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Ultrasonic tagging of light: Theory

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ABSTRACT **A theory is provided for the detection efficiency of diffuse light whose frequency is modulated by an acoustical wave. We derive expressions for the speckle pattern of the modulated light, as well as an expression for the signal-to-noise ratio for the detector. The aim is to develop a new imaging technology for detection of tumors in humans. The acoustic wave is focused into a small geometrical volume, which provides the spatial resolution for the imaging. The wavelength of the light wave can be selected to provide information regarding the kind of tumor.**

There is a need to develop new imaging technology to detect cancers and tumors in humans. The present technology needs to be improved, and many different techniques have been suggested (1–6). For example, x-rays and ultrasound show tumors and provide accurate spatial information, but they do not provide information on the tumors' nature. Infrared light diffuses in humans and cannot provide information on spatial positions unless the tumor is very near to the surface. Light can, however, provide information on the nature of the tumor, because different tumors have different absorption bands.

Recently, a new imaging technology that combines the benefits of ultrasound and diffuse light was proposed (7). Since then it has been under continuous development (8–13). By having the ultrasound and light present simultaneously, and by detecting the light intensity at the frequency modulated by the sound wave, one detects photons that have interacted with both sound and light. We use the phrase "tagging" to denote the process of modulating the light wave by the ultrasound frequency. UTL is ultrasound tagging of light. By focussing the sound wave into a small geometrical volume, one can provide spatial information regarding where the tagging occurred. The spatial resolution is about the wavelength of the ultrasound, which is about 1 mm, depending on the sound frequency. High frequency sound is used because of its shorter wavelength. Pulsing the laser light allows multiple images as the slow sound pulse travels through the medium. Phased arrays can be used to send ultrasound in different directions, which allows for sweeping over the complete volume. The tagged signal should be different in the volume occupied by a tumor. Varying the wavelength of the light can then provide information about the nature of the tumor, and the ultrasound can provide information regarding the tumor's location. When finally developed, this UTL technology could provide a new method of imaging.

Although this idea is attractive, the UTL technology is still in its embryonic stage. Here we develop a mathematical model of the imaging process. We consider what kind of signal will be detected in this measurement. Several related issues bear upon this answer. First, what is the nature of the speckle pattern for a modulated signal? Diffuse light from a coherent source is known to be emitted with a interference pattern that is nearly random in nature. Its speckle properties are well documented (14). In the present experiment, the light will be provided by a laser. If the laser has frequency ω_L and the ultrasound has frequency ω_S, the modulated light has a frequency of $\omega_L + \omega_S$ or else $\omega_L - \omega_S$. What are the speckle properties of the modulated signal? As far as we can tell, this issue has never been discussed. In the next section, we solve this problem and derive the distribution of intensities for a modulated signal. A second theoretical question is the nature of the tagging process. We propose that it is Brillouin scattering. However, because the photons are diffusing, the theory for the intensity of Brillouin scattering is quite different from the theory in the usual formulas, which assume ballistic photons. Finally, the third issue is to derive an expression of the signal-to-noise ratio (SNR) of the modulated signal. We show that this ratio is independent of the area of the detector but is proportional to the intensities of the laser and the ultrasound.

1. Modulated Signal

Here we develop a theoretical model for the ultrasonic modulation of light in random media. Our final goal is an expression for the SNR. This model has been tested by experiments that are not described here. We have a detector of area A that captures the light intensity (denoted by I) with an efficiency η and converts it into a current (denoted by i). These are proportional, $i = \zeta I$, where $\zeta = e\eta A/\hbar\omega_L$. The light intensity and the current are regarded as having two components, which are called the unmodulated and modulated signals. These are understood by noting that the light intensity is given by the square of an electric field, and the field has two components:

$$I = |E|^2 \quad [1]$$

$$E(t) = e^{-i\omega_L t}[E_U + E_T \cos(\omega_S t)] \quad [2]$$

$$I(t) = I_U(t) + I_M(t)\cos(\omega_S t) + I_T \cos(2\omega_S t) \quad [3]$$

$$I_U = |E_U|^2 + \frac{1}{2}|E_T|^2 \quad [4]$$

$$I_M(t) = 2\Re\{E_U^* E_T\} \quad [5]$$

$$I_T = \frac{1}{2}|E_T|^2. \quad [6]$$

The electric field E_U is from the unmodulated light. The electric field E_T is from the light tagged by the ultrasound. Its frequency is shifted by the ultrasound frequency. The signal from I_T is too small to detect, and the modulated signal is from $I_M \cos(\omega_S t)$. It comes from the cross-term between the unmodulated and the tagged electric fields. An important feature of these intensities is that I_U is strictly positive, while I_M can have either sign. I_M is not positive because it is a cross-term between two different electric fields, which are statistically independent. The signal comes from $i_M = \zeta I_M$, while the noise comes from $i_U = \zeta I_U$. The tagging process is very weak,

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Abbreviations: UTL, ultrasound tagging of light; SNR, signal-to-noise ratio.

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so that $i_M \ll i_U$. We introduce the dimensionless parameter r, which is the fraction of light that is tagged:

$$|E_T|^2 = r|E_U|^2, \quad [7]$$

which is averaged over the area of the detector. Below we give a theory for this important parameter. It depends on the power in the ultrasound source, as well as on various geometrical factors due to the size and shape of the system.

The intensities $I_{U,M}$ are random variables with statistical properties (14). When coherent light from a laser undergoes diffusion, the emitted light has a speckle pattern (14). The speckle is a pinpoint of light over which the signal is coherent. The dimension of a speckle is $d_{\text{speckle}} = 1.2\lambda_L \ell / d_a$, where λ_L is the wavelength of light, ℓ is the distance from the detector to the sample surface, and d_a is the aperture of the light at the surface. The area of a typical speckle is about $A_S = \alpha \lambda_L^2$, where $\alpha = 3 - 5$. This estimate should be the same for the modulated and unmodulated light, because their wavelengths are nearly identical. We also define the number of speckles $N_S = A/A_S$ on the detector. It should be about the same number for the modulated and unmodulated light. The speckle patterns of these two signals will have different arrangements and will not coincide, but they are similar in area and number.

We are treating each speckle as being statistically independent. This is the conventional model for the unmodulated signal. We apply the same model to the modulated signal, as is discussed below.

1.1. Untagged Light. Write the current from the untagged light as a summation over the individual speckles

$$i_U = \xi \sum_{j=1}^{N_s} I_j \quad [8]$$

$$\xi = \frac{e\eta A_S}{\hbar \omega_L}, \quad [9]$$

where I_j is the light intensity of the individual speckles. The individual speckles have as their probability distribution (14)

$$P_s(I) = \frac{1}{I_0} e^{-I/I_0}, \quad [10]$$

where I is always positive. Here I_0 is the average intensity of the light at the surface. Using this distribution, we use Markov methods (15) to calculate the average value of i_U. First we calculate the distribution of values for this current. It has a probability distribution

$$\bar{P}_s(i_U) = \left\langle \delta\left(i_U = \xi \sum_{j=1}^{N_s} I_j \right) \right\rangle \quad [11]$$

$$= \int \frac{ds}{2\pi} e^{isi_U} m(s)^{N_s} \quad [12]$$

$$m(s) = \int_0^\infty P_s(I) dI e^{-is\xi I} \quad [13]$$

$$= \frac{1}{1 + is\xi I_0} \quad [14]$$

$$\bar{P}_s(i_U) = \frac{(i_U/i_0)^{N_s - 1}}{i_0 (N_s - 1)!} e^{-i_U/i_0} \quad [15]$$

$$i_0 = \xi I_0. \quad [16]$$

The distribution of current in the detector $\bar{P}(i_U)$ is given by the above expression. The current i_U has a Poisson distribution. The averages for this distribution are well known:

$$\langle i_U \rangle = \int_0^\infty dJ J \bar{P}_s(J) = N_s i_0 \quad [17]$$

$$\langle i_U^2 \rangle = \int_0^\infty dJ J^2 \bar{P}_s(J) = N_s(N_s + 1)(i_0)^2. \quad [18]$$

We shall use these results below. The term $\langle i_U \rangle = N_s i_0$ is the shot noise, and i_0 is the average current from one speckle.

