THE ELECTRIC RESISTANCE AND CAPACITY OF BLOOD FOR FREQUENCIES BETWEEN 800 AND 4½ MILLION CYCLES.

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GENERAL.

In this report a series of measurements is presented upon the electric resistance and capacity of blood (as defined in the preceding paper) at from 800 up to 4½ million cycles. For low frequencies the resistance and capacity are found to be independent of the frequency of the electric current; above a certain frequency (ω), however, they both begin to decrease. It is found that the experimental values ($R(\omega)$; $C(\omega)$) approximately correspond to the two simple equations:

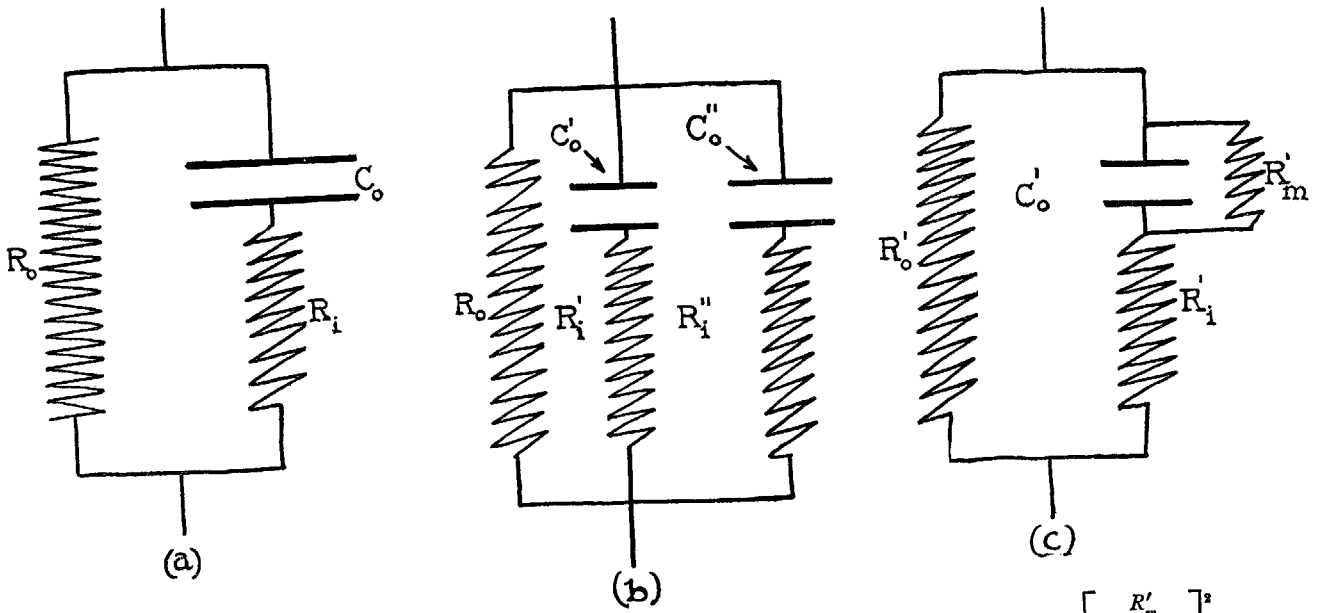
$$\frac{1}{R(\omega)} = \frac{1}{R_o} + \frac{1}{R_i}\frac{C_o^2\omega^2R_i^2}{1 + C_o^2\omega^2R_i^2} \tag{1}$$

$$C(\omega) = \frac{C_o}{1 + C_o^2\omega^2R_i^2} \tag{2}$$

These equations are derived by considering the blood as equivalent to the system shown in the diagram (*a*) of Fig. 1 $\left(\frac{1}{R_o} + \frac{1}{R_i}\right)^{-1}$ representing the resistance of the blood at infinite frequency.

The diagram expresses the fact that the current which passes through the capacity of the membrane has to go through a certain resistance (R), composed of the resistance of the interior of the corpuscle and of the resistance of a certain mass of the intercellular liquid which lies in front of and behind the corpuscle.

