# Committee proposals and restrictive rules

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ABSTRACT I analyze a game-theoretic model of committee-legislature interaction in which a majority decision to adopt either an open or closed amendment rule occurs following the committee's proposal of a bill. I find that, in equilibrium, the closed rule is almost always chosen when the dimension of the policy space is >1. Furthermore, the difference between the equilibrium outcome and that which would have occurred under the open rule can be arbitrarily small.

Congressional scholars are in widespread agreement that the first nontrivial stage of the lawmaking process is committee action on legislation introduced by members and referred by the chamber's parliamentarian. For major legislation in the House of Representatives, a key ingredient in this process is the special order or, in more common terms, the rule, which, among other things, can limit the number and scope of allowable changes to any bill reported out of committee. In terms of constraints on amendments, rules range from the closed rule, in which no amendments are allowed, to the open rule, in which any and all (germane) amendments are allowed. Such rules are bill-specific and do not take effect automatically: Subsequent to the reporting of a bill, the Rules Committee makes a recommendation on the appropriate rule. This rule is then subject to majority approval prior to the House taking up the legislation to which the rule applies.

The adoption of restrictive (i.e., nonopen) rules in the House has seen a dramatic increase in the last two decades, from 15% in the late 1970s to 70% in the early 1990s (1). The theoretical literature on the use of such rules has sought to understand why a majority would willingly constrain its ability to amend legislation produced by a committee whose interests need not necessarily coincide with those of the legislature. One of the more popular explanations comes from the structure-induced equilibrium approach, which views restrictive rules either as devices for avoiding the chaos that would presumably occur under the open rule (2) or as part of an institutional arrangement for exchanging support across the various committee jurisdictions (3). A competing explanation asserts that restrictive rules will, in certain situations, provide greater incentives for a committee to both acquire relevant information and to signal such information through its choice of bill (4) (see ref. 5 for a review of these and other theories of legislative institutions).

While different in many respects, the structure-induced and informational approaches have in common a pair of critical features, the alteration of which provides the focus of the current paper. The first is that they typically have the legislature voting on the amendment rule prior to the bill being delivered by the committee, in contrast to the actual practice in Congress described above (see refs. 6 and 7 for exceptions, the former structure-induced and the latter informational). Hence, one of my motivations is to analyze a more descriptively

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accurate model of the process by which legislation in Congress is actually produced.

The second common feature has to do with structural assumptions on the set of possible outcomes or policies the legislature might enact. The existing models are examples of "spatial" models in which the policy domain is assumed to be a compact and convex set in some finite dimensional Euclidean space. Thus, for instance, the domain could be the interval [0, M and so be one-dimensional, which might usefully model situations in which the policy under consideration is, for example, spending on health care or the size of a particular defense project (with M being the maximum feasible amount of each). Or the domain might be the set of all k non-negative real numbers adding up to M, with the policy choice being how to distribute a budget of M among k different programs; the policy dimension here would then be k-1. Nearly all structure-induced and informational models assume the policy dimension is equal to one whereas here I will allow this dimension to be arbitrary so as to cover a wider class of

For reasons of analytical tractability, my model, like most others in the literature, limits the choice of amendment rules to the two extremes, namely, the open rule and the closed rule, and assumes a single-member committee. Further, decision making under the open rule is modeled here as an explicit bargaining process among the legislators; in particular, results from the bargaining model found elsewhere ("A Bargaining Model of Collective Choice," J.B. and J. Duggan, unpublished work) (which generalizes the model in ref. 8) are employed. While an admittedly crude description of what actually goes on under the open rule, as will be seen, I only require a few of the qualitative results from the bargaining model for my general conclusions to hold.

I find that, in equilibrium, it is almost always the case that the bills considered by the legislature are assigned the closed rule when the policy dimension is >1 (where "almost always" is given a mathematically precise definition). The underlying logic of this result is that, with multiple dimensions, the committee has the flexibility to craft a bill in such a way as to avoid the imposition of an open rule and subsequently having its bill altered through amendments. This flexibility is noticeably absent in the one-dimensional case, which leaves the equilibrium prediction more opaque: There exist equilibria in which the open rule is adopted, but this need not be true of all equilibria.