1.2. Modulated Light. Now we consider the statistical properties of the modulated light I_M, which gives the modulated current i_M. As a modulation, it can be either plus or minus with equal probability. Thus its statistical properties are different from those of the unmodulated signal, which is always positive. We have derived the results by using the methods in ref. 14. From Eq. **7** we note that the size of the tagged electric field is $E_T \sim \sqrt{r} E_U$, so that the size of the modulated intensity is $I_M \sim \sqrt{r} I_U$:

$$P_M(I_M) = \frac{1}{2\sqrt{r} I_0} e^{-|I_M|/(\sqrt{r} I_0)} \quad [19]$$

Again I_0 is the average intensity of the unmodulated light. The modulated current i_M is written as a summation over the speckles, as we did in Eq. **8** for the unmodulated light. Then we start the statistical development as we did in Eq. **11**. The function $m(s)$ has a different value because even moments of i_M vanish because it can have either sign.

$$\bar{P}_M(i_M) = \int \frac{ds}{2\pi} e^{isi_M} m(s)^{N_s} \quad [20]$$

$$m(s) = \int_{-\infty}^\infty P_M(I) dI e^{-is\xi I} \quad [21]$$

$$m(s) = \frac{1}{1 + r(s\xi I_0)^2} \quad [22]$$

This distribution can be given by a complicated series. However, at large values of N_s it is well approximated by a Gaussian:

$$\bar{P}_M(i_M) = \frac{1}{\sigma\sqrt{2\pi}} e^{-i_M^2/2\sigma^2} \quad [23]$$

$$\sigma^2 = 2N_s r i_0^2. \quad [24]$$

The averages for this distribution are

$$\langle i_M \rangle = \int_{-\infty}^\infty J dJ \bar{P}_M(J) = 0 \quad [25]$$

$$\langle i_M^2 \rangle = \int_{-\infty}^\infty J^2 dJ \bar{P}_M(J) = \sigma^2. \quad [26]$$

The average value of the square of the modulation current $\langle i_M^2 \rangle$ is proportional to the number of speckles. The average value of the square of the unmodulated current $\langle i_U^2 \rangle$ is proportional to the square of the number of speckles. This difference occurs because the unmodulated intensity is always positive, while the modulated current can have either sign.

One can think of the modulated signal as having a random walk in intensity space. The average intensity is then zero, while the average of the square is proportional to the number of steps, which here is the number of speckles.

To give some feel for the magnitude of these quantities, we give approximate values for the laboratory measurements. The laser wavelength was $\lambda_L = 1064$ nm, which gives the speckle $A_s = 3.14 \times 10^{-6}$ mm^2. The detector had an area of $A = 0.3$ mm^2, so the number of speckles is $N_s = A/A_s = 10^5$. The current in the detector for one measurement was $\langle i_U \rangle = 30\mu$A so that $i_0 = \langle i_U \rangle / N_s = 0.3$ nA.

2. Power Spectrum

The modulated signal (from the tagged light) and the noise (from the unmodulated light) are both found from the power spectrum (16, 17). Both can be derived from the current–current correlation function. We use the definition of current in ref. 3, keep only the first two terms, and assume that the current is proportional to intensity:

$$\begin{aligned} A(\tau) &= \langle i(t+\tau)i(t)\rangle \\ &= \langle i_U(t+\tau)i_U(t)\rangle \\ &\quad + \langle i_M(t+\tau)i_M(t)\cos(\omega_S t)\cos(\omega_S t+\omega_S\tau)\rangle. \end{aligned} \quad [27]$$

