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## statnet: Software Tools for the Representation, Visualization, Analysis and Simulation of Network Data

**Mark S. Handcock,**  
*University of Washington*

**David R. Hunter,**  
*Penn State University*

**Carter T. Butts,**  
*University of California, Irvine*

**Steven M. Goodreau, and**  
*University of Washington*

**Martina Morris**  
*University of Washington*

### Abstract

**statnet** is a suite of software packages for statistical network analysis. The packages implement recent advances in network modeling based on exponential-family random graph models (ERGM). The components of the package provide a comprehensive framework for ERGM-based network modeling, including tools for model estimation, model evaluation, model-based network simulation, and network visualization. This broad functionality is powered by a central Markov chain Monte Carlo (MCMC) algorithm. The coding is optimized for speed and robustness.

### Keywords

exponential-family random graph model; Markov chain Monte Carlo; networks; random graph models

## 1. Introduction

**statnet** is a suite of software packages for statistical network analysis in R (R Development Core Team 2007) that implements recent advances in the statistical modeling of random networks. The models, based on statistical exponential families (“exponential-family random graph models,” or ERGMs), generalize the  $p_1$  (Holland and Leinhardt 1981) and Markov random graph (Frank and Strauss 1986) models first developed in the social network literature. These in turn derive from developments in spatial statistics (Besag 1974). The general form is sometimes referred to as “p-star” or “p\*” in the social network literature (Wasserman and Pattison 1996), reflecting its early origins.

Networks are a form of “relational data,” i.e., data whose properties cannot be reduced to the attributes of the individuals (nodes) involved. The relation, or “tie,” is the object of (and unit of) analysis. Relational data arise in many fields, and are central to the concept of “social” in the social sciences. In typical applications, the nodes in a network represent individuals, and

the links (edges) represent a specified relationship between individuals. Nodes can also be used to represent larger social units (groups, families, organizations), objects (physical resources, servers, locations), or even abstract entities (concepts, texts, tasks, random variables).

In network modeling, our goal is to predict the joint probability that a set of edges exists on nodes in a network. An edge can be binary or valued, directed or undirected. In general, these edges are not independent, and that is the challenge for both model specification and estimation. ERGMs were developed to address the complex dependencies within relational data structures and provide a flexible framework for representing them. Simple examples include degree distributions, stars, nodal attribute-based mixing, triangles and other higher order cycles that lead to clustering, and mutuality and transitivity in directed networks. These observed network statistics are properly regarded as outcomes, and the goal of the model is to specify the process that leads to their joint distribution. The terms in an ERG model that represent this process are also network statistics. The statistics for process and outcome may sometimes coincide – for example, a propensity for triad closure leading to a large number of triangles in a network would lead to a model with a term for the number of triangles. But the same outcome may arise from an alternative underlying process – for example, a propensity for within-group partners leads to triad closure when groups are small (Goodreau, Kitts, and Morris 2008b), and this would lead to a model with terms for the relative frequency of within-group ties. ERGMs provide a statistical framework for evaluating alternative hypotheses about the processes that lead to the observed outcomes.

The **statnet** suite of packages provides a comprehensive framework for ERGM-based network modeling: tools for model estimation, for model evaluation and for model-based network simulation. This broad functionality is powered by a central Markov chain Monte Carlo (MCMC) algorithm that can easily handle networks of several thousand nodes or more, though the size of the problem is dictated more by the number of edges (and edge attributes) than by the number of nodes. See Hunter, Goodreau, and Handcock (2008a); Goodreau (2007); Goodreau *et al.* (2008b) for examples of large networks that may be handled by **statnet**. For statistical inference, MCMC is used to approximate the likelihood function by sampling the space of possible networks. The sample is obtained by sequentially updating the values of dyads in the network according to a random schedule using a Metropolis-Hastings algorithm, creating a Markov chain of networks with the appropriate statistical properties (Hunter, Handcock, Butts, Goodreau, and Morris 2008b). The sequence produced by this algorithm can also be used as a method for simulating realizations of networks from the model, which in turn can be used for model evaluation. The algorithm can also be used to produce a dynamically changing network over time. The broad functionality of this MCMC algorithm provides a unifying coherent framework for modeling, and it extends the potential scope of network analysis: providing greater flexibility, range and insight into the generative principles of network formation, and a foundation for the analysis of diffusion across network structures.

Network analysis is a rapidly growing field, and there are now a number of computer packages available that provide a wide range of analytical tools. The methodology in these packages falls into three general classes: descriptive techniques, permutation methods, and generative models. The classes range roughly along a continuum, from capturing static regularities in network structure to testing models for the emergence of that structure.