The model also sheds light on the effect of restrictive rules on policy outcomes. It might be thought that the use of such rules necessarily leads to a significant policy bias in favor of the committee and at the expense of the legislature's interests (however defined). Here, on the other hand, since the amendment rule is voted on after the bill is proposed, the outcome that would occur under the open rule acts as a constraint on the degree to which the committee can bias the bill toward its preferred outcome. In fact, I show by way of example that the

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difference between the equilibrium bill, which receives the closed rule and is subsequently accepted, and the policy that would have occurred under the open rule can be arbitrarily small. This suggests that merely observing the closed rule being employed is not sufficient to infer any substantial degree of bias in legislative outcomes.

### The Model

Let  $X \subset \mathcal{R}^d$  denote a nonempty, compact, and convex set of feasible policies,  $N = \{1, \ldots, n\}$  the set of legislators, with  $n \geq 3$  and odd, and let each  $i \in N$  have preferences on X representable by a smooth and strictly concave utility function  $u_i: X \to \mathcal{R}$ . Let  $x_i$  denote the unique solution to  $\max\{u_i(x): x \in X\}$ , referring to this as i's ideal policy, and assume the utilities  $u = (u_1, \ldots, u_n)$  are common knowledge among the members of N. Let  $s \in X$  denote the status quo policy and  $c \in N$  a distinguished member of the legislature we refer to as the committee.

A policy is chosen from the set *X* according to the following procedure (see Fig. 1): Initially, c proposes a bill  $b \in X$ (decision node B), after which a motion is automatically brought forth on whether the bill should be considered (decision node M). All  $i \in N$  vote on this motion, and, if the motion fails to receive a majority, the process ends with the status quo policy s remaining in place. If the motion receives a majority, a second vote is taken, on whether the bill should be considered under the open or the closed rule (decision node R). If a majority votes is favor of the closed rule, the bill b is paired against the status quo s in a final vote (decision node F), with a majority determining the outcome. If, on the other hand, the open rule is adopted, members of N are assumed to bargain over the final policy. This bargaining process is modeled as a discrete time, infinite horizon game in which each period  $i \in N$  is recognized as proposer with probability  $\rho_i >$ 0; when recognized, i makes a proposal  $p \in X$ , and then all  $j \in X$ N vote either to accept or to reject p. If a majority accepts, then the process ends with p being implemented; otherwise, the process moves to the next period. The payoff to  $i \in N$  from  $x \in$ X being implemented after t periods of bargaining is simply equal to  $u_i(x)$ ; that is, legislators do not discount future payoffs.

The collective decision-making procedure outlined above generates a noncooperative, extensive form game of complete information among the members of *N*. From the set of Nash

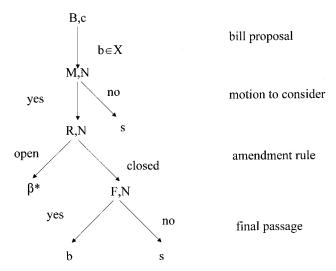


Fig. 1. The sequence of decisions: a pair D,I lists, respectively, the decision nodes' label (B, M, R, or F), and the individuals involved in the decision (c or N).

equilibria (9) to this game, I identify a subset of strategy profiles satisfying various appealing properties. For convenience, I partition the description of strategies and equilibrium behavior into (i) the bargaining subgame following a vote to adopt the open rule, and (ii) the rest of the game, under the assumption that behavior in the former is independent of behavior in the latter (this assumption is nontrivial if there are multiple equilibria in the bargaining subgame). In the bargaining subgame, a history of length l identifies all that has transpired in previous bargaining periods (who the previous proposers were, what they proposed, how individuals voted) as well as whether, in the current period, the process stands prior to the proposer being recognized, after such recognition but prior to the proposal being made, or after the proposal but prior to the vote. Therefore, in general, a strategy for a player will be a map specifying her intended action (what to propose, how to vote) as a function of the history to date. I focus here only on equilibria in stationary strategies, and so, to avoid irrelevant generality, I provide a formal definition only of such strategies. A stationary strategy for  $i \in N$  consists of a proposal  $p_i \in X$  offered anytime i is recognized and an acceptance rule  $\alpha_i: X \to \{accept, reject\}$  specifying which proposals she accepts. It turns out that mixed proposals are required to prove existence of equilibria in the bargaining game; that is, an individual randomizes over which alternative to propose (however, at the voting stage individuals observe the realized proposal). Let P(X) denote the set of (Borel) probability measures on X, endowed with the topology of weak convergence, and let  $\pi_i \in P(X)$  be a stationary mixed proposal for i. Thus,  $\beta_i = (\pi_i, \alpha_i)$  constitutes a stationary strategy for i in the bargaining subgame, and  $\beta = (\beta_1, \ldots, \beta_n)$  constitutes a profile of stationary strategies.