Diagram (*a*) is of course only a rough representation of actual facts, and equations (1) and (2) are accordingly only approximations. We expect in another publication to treat the problem by a more exact



$$R(\omega) = \left[\frac{1}{R_o} + \frac{1}{R_i}\frac{C_o^2\,\omega^2\,R_i^2}{1 + C_o^2\,\omega^2\,R_i^2}\right]^{-1} \quad (1)$$

$$C(\omega) = \frac{C_o}{1 + C_o^2\,\omega^2\,R_i^2} \quad (2)$$

$$R(\omega) = \left[\frac{1}{R_o} + \frac{1}{R'_i}\frac{C'^2_o\,\omega^2\,R'^2_i}{1 + C'^2_o\,\omega^2\,R'^2_i} + \frac{1}{R''_i}\frac{C''^2_o\,\omega^2\,R''^2_i}{1 + C''^2_o\,\omega^2\,R''^2_i}\right]^{-1} \quad (3)$$

$$C(\omega)\ \frac{C'_o}{1 + C'^2_o\,\omega^2\,R'^2_i} + \frac{C''_o}{1 + C''^2_o\,\omega^2\,R''^2} \quad (4)$$

$$C_o = C'_o\left[\frac{R'_m}{R'_i + R'_m}\right]^2$$

$$R_i = \frac{(R'_m + R'_i)}{R'_m}\,R'_i$$

$$R_o = \left[\frac{1}{R'_o} + \frac{1}{R'_m + R'_i}\right]^{-1}$$

FIG. 1. Electrical diagrams of blood.

method. It would undoubtedly be more logical to consider the capacity as composed of different parts, each of which has a different value for $C_o \cdot R_i$. For the case in which the capacity is composed of two parts, C_o' and C_o'' as shown in diagram (*b*) of Fig. 1, we obtain the following substitutes for the equations (1) and (2)

$$\frac{1}{R(\omega)} = \frac{1}{R_o} + \frac{1}{R_i'} \frac{(C_o'\omega R_i')^2}{1 + (C_o'\omega R_i')^2} + \frac{1}{R_i''} \frac{(C_o''\omega R_i'')^2}{1 + (C_o''\omega R_i'')^2} \tag{3}$$

$$C(\omega) = \frac{C_o'}{1 + (C_o'\omega R_i')^2} + \frac{C_o''}{1 + (C_o''\omega R_i'')^2} \tag{4}$$

We would certainly expect that equations of this form, (3) and (4), rather than of the simple form, (1) and (2), would have to be used for blood since here generally $C_o R_i$ depends on the orientation of the single corpuscle. The application of (3) and (4) rather than of (1) and (2) would of course be imperative if there existed a considerable number of corpuscles in the blood essentially different from the majority, and also if there should exist a region in each corpuscle which, as regards $C_o R_i$, is very different from the rest of the corpuscle.

It may be well to point out that equations of the form (1) and (2) (or (3) and (4)) will also hold for the case in which the cell membrane is conducting; this condition is shown in diagram (*c*) of Fig. 1: the corresponding formulæ for resistance and capacity are obtained from (1) and (2) by using:

$$C_o = C_o' \left(\frac{R_m'}{R_i' + R_m'} \right)^2$$

$$R_i = \frac{R_m'}{R_m' + R_i'} R_i'$$

and

$$\frac{1}{R_o} = \frac{1}{R_o'} + \frac{1}{R_m' + R_i'}$$

In order to simplify the problem, we have assumed in deriving the foregoing formulæ that the capacity of the blood is due solely to the static capacity of the membrane around the red corpuscle. This assumption is strongly supported by our experimental results; however, the formulæ will hold equally well for the case in which the

capacity is due to polarization at the interphases, when C_0 and R_i are considered to be dependent on the frequency.

Calculation of Specific Resistance of Corpuscle Interior.

By the application of formula (1) (or (3)) to our experimental data, the resistance of the blood for infinite frequency can be extrapolated with considerable accuracy. When $\omega = \infty$ formula (1) reads:

$$\frac{1}{R(\infty)} = \frac{1}{R_0} + \frac{1}{R_i}$$

In an earlier paper (1) the following formula was derived for the case of a suspension of homogeneous spheroids:

$$\frac{1 - \frac{r}{r_1}}{1 - \frac{r}{r_2}} \left(\frac{r_1}{r_2} - 1 \right) = \beta \frac{\rho}{1 - \rho} \tag{5}$$

in which r, r_1, and r_2 are the specific resistances of the suspension and of the suspending and the suspended phases respectively, ρ is the volume concentration of the suspension, and β is a constant which depends on the ratio $\left(\frac{a}{b}\right)$ of the thickness to the width of the spheroid and on $\frac{r_1}{r_2}$, and which is represented graphically in the paper referred to above. By means of this equation, using for r the value $(R(\infty))$ of the resistance of the blood at infinite frequency, we can calculate r_2, which is the specific resistance of the interior of a red corpuscle.

