5	1.35
692	31.2	0.0420	0.0315	75	75	75.0	1.33
722	19.0	0.0465	0.0355	71	76	74.5	1.31
383	17.5	0.0500	0.0387	69	77	74.0	1.29
796	8.9	0.0550	0.0436	72	80	72.0	1.26
715	9.5	0.0700	0.0595	65	85	66.0	1.20

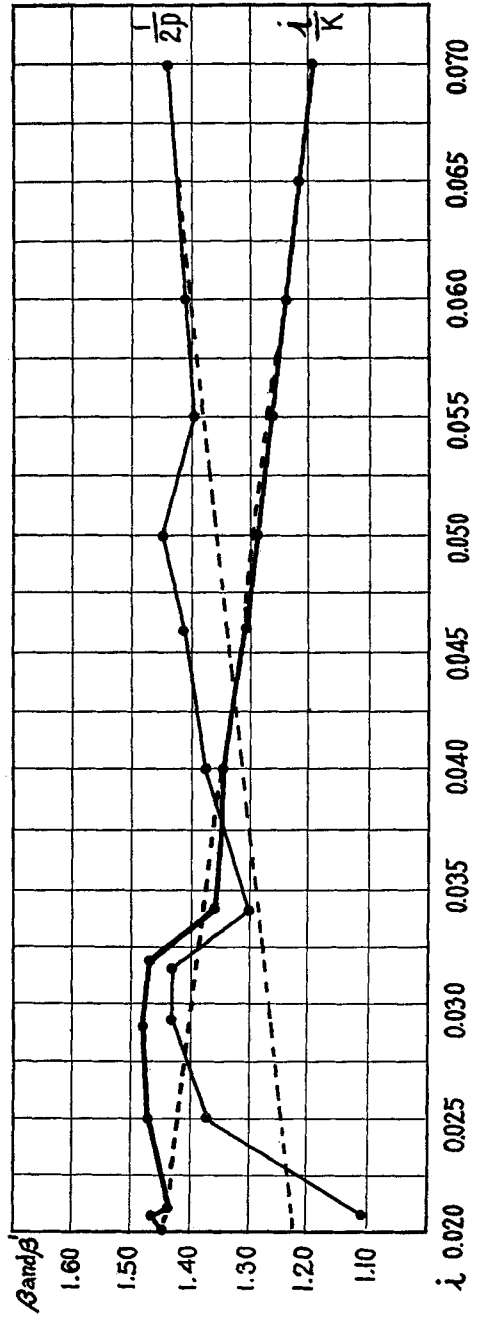
to exist between  $K$ ,  $i$ , and  $2p$ . This is clearly shown by the seventh column in which is reported the term

$$\delta = 100 \frac{K}{i}$$

The value of  $\delta$  for each wound is near enough to  $2p$  to allow its substitution for  $2p$ , approximately. This result is a natural conclusion of

the first two remarks, since, if we call  $\beta$  the ratio  $\frac{i}{K}$ , we can write

$$2p = \frac{1}{\beta} = \frac{\delta}{100}$$



TEXT-FIG. 6. Relations between  $i$ ,  $K$ , and  $2p$ . The ordinates represent  $\frac{i}{K} = \beta$  and  $\frac{1}{2p} = \beta'$ . Light lines,  $\beta'$ . Heavy lines,  $\beta$ .