A profile  $\beta^*$  constitutes an equilibrium of the bargaining subgame if, for all  $i \in N$ , (i)  $\pi_i^*$  is optimal given the acceptance rules  $(\alpha_1^*, \ldots, \alpha_n^*)$  of the others, and (ii)  $\alpha_i^*$  is optimal given that  $\beta^*$  describes what would happen subsequent to the current proposal being rejected. Specifically, condition (ii) requires that i's vote on a proposal  $x \in X$  depend on which is greater,  $u_i(x)$  or her continuation value, defined as her expected utility if the current proposal is rejected. By stationarity and the assumption of no discounting, this value is simply equal to her expected utility under the profile  $\beta^*$ ; with some abuse of notation, denote this  $u_i(\beta^*)$  and assume that, when  $u_i(x)$ equals  $u_i(\beta^*)$ , i votes in favor of x. Condition (i) requires that, when recognized, i place positive probability only on utilitymaximizing proposals from those acceptable to a majority, with strict concavity of the utility functions implying both that there will always exist proposals acceptable to a majority and that i prefers to offer some acceptable proposal to having her proposal be rejected. Hence, in any equilibrium, the first proposal offered will be accepted, with the location of this proposal potentially depending on the identity of the proposer. Further, the set of equilibrium outcomes coincides with the set of equilibrium proposals.

Embedded in these conditions are two refinements of the Nash equilibrium concept. The first is found in both (i) and (ii)and requires that proposals and voting be optimal after any possible history of the game, not just the equilibrium history; this is known as subgame perfection (10). Thus, for example, even if the first proposer offers something other than what was expected, individuals' first-period voting behavior should be optimal given the prediction that future proposals (and voting) will play out according to the equilibrium strategies. The second is found only in (ii), and stems from the fact that in majority voting over a pair of alternatives there exist Nash equilibria in which individuals vote for a common alternative regardless of their preferences. Such behavior is consistent with Nash by the fact that, if a majority is already voting in favor of one alternative, any one individual's vote cannot effect the outcome; hence, they are indifferent and so find it optimal to go along with the majority. On the other hand, an individual's weakly dominant strategy in such a game is to vote for her preferred alternative, owing to the possibility that the others will split their votes between the two, thereby rendering her pivotal: that is, her vote is decisive in determining the outcome. Thus, (ii) requires each  $i \in N$  to consider themselves pivotal in the vote on whether to accept a proposal.

For the remainder of the game, a strategy for  $i \in N$  includes three voting rules,  $v_i = (v_i^m, v_i^r, v_i^f)$ , specifying, respectively, (i) how i votes on the motion to consider the bill bproposed by c; (ii) how i votes with respect to the open vs. closed rule, as a function of the bill b and the vote on the motion to consider; and (iii) how i votes on final passage under the closed rule, as a function of the bill b, the vote on the motion to consider, and the vote on the amendment rule. As in the bargaining subgame, the equilibrium conditions on the voting rules embody the notions of subgame perfection and weak dominance, in the form of sophisticated voting (11) (which is equivalent to iteratively deleting dominated strategies). An advantage here is that, once an equilibrium  $\beta^*$  for the bargaining subgame has been selected and so the utilities  $\{u_i(\beta^*)\}_{i\in\mathbb{N}}$  are well defined, the game becomes one of a finite length. Thus, I can solve for the equilibrium strategies of the players for the remainder of the game by first solving for the optimal behavior at decision node F and then working backwards up the (collective) decision tree. On a vote between b and s at node F, i bases her decision on the relative values of  $u_i(b)$  and  $u_i(s)$  (by weak dominance), with indifference assumed to favor b. On a vote between the open and closed rules at node R, her decision is based on  $u_i(\beta^*)$  and either  $u_i(b)$  or  $u_i(s)$ , depending on whether b or s is predicted to prevail at node F; refer to this as the sophisticated equivalent outcome at node F, and note that common knowledge of u implies each  $i \in N$  can perfectly predict this outcome. If indifferent at decision node R, I assume i votes for the closed rule. On the motion to consider, her decision is based on  $u_i(s)$  and one of  $\{u_i(\beta^*), u_i(b), u_i(s)\}\$ , depending on the sophisticated equivalent outcome at node R ( $\beta^*$ , b, or s, respectively), which again i can perfectly predict. If the sophisticated equivalent outcome at R is  $b \neq s$  and i is indifferent between s and b, I assume she votes in favor of the motion whereas if the sophisticated equivalent outcome at R is s, I assume she votes against the motion. To these behavioral assumptions, I add the preference assumption that, for all  $i \in N$ ,  $u_i(s) \neq u_i(\beta^*)$ , so that (because n is odd) there exists a majority that either strictly prefers s to  $\beta^*$  or else strictly prefers  $\beta^*$  to s.