HISTORICAL NOTE.

The electric resistance of blood for currents of very high frequencies has already been investigated by R. Höber (2). From his experimental results he drew the conclusion that the conductivity of the interior of a red corpuscle lies between that of a 0.1 and a 0.4 per cent NaCl solution, the limits being determined by the accuracy of his experimental methods. (We may mention that the present measurements lead to a value of about 0.17 per cent.)

Recently M. Philippson (3) has made a single series of determina-

tions of the impedance of a suspension of the red corpuscles of a horse for frequencies between 1000 and $3\frac{1}{2}$ million cycles. Philippson's method consisted in a simultaneous measurement of the current and of the potential across the electrolytic cell by means of a tube voltmeter. The suspension was obtained by a 1 hour centrifugation. The value of the specific resistance of the interior of a corpuscle (3.15 times the specific resistance of the serum) as calculated from his experimental results, is in fair agreement with our own values for other animals; on account of the prolonged centrifugation before the measurement, the value derived from Philippson's data is probably somewhat lower than the normal value.

EXPERIMENTAL.

The method of measurement has already been described in earlier reports (4) regarding the capacity of blood (see especially the preceding paper), and we shall add here only some later developments and certain details which are of especial importance in measurements at very high frequencies.

Fig. 2 is a diagram of the electrolytic cell used in most of the experiments presented here. Several other types of cell, however, have been used in the course of the investigation for the sake of comparison or of convenience. Different sizes of the cell shown in Fig. 2 hold from 10 to 75 cc. of blood. The cell is so designed as to have a low polarization at the electrodes, which are made of platinum and covered with platinum black and are fused into the glass. The cell is divided into two cup-shaped halves, between which a celluloid diaphragm is inserted. The edge of each half which is in contact with the diaphragm is broadened and ground flat and circular. The diaphragm, which is also circular and of the same size, can thus be easily centered and the whole cell when clamped together is easily made water-tight. We use a set of these diaphragms, in the centers of which are holes of different areas. By means of these diaphragms varying and readily reproducible cell constants can be obtained over a considerable range. For the measurement of blood we usually employ a cell constant of about 1.

Stirring is attained by displacing the suspension back and forth between the two bulbs which are attached to the inlet tubes of the

