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*COLOR VISION AND THE NATURAL IMAGE PART II**

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In Part I we proposed a new co-ordinate system to describe the colors we obtained in images, since these colors were not those predicted by classical color theory. As a strong working hypothesis we proposed that the color, at least in images derived from two primaries, depends neither on the wave lengths of these primaries nor on the relative energy of these primaries at a given point in the image. This new co-ordinate system implies instead that color at a point in an image depends on a ratio of ratios; namely, as numerator, the amount of a long-wave stimulus at a point as compared with the amount that might be there; and, as denominator, the amount of a shorter wave stimulus at that point as compared with the amount that might be there. (This was Fig. 13 in Part I and is reproduced here as Fig. 1.)

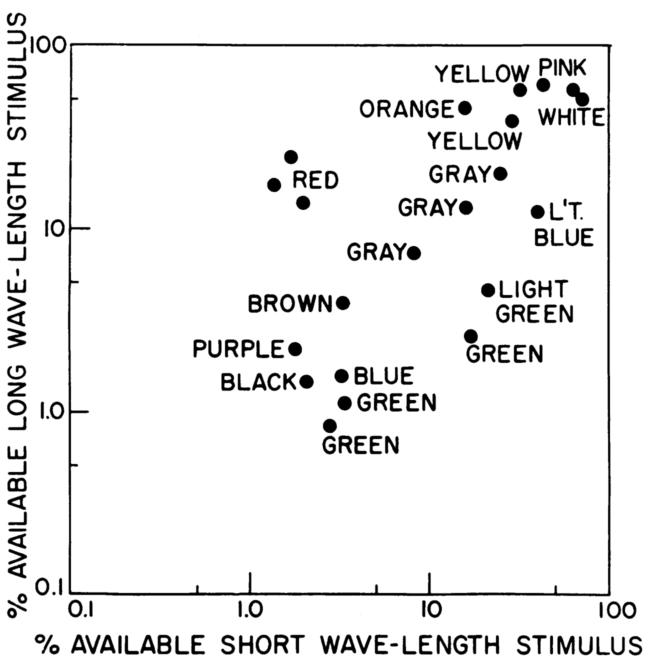

FIG. 1.—Co-ordinate system.

In order to conduct experiments which would delineate the boundaries of this hypothesis we built a dual-monochromator (Fig. 2). With this instrument we may transilluminate a pair of photographic transparencies, each with a narrow spectral band of light or with white light. It is a simple matter for the observer to change the wave-length or the energy of either of the stimuli, and to watch the effect these changes have on the colors in the composite image.