But this is of immediate value for calculating the curve by equation (9), because, if the coefficient  $K$  can be determined by giving two experimental dates 4 days apart, the same process cannot be used for determining the parameter  $2p$ , for the parabola

$$\alpha = \frac{T^2}{2p}$$

can only be determined when  $T$  is great enough; *viz.*, 12 or 16 days. Otherwise the ordinates  $\alpha$  are smaller than 1 and the curve is not accurately defined. It is therefore worth while to be able to make an approximate calculation first. If this shows a noticeable error, it is easy to make a correction, as soon as several days have passed. Consequently, it is clear from the above paragraphs that it is possible to calculate the curve resulting from the equation

$$(9) \quad S_T = S_0 e^{-K \left( T + \frac{T^2}{2p} \right)}$$

by simply starting from a single measure of the wound and the age of the patient, that is from the index, since

$$K = \frac{i}{\beta}$$

Text-fig. 6 shows the relations between  $K$  and  $i$ . In order to make it clearer, I have plotted close to each point on the observed curve (dotted line) the inverse value of  $\beta$ ; that is,  $\frac{1}{\beta}$ . The light curve which expresses the observed variations of  $\frac{1}{2p} = \beta'$  shows that  $2p$  varies approximately inversely to  $\beta$  since every point on the curves can be expressed by the inverse value of the ordinate. This means that  $2p$  varies inversely to  $\delta$ . If we admit as a possibility that the light dotted line corresponds to the average mean value of  $2p$  and that the values which deviate from it are due to errors of calculation, the probable values of  $2p$  can be computed for a certain value of  $i$ . The figures in the column marked  $\frac{1}{\beta'}$  (Table II) may be used for the first approximation of  $2p$ .



*Calculation of the Coefficients.*

These analogies may be of use in determining  $2p$ , but sometimes a less accurate approximation is obtained when both  $K$  and  $2p$  are inferred from the index. In such a case the direct calculation of  $K$ , which is extremely simple, is of more value. It is deduced, as stated above, from equation (9) which gives

$$K = \frac{1}{T} \text{Log} \frac{S_0}{S_t}$$

$S_0$  being the first measure of the area of the wound,  $S_t$  the area at the time  $t$  (practically 4 days). After  $K$  has been determined, at least two values of  $\alpha$  must be calculated unless the relations between the coefficients, mentioned above, are employed. We have stated

$$(7) \quad \alpha = \frac{1}{K} \text{Log} \frac{S_0}{S_T} - T$$

The values of  $S$  corresponding to 12 and 20 days, for instance, are taken. (The greater  $T$  is, the more accurate the values of  $2p$  will be within the limits of 20 to 40 days.)  $2p$  is immediately obtained by means of the formula

$$(8) \quad 2p = \frac{T^2}{\alpha}$$

Since we have two values of  $\alpha$ , we obtained two values of  $2p$  and the mean value is taken.

The coefficient  $K$  is smaller than  $i$  and the quantity  $\alpha$  must remain positive. If the contrary happens ( $\alpha < 0$ ), a determination of  $K$  for a longer period of time (5, 6, or 8 days) must be made. This rarely happens.

*Use of the Equation. Calculation of the Curves; Numerical Examples.*

In order to enable the reader who is unfamiliar with the use of mathematical formulas, to use this equation, we shall make the complete calculation of one curve by using successively the direct calculation, or ordinary method,  $i$  being the supposed unknown, and then the method based upon the analogies existing between the coefficients. The difference in accuracy of both methods will thus be noted. Whenever the index  $i$  varies around 0.04 the results obtained by the second

technique are excellent. The reason is evident from Text-fig. 6 in which the values of the coefficients are in accordance with this particular value of  $i$ .

(1) *Direct Calculation.*—Only the initial area and that after 4, 8, or 20 days are given (Table III).

TABLE III.  
*Example of Direct Calculation.*

$T$	$S_0$ observed area.	$\text{Log } S$	$\text{Log } \frac{S_0}{S}$	$\alpha$	$2p$	Calculated area.
	<i>sq. cm.</i>					<i>sq. cm.</i>
0	40.4	1.602				
4	33.5	1.525	0.077	0		
8	27.0	1.431	0.171	0.9	71	27.0
20	12.5	1.097	0.505	6.2	65	12.8
28	6.8					7.0
36	3.5					3.5
44	1.7					1.7
54	0.6					0.6

