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Quasi-axially symmetric stellarators

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ABSTRACT **Confinement of a plasma for controlled thermonuclear fusion is studied numerically. Toroidal equilibria are considered, with an emphasis on the Modular Helias-like Heliac 2 (MHH2), which is a stellarator of low aspect ratio with just two field periods surrounded by 16 modular coils. The geometry is fully three-dimensional, but there is an axial symmetry of the magnetic structure that is calculated to give confinement competitive with that in circular tokamaks. Additional vertical and toroidal field coils, together with a current drive, provide the flexibility and the control of rotational transform necessary for a successful experiment. An MHH3 device with three field periods and comparable quasi-axial symmetry is presented, too, and because of reversed shear, its physical properties may be better. Variational analysis of equilibrium and stability is shown to give a more reliable prediction of performance for these stellarators than linearized or local theories that suffer from a failure of differentiability and convergence.**

1. Introduction

It is natural to choose toroidal geometry for the magnetic confinement of plasma in a fusion reactor because the orbits of hot ions and electrons should not be allowed to escape. In an axially symmetric tokamak the poloidal field required for equilibrium is produced by net toroidal current that is hard to control and may cause disruptions. Stellarators instead provide for the poloidal field by introducing three-dimensional asymmetries that cause the magnetic lines to spiral around on nested flux surfaces where they acquire nonzero rotational transform in a natural way. The advantage of stellarators is that they can be operated in a steady state and are relatively stable, but complicated geometry makes them difficult to visualize and construct.

The conventional way to generate a stellarator field is by means of helical coils that become interlocked as they rotate around the torus. However, better configurations have been found by shaping the plasma so that the external magnetic field confining it is produced by a system of modular coils wound on a desirable outer control surface. Numerical calculations enable one to design modular stellarators with twisted coils not unlike the toroidal field coils of a typical tokamak. This new approach opens up opportunities to optimize equilibrium, stability, and transport of the plasma.

Modern methods of computational physics have been applied to improve the equilibrium, stability, and transport properties of modular stellarators. One of the best new configurations is the helias (1), which was discovered by running the BETA code written at New York University (2). The Wendelstein 7-X (W7-X) experiment authorized for construction in Germany is one implementation of this concept (3). Another is the Helically Symmetric Experiment (HSX) at the University of Wisconsin, which has been designed so that the magnetic structure has a helical symmetry leading to good confinement of hot elecrons (4). A related configuration, called the Modular Helias-like Heliac, was developed; it has been optimized by lowering the number of field periods from 4 to 2 and reducing the aspect ratio of the plasma to 3.5 (5). This stellarator, known as the Modular Helias-like Heliac 2 (MHH2), has a magnetic structure approximating remarkably well the two-dimensional symmetry of a tokamak. Here we shall study physics problems for the MHH2 that have arisen during discussions of a new experiment. We also introduce a new MHH3 configuration with three field periods and aspect ratio 4.5 whose magnetic spectrum has axial symmetry almost as good as that of the MHH2 and whose rotational transform has reversed shear (cf. Fig. 1).

For toroidal equilibria the matrix of Fourier coefficients of the magnetic field strength in a flux coordinate system is known as the magnetic spectrum (6). The MHH2 and MHH3 are called quasi-axially symmetric stellarators because their spectra consist to a good approximation of just one row of elements with the same indices as those of a tokamak. Because of this symmetry property, the orbits of trapped particles lie on closed drift surfaces and remain well confined. The favorable transport in quasi-axially symmetric stellarators has been substantiated by Monte Carlo calculations employing a variety of codes (7).