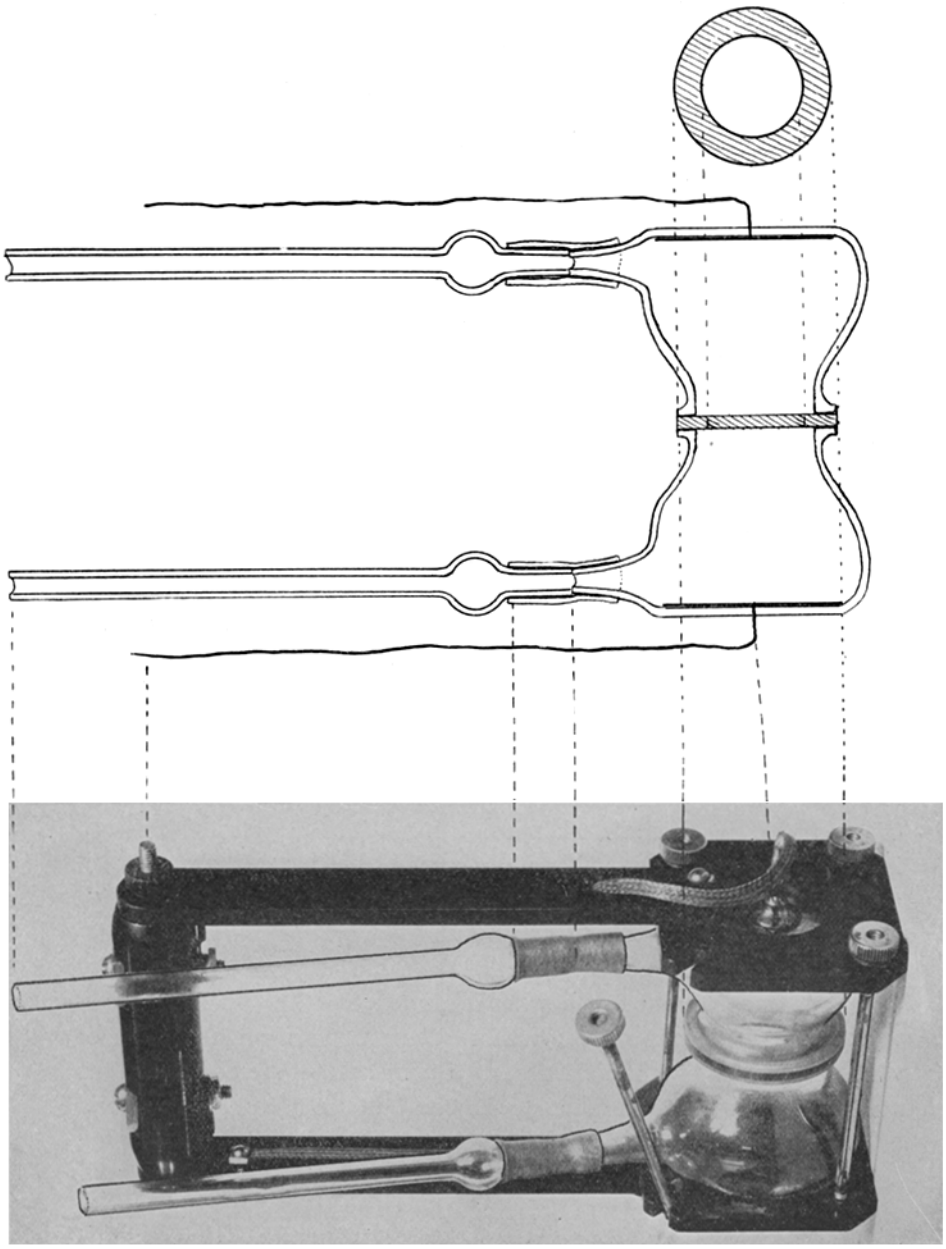


FIG. 2. Conductivity cell for measuring electric resistance and capacity of blood

cell. The stirring is a very important procedure. As soon as it is stopped, the resistance and capacity usually rise rapidly to new and rather fixed values. This change, which amounts to a few per cent and is accomplished within a minute or less, can hardly be due to an ordinary settling of the corpuscles. We have observed a quite similar effect with cream (5) and we believe that both of these effects are due to the establishment of some kind of an ordered arrangement of the particles of the suspensions, independent of gravity. In this connection it is interesting to note that for certain specimens of blood this effect was much more pronounced than for others; the same was true with cream. In the investigations reported here we always endeavored to measure the resistance and the capacity exactly at the moment when the stirring was stopped, for the reason that in this work we use a formula derived under the assumption that the suspended particles are distributed at random. In our work with cream (5) we found that the formula for this case is exact only when the resistance is taken immediately after stirring and consequently conclude that the assumption of random distribution is only fulfilled at that moment. That the same holds for blood is not certain, of course; in any case the effect is so small that it is of minor importance in the present work.

For the control of the temperature during the measurements the cell is placed in an air bath, at the bottom of which is a container filled with mercury. This container can be raised so that the cell may be immersed in the mercury, thus accelerating the establishment of temperature equilibrium.

For the measurements at the higher frequencies the following point must be noted. When a homogeneous liquid is measured by substituting it in a bridge against the units of a resistance box, it will usually be found that the resistance of the liquid apparently changes with the frequency. For the bridge here used the effect is negligible for frequencies under 1 million cycles and amounts to a few per cent for higher frequencies up to about 3 million cycles. For still higher frequencies it may amount to as much as 10 per cent. This effect is principally due to the difference of the coupling between the cell and the bridge on the one hand and the substituted resistance box and the bridge on the other, and, at the very

highest frequencies, in part to the fact that the resistance of the coils of the resistance box is not independent of the frequency, owing to the presence of an appreciable amount of inductance and distributed capacity in the coils.