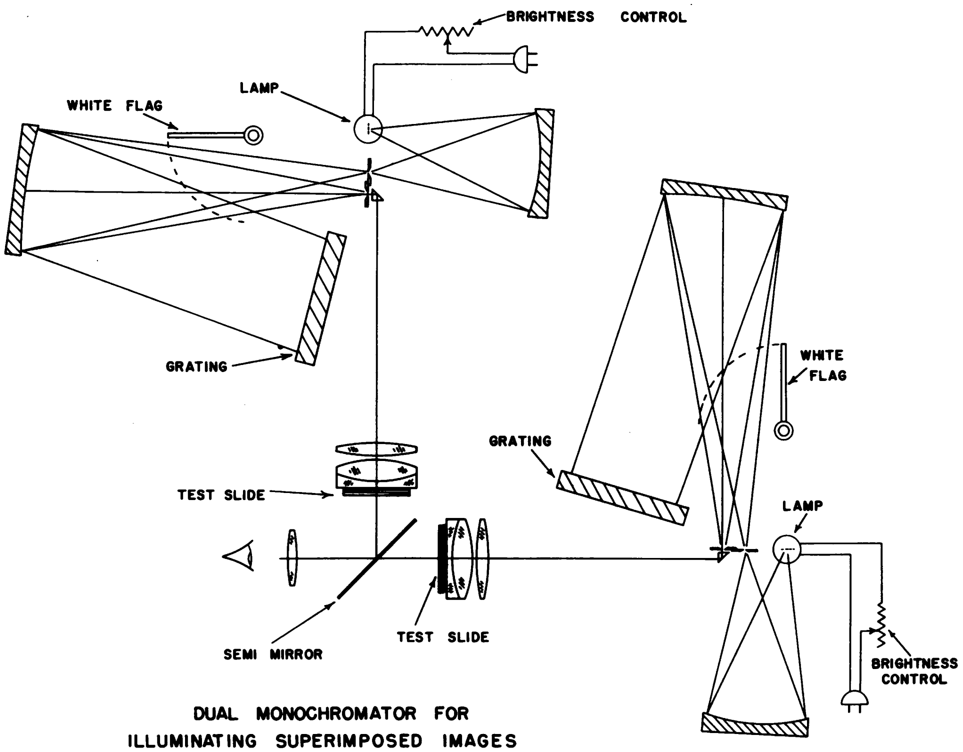

FIG. 2.—The dual monochromator viewer superposes, via the semimirror, two transparent images from a color separation camera. These images can be illuminated with narrow bandwidths of any two desired wave-lengths which are selected by rotating the gratings of Littrow-type monochromators. Compact filaments are imaged at the entrance slits of the monochromators, and the spectra are imaged at the exit slits. Images of the exit slits are superposed at the exit pupil by condenser systems corrected for spherical and chromatic aberration. A white flag can be inserted into either path to provide white light. (The dual monochromator was designed for us by David S. Grey and executed by him and Stanley W. Haskell.)

These experiments have supported our original hypothesis, and the first part of that hypothesis may now be restated as follows: Color in images cannot be described in terms of wave-length and, in so far as the color is changed by alteration of wave-length, the change does not follow the rules of color-mixing theory. The chart in Figure 3 describes the results of our experiments, and is a graphic presentation of the degree to which color is independent of wave-length, as well as of the way in which the range of colors is limited with some combinations of wave-lengths.[1] *For in this new approach to the problems of color, two questions keep coming to the forefront: (1) What variety of sensations can be elicited by a pair of stimuli in the image situation? (2) How does the sensation elicited by the stimuli at a given place in the*