The most satisfactory way to study magnetohydrodynamic equilibrium and stability of complicated configurations like the MHH2 and MHH3 is to run three-dimensional computer codes (8, 9). In this work, we rely on the NSTAB code written by Mark Taylor (10), a code that constructs weak solutions of the differential equations and captures islands or current sheets that are sometimes poorly modelled. The variational principle of ideal magnetohydrodynamics enables us to treat questions of equilibrium and stability simultaneously by a single method. In Section 2, we shall use this procedure to analyze global stability of the MHH2 and MHH3. In Sections 3 and 4, we discuss specifications for a tentative experiment, with an emphasis on flexibility. We show how modular coils can be found that are only moderately twisted and would not be excessively hard to manufacture. We analyze the dependence of the equilibrium on auxiliary vertical and toroidal fields and suggest how these can be used to obtain a family of interesting configurations within the framework of a single experiment. Parameters defining the new quasi-axially symmetric MHH3 stellarator that has been discovered are also given.

2. Equilibrium and Stability

The NSTAB code provides a computer implementation of the variational principle

$$\int\int\int [B^2/2 - p]dx_1dx_2dx_3 = \text{minimum}$$

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Abbreviations: MHH2, Modular Helias-like Heliac 2; W7-X, Wendelstein 7-X; HSX, Helically Symmetric Experiment.

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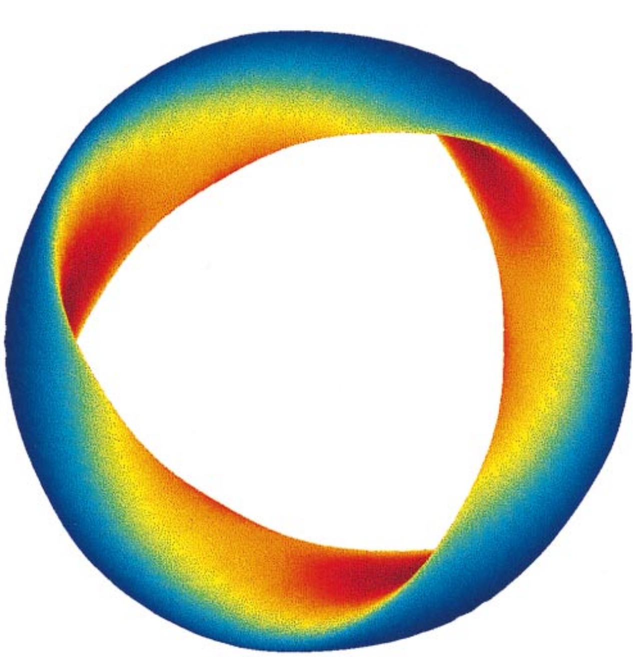

FIG. 1. Top view of the plasma in an MHH3 stellarator colored to display quasi-axial symmetry of the magnetic spectrum.

of magnetohydrodynamics, where B is the strength of the magnetic field and p is the pressure (10). Excellent resolution is obtained by combining a spectral representation in the toroidal and poloidal angles with a low order but exceptionally accurate finite difference scheme in the radial direction. Conservation form of the magnetostatics equations is used to capture islands and current sheets effectively on crude grids (11). This method enables one to discuss global stability by looking for bifurcated solutions of the equilibrium problem. We shall apply it to investigate physical properties of the MHH2 quasi-axially symmetric stellarator.

In our formulation of the problem, the toroidal equilibria calculated by the variational principle lend themselves in a natural way to representations

$$\mathbf{B} = \nabla s \times \nabla \theta = \nabla \phi + \zeta \nabla s$$

of the magnetic field in terms of Clebsch potentials (2). It is convenient to choose the toroidal flux s as a radial coordinate and to renormalize θ and ϕ so they become poloidal and toroidal angles on each flux surface $s = \text{const}$. In this invariant coordinate system, we can expand the magnetic field strength in a Fourier series

$$1/B^2 = \sum B_{mn} \cos(m\theta - [n - \iota m]\phi),$$

where ι is the rotational transform. The coefficients B_{mn}, which are functions of s alone, comprise the magnetic spectrum of the equilibrium (6). The axial symmetry property of the MHH2 and MHH3 stellarators simply asserts that the terms with $n \neq 0$ are relatively small. More specifically, in the case of the MHH2 they contribute little more than 1% to the total field strength, which suffices to guarantee confinement times comparable to those in standard tokamaks (5).