Descriptive techniques include the traditional social network summaries drawn from the graph theoretic literature and reviewed in Wasserman and Faust (1994). These seek to characterize the systematic patterns observed in networks (e.g., the degree distribution, the number of triangles, the centrality of nodes, or the centralization of the network as a whole), but there is no real statistical inference associated with these methods. Descriptive techniques are featured

in established packages like **UCINET** (Borgatti, Everett, and Freeman 1999) and **Pajek** (Batagelj and Mrvar 2007), and they are included in the **sna** (Butts 2007) package in R.

Permutation methods employ computationally intensive resampling to perform statistical inference for traditional statistical models on networks (e.g., the quadratic assignment procedure, QAP, for matrix regression). In this approach, the dependence among observations is treated simply as an obstacle to statistical inference that the permutation distribution allows one to ignore. Such procedures are available in **UCINET** (Borgatti *et al.* 1999), **sna** and **netperm** (Butts 2006), and also in some traditional statistical environments like Stata (StataCorp. 2007).

Generative models provide a full stochastic representation of the process of network formation, which allows the dependence among observations to become the focus of the model. Simple examples include the Bernoulli model and the preferential attachment model, each of which represents a single type of network generating process, and the log-linear models for nodal attribute mixing that provide a class of generative mixing models. ERGMs are a very general class of generative models, which includes the Bernoulli, preferential attachment and mixing models as special cases. When fully specified, generative models also provide a framework for model evaluation and inference. Such fully specified ERGMs are currently available in **statnet** and **stocnet** (Boer, Huisman, Snijders, and Zeggelink 2003).

## 2. Overview of statnet components

**statnet** (Handcock, Hunter, Butts, Goodreau, and Morris 2003b) is written in a combination of the open-source statistical language R (R Development Core Team 2007) and (ANSI standard) C (Kernighan and Ritchie 1988). It is usually used interactively, via a command line, from within the R graphical user interface. It can also be used in non-interactive (or “batch”) mode to allow longer or multiple tasks to be processed without user interaction.

The **statnet** suite of packages, which includes two required components and several optional components, is available on the Comprehensive R Archive Network (CRAN) at <http://CRAN.R-project.org/> and also on the **statnet** project Web site at <http://statnetproject.org/> The suite can be installed directly over the Internet using the `install.packages` command within R as described in Goodreau, Handcock, Hunter, Butts, and Morris (2008a). To obtain a list of the functions available in any of the **statnet** packages, type `help(package = “<pkg name>“)`, as in

```
R> help(package = "ergm")
```

To obtain further information about a particular function, such as the `ergm` function within the **ergm** package, type `help("ergm")` or simply `?ergm` for short.

The individual component packages of the **statnet** suite are listed below. Most of these packages are described in detail in the subsequent articles in this volume.

### 2.1. Required packages: ergm and network

- **ergm** is a collection of functions to fit, simulate from, plot and evaluate exponential-family random graph models. The main functions within the **ergm** package are `ergm`, a function to fit exponential-family random graph models in which the probability of a network is dependent upon a vector of network statistics specified by the user; `simulate`, a function to simulate random networks using an ERGM; and `gof`, a function to evaluate the goodness of fit of an ERGM to the data, **ergm** contains many other functions as well; for a guide to the basic types of functionality these

functions provide, see Hunter *et al.* (2008b), Morris, Handcock, and Hunter (2008), and Goodreau *et al.* (2008a) in this volume.

- **network** is a package to create, store, modify and plot the data in network objects. The network object class, defined in the **network** package (Butts, Handcock, and Hunter 2007; Butts 2008a), can represent a range of relational data types and it supports arbitrary vertex/edge/network attributes. Data stored as `network` objects can then be analyzed using all of the component packages in the **statnet** suite.

## 2.2. Optional packages

The optional packages **sna**, **degreenet**, **latentnet**, and **networksis** are all available on CRAN:

- **sna**: A set of tools for traditional social network analysis (Butts 2008b).
- **degreenet**: This package was developed for the degree distributions of networks. It implements likelihood-based inference, bootstrapping, and model selection, and it includes power-law models such as the Yule and Waring as well as a range of alternative models that have been proposed in the literature. (Handcock 2003b). The theory behind these methods is described in Jones and Handcock (2003a,b); Handcock and Jones (2004, 2006).
- **latentnet**: A package to fit and evaluate latent position and cluster models for statistical networks based on Hoff, Raftery, and Handcock (2002) and Handcock, Raftery, and Tantrum (2007). The probability of a tie is expressed as a function of distances between these nodes in a latent space as well as functions of observed dyadic level covariates. For details about this package, see Krivitsky and Handcock (2007) in this volume.
- **networksis**: A package to simulate bipartite networks with fixed marginals through sequential importance sampling (Admiraal and Handcock 2007).