We have assumed that the correlation of the currents $\langle i_U i_M\rangle = 0$. The time dependence of the intensity $I_M(t)$, and $i_M(t)$, is due to variations in the amplitudes of the electric fields. These may be caused by variations in the intensity of the laser or by the motion of the scattering centers in the medium. In our intended application, the scattering centers that produce the speckles are not fixed in position but are cells that are diffusing or flowing in biological tissue. The speckle pattern will change because of slow variations in the light intensity. This change affects the power spectrum of both the modulated and the unmodulated light. A measure of this effect is the time constant for the decay in the autocorrelation function of a single speckle. We assume that the time decay of the correlation is of the form $\exp(-|\tau|/\tau_0)$, where τ_0 is the decay time. The important correlations are then

$$\langle i_U(t+\tau)i_U(t)\rangle = N_s(N_s + e^{-|\tau|/\tau_0})i_0^2 + eN_s i_0\delta(\tau) \quad [28]$$

$$\langle i_M(t+\tau)i_M(t)\rangle = \langle i_M^2\rangle e^{-|\tau|/\tau_0} \quad [29]$$

so that

$$A(\tau) = N_s^2 i_0^2 + N_s i_0[e\delta(\tau) + i_0 e^{-|\tau|/\tau_0}(1 + r\cos(\omega_S\tau))]. \quad [30]$$

The term in $A(\tau)$ containing a delta function $\delta(\tau)$ is due to the shot noise.

From the Wiener–Khinchin Theorem (16, 17), the power spectrum is the Fourier transform of $A(\tau)$

$$P(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} A(\tau) \quad [31]$$

$$\begin{aligned} &= 2\pi N_s^2 i_0^2\delta(\omega) + ei_0N_s \\ &\quad + \tau_0 N_s i_0^2\left[\frac{2}{1+(\tau_0\omega)^2} + \frac{r}{1+\tau_0^2(\omega-\omega_S)^2}\right. \\ &\qquad \left. + \frac{r}{1+\tau_0^2(\omega+\omega_S)^2}\right]. \end{aligned} \quad [32]$$

The first term in $\delta(\omega)$ is unimportant. The second term $eN_s i_0$ is the shot noise and provides the basic noise spectrum. The last three terms are the signal from the fluctuations in the untagged and the tagged light.

In the laboratory experiment, the power spectrum is recorded on a Hewlett–Packard spectrum analyzer. Let B be the bandwidth of the spectral analyzer in hertz. We assume that we evaluate $P(\omega)$ in the vicinity of the ultrasound frequency ω_S:

$$\tilde{P} = \int_{\omega_S-\pi B}^{\omega_S+\pi B} d\omega P(\omega) \quad [33]$$

$$\approx eBi_0N_s + \frac{N_s i_0^2 r}{\pi}\tan^{-1}(\pi B\tau_0). \quad [34]$$

The only noise term is the shot noise $eN_s i_0$. The second term is the signal from the ultrasound tagging. The SNR of these two terms is

$$\frac{S}{N} = \frac{ri_0}{eB_{\text{eff}}} \quad [35]$$

$$\frac{1}{B_{\text{eff}}} = \frac{\tan^{-1}(\pi B\tau_0)}{\pi B}. \quad [36]$$

We have defined an effective band width B_{eff}. In a laboratory experiment on static, phantom tissue, τ_0 is very long, $\pi B\tau_0 \gg 1$, so that $B_{\text{eff}} = 2B$. However, on living biological tissue, where cells diffuse or drift in blood flow, the value of τ_0 will be must smaller. The actual value must be measured for each case. However, we expect that $\pi B\tau_0 \ll 1$ so that $B_{\text{eff}} = 1/\tau_0$. Thus the SNR in a laboratory experiment on phantom tissue should be much larger than that found in a measurement on living biological tissue. The latter value can not be determined without a measurement of τ_0.

An important feature of our formula (Eq. **35**) is that it is independent of the area of the detector: the dependence on the number of speckles has cancelled out of the numerator and denominator. It is proportional to the intensity of light because i_0 has this dependence. The factor of r is also proportional to the intensity of the ultrasound, as is derived below.