The equilibrium conditions

$$\nabla \cdot \mathbf{B} = 0, \quad \mathbf{J} \times \mathbf{B} = \nabla p$$

can be manipulated to arrive at a simple magnetic differential equation

$$\frac{\partial \lambda}{\partial \phi} = p' \frac{\partial}{\partial \theta} \frac{1}{B^2}$$

for the parallel current λ in terms of the field strength B. Formal integration then yields the remarkable Fourier expansion

$$\lambda = \frac{\mathbf{J} \cdot \mathbf{B}}{B^2} = p' \sum \frac{m B_{mn}}{n - \iota m} \cos(m\theta - [n - \iota m]\phi),$$

provided that the relevant differentiations can be performed. The small denominators that appear on the right exhibit dramatically the resonances that occur at rational surfaces where $\iota = n/m$. The resulting failure of convergence of the series is important in the KAM theory, which shows that smooth solutions of the equilibrium problem cannot exist in three dimensions because the coefficients B_{mn} do not in general vanish (2).

The success of stellarator experiments makes it imperative to find a formulation of the toroidal equilibrium problem in three dimensions that overcomes the difficulty about nonexistence of continuously differentiable solutions. The answer furnished by the NSTAB code is to calculate weak solutions determined by equations in a conservation form that is associated with the variational principle of magnetohydrodynamics and requires less differentiation. Because the dependence of the magnetic field on the poloidal and toroidal angles is relatively smooth, good resolution in those variables can be obtained by the spectral method. However, in the radial coordinate s we use a special finite difference scheme that captures islands accurately on grids with a mesh size comparable to the island width. Detailed calculations have demonstrated that this mathematical model simulates the physics of the plasma remarkably well (10).

It has been proposed to construct a compact tokamak with stellarator sidebands that might contribute very little to the rotational transform. We applied the NSTAB code to such a configuration with ι decreasing over the full torus from 1.1 at the magnetic axis down to 0.2 at the separatrix. Significant $m = 3$, $n = 2$ islands appear at the rational surface $\iota = 2/3$. We performed calculations of the relevant equilibrium at

$$\beta = 2\langle p \rangle / B^2 = 0.007$$

by using 15 mesh intervals in the radial coordinate s and by using spectral terms of degree up to 20 in the angle coordinates. Despite the nested surface hypothesis implicit in our method, the presence of islands became clearly visible in the computation.

Even for true tokamaks with two-dimensional solutions we have been able to find a variety of bifurcated equilibria that have islands breaking the axial symmetry. An example is displayed in Fig. 2 for a spherical tokamak of aspect ratio $A = 1.8$ with rotational transform in the interval $1.1 \geq \iota \geq 0.2$. The computation is similar to what we described above and confirms the ability of the NSTAB code to capture small islands on coarse grids in toroidal equilibria. To draw correct physical conclusions, it is necessary to allow for this kind of complication in the structure of the magnetic surfaces.

The NSTAB code solves the magnetostatic equations numerically by means of an accelerated method of steepest descent. In significantly unstable cases, the potential energy has access to a lower level and a second bifurcated solution can be constructed. Calculation of the bifurcated equilibrium is facilitated by introducing a perturbation δf in the equations that is associated with some mode deemed to be dangerous. A quotient of norms $\| \delta U \| / \| \delta f \|$ of the corresponding change δU of the solution U and of δf itself serves as one measure of the instability (11). More convincing evidence is obtained when there is convergence to a bifurcated solution different from the original equilibrium after the perturbation has been removed. That is a result relatively free of details about the specific formulation of the variational principle that is used.