The difference between the true resistance and the apparent resistance which depends not only on the frequency but also on the resistance, we shall term the resistance defect. A table of the resistance defects for different resistances and frequencies is made up in the following way. The cell is filled with serum and a series of different diaphragms is placed consecutively in it, the diaphragms being so chosen, that the resistances obtained are distributed regularly over the range inside which our experimental resistances fall. Measurements of resistance and capacity are made at all experimental frequencies for each diaphragm; the difference between the resistance values obtained at such a low frequency that the defect is inappreciable, and at a certain high frequency, gives the resistance defect for that resistance and frequency. By employing the procedure of the preceding paper, these measurements will also give the zero capacity. Practically the same values for the resistance defect and the zero capacity are obtained when the resistance is varied by using different dilutions of the serum, as was done in the preceding paper. The resistance defect and the zero capacity are measured only once for each cell, perhaps with an occasional rechecking; their values are nearly the same for all the cells which we have used.

An abstract of the protocol of measurements is given in Table I.

The method described is not practical at the very highest frequencies (over about 3 million cycles), because at these frequencies the resistance and inductance of the coils of the resistance box change with the frequency. We employ here a strict substitution method. After having finished the measurements on the blood at all the different frequencies, the cell is filled with serum and consecutive measurements are made with different specially chosen diaphragms in the cell. Each diaphragm is so chosen that the resistance of the cell at one of the frequencies in question, will be practically equal to the resistance of the blood at the same frequency. With each of these diaphragms we make a measurement at the high frequency and repeat at a low frequency, the difference between the values of the resistances thus

secured being the resistance defect, which must be added to the observed value for the resistance of the blood at the same frequency. The correct capacity of the blood is the difference between the settings of the right condenser (C_r) for the blood and for the serum respectively.

It may be well to note here, also, that the procedure which we have described does not completely eliminate the influence of polarization at the electrodes of the cell. Therefore it is always necessary to ascertain that the polarization is negligible at all frequencies with which we desire to work. The frequency at which the polarization becomes appreciable is easily found by measuring the serum at decreasing frequencies. The setting of the condenser C_r stays very nearly constant until the critical frequency is reached when an abrupt change begins.

TABLE I.

Capacity and Resistance of Blood of Calf for Frequencies from 87,000 to 4½ Million Cycles.

Jan. 19, 1925.

Cycles per sec. ($\omega/2\pi$).	Capacity (m.m.f.).	Resistance (ohms).	$\frac{146}{1+(146\cdot10^{-12}\cdot\omega\cdot370)^2}$	$\left[\frac{1}{370}\cdot\frac{(146\cdot10^{-12}\cdot\omega\cdot370)^2}{1+(146\cdot10^{-12}\cdot\omega\cdot370)^2}+\frac{1}{191}\right]^{-1}$
87,000	146	191	146	191
833,000	130	181	135	183
$1.17\cdot10^6$	118	174	126	178
$1.52\cdot-$	106	168	115	172
$2.04\cdot-$	90	159	98	163
$3.04\cdot-$	68	148	71	151
$3.82\cdot-$	60	144	55	144
$4.52\cdot-$	39	138	43	140
∞	—	(124) extrapolated.	—	126

Blood of calf, defibrinated, concentrated by centrifugation.

Volume concentration of suspension: **46.0 per cent** $\left[\frac{a}{b}=\frac{1}{3}\right]$.

Constant of electrolytic cell: **.856.**

Capacity of 1 cc. of blood for a volume concentration of 100 per cent: $\frac{146\cdot.856}{.60}=$ **208 m.m.f.**

Temperature: 21.6°C. Specific resistance of serum: **89.4 ohms.**

TABLE I—*Concluded.*

Specific resistance of inside of corpuscles is **3.5 ± 5 per cent** *times specific resistance of serum.*

Abstract of Protocol.

	Blood.	Serum.	
Diaphragm	1.50 cm.	.88 cm.	—
Cycles per sec	$2.04 \cdot 10^6$	—	87,000
R_1' (ohms)	158.7	155.3	
C_r' (m.m.f.)	103	198	
R_r'' (ohms)	157.5	154.2	156.0
C_r'' (m.m.f.)	256	257	
Temperature	21.60	—	
$C_r''-C_r'$	153	59	
Inductance of coils (10^{-10} henry) L_{100}*	2270	2270	
Inductance of coils (10^{-10} henry) L_{10}	5220	5220	
Inductance of coils (10^{-10} henry) L_1	4130	2460	
Inductance of coils (10^{-10} henry) $L_{.1}$	1120	560	
Inductance of leads (10^{-10} henry)	-900	—	
Total inductance	11,800	9600	
Equivalent capacity (m.m.f.) $\left(\frac{\text{total inductance}}{R_r''^2}\ 10^2\right)$	48	40	
Capacity corrected for inductance	105	19	
Serum capacity subtracted	86		
Correction due to difference in static capacity of diaphragms used for blood and serum	4		
Capacity of blood (m.m.f)	90		
Resistance of blood (ohms)	159		

* L_{100}, L_{10}, L_1, and $L_{.1}$ are the total inductances of the coils used in the hundreds, tens, units, and tenths of units, decades of the resistance box.