3. Ultrasonic Tagging

The interaction between sound and light is weak. Here we estimate this interaction and derive a formula for the modulation efficiency r.

There are two different mechanisms of scattering light from a collection of particles. The first is Brillouin scattering, which is the *coherent* scattering from the density fluctuations. The density modulation of the sound wave sets up a diffraction scattering. It strongly scatters light in transparent media. Here we discuss the intensity of scattering when the light is diffusing with a mean-free-path that is less than the wavelength of sound λ_S. The second mechanism of scattering is when the light scatters from the individual cells in the biological media. This *incoherent* scattering we estimate to be much weaker than the Brillouin scattering, and we neglect it.

We start with the standard Boltzmann equation (18–20) for the density of photons $f(\vec{r},\hat{s})$ at the position $\vec{r}$ that are going in the direction $\hat{s}$. The media provides the two attenuation coefficients for scattering (μ_s) and absorption (μ_a).

$$[\hat{s}\cdot\vec{\nabla} + \mu_s + \mu_a]f(\vec{r},\hat{s}) = \mu_s\int\frac{d\Omega_{s'}}{4\pi}p(\hat{s}\cdot\hat{s}')f(\vec{r},\hat{s}') \quad [37]$$

The term on the right give the scattering into various directions, where $p(\hat{s}\cdot\hat{s}')$ is the probability of scattering of the particles, by the media, from $\hat{s}$ to $\hat{s}'$. The light intensity I and photon flux $\vec{S}$ are defined as

$$I = \int\frac{d\Omega_s}{4\pi}f(\vec{r},\hat{s}) \quad [38]$$

$$\vec{S} = c\int\frac{d\Omega_s}{4\pi}\hat{s}f(\vec{r},\hat{s}). \quad [39]$$

The first moment of Eq. **37** is obtained by taking the integral $d\Omega_s$ over all directions

$$\vec{\nabla}\cdot\vec{S} + \mu_a cI = 0. \quad [40]$$

Our primary interest is in the distribution function for the modulated light. Denote its distribution by $f'(\vec{r},\hat{s})$. We assume that in a small region V_{US} the scattering rate is altered

$\mu_s' \to \mu_s' + \delta\mu_s V_{US}\delta^3(\vec{r}-\vec{r}_0)$. Then the tagged light obeys the Boltzmann equation

$$[\hat{s}\cdot\vec{\nabla}+\mu_s+\mu_a]f'(\vec{r},\hat{s}) = \mu_s\int\frac{d\Omega_{s'}}{4\pi}p(\hat{s}\cdot\hat{s}')f'(\vec{r},\hat{s}') - \delta\mu_s V_{US}\delta(\vec{r}-\vec{r}_0) \times\left[f(\vec{r}_0,\hat{s})-\int\frac{d\Omega_{s'}}{4\pi}p(\hat{s}\cdot\hat{s}')f(\vec{r}_0,\hat{s}')\right]. \quad [41]$$

Take the first two moments of this equation. Primed variables refer to modulated light

$$c\mu_a I' + \vec{\nabla}\cdot\vec{S}' = 0 \quad [42]$$

$$\frac{c}{3}\vec{\nabla}I' + (\mu_s'+\mu_a)\vec{S}' = g\delta\mu_s V_{US}\delta(\vec{r}-\vec{r}_0)\vec{S} \quad [43]$$

$$\mu_s' = \mu_s(1-g) \quad [44]$$

$$g\hat{s}' = \int\frac{d\Omega_s}{4\pi}\hat{s}\,p(\hat{s}\cdot\hat{s}'). \quad [45]$$