The calculation requires accordingly (a) the determination of  $K$

$$\left( K = \frac{\text{Log } S_0 - \text{Log } S_T}{T} \right), \quad K = \frac{0.077}{4} = 0.0192$$

(b) the determination of  $\alpha$  (8th day)

$$\left( \alpha = \frac{\text{Log } S_0 - \text{Log } S_T}{K} - T \right), \quad \alpha = \frac{0.171}{0.0192} - 8 = 0.9$$

(c) the determination of  $2p$  (8th and 20th days)

$$\left( 2p = \frac{T^2}{\alpha} \right), \quad 2p = \frac{64}{0.9} = 71$$

and finally the calculation of the points of the curve for the given times by formula (9)

$$\text{Log } S_T = \text{Log } S_0 - K \left( T - \frac{T^2}{2p} \right)$$

(2) *Indirect Calculation.*—This is based only upon the relations previously stated between  $i$ ,  $K$ , and  $2p$ .  $2p$  may be taken either from

Table II or from Text-fig. 6, for the given value of  $i$ . In the preceding example  $i = 0.0285$  (Table V). For this value, the table indicates  $\frac{1}{\beta'} = 2p = 78.5$ . The factor given by Text-fig. 6 is 1.41. Hence  $K = \frac{i}{1.41} = 0.0202$ . By applying the formula the areas given in Table IV are calculated (compare with Table III).

This shows that the values are as good as those obtained by the direct method, sometimes even better, because in the latter method  $2p$  has only been determined from two points on the curve, which is a cause of error.

A direct determination of  $K$  can also be made and only the value of  $2p$  read in the table. The results obtained by this intermediate technique are good, but it shows no particular advantage, and, on the contrary, introduces a new factor of error and requires more time.

TABLE IV.

Area (S).					
8th day.	20th day.	28th day.	36th day.	44th day.	54th day.
<i>sq. cm.</i>	<i>sq. cm.</i>	<i>sq. cm.</i>	<i>sq. cm.</i>	<i>sq. cm.</i>	<i>sq. cm.</i>
27.0	12.4	6.8	3.5	1.6	0.5

It must be borne in mind that the determination of a curve by a single measure of the area and the normal index presents many advantages which may be of greater interest than the perfect coincidence between two curves obtained by two equations of different form. The advantages are: (1) the possession of the normal curve of cicatrization corresponding to the normal index, characterizing the age of the patient; this curve is used as a standard with which the individual curve is compared, if they do not agree; (2) the elimination of errors due to two measures of the wound, because during the time elapsed between the measures—4 days for example—a slight acceleration or a slight lessening in the rate might have occurred. The so called indirect method, therefore, should be used when the normal curve of a wound is to be calculated. In order to facilitate this calculation, I have drawn Text-fig. 7 similar to Text-fig. 6, except that the ob-

TABLE V.  
*Calculation of the Curve of Cicatrization.*  
*The Two Coefficients of the Formula*  
 $S' = S [1-i (t + \sqrt{nt})].$

Area of wound.	1st coefficient—index of cicatrization $i$ .					2nd coefficient (time coefficient) $t + \sqrt{nt}$ .
	Age of patient.					
	20 yrs.	25 yrs.	30 yrs.	32 yrs.	40 yrs.	
sq. cm.						
150 and over.	0.0200	0.0200	0.0200	0.0200	0.0200	6.00
						6.81
						7.43
140	0.0210	0.0200	0.0200	0.0200	0.0200	8.00
						8.45
130	0.0220	0.0200	0.0200	0.0200	0.0200	8.90
						9.30
120	0.0225	0.0200	0.0200	0.0200	0.0200	9.65
						10.00
110	0.0240	0.0200	0.0200	0.0200	0.0200	10.32
						10.64
100	0.0250	0.0200	0.0200	0.0200	0.0200	10.93
						11.21
90	0.0275	0.0220	0.0200	0.0200	0.0200	11.48
						11.75
80	0.0300	0.0230	0.0200	0.0200	0.0200	12.00
						12.25
70	0.0325	0.0250	0.0200	0.0200	0.0200	12.48
						12.72
60	0.0355	0.0300	0.0225	0.0200	0.0200	12.95
						13.16
50	0.0400	0.0340	0.0265	0.0230	0.0200	13.37
						13.60
40	0.0445	0.0400	0.0310	0.0270	0.0220	
30	0.0500	0.0450	0.0375	0.0330	0.0260	
25	0.0540	0.0500	0.0400	0.0375	0.0290	
20	0.0580	0.0540	0.0465	0.0425	0.0325	
15	0.0645	0.0600	0.0525	0.0475	0.0380	
10	0.0700	0.0660	0.0625	0.0550	0.0450	
5 and under.	0.0800	0.0750	0.0700	0.0700	0.0700	