A test of the reliability of this method described was obtained by making several series of measurements on cream, where the capacity is zero and the resistance constant.

The following series of experiments will illustrate the procedure and will give a preliminary value for the specific resistance of the interior of a red corpuscle. It is hoped that later publications will show the application of the method to different problems of biological interest and will give also a more extended series of control measurements.

The two experiments here reported were made on the blood of a calf. In the first the original blood was measured and the interior conductivity calculated. In the second the corpuscles of this blood were suspended in an isotonic sugar solution and a new series of measurements was made from which the interior conductivity was again calculated. A control of the method was thus obtained.

These experiments are typical of several others in which different volume concentrations have been used. We have also made a few experiments on the blood of a sheep, for which the specific conductivity of the interior of the corpuscles seems to be about the same as for the corpuscles of a calf—namely 3.5 times the specific resistance of the serum. The variation for different animals of the same species is about ± 10 per cent.

The defibrinated blood was obtained from the slaughterhouse. The volume concentrations given in the tables were obtained from the ratio of the resistance of the suspension to that of the suspending liquid using formula (5). In this formula, as we have already stated, there enters the ratio $\frac{a}{b}$ of the thickness of the corpuscle to its diameter. For this ratio we have used a value of $\frac{1}{3}$[1], a value which in other experiments we found gave volume concentrations in good agreement with those obtained by the hematocrit. In the experiments presented in the tables resistance and capacity were not measured at frequencies under 8700 cycles; in other experiments we have measured at frequencies above 3600 cycles and have found the resistance and capacity to be constant over this whole range of frequencies.

In the tables are given the values of the resistance and capacity which are calculated by equations (1) and (2), selecting the value for R_i which gives the best agreement. There is a deviation between the observed and calculated values which is especially marked in the case of capacity, consisting in a too rapid decrease of the experimental values for the lower frequencies. The character of the deviation suggests the application of equations (3) and (4). As a matter of fact a fairly satisfactory agreement can be obtained by these equations.

[1] For sheep we used the value $\frac{1}{2}$.

choosing rather small values for $\frac{C_o'}{C_o''}$ and rather large for $\frac{C_o' R_i'}{C_o'' R_i''}$. However, an exact agreement does not seem to be obtained by these equations and since, furthermore, it does not seem possible to attach any definite theoretical meaning to the values of C', C'', R_i', and R_i'' which furnish the best agreement (these values are quite different from what would be expected if C' and C'' corresponded to the two main orientations of a red corpuscle), it is uncertain whether this agreement is of any significance. We have consequently restricted ourselves to the use of formulæ (1) and (2). As stated above it is doubtful if any of the formulæ (1), (2), (3), or (4) is theoretically correct.

The application of (1) and (2) is quite sufficient to obtain a very satisfactory extrapolation of the value of the resistance to infinite frequency. This value is given in the tables as is also the specific resistance of the interior of the red corpuscle as calculated by equation (5) using $\frac{a}{b} = \frac{1}{3}$.

In the first experiment (Table I) the blood was concentrated to 46 per cent by centrifugation and measurements were made with this suspension. In the second experiment, which was made the next day, 145 cc. of the suspension were diluted with 227 cc. of a 5.4 per cent dextrose solution, the volume concentration of this suspension being consequently $\frac{46 \cdot 145}{372} = 17.9$ per cent. The same value of the concentration was also obtained from the resistances of the suspension and of the suspending liquid (using $\frac{a}{b} = \frac{1}{3}$), showing that the shape of the corpuscles was not changed appreciably by the addition of the dextrose solution. The resistance of the suspension did not change appreciably while the experiment lasted (about 1 hour), being 306.3 ohms at $1.17 \cdot 10^6$ cycles at the beginning of the experiment and 307.8 ohms at the end, values which are identical within the accuracy with which the resistances can be reproduced. We may therefore conclude that no interchange of electrolytes took place between the corpuscles and the dextrose solution during the time of the experiment.[2]

[2] In another similar experiment with sheep blood, when we concentrated the suspension to 72 per cent after the addition of the same dextrose solution to the original blood, we found also that the resistance of the suspension remained constant to within 1 or 2 per cent over an interval of about 1 hour.