Finally, given a profile of equilibrium voting strategies  $v^* = (v_1^*, \ldots, v_n^*)$  and bargaining strategies  $\beta^* = (\beta_1^*, \ldots, \beta_n^*)$ , the committee  $c \in N$  can solve for the resulting policy associated with any bill  $b \in X$  she could propose; denote this policy  $x(b; v^*, \beta^*)$ . I require her to make a utility-maximizing choice of bill; that is,  $b^*$  should solve

$$\max\{u_c(x(b;v^*,\,\beta^*)):b\in X\}.$$

# **Equilibrium Behavior**

I begin with a description of three qualitative properties of equilibria in the bargaining subgame, properties which, while not completely characterizing equilibrium behavior there, are nevertheless sufficient for the tasks at hand. Two of the three properties are related to the majority rule core associated with the utility profile u, defined as

$$K(u) = \{x \in X : \nexists y \in X \text{ s.t.} | \{i \in N : u_i(y) > u_i(x)\} | > n/2 \}.$$

Thus, a policy is in K(u) if no majority can find an alternative they all prefer to it. Since  $u_i(.)$  is strictly concave for all  $i \in N$  and n is odd, the core K(u) is either empty or a singleton;

denote this unique core point by  $x^*$ . When d=1, the "median voter theorem" (12) states that K(u) is nonempty for all u, with  $x^*$  equal to the median of the legislators' ideal policies. On the other hand, if d>1, the necessary conditions on u to guarantee the nonemptiness of K(u) are quite severe; I take up this issue in more detail below (for more on the core see ref. 13). A proof of the following result can be found elsewhere (J.B. and J. Duggan, unpublished work):

LEMMA 1. In the bargaining subgame, (a) there exists an equilibrium  $\beta^*$ ; (b)  $p_1 = \ldots = p_n^* = p^*$  is an equilibrium if and only if  $p^* = x^*$ ; (c) If d = 1, then the unique equilibrium is where  $p_1^* = \ldots = p_n^* = x^*$ .

By Lemma 1(a), there exists an equilibrium to the bargaining subgame for any profile u of utilities, and, therefore, (selecting any one if there are multiple equilibria) the utilities  $\{u_i(\beta^*)\}_{i\in \mathbb{N}}$  associated with selecting the open rule are well defined. By Lemma 1(b), the only time it is an equilibrium for all individuals to make the same proposal is when there exists a majority rule core point; further, everyone proposing this point constitutes an equilibrium. There may exist other equilibria; however, Lemma 1(c) says that, when the policy dimension is one, this "core" equilibrium is the only equilibrium.

I turn next to a characterization of the equilibrium behavior at the voting stages. Fix an equilibrium  $\beta^*$  in the bargaining subgame, and let  $x(\beta^*)$  denote the expected outcome generated by  $\beta^*$ :

$$x(\beta^*) = \sum_{i \in N} \rho_i \left[ \int_X x \, \pi_i^*(dx) \right].$$

Since each  $u_i$  is strictly concave,  $u_i(x(\beta^*)) \ge u_i(\beta^*)$  for all  $i \in N$ , with this inequality strict unless all equilibrium proposals coincide. Next, define

$$A^{\circ}(u) = \{x \in X : |\{i \in N : u_i(x) \ge u_i(\beta^*)\}| > n/2\}$$

as the set of policies that are weakly preferred by a majority to the utility associated with the open rule. This set will be nonempty (in particular,  $x(\beta^*) \in A^o(u)$ ) and compact (by the continuity of the  $u_i$ s). Likewise, define

$$A^{s}(u) = \{x \in X : |\{i \in N : u_{i}(x) \ge u_{i}(s)\}| > n/2\}$$

as the set of policies that are weakly preferred by a majority to the status quo; this set will as well be nonempty  $(s \in A^s(u))$  and compact.