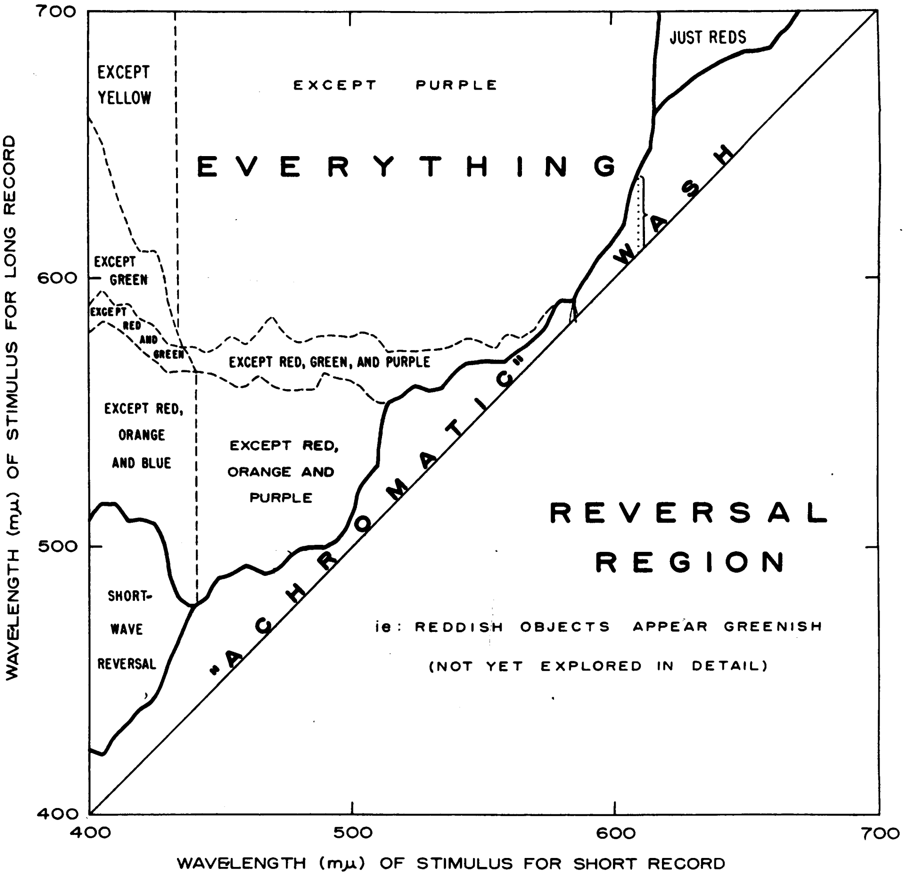

FIG. 3.—*Range of colors produced by different pairs of wave lengths.* "Everything" is used to mean red, orange, yellow, green, blue, purple, brown, black, white and gray. Broken lines are used to define subdivisions in which the range of colors obtained differs only slightly from the gamut in the larger area.

image compare with the sensation the observer would have had when he was looking at the subject being photographed; are the sensations in the synthetic situation in the same color-order as they were in the original situation? The area above the 45° line includes, by definition, all combinations of wave-lengths where the stimulus for the long record is longer than the stimulus for the short. In almost every case this arrangement causes the colors to appear in correct hierarchical order. In the area below the 45° line, the stimuli, and therefore the colors, are reversed. The only exception to this general rule occurs in the area marked "Short-wave Reversal" and this region will be discussed later in more detail.

The largest area on the graph is the one in which the two stimuli will give "Everything"—i.e., red, orange, yellow, green, blue, purple, brown, black, white, and gray. These colors persist with a wide variety of pairs of wave-lengths. There are places where the gamut of colors is restricted, but even in these areas the range of color which *is* produced is far greater than would be predicted by classical theory. These areas of limitation are indicated by broken line divisions at the edges of the larger region of good color. There is, in addition, a small area where the range of color is quite limited: two very long wave-lengths (e.g. 660–690) give only different shades of red.

When a wave-length from 400 to 435 is used as a short stimulus, with one from 440 to 510 as the long, there is the reversal noted in the lower left hand corner of the diagram; the objects which were red in the original scene are a vivid green, and the green objects appear to be a somewhat purplish red. When the stimuli here are reversed, making the shorter wave-length the stimulus for the long record, the colors appear in the correct hierarchical order. The remarkable phenomenon consequent on this observation requires a great deal of further study; the significant point at the moment is that the shorter wave-length acts as the long stimulus in producing these subtle and rich colors.

The long area running through the diagram just above the 45° line is labeled "'Achromatic' Wash." The pairs of wave-lengths which fall within this area will not produce any color in an image. There will be a wash of color reminiscent of what would be expected classically, but so pale that it falls in an entirely different visual category from the color obtained on *objects* in images. The width of the Achromatic Wash region can be read as a function of either the long or short stimulus; and the curve defining that region tells the separation required between the long and short stimuli in order to give color to the objects in the image. The separation necessary at any point may be determined very simply. For example, with a short stimulus of 610 mμ an imaginary vertical line may be drawn which will intersect the solid line curve that marks the first appearance of color. Reading from that point of intersection across to the long stimulus axis we find that that point is 640 mμ. The difference between the two wave-lengths (a distance here represented by the dotted line between the 45° line and the curve) is 30 mμ; this is the separation required to see color at that point. The colors which will be seen with that separation are listed in the area on whose boundary the junction point lies. With the pair 610-640 the full gamut of color appears at once, although with this extreme combination the colors are unsaturated; with certain other wave-lengths the minimum separation gives only a limited range of colors. If, for instance, we use 475 as the short stimulus, the minimum separation is 20 mμ: at this point, with a long stimulus of 495, yellow, green, blue, brown, black, white, and gray will appear. If the long wave-length is moved up to 560, the green will disappear and orange will be added; and at 570 the gamut is complete: "everything" (except purple) is in the image. There is a particularly interesting region between 570 and 590, for here, as the diagram indicates clearly, very little separation of wave-lengths is required. Given a short wave-length of 580, the long wave-length need be made no longer than 590 before the full gamut of color appears. (This small separation can best be studied by passing the wave-lengths across each other, noting the sharp reversal in the hierarchical order of the colors.) With this separation of only 10 mμ the cool colors are unsaturated but the blues and greens are recognizable.