Our most successful analysis of stability for the new MHH3 configuration has resulted from performing perturbations δf

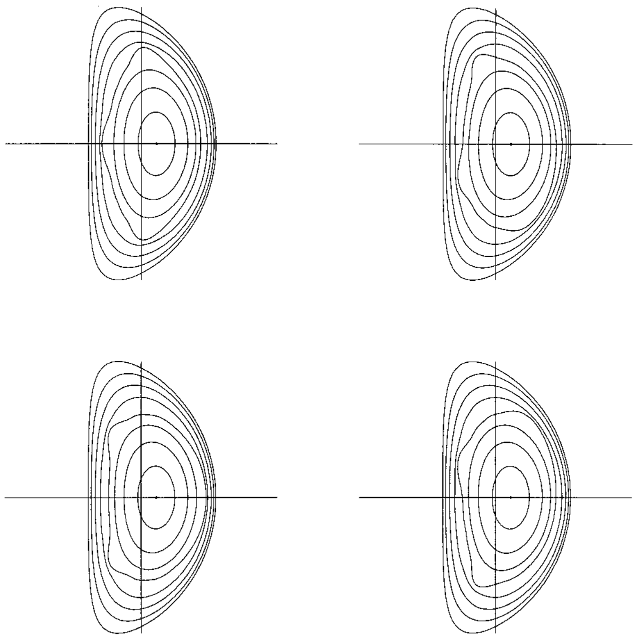

FIG. 2. NSTAB calculation of four Poincaré sections of the flux surfaces of a spherical tokamak showing how the code captures small islands at the resonance 2/3 in a bifurcated equilibrium without two-dimensional symmetry.

that break the stellarator symmetry of the equilibrium over a single period that is characterized by the appearance of exclusively cosine terms in the Fourier series for $1/B^2$. A bifurcated solution found this way is displayed in Fig. 3 for a mode triggered by trigonometric functions with $m = 6,\ n = 2$. The instability has been induced by adding a large net current so that the rotational transform ι over one field period falls from 0.28 at the magnetic axis down to 0.21 at the edge of the plasma. In this computation, we used a pressure profile $p = p_0(1-s^{1.5})^{2.0}$

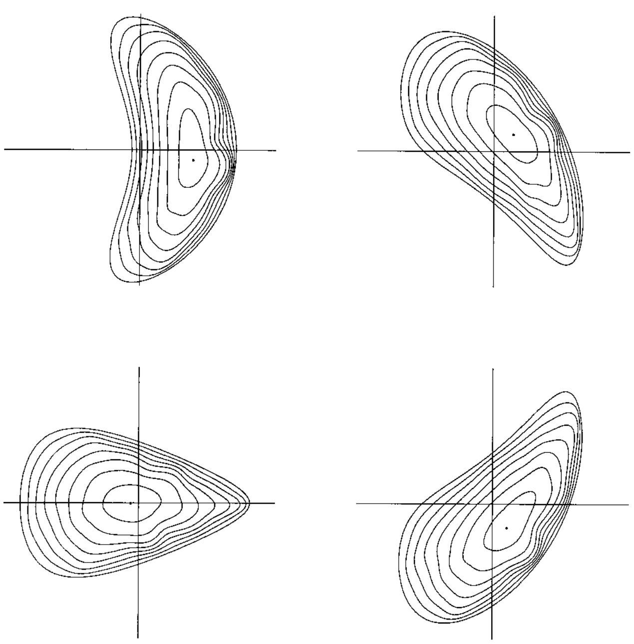

FIG. 3. Four Poincaré sections of a bifurcated MHH3 equilibrium with magnetic surfaces that do not have an expected stellarator symmetry. This establishes nonuniqueness and consequent instability at an average β of 5.5% because there is large net current in the solution. The ripple of the flux surfaces is localized in an outer region of bad curvature and follows the magnetic lines, like a ballooning mode.

with $\beta = 0.055$. Once more the mesh had 15 radial intervals and the spectral approximation was truncated at terms of degree 20. Observe that the asymmetric ripple of the bifurcated flux surfaces is concentrated in a region of bad curvature and has a ballooning structure that follows the magnetic lines. Individual bulges are confined to little more than two field periods.