Eliminating I' from the above equations gives

$$[\nabla^2-\gamma^2]\vec{S}' = 3g\mu_a\delta\mu_s\delta(\vec{r}-\vec{r}_0)\vec{S} \quad [46]$$

$$\gamma^2 = 3\mu_a[\mu_s'+\mu_a]. \quad [47]$$

We have rapidly introduced a series of constants that may be unfamiliar to the reader. The effective scattering rate μ_s' enters into the diffusion equation. The attenuation of the intensity of light is given by the parameter γ. In Table 1, we have collected some typical published data on light diffusion through various types of biological tissue. Most of these values are in the range of $\gamma = 2$–$4\ \text{cm}^{-1}$. For $\gamma = 3\ \text{cm}^{-1}$ and $d = 5$ cm, then $\exp(-\gamma d) = 3\times10^{-7}$. Although this value is small, one can detect the light coming through the tissue. The value of g is typically estimated to be $1 < g < 0.9$ so that μ_s is quite large. The value of the absorption coefficient μ_a is quite small, typically 10–100 times smaller than μ_s'. The small value of the absorption coefficient is why the light diffuses. The large value of the scattering coefficient μ_s predicts a small value for the mean-free-path of the diffusing light. The values for blood are noticeably different. Other data are in refs. 24 and 25.

In solving this equation, we make the following assumptions. The sample is in a slab geometry. The incident light of intensity I_i uniformly illuminates one side of the sample. It reaches the scattering center at position $\vec{r}_0$ (depth z_0) with an intensity $I_i r_t \exp(-\gamma z_0)$, where r_t is the transmission coefficient for light to enter the tissue. From Eq. **40** one finds that $\vec{S}(\vec{r}_0) = \hat{z}(\mu_a c/\gamma)I_i r_t\exp(-\gamma z_0)$. At the scattering center, the scattered light is directed isotropically outward in all radial directions. The scattered light that reaches the other side of the sample at a distance R from the scattering center is as follows:

$$\delta I = r_t I_i\frac{3V_{US}g\mu_a\delta\mu_s}{4\pi R}e^{-\gamma(R+z_0)} \quad [48]$$

$$\vec{R} = \vec{r}-\vec{r}_0. \quad [49]$$

Table 1. Light absorption and scattering

Type of tissue	λ, nm	μ_a, cm^{-1}	μ_s', cm^{-1}	γ, cm^{-1}	Ref.
Human brain	1064	0.4	5.5	2.7	21
Canine prostrate	1064	0.4	4.4	2.4	21
Pig liver	1064	0.5	2.4	2.1	21
Pig stomach	805	0.16	20	3.1	22
Human skin	805	0.09	29	2.7	22
Rat muscle	633	0.36	11	3.5	22
Blood	633	25	8	50	23
subcutaneous fat	633	0.2	4	1.6	23

We estimate $R = d - z_0$ in the exponent, and $R \approx d/2$ in the denominator, where d is the thickness of the slab. These approximations yield the following expression for the tagging fraction:

$$r = \frac{3V_{US}g\mu_a\delta\mu_s}{2\pi d}. \quad [50]$$

Another possible experimental setup is to use a continuous wave ultrasound pulse. Then the ultrasound occupies a cylinder of area A_{US} in the sample. In our analysis, we assume the cylinder is perpendicular to the direction that light defuses. A similar analysis for this case yields

$$r = \frac{3}{2\pi}A_{US}g\mu_a\delta\mu_s. \quad [51]$$

These expressions provide the basic geometrical factors relating to the tagging. Next we derive $\delta\mu_s$ from the interaction between sound and light.

The ultrasound has a wave vector $\vec{q}$. If Δ is the displacement of the particles, then the variation in density caused by the ultrasound is

$$\frac{\delta\rho}{\rho} = (q\Delta)\cos(\vec{q}\cdot\vec{r}-\omega_S t), \quad [52]$$

where ρ is the density of the system. In the standard theory of Brillouin scattering, the variation in dielectric constant ε caused by the density variations is

$$\delta\varepsilon = \rho\frac{\delta\varepsilon}{\delta\rho}\frac{\delta\rho}{\rho} = (\varepsilon-1)(q\Delta)\cos(\vec{q}\cdot\vec{r}-\omega_S t), \quad [53]$$