A second interesting feature of the area between 570 and 590 becomes apparent when a single wave-length is used as one stimulus and white is used as the other. With a wave-length from the "blue" part of the spectrum, white acts as the long stimulus; and with a wave-length from the "red" part of the spectrum, white becomes the short member of the pair. What then is the wave-length which is equivalent to white, the wave-length than which white is neither longer nor shorter? We have found that 588 is that wave-length. We can use 588 as one stimulus and white as the other, and there will be essentially no color in the image. But the slightest shift of that wave-length to a position above or below 588 will bring color into the image. The hierarchical order of the colors will reverse as the white is made to act first as the long and then as the short stimulus. The remarkable fact is that many observers of various ages and races, given the simple instruction to find the point where the red object turns to green, have all set the wave-length dial to within one or two millimicrons of 588. From this consistency and precision we have learned that the eye must have a fantastic mechanism for finding a balance point within a band of wave-lengths.

The second part of our original hypothesis was that the colors in an image are largely independent of the relative energies of the two stimuli. Experiments with the dual-monochromator have shown that this is true; furthermore, when the colors finally change because of extreme changes in relative energy, they do not alter in the way that the classical laws of color-mixing predict.

This is shown by the following tables which describe the stability of colors for various pairs of wave-lengths as the relative energy is altered. (Tables 1, 2, 3, and 4.) The values for the range over which the color was seen were obtained by altering the energy of one stimulus and measuring its energy at various levels. Because the measurements were made in this way, the responses of the eye and of the photometer to that stimulus, in relation to their response to other stimuli, did not need to be considered. However, it was assumed that increasing the energy of one stimulus by X was equivalent to decreasing the energy of the other by X; (the results of other experiments, to be described elsewhere, give this assumption some justification). These are preliminary figures, the result of averaging the observations recorded by only a few observers. There is no doubt, however, of the validity of the conclusion that for many wave-length pairs color is stable over a very great range of relative brightnesses of the two stimuli.

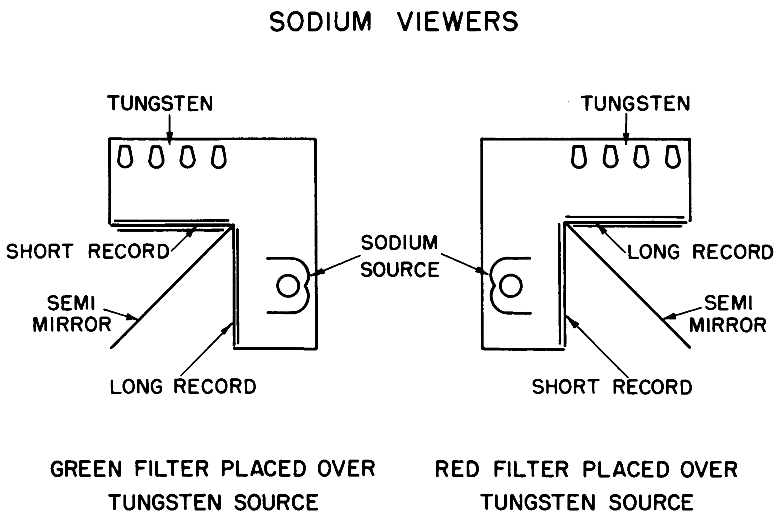

FIG. 4.—Sodium viewers.

TABLE I

Using Wave-lengths 560 mμ and 615 mμ as Projection Stimuli

Colors Seen	Range Over Which Seen	Variation in Color Over This Range
Red	300 *to 1*	Very dark red to red-orange
Brown	200	Red-brown to green-brown
Green	100	Green to dark green with a touch of blue
Orange	100	Light orange to red-orange
Gray } White }	30	Cool gray to warm gray
Yellow	30	Green-yellow to orange-yellow
Blue	6	A dark blue never easily distinguishable from black

TABLE 2

Using Wave-lengths 450 mμ and 575 mμ as Projection Stimuli

Colors Seen	Range Over Which Seen	Variation in Color Over This Range
Gray	200 *to 1*	Little variation
Brown	100	Yellow-brown to dark brown
White	100	Yellowish-white to bluish-white
Yellow	30	Yellow to off-white
Yellow-green	30	Yellow-green to yellow-orange
Blue	10	Blue-violet to blue-green
Green	6	Blue-green to gray-green
Red	5	Dark red to dark orange-red
Orange	5	Yellow to red-orange