Because the variational principle of magnetohydrodynamics furnishes an accepted mathematical model of stability in plasma physics, the key issue in our approach becomes the numerical accuracy of the NSTAB code. Extensive comparisons with two-dimensional theory and with laboratory measurements suggest that NSTAB computations of the kind we have described do have the necessary resolution, and the results seem to provide a realistic simulation of the most essential phenomena. More specifically, it has been found that recent estimates of β limits for the Compact Helical System experiment in Japan agree well with our prediction of nonlinear stability for modes of moderately high order (12). Only a limited number of harmonics are retained in the spectral method when differentiability fails because convergence is only asymptotic, but adequate force balance is achieved before applying the stability test.

None of the methods that have been proposed to solve the three-dimensional equilibrium problem are completely successful. The principal difficulty with the NSTAB approach is to filter out erroneous harmonics in the neighborhood of the magnetic axis. In the PIES code, the interface between a region of nested surfaces and an island region at constant pressure ought to be a flux surface, but the surface current there caused by discontinuity of the magnetic field at any sharp boundary is neglected (13). Most linearized stability calculations build on runs of the VMEC code that may not be fully converged. The VMEC method is closely related to ours (14), but a compatibility condition associated with nonuniqueness of the Clebsch angle θ in the specification of $\mathbf{B}$ is treated in a different fashion, and it is unclear whether the discrete system of equations in the code has an exact solution. Moreover, a question of reliability arises about the evaluation of derivatives occurring in the linearized problem whose existence is placed in doubt by the KAM theorem (2). In changing coordinates for an analysis of ballooning modes, a problem can also occur if infinite series calculated by means of the finite Fourier transform are not filtered appropriately to arrive at meaningful results.

The Mercier local stabiity criterion can be written in the form

$$\Omega = C_1 + C_2 p'^2\left[\left(\int \lambda dF\right)^2 - \int dF \int \lambda^2 dF\right] > 0,$$

where the coefficients C_k and the measure dF have been defined elsewhere (2). Because of Schwarz's inequality, the contribution in square brackets is always negative. The divergence of the Fourier series for the parallel current λ in fully three-dimensional equilibria therefore signifies that the criterion will predict instability if too many terms are included in the calculation. However, the series has been truncated in the NSTAB code to arrive at a more practical version that turns out to be well correlated for stellarators with our nonlinear analysis. In this context a physically more realistic criterion for stability seems to be $\Omega > -0.01$.

Ballooning mode algorithms that have been presented in the literature involve a system of ordinary differential equations obtained from an asymptotic expansion of perturbations in the neighborhood of some magnetic line (15). The Mercier criterion emerges as a limiting case of the ballooning theory, so the latter may also predict erroneous instability for stellarators if it is carried to an extreme. Of more concern, however, are computations restricted to a shorter arc of the magnetic line. The answer depends on the length of the arc, so results may become biased to produce a misleading conclusion. Moreover, the ordinary differential equations for ballooning modes have features in common with linearized stability theory, which means that derivatives have to be computed that may not exist. We circumvent these difficulties by appealing in the last analysis to the nonlinear stability test based on an NSTAB computation of bifurcated solutions, and we only rely on a truncated version of the Mercier criterion for quick parameter searches. On the whole, the Mercier results seem to fit stellarator data well, whereas the ballooning theory is preferred for tokamaks.

Net current in toroidal equilibria can be a cause of instability. Because our MHH2 and MHH3 stellarators have a magnetic structure with the two-dimensional symmetry of a tokamak, they are expected to have significant bootstrap current. This means that the size of the bootstrap current may well become the decisive factor determining the β limit. A small amount of bootstrap current seems to improve the equilibrium, but, as we have indicated in Fig. 3, larger amounts can be damaging to stability. On the whole equilibrium seems to be more of a problem than stability for stellarators, so if sensible estimates of the bootstrap current turn out to be correct, we predict that the average β limit of an optimized MHH2 configuration might be as high as 4%, and results for the new MHH3 are comparable.