Our next result is an immediate application of the sophisticated voting logic described above:

LEMMA 2. (a) If  $b \in A^{\circ}(u) \cap A^{s}(u)$ , the sophisticated voting outcome is b; (b) if  $b \notin A^{\circ}(u) \cap A^{s}(u)$ , the sophisticated voting outcome is the majority-preferred choice from  $\{s, \beta^*\}$ .

In particular, as I assume that when s is the sophisticated equivalent outcome at R individuals simply reject the motion to consider the only time the closed rule is adopted is when the committee proposes a bill that majority-defeats both the status quo and the open rule.

To see that an optimal policy choice  $b^*$  for c always exists and, hence, that I have existence of equilibria for the entire game, first note that the set  $A^o(u) \cap A^s(u)$  is always nonempty; this follows since  $x(\beta^*) \in A^o(u)$  and  $s \in A^s(u)$ , and so, if a majority weakly prefers  $x(\beta^*)$  to s, the former will be in  $A^s(u)$ , and conversely. Next, since the intersection of compact sets is compact, the continuous function  $u_c(.)$  achieves a maximum on the set  $A^o(u) \cap A^s(u)$  by Weierstrass' Theorem; denote this maximum  $\bar{v}(u)$ . Therefore, c's choice of optimal proposal reduces to two options: choose (optimally) from  $A^o(u) \cap A^s(u)$  and receive  $\bar{v}(u)$  or choose from outside of  $A^o(u) \cap A^s(u)$  and receive either  $u_c(\beta^*)$  or  $u_c(s)$ , depending on which of  $\beta^*$  and s is majority-preferred to the other. Among these two options, one will obviously give at least as high a utility as the other, and

so, an optimal choice  $b^*$  for c always exists. Note that, since  $A,^{o}(u) \cap A^{s}(u)$  need not be convex, even when c prefers to choose from this set, her utility-maximizing choice need not be unique; therefore, we may have multiple equilibria.

Finally, I turn to identifying when  $b^*$  will lie in  $A^o(u) \cap A^s(u)$ , thereby [via Lemma 2(a)] triggering the adoption of the closed rule by the legislature. If I can show that there exists a policy in  $A^o(u) \cap A^s(u)$  that gives the committee a strictly higher payoff than that from the majority-preferred choice between s and  $\beta^*$ , so too must c's optimal proposal from within this set give a strictly higher payoff, implying  $b^* \in A^o(u) \cap A^s(u)$ . Thus, in what follows, I can simply focus on identifying when such better proposals exist. The answer turns out to depend on whether the core K(u) is empty or not and, when empty, on which of  $\beta^*$  and s is majority-preferred to the other. Write  $\beta^*Ps$  for when  $\beta^*$  is majority-preferred to s, and similarly define  $sP\beta^*$ .

If  $K(u) = \emptyset$ , then, by Lemma 1(b), the equilibrium proposals (and, hence, the outcomes) cannot all coincide, and so  $u_i(x(\beta^*)) > u_i(\beta^*)$  for all  $i \in N$ . Furthermore, if  $\beta^*Ps$ , then  $u_i(\beta^*) > u_i(s)$  for a majority of individuals. Therefore,  $x(\beta^*)$  is in  $A^s(u)$ ; and, since  $u_c(x(\beta^*)) > u_c(\beta^*)$ , c's optimal proposal  $b^*$  will be in  $A^o(u) \cap A^s(u)$ , and the closed rule will be adopted.