TABLE 3

Using Wave-lengths 575 mμ and 595 mμ as Projection Stimuli

Colors Seen	Range Over Which Seen	Variation in Color Over This Range
Yellow	100 *to 1*	Always near yellow
Gray	100	Always gray
Brown	30	Orange-brown to green-brown
Blue-green	30	Grayish blue-green to dark blue-green
Orange	30	Yellow to orange
Red	20	Dark brick-red to red-orange
White	10	Yellowish-white to white

TABLE 4

Using Wave-lengths 410 mμ and 460 mμ as Projection Stimuli

Colors Seen	Range Over Which Seen	Variation in Color Over This Range
White	40 *to 1*	Always near white
Green	40	Dark green to light blue-green
Violet-red	20	Dark violet-red to rosy-pink
Gray —	6	Violet-gray to gray
Light blue	6	Light blue to aqua

SODIUM VIEWERS

Experiments 13, 14, and 16 (Part I) showed that a narrow band of wave lengths (Wratten No. 73) may be the shorter member of one pair of stimuli, and then the longer member of another pair; the capacity in which this narrow band acts is determined by the length of the other stimulus used with it. This versatility of wave length was difficult to demonstrate to an audience using projected images because much of the light was not transmitted by the dark filters. In order to make the demonstration more effective we built a pair of viewers (Fig. 4), each employing a large sodium-vapor street light as one of the illuminants.[2] In each