3. Configuration Parameters

In the design of the MHH2 stellarator, an important role has been played by Monte Carlo transport calculations and by line tracing in addition to the work we are describing here with the NSTAB code. In a cylindrical coordinate system r, ϑ and z the geometry of the plasma is defined by the formula

$$r + iz = r_0 + iz_0 + R[r_1 + iz_1 - r_0 - iz_0],$$

where R depends on the toroidal flux s and a pair of poloidal and toroidal coordinates u and $v = \vartheta/\pi$. The magnetic axis is described by the two functions, r_0 and z_0, of the toroidal angle v, whereas r_1 and z_1 specify the shape of the plasma boundary. For this separatrix, we introduce the Fourier representation

$$r_1 + iz_1 = e^{iu} \sum \Delta_{mn} e^{-imu+inv},$$

whose form is suggested by conformal mappings of the exterior of the unit circle in the complex plane. Improved zoning can be obtained by performing an additional substitution

$$u = u_1 - \sum Z_{mn} \sin(mu_1 - nv)$$

on the poloidal angle u. Rezoning helps to achieve adequate resolution in harder cases like the helias. In the NSTAB code, the potential energy is minimized as a functional of the unknowns R, θ, r_0, and z_0.

The conventional aspect ratio of the plasma has the value $A = \Delta_{10}/\Delta_{00}$, and we usually impose the normalization $\Delta_{00} = 1$ on the small plasma radius. Conformal mapping shows that shape factors Δ_{mn} with large negative m cause unrealistic cusps to penetrate the plasma, so we choose to leave them out when $m < -1$. The terms with $m = -1$, which define catenoids or crescents, are helpful because they contribute significantly to the magnetic well. Analytic geometry shows that the coefficients Δ_{1n} specify the helical excursion of the magnetic axis, whereas Δ_{2n} makes the plasma shape elliptical, Δ_{3n} makes it triangular, Δ_{4n} makes it rectangular, and so forth. Experience with the computations establishes that each Δ_{mn} has a strong influence on the corresponding coefficient B_{mn} in the magnetic spectrum, which makes it easier to design stellarators with approximate two-dimensional symmetry.

Resonant surfaces $\iota = m/n$ are dense throughout the plasma, and at each of them there is singular behavior of the parallel current λ unless the corresponding term B_{mn} in the magnetic spectrum vanishes. These coefficients seem to become large when islands form at the resonances, so it is desirable to reduce them as much as possible in the design of a stellarator. Unfortunately, the problem of eliminating unwanted terms in the spectrum does not respond well to systematic treatment in resonant cases, but we have been successful anyway in the development of specifications for quasi-axially symmetric stellarators. What matters most is that in any helias the coefficient B_{21} is extremely small despite the fact that the ellipticity Δ_{21} contributes the bulk of the rotational transform.

After equilibrium, stability and transport computations have produced satisfactory values of the shape factors Δ_{mn} defining the separatrix, we find modular coils to generate the external magnetic field by applying a modified version of the NESCOIL code (16). This method is based on the Biot–Savart formula

$$\mathbf{B} = \nabla \times \int\int \nabla\varphi \times \mathbf{N}\, d\sigma/r,$$

which represents the magnetic field in terms of a potential function

$$\varphi = v/(2\pi) + \sum \varphi_{mn} \sin(mu - nv)$$

defined over an outer control surface given by rules just like those for the separatrix (5). In Table 1, values of coefficients Δ^b_{mn} for the plasma boundary, coefficients Δ^c_{mn} for the control surface, and coefficients φ_{mn} for the potential are all presented for a choice of the MHH2 configuration designed to