Next, suppose  $K(u) = \emptyset$  but  $sP\beta^*$ . The analysis here depends on whether the following condition holds:

$$\exists y \in A^s(u), y \neq s, s.t. u_c(y) \geq u_c(s);$$

equivalently, there exists a coalition  $L\subseteq N$  such that |L|>n/2,  $c\in L$ , and  $u_i(y)\geq u_i(s)$  for all  $i\in L$ . For any  $\alpha\in(0,1)$ , define  $z_\alpha=\alpha s+(1-\alpha)y$ ; by strict concavity,  $u_i(z_\alpha)>u_i(s)$  for all  $i\in L$  and all  $\alpha\in(0,1)$ , and so  $z_\alpha$  is in  $A^s(u)$  for all  $\alpha\in(0,1)$ . Since  $sP\beta^*$ , by continuity  $z_\alpha$  will be in  $A^o(u)$  for  $\alpha$  sufficiently close to 1. Hence, there exists  $\alpha\in(0,1)$  such that  $z_\alpha\in A^o(u)\cap A^s(u)$  and  $u_c(z_\alpha)>u_c(s)$ , implying as above that  $b^*\in A^o(u)\cap A^s(u)$ , and, in equilibrium, the closed rule is adopted.

Conversely, suppose the above condition is not satisfied; that is, for all  $y \in A^s(u)$ ,  $y \neq s$ ,  $u_c(y) < u_c(s)$ . This implies  $s = \arg\max\{u_c(x) : x \in A^o(u) \cap A^s(u)\}$ , and so c is indifferent between proposing b = s and proposing any  $b \notin A^o(u) \cap A^s(u)$ . Thus, the equilibrium outcome will be equal to s, with the voting assumption above implying that the motion to consider is denied.

We can actually turn the above condition into a "core"-type of condition: define the *c-core*,  $K_c(u)$ , as

$$K_c(u) = \{x \in X : \not\exists (y, L) \text{ s.t. } |L|$$
  
>  $n/2, c \in L \& u_i(y) > u_i(x) \forall i \in L\}.$ 

Thus, elements of  $K_c(u)$  are such that no majority coalition including c can find a preferred alternative. The above condition is then satisfied precisely when s is not in  $K_c(u)$ , and so c can find a majority (including herself) that prefers some movement away from the status quo.

What we have shown, then, is the following:

PROPOSITION 1. Suppose  $K(u) = \emptyset$ . (a) If  $\beta^*Ps$ , then, in every equilibrium, the closed rule is adopted and the committee's bill is accepted; (b) if  $sP\beta^*$  and  $s \notin K_c(u)$ , then, in every equilibrium, the closed rule is adopted and the committee's bill is accepted; (c) if  $sP\beta^*$  and  $s \in K_c(u)$ , then, in every equilibrium, the motion to consider is denied.

Thus, when the core is empty, any committee bill considered by the legislature, that is, for which the motion to consider is approved, is assigned the closed rule. From above, I know that, when d=1, the core is nonempty for all utility profiles, and so this result is not relevant when the policy dimension equals one. On the other hand, when d>1, the core can be empty,

and, indeed, existing results show that it is empty for "almost all" utility profiles.

In ref. 14, core emptiness is addressed under the assumption that utility functions are strictly quasiconcave, a weaker condition than the strict concavity assumed here. It is shown that, when d > 1, the core is empty on the interior of X for an open and dense set of utility profiles under the Whitney  $C^{\infty}$  topology [a similar conclusion holds on the boundary of X when d > 2and this boundary is smooth (15)]. That is, if  $K(u) = \emptyset$ , then there exists an open set U of utility profiles containing u such that  $K(u') = \emptyset$  for all  $u' \in U$ ; and, if  $K(u) \neq \emptyset$ , then, for all open sets U of utility profiles containing u, there exists  $u' \in$ U such that  $K(u') = \emptyset$ . Here, I have a subset of the allowable profiles considered in ref. 14; yet, one can show that the core is almost always empty on this restricted class of profiles (under the relative topology) as well. Openness will be inherited from the openness on the larger set of profiles. Denseness is not inherited but can be shown constructively from the necessary conditions for an alternative to be a majority rule core point found in ref. 16. For instance, if  $x_i \neq x_j$  for all  $i, j \in N$ , these conditions are met only at some individual's ideal policy, and then only if the remaining individuals can be paired up in such a way as to make the utility gradients of the members of each pair point in exactly opposite directions. If a core point exists at  $x_i$ , slightly shifting the function  $u_i$  in any direction will render the core empty.

Combining this result on the core with Proposition 1 produces our main conclusion: For almost all utility profiles, the closed rule will be assigned to any committee bill considered by the legislature when the policy dimension is greater than one.