served points are suppressed and the scale of the ordinates is larger, so that a greater accuracy may be obtained. To show the degree of approximation obtained by the new technique I have collected the calculations of four wounds. The figures calculated according to the extrapolation and exponential equations correspond to every observed area. These curves have been chosen intentionally, so that their indices are different. The calculation of the coefficients  $K$  and  $2p$  is then simply done by looking for the index in Table V in function of the age of the man and of the area of the wound; then by using the relations (Table II)

$$K = \frac{i}{\beta} \text{ and } 2p = \frac{100}{\beta'}$$

calling  $\beta$  the observed values of  $\frac{i}{K}$  (solid line) and  $\beta'$  the observed values of  $\frac{1}{2p}$  (dotted and broken line), the values of  $2p$  can also be found in Text-fig. 7, since the values of Table II have been calculated from this straight line. Text-fig. 7 is only used in order to give two conversion factors  $\beta$  and  $\beta'$ , to be applied for computing  $K$  and  $2p$ .

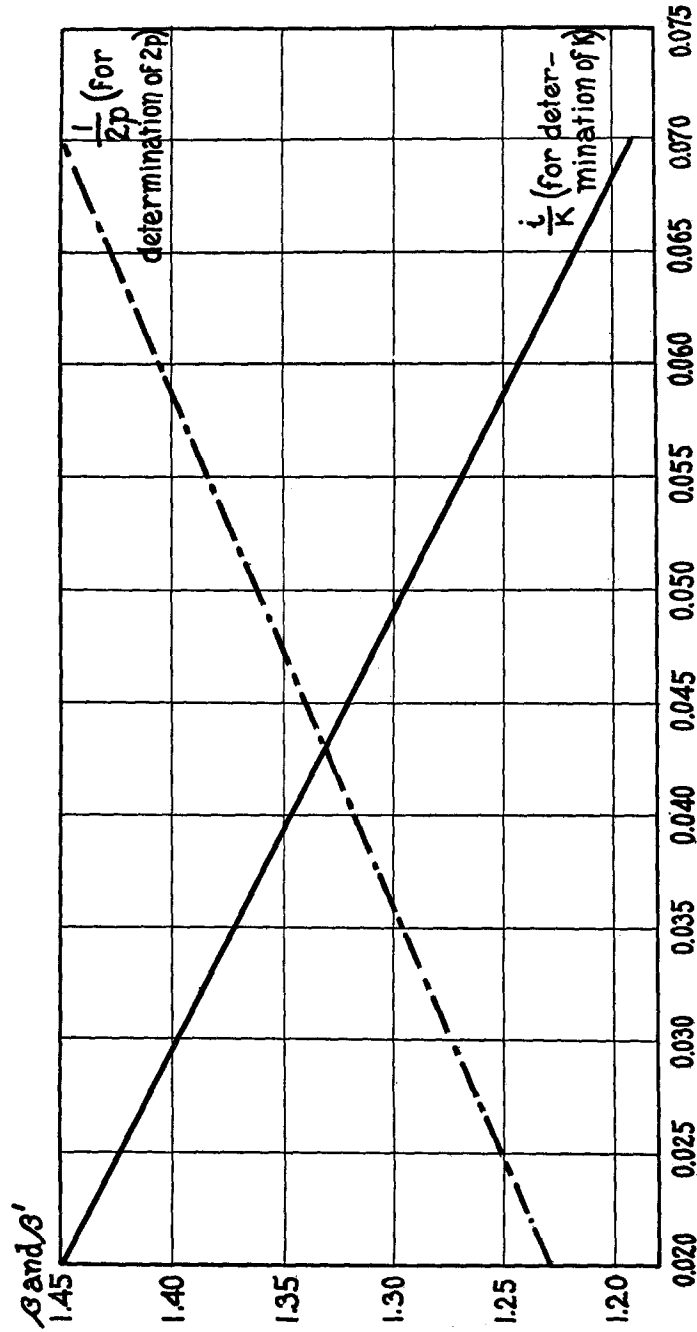
#### Comparative Examples.