Table 1. Specifications for an MHH2 stellarator whose separatrix is defined by the parameters Δ^b_{mn}, whereas the control surface for the coils is defined by the coefficients Δ^c_{mn}

m	n	Δ^a_{mn}	Δ^b_{mn}	Δ^c_{mn}	φ_{mn}	Δ^d_{mn}
−2	−1	0.00	0.00	−0.16	0.000	0.00
−1	−1	0.18	0.19	0.37	0.000	0.14
−1	0	0.16	0.15	0.23	0.000	0.10
−1	1	−0.02	−0.02	−0.10	0.000	0.02
0	0	1.00	1.00	1.95	0.000	1.00
0	1	−0.03	−0.03	0.00	−0.694	−0.01
0	2	0.00	0.00	0.00	−0.088	−0.01
1	−1	0.05	0.05	0.03	−0.092	0.05
1	0	3.20	3.20	3.35	0.196	4.50
1	1	0.30	0.27	0.20	0.539	0.07
1	2	0.05	0.05	0.10	0.098	0.01
1	3	0.00	0.00	0.00	−0.019	0.00
2	−1	0.00	0.00	0.00	−0.024	0.02
2	0	0.00	0.00	0.00	0.036	−0.08
2	1	−0.49	−0.45	−0.32	−0.556	−0.36
2	2	−0.07	−0.06	0.00	0.028	0.00
2	3	0.00	0.00	0.00	0.031	0.00
3	0	0.00	0.00	0.00	0.018	0.02
3	1	−0.04	−0.04	0.00	0.072	−0.04
3	2	0.09	0.08	0.12	0.118	0.09
3	3	0.00	0.00	0.00	−0.133	0.02
4	−1	0.00	0.00	0.00	0.000	0.01
4	0	0.01	0.02	0.00	−0.067	0.00
4	1	0.03	0.02	0.00	0.067	0.02
4	2	−0.02	−0.02	0.00	−0.069	0.00
4	3	−0.02	−0.02	0.00	−0.100	−0.02
4	4	0.00	0.00	0.00	0.089	0.00
4	5	0.00	0.00	0.00	−0.030	0.00
5	1	−0.01	0.00	0.00	0.000	0.00
5	2	0.00	0.00	0.00	0.037	0.00
5	3	0.00	0.00	0.00	0.080	0.00
6	2	0.00	0.00	0.00	−0.046	0.00

The shape factors Δ^a_{mn} were obtained from the magnetic lines of an altered equilibrium found by adding auxiliary toroidal and vertical fields. The coefficients φ_{mn} determine the location of coil filaments on the control surface. The last column, Δ^d_{mn}, defines the separatrix of a quasi-axially symmetric MHH3 stellarator with three field periods.

have a high β limit (11). The shape factors Δ^c_{mn} have been filtered judiciously to ensure that the modular coils, which are defined by level curves of φ, do not overlap or become excessively twisted. The first column Δ^a_{mn} in the table was calculated afterwards from the separatrix of another configuration obtained by activating auxiliary toroidal and vertical field coils. The off-design case also has desirable physical properties and exhibits the flexibility of the MHH2 concept. The last column lists the Fourier coefficients Δ^d_{mn} specifying the separatrix of the new quasi-axially symmetric MHH3.

The NESCOIL code solves for the potential φ by fitting the external harmonic field to the field inside the plasma in a sense of least squares. This is not a well posed problem because there may not be a global analytic continuation of **B** outside the plasma. Therefore, in practice we examine magnetic lines produced by the coils to make sure that some flux surface of the Biot–Savart field does have the originally prescribed values for the Fourier coefficients Δ^b_{mn}. In the calculations, it is easy to include an additional vertical field and an additional toroidal field. That enables one to investigate the influence corresponding auxiliary coils might have on the configuration and to correct for any outward shift of the plasma column as β increases. From our work, it becomes clear that there is flexibility in the implementation of the MHH2 concept and that a family of good configurations can be studied with one basic set of modular coils. Smaller toroidal coils can be used to control the rotational transform, perhaps in conjunction with a current drive.