When the core is nonempty, as, for example, when the policy dimension equals one, I know from Lemma 1(b) that there exists an equilibrium of the bargaining game in which  $p_1^* = \ldots = p_n^* = x^*$ ; furthermore, by Lemma 1(c), this is the unique equilibrium when d=1. It is easily seen that, here,  $x^*$  is the only element in  $A^o(u) \cap A^s(u)$ , and so (since  $x^*$  is majority-preferred to s), regardless of what c proposes, the resulting equilibrium outcome will be  $x^*$ . Therefore, c is indifferent over all possible bills she could propose, and, hence,  $b^* \neq x^*$ , with subsequently the open rule being adopted, constitutes an equilibrium. Of course,  $b^* = x^*$ , with then the closed rule being adopted, is an equilibrium as well, and so we are left without a clear prediction of rule assignment when the core is nonempty.

One escape route from this indeterminacy is to employ a selection argument in the spirit of trembling hand perfection (17), in which each suboptimal strategy for an individual receives some vanishingly small probability of being played. If these "error" probabilities only occur on the vote concerning the amendment rule, it is easily seen that c's optimal proposal will not be  $x^*$  but instead will be some element b of  $A^s(u)$ preferred by c to  $x^*$ : In making such a proposal, c gains a small but positive probability of generating b as the outcome, with  $x^*$  being the outcome under the complementary probability. This convex combination is clearly superior to proposing  $x^*$ , which generates  $x^*$  as the outcome with certainty, and so we would resolve the above indeterminacy in favor of the open rule. (Of course, the proper application of trembling hand perfection requires positive errors throughout the entire game.)

Alternatively, instead of modifying the solution concept, one could modify slightly the committee's preferences in the game. Suppose that, for a fixed policy outcome, c prefers fewer stages in the process. Thus, c, would rather propose  $x^*$ , so that the closed rule is adopted and  $x^*$  is voted over s, than offer some other policy that triggers the open rule, a new proposal (of  $x^*$ ), and then a final vote. (A similar conclusion would hold if, ceteris paribus, c prefers to offer a bill that is accepted to one that is not.) In this case, the indeterminacy is again resolved,

but in the opposite direction. In fact, the prediction now would be that the closed rule should be the only rule ever observed, regardless of the policy dimension.

In general, then, the one-dimensional model is less clear than its multiple dimension counterpart with regards the amendment rule assigned to the committee's bill. It is important to point out, however, that this lack of determinacy with respect to the rule assignment when d=1 is caused precisely by the stark determinacy with respect to the policy outcome, namely, the fact that, regardless of the bill proposed by c, the resulting policy will necessarily be equal to the majority core point,  $x^*$ .

## Policy Bias and the Closed Rule

From the above proposition, we see that, in multiple dimensions, closed rules are almost always assigned to any committee bill considered by the legislature, with this bill ultimately being adopted. This is the same sequence that commonly occurs in the structure-induced equilibrium models as well (although the actual policies implemented will most likely differ). In the current model, however, the degree to which the committee's bill, and, hence, the final policy, differs from that which would have occurred under the open rule is constrained by the fact that, by subgame perfection, a majority must find the former to be weakly preferable to the latter. This constraint has the effect of potentially rendering this policy difference infinitesimal, depending on the preferences of the legislators. I demonstrate this claim through the use of a simple example.

Suppose each individual's utility function is quadratic:  $u_i(x) = -||x - x_i||^2$ , so that we can completely characterize the preferences of each  $i \in N$  by their ideal policy  $x_i \in X$ . Let  $E(x_1, \ldots, x_n)$  denote the set of equilibrium proposals in the bargaining subgame when the preferences are given by  $(x_1, \ldots, x_n)$ ; thus, I can view E(.) as a correspondence from the set of profiles of ideal points,  $X^n$ , into the set of profiles of proposal strategies,  $[P(X)]^n$ . It has been shown (J.B. and J. Duggan, unpublished work) that this correspondence is upper hemicontinuous, so that, if  $\{(x_1^k, \ldots, x_n^k)\}_{k=1}^\infty$  is a sequence of ideal point profiles converging to  $(x_1, \ldots, x_n)$ , and  $\{\beta^{*k}\}_{k=1}^\infty$  is a sequence of equilibria such that, for all k, I have  $\beta^{*k} \in E(x_1^k, \ldots, x_n^k)$ , then there is a subsequence of  $\{\beta^{*k}\}_{k=1}^\infty$  that converges to some  $\beta^* \in E(x_1, \ldots, x_n)$ .