##### Patient 360.

$$S_o = 129.4 \text{ sq. cm.}, i = 0.021, \beta = 1.44, \beta' = 1.23, K = \frac{i}{\beta} = 0.01458,$$

$$2p = \frac{1}{\beta'} = \frac{1}{1.23} \times 100 = 81.$$

Area.	8th day.	20th day.	44th day.	60th day.	76th day.	84th day.
	sq. cm.	sq. cm.	sq. cm.	sq. cm.	sq. cm.	sq. cm.
Observed .....	105.0	57.0	13.8	4.1	1.8	0.6
Calculated { Equation (1).....	96.8	55.9	14.0	4.7	1.9	1.0
" (9).....	96.5	56.0	13.4	3.9	1.4	0.4



TEXT-Fig. 7. Relations between  $i$ ,  $K$ , and  $2p$ . The chart gives the ratio  $\beta = \frac{i}{K}$  and  $\frac{1}{2p} = \beta'$ , by means of which  $K$  and  $2p$  can be calculated.

*Patient 488.*

$$S_0 = 34.5 \text{ sq. cm.}, i = 0.03, \beta = 1.40, \beta' = 1.27, K = 0.0214, 2p = 79.$$

Area.	8th day.	20th day.	32nd day.	44th day.	48th day.	
	sq. cm.	sq. cm.	sq. cm.	sq. cm.	sq. cm.	
Observed.....	20.6	11.0	4.0	1.7	0.9	
Calculated {	Equation (1).....	22.4	9.9	3.7	1.3	0.8
	" (9).....	22.2	10.0	3.8	1.2	0.7

*Patient 694.*

$$S_0 = 44.3 \text{ sq. cm.}, i = 0.0425, \beta = 1.33, \beta' = 1.33.$$

Area.	8th day.	16th day.	20th day.	28th day.	
	s	sq. cm.	sq. cm.	sq. cm.	
Observed.....	23.6	11.2	8.5	2.5	
Calculated {	Equation (1).....	23.5	10.5	6.9	2.4
	" (9).....	23.1	10.7	6.8	2.6

*Patient 519.*

$$S_0 = 19.0 \text{ sq. cm.}, i = 0.0570, \beta = 1.26, \beta' = 1.39.$$

Area.	4th day.	12th day.	20th day.	28th day.	
	sq. cm.	sq. cm.	sq. cm.	sq. cm.	
Observed.....	12.2	4.2	1.0	0.4	
Calculated {	Equation (1).....	12.5	4.4	1.2	0.3
	" (9).....	12.2	4.4	1.3	0.3

## CONCLUSION.

1. The law of cicatrization of surface wounds may be expressed by an exponential formula in which the two coefficients may be determined.

2. A simple relation exists between these coefficients and the index,  $i$ , of cicatrization, previously established in function of the age of the patient and of the area of the wound.

3. The proposed equation with a simplified exponent, reduced to a single coefficient, expresses satisfactorily the phenomenon of contraction.