The rotational transform ι over one field period of the MHH2 device specified in Table 1 drops from just above 1/4 at the magnetic axis to just above 1/5 at the separatrix. Numerical studies of the effect of changes in the definition of the coils show that the magnetic surfaces are robust. An $m = 4$, $n = 1$ island located near the magnetic axis can be controlled by means of variations in the corresponding harmonic associated with the winding law. When the surface they lie on is placed properly, 16 modular coils are enough to avoid unwanted toroidal ripple. There is ample space between the coils, which seem easier to construct than those of more conventional stellarators. At lower β it might be desirable to construct a device with less shear and a bigger magnetic well. That can be achieved by reducing the crescent shape factor $\Delta_{-1,0}$.

In the vacuum magnetic field of the MHH3 we have defined, the rotational transform over one period rises from approximately 0.12 at the magnetic axis to 0.16 at the separatrix. This leaves plenty of room for a significant contribution from bootstrap current without crossing resonances at $\iota = 1/4$ or $\iota = 1/3$ when β increases. Possible improvement in the performance of the MHH3 over that of the MHH2 associated with reversed shear is under investigation, and ballooning mode calculations predict a noticeably higher β limit for the MHH3 than for the MHH2.

4. Proposal for an Experiment

Research thus far on the helias concept has all been theoretical and relies heavily on large-scale computer codes. There are so many questions about the mathematical model that the calculations are perhaps best viewed as a simulation of the physics. Therefore, it is of interest to plan a proof of principle experiment to verify the predictions of the numerical work. This might be feasible at a cost of tens of millions of dollars for construction and operation over a period of several years. The large radius of the device could be as small as 1.5 m and still allow for a minimum radius of 20 cm to deal with impurities. With a magnetic field of only 0.5 tesla, one could test the β limit and address concerns about the role of ballooning modes. A goal of the experiment would be to get information about the level of bootstrap current to be expected in toroidal configurations, and it might be possible to investigate the control of islands and the behavior of a divertor in this geometry.

Computerized machining now makes it possible to build the kind of modular coils required for an MHH2 experiment with frames meeting the necessary tolerances (3). Smaller toroidal and vertical field coils placed at some distance from the plasma should supply adequate flexibility and provide for control of the rotational transform. The addition of a current drive would be desirable, especially because bootstrap current is expected to be important in a quasi-axially symmetric stellarator. It should be possible to study the quality of the magnetic surfaces and learn more about islands, but our computations indicate that one should not expect to raise the rotational transform much above $\iota = 0.3$ over one field period. Because the problem of constructing a successful stellarator experiment is technically challenging, the plan of the physics should be as simple as possible.

Because it is hard to predict the precise size of the bootstrap current, we have investigated situations where the associated rotational transform becomes as large as a third of the total (17). In that case, the MHH2 stellarator seems to have a β limit of only 4%, and a certain amount of profile optimization is required to arrive at this estimate. Small values of the bootstrap current help to achieve good equilibrium, but instability of low order modes may be triggered by more substantial amounts. In a flexible experiment, the increase of rotational transform caused by bootstrap current can be compensated for by using toroidal field coils to reduce the stellarator contribution.

The MHH2 concept offers the possibility of assessing properties of a helias within the framework of a compact stellarator program. The W7-X experiment under construction in Europe is a much more ambitious proposal to achieve comparable goals. Because of the substantial elongation of its plasma cross-sections, the minimum small radius will only be 20 cm for a planned large radius of 5 m. On the other hand, a four field period MHH4 configuration modelled on the HSX device at the University of Wisconsin seems quite competitive because it would have much less bootstrap current and yet not need to be as big as the W7-X. Among the compact stellarators that have been proposed, only the MHH2 and MHH3 seem to have good transport at reactor conditions. Moreover, a hybrid tokamak with only small stellarator sidebands may not be attractive because of the tendency of resonant islands to form and because of difficulties in controlling the net current. However, if the bootstrap current turns out to be very big, an MHH3 configuration with relatively little rotational transform, like the one in Table 1, might become quite attractive.

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