Now, let  $X = [-1, 1]^2$ , s = (1, 1), n = 3, c = 1, and let ideal points be  $x_1 = (-1, 0), x_2 = (0, y), \text{ and } x_3 = (1, 0).$  I will hold fixed  $x_1$  and  $x_3$ , and vary y in  $x_2$ . Consider a sequence of ys converging to zero, and denote by  $\beta^*(y)$  the equilibrium in the convergent subsequence associated with y. By Lemma 1(c), I have a unique equilibrium in the bargaining game when y = 0, namely  $\beta^*(0) = (0, 0)$  (since, in this instance, the origin is the majority rule core point in one dimension), and so, by upper hemicontinuity,  $\beta^*(y)$  converges to (0, 0). Thus, for y sufficiently close to zero, the expected outcome if the open rule were to be adopted is close to the origin. Now, for all  $y \neq 0$ , the majority rule core is empty, and, therefore, by Proposition 1, in equilibrium the closed rule is adopted and the committee's proposal  $b^*(y)$  is passed. To locate this proposal, note that, since  $u_i(\beta^*(y))$ , being i's expected utility in the equilibrium  $\beta^*(y)$ , is continuous in  $\beta$ , I have that, for all  $i \in N$ ,  $u_i(\beta^*(y))$  is converging to  $u_i(0, 0)$ . But then the set of proposals weakly preferred by individual 2 to  $\beta^*(y)$  is converging to the set  $\{(0, 0)\}$  (i.e., her ideal point) while the analogous set for individual 3 is converging to  $\{x \in X : d(x, x) \in X : d(x, x) \in X : d(x, x) \in X\}$  $(x_3) \le 1$  [where d(.) denotes Euclidean distance]. Therefore, since the committee is capturing at least one of these other voters at each y, it must be that the committee's proposal  $b^*(y)$ is converging to (0,0) as well. Hence, I can make  $b^*(y)$  as close

to  $\beta^*(y)$  as I wish by choosing y suitably close to zero since each of these sequences is converging to (0, 0).

This exercise demonstrates how the difference between the equilibrium outcome under the closed rule and that which would have occurred under the open rule can be arbitrarily small. Conversely, any argument to the effect that such a difference is substantial must, in the current model, be predicated on an additional assumption concerning the heterogeneity or dispersion of the individuals' preferences.

#### Discussion

The analysis above has demonstrated how the dimension of the policy space can be an important factor in determining the amendment rule governing a committee's bill, in that, with multiple dimensions, we should almost always observe the closed rule being employed. With regard to testable implications, and consciously side-stepping the issue of measuring the policy dimension, the model thus predicts a central tendency toward the use of closed rules when this dimension is greater than one. However, in terms of explaining variation in the use of restrictive rules—for example, the intertemporal variation described in the first section—the model has less potential, as any theoretical variation requires the selection of an "open rule" equilibrium in one dimension in the presence of competing arguments.

A number of extensions of the model immediately suggest themselves, some combination of which might generate a more significant amount of variation. The first has to do with the bargaining model of the open rule employed here, which looks much more like a free-for-all than the structured procedure found in Congress. In particular, a more realistic model of the open rule would have at least some role for the committee's bill, with any such role adding to the strategic considerations of the committee in formulating an optimal bill. Second, the current model ignores the role of the Rules Committee, whose influence emanates from its ability to propose amendment rules (in much the same way the committee's influence emanates from its ability to propose the bill). With only two possible rules, any such influence is quite limited; however, expanding the set of rules by allowing for intermediate levels of amending would both increase this influence as well as, again, provide a more accurate picture of existing processes. Finally, it would be nice to have multiple members of the committee, who might, for example, engage in their own bargaining over the proposal to make it to the floor. The key technical difficulty with applying my own results (J.B. and J. Duggan, unpublished work) to such a model is that any proposal would, as here, have to lie in the set of alternatives defeating both the status quo and the open rule. This set will typically be nonconvex in multiple dimensions whereas, in the aforementioned results, the proposal space is required to be convex.

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