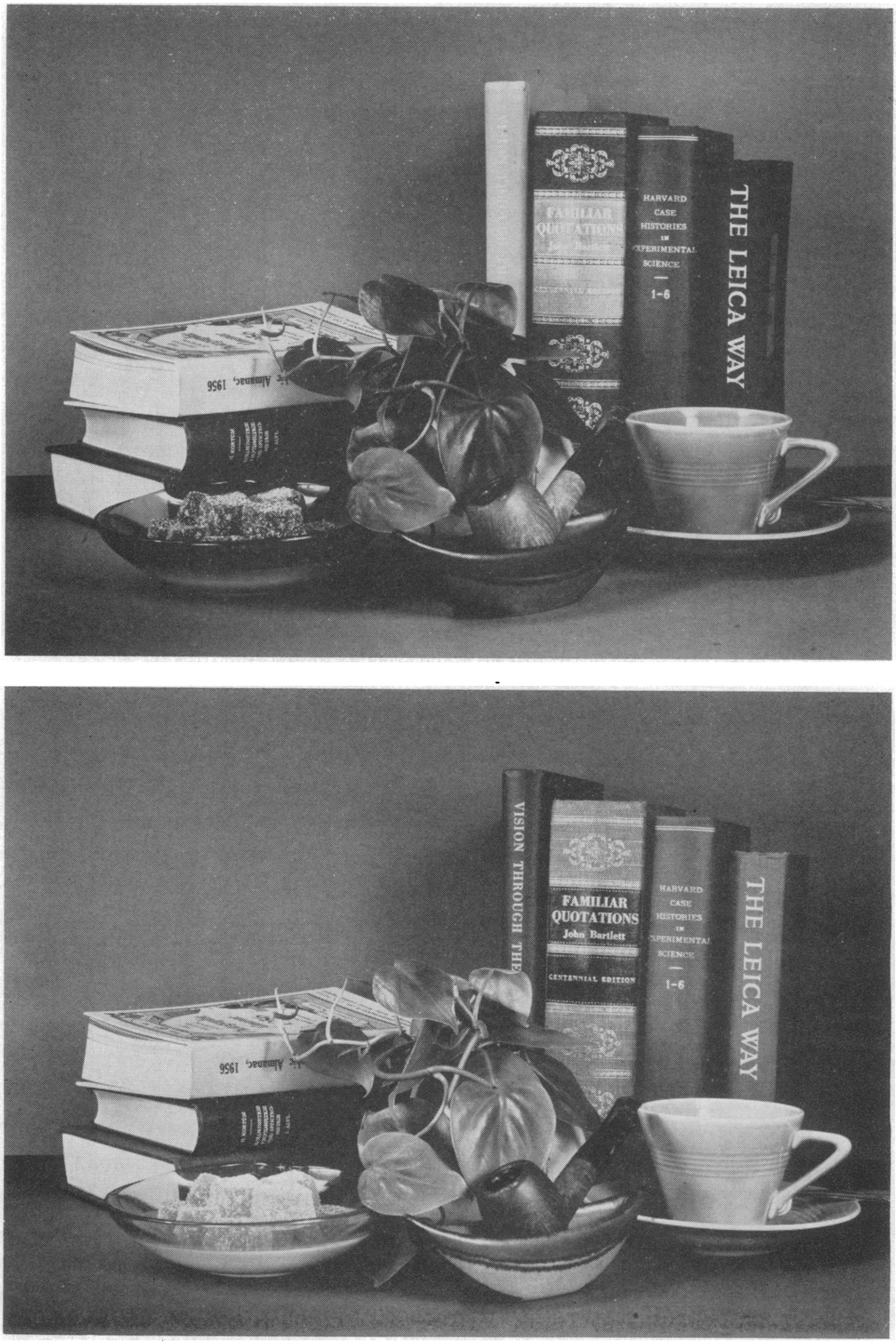

FIG. 5.—Still life of books and plants. Upper image transluminated with longer wave-length stimulus; lower image with shorter wave-length stimulus. This applies also to Figure 6.

FIG. 6.—Still life of groceries.

viewer the long and short records are placed at right angles to one another and the images are superposed, as they are in the dual monochromator, by means of a semi-mirror at 45°. The photographic transparencies are two feet square, making the image easily visible to a large audience; and the image is many times brighter than it would be if projected on the screen through narrow band filters.

In the viewer on the right in Figure 4 sodium is the short stimulus, while tungsten light through a red filter is the long (Experiment 23). The resulting composite image (Fig. 5) covers a range of color sensations, including green and some blue.

In the viewer on the left in Figure 4 the long record is illuminated by sodium; tungsten light through a green filter is the short stimulus (Experiment 24). The resulting picture (Fig. 6) is fully colored, covering the range from red to blue-green. It is interesting to note that in some areas here the sodium D-line is being seen as a red, whereas in the previous experiment the D-line was seen as a green. These two experiments prove conclusively to an audience that a single wave-length may be used as the short member of one pair of stimuli and as the long member of another.

The two sodium viewers used in Experiments 23 and 24 may be set up next to each other (Experiment 25). It then becomes clear that sodium can act as the long stimulus in one and as the short in the other *simultaneously;* that, as the observer looks quickly from one to the other, each of the pictures is quite satisfactory. The observer may also move away from the viewers until the angle they subtend is so small that both images are seen essentially "at the same time," and again the adjacency of the two displays does not destroy the color in either one of them.

The room lights are turned on, making the sodium viewers a part of the room scene (Experiment 26). The colors of objects in the mirror viewer are as real as the colors of the objects in the room.

There is a further test (Experiment 27) to which the colors in the images may be subjected: If the observer stands slightly to one side, by the front edge of the mirror, he will be able to see the sodium light, the filtered tungsten light, and the mixture of the two in the colored image on the mirror—all at the same time. Thus the colors in the image prove to be stable, not only with respect to the colors of other objects in the room (Experiment 26), but also with respect to the spectral colors of the stimuli producing the image.

We have learned from our recent work that the discussion of the negative-positive combination, and also the description of "Johnny's Magic Bookcase," belong most properly to the study of visual fields. They will, therefore, be included in Part III where the nature of the field in the image situation will be discussed.

* Part I appeared in the January, 1959, issue of these *Proceedings* and is a necessary introduction to Part II.

[1] We hesitate to idealize the boundaries of color areas on the chart because we would need measurements from hundreds of observers in order to do this accurately. We have been able to test enough observers to know that the representation given here is essentially correct for all of them, but we have chosen to maintain maximum accuracy by presenting curves which are precise for one of these observers. In making the measurements the following method was used: With each pair of stimuli the best brightness ratio was determined, and the colors seen at that relative brightness are the ones recorded in Figure 3.

[2] This apparatus was first demonstrated at a meeting of the Optical Society of America on October 9, 1958, in Detroit.