The serum of the original blood was diluted by the addition of the dextrose solution to the same extent as was the serum of the blood used in the foregoing procedure. The resistance of this diluted serum was measured and was found to be equal to the resistance of the suspending liquid which was drawn from the suspension after centrifugation. This fact proves that no appreciable interchange of electrolytes took place between the corpuscles and the suspending liquid at the moment when the dextrose solution was added.[3]

TABLE II.

Capacity and Resistance of Red Corpuscles of Calf Suspended in a Dextrose Serum Mixture for Frequencies from 87,000 to 4½ Million Cycles.

Jan. 20, 1925.

Cycles per sec. ($\omega/2\pi$).	Capacity (m.m.f.).	Resistance (ohms).	$\frac{73}{1+(73\cdot10^{-12}\cdot\omega\cdot942)^2}$	$\left[\frac{1}{942}\cdot\frac{(73\cdot10^{-12}\cdot942)^2}{1+(73\cdot10^{-12}\cdot\omega\cdot942)^2}+\frac{1}{332}\right]^{-1}$
87,000	73	332	73	332
833,000	62	318	64½	319
$1.17\cdot10^6$	55	308	58	310
$1.52\cdot-$	47	299	51	300
$2.04\cdot-$	37	288	41	288
$3.04\cdot-$	25	273	26½	271
$3.82\cdot-$	20	263	19½	264
$4.52\cdot-$	17	258	15	259
∞	—	(244) extrapolated.	—	245

227 cc. of 5.4 per cent dextrose solution added to 145 cc. of 46.0 per cent suspension from experiment of Table I.

Volume concentration of new suspension: **17.9 per cent.** Constant of electrolytic cell: **.826.**

Capacity of 1 cc. of suspension for a volume concentration of 100 per cent: $\frac{73\cdot.826}{.275}$

= 220 m.m.f.

Temperature: **21.8°.** Specific resistance of suspending liquid: **291.0.**

Specific resistance of inside of corpuscle is **3.5 ± 5 per cent** *times specific resistance of original serum.*

[3] In the case described in Foot-note 2, however, a definite loss of electrolytes took place at the moment when the dextrose solution was added, as was shown by the same test. Correspondingly the resistance of the interior of the corpuscle was in this experiment found to be somewhat lower than for the normal blood.

It will be seen that we obtain the same value for the specific resistance of the interior of the corpuscle in both experiments—namely 3.5 times the specific resistance of the original serum.

The values given in the tables for the capacity of 1 cc. of blood for a volume concentration of 100 per cent is obtained by the formula given in the previous paper (formula (1)). The values obtained in the two cases are the same within the experimental errors, showing that the capacity is not dependent (or at least is dependent only to a very slight degree) on the suspending liquid.

SUMMARY.

1. The variation of the experimental values ($R(\omega)$), ($C(\omega)$) of the resistance and capacity of blood for increasing frequencies is approximately represented by the equation:

$$\frac{1}{R(\omega)} = \frac{1}{R_0} + \frac{1}{R_i} \cdot \frac{C_0^2 \omega^2 R_i^2}{1 + C_0^2 \omega^2 R_i^2} \tag{1}$$

$$C(\omega) = \frac{C_0}{1 + C_0^2 \omega^2 R_i^2} \tag{2}$$

in which R_0 and C_0 are the resistance and capacity of the blood at low frequency and $\left(\frac{1}{R_0} + \frac{1}{R_i}\right)^{-1}$ is the resistance of the blood at infinite frequency. Formulæ (1) and (2) are derived by considering the blood as equivalent to the system shown in the diagram (*a*) of Fig. 1.

2. By the application of formula (1) to our experimental data the value of $R(\infty)$ can be extrapolated with high accuracy. $R(\infty)$ represents the resistance, which would have been obtained at low frequency, if the membranes around the corpuscles could have been removed.

3. The specific resistance of the corpuscle interior can be calculated by equation (5), using experimental values for $R(\infty)$, for the volume concentration of the blood and for the specific resistance of the serum.

4. The specific resistance of the interior of the red corpuscle of the calf is found to be 3.5 $\pm$ 10 per cent times the specific resistance of the serum.

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