where we assumed that $\varepsilon = 1+4\pi\alpha\rho$ because local field corrections are small in systems where the cells are larger than the wavelength of light. By using the Golden Rule of quantum mechanics, we derive the tagging rate for a photon of wave vector $\vec{k}$:

$$\delta\mu_s(\vec{k}) = \frac{\pi\omega_L^2}{4cn^2}(\varepsilon-1)^2(q\Delta)^2\delta(\omega_L-\omega_L'\pm\omega_S) \quad [54]$$

$$(q\Delta)^2 = \frac{I_{US}}{I_S} \quad [55]$$

$$I_S = \frac{1}{2}\rho C_s^3, \quad [56]$$

where n is the refractive index, c is the velocity of light, and C_s is the velocity of sound. The ultrasound intensity is I_{US}, while I_S is a reference intensity, which is $1.7\ \text{W/m}^2$ for water. The tagged light has a frequency $\omega_L' = c|\vec{k}+\vec{q}|$. Because $q \ll k$, one can approximate $\omega_L' \approx \omega_L + c\hat{k}\cdot\vec{q} = \omega_L+\omega_S$.

One must average over the directions $\vec{k} = k\hat{s}$ of the photons. In diffusion this is given by $f(\hat{s},z) = n(z)+3j(z)\hat{s}\cdot\hat{z}/c$. The first term contains the density $n(z)$ of unmodulated photons, while the second contains their current ($j = -Ddn(z)/dz$). The current term gives a negligible contribution in the present geometry, because the ultrasound direction $\vec{q}$ is perpendicular to $\hat{z}$.

We must also take into account the important fact that the photons have a mean-free-path that is short compared to the wavelength of the ultrasound. This provides an uncertainty for the wave vectors of the photons. The uncertainty can be included in the present analysis by replacing the delta function for frequency conservation by a Lorentzian for the conservation of wave vector ($\omega_L' = ck'$):

$$\delta(\omega_L'-\omega_L\pm\omega_S) \to \frac{1}{2\pi c}\frac{\mu_T}{(k-k')^2+(\mu_T/2)^2} \quad [57]$$

$$\mu_T = \mu_s+\mu_a. \quad [58]$$

By using expressions **55** and **57** in Eq. **54** gives

$$\delta\mu_s = \frac{\pi}{2\mu_T}\left(\frac{\omega_L}{c}\right)^2(\varepsilon-1)^2\frac{I_{US}}{I_S}F(2q/\mu_T) \qquad [59]$$

$$F(x) = \int_0^1 \frac{d\nu}{1+x^2\nu^2} = \frac{\tan^{-1}(x)}{x}. \qquad [60]$$

The final expression for r in the CW limit **51** is

$$r = \frac{3}{4\pi}\left(\frac{g\mu_a}{\mu_T}\right)\left(\frac{\omega_L}{cn}\right)^2(\varepsilon-1)^2\frac{A_{US}I_{US}}{I_S}F\left(\frac{2\omega_S}{C_s\mu_T}\right). \qquad [61]$$

A similar expression is derived for the pulsed ultrasound; the area A_{US} is replaced by the factor V_{US}/d. These expressions are our final result for the tagging due to Brillouin scattering in a turbid medium. The tagging fraction depends on the ultrasound power $P_{US} = I_{US}A_{US}$. The factor of area cancels away, showing that in the present geometry the power is the important quantity.

4. Summary

We have considered the process where light is diffusing through a material while a pulsed sound wave is focused into a small volume V_{US}, or else a continuous wave source sends the sound along a small a path with a small area A_{US}. The diffusing light waves that enter the ultrasound region have a small probability of inelastically scattering from the sound, which shifts their frequency by the sound wave frequency. This modulated light is then detected after it transmits the sample. We derive an expression for the intensity of the modulated signal, as well as the noise spectrum. Thus we derive an expression for the SNR, which is a key parameter when deciding whether there is enough signal to take an image. The SNR depends on the fraction r of photons that are modulated by the sound frequency. An expression was derived for r by assuming that the important scatttering process was Brillouin scattering of the photons by the sound waves.

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