Additional optional packages are available on request, as described below.

- **dynamicnetwork**: A set of tools for visualizing dynamically changing networks (Bender-deMoll, Morris, and Moody 2008).
- **netperm**: A package for permutation Models for relational data (Butts 2006). It provides simulation and inference tools for exponential families of permutation models on relational structures.
- **rSoNIA**: Provides a set of methods to facilitate exporting data and parameter settings and launching **SoNIA**, which stands for Social Network Image Animator (Bender-deMoll and McFarland 2003). **SoNIA** facilitates interactive browsing of dynamic network data and exporting animations as a **QuickTime** (Apple Inc. 1999) movies.

## 3. Principles of ERGM-based network modeling

ERGMs represent the generative process of tie formation in networks, and there are two basic types of processes: dyadic dependent and dyadic independent. A dyad refers to a pair of nodes and the relations between them. Dyadic dependent processes are those in which the state of one dyad depends stochastically on the state of other dyads. A classic example is the concept that “the friend of my friend is my friend” — the presence of a friendship tie in dyads  $(i,j)$  and  $(j,k)$  increases the probability of a friendship tie in dyad  $(i,k)$ . Dyadic independent processes exhibit no direct dependence among dyads: An example is the related social concept that “birds of a feather flock together” — if the two nodes in a dyad have similar attributes, the probability of a friendship tie is increased. The state of the dyad depends on the attributes of the two nodes, but not on the state of other dyads.

The distinction between these two types of processes affects the specification, estimation and behavior of ERG models. Models with only dyadic independent terms have a likelihood function that simplifies to a form that can be maximized using standard logistic regression methods. Intuition about how these models behave is usually straightforward, as for logistic regression models. By contrast, models for processes with dyadic dependence require computationally intensive estimation and imply complex forms of feedback and global dependence that confound both intuition and estimation.

It can be difficult to specify a sensible model that contains dyadic dependent terms, since a model that seems reasonable may in fact have very counterintuitive implications. A good example comes from the recent literature on clustering in networks. Clustering is often represented as a tendency to form closed triangles, and a common summary measure is the “mean clustering coefficient,” the number of triads with all three ties present divided by the number with at least two ties present. A natural ERGM analog for this is a model with two terms — the edge density and the mean clustering coefficient — and corresponding parameters  $\theta = (\theta_1, \theta_2)$ . The intuition seems straightforward; the density term governs the overall number of ties and the clustering term captures the propensity for, or against, triad closure relative to a Bernoulli random network. In most social networks, we would expect  $\theta_2$  to be positive. But the distribution of networks produced by this model (obtained through simulation) displays a bizarre pattern. First, for many combinations of possible values of the parameters in  $\theta$ , the model produces networks that are either full (every tie exists) or empty (no ties exist) with probability close to one. Second, even for parameters that do not produce these extremal networks, such as those illustrated in Figure 1, the distribution of networks produced by the model is often bimodal; one mode has low density and high triad closure, the other has high density and low triad closure. The model almost never produces networks at the average density and triad closure.

The technical term for this behavior is “model degeneracy,” and a detailed exposition of the topic and the example above can be found in Handcock (2003c) or Handcock (2003a). The intuition behind this problem is relatively simple. If we specify a model that is unlikely to produce the observed network, then one of two things can happen when this model is fit to the data: the maximum likelihood estimator (MLE) may exist, but it will not provide a good fit to the data, as in Figure 1; or the MLE may not exist, and the estimation will not converge properly (as in the extremal case above). Degeneracy is an indication of model mis-specification – not a shortcoming of the MCMC estimation procedure.

The solution to the degeneracy problem is to specify a model that is a better fit to the data, but this is often more difficult than usual. With linear models, for example, the estimated coefficients are linear functions of the observed data. These closed-form solutions can be used to construct predicted values, and mis-specification can be diagnosed by comparing observed to predicted values. With ERGMs, if the model is mis-specified and fails to produce an MLE, the analyst can be left with little information to help guide the re-specification of the model.

A technique that commonly leads to degeneracy when representing dyad dependent processes is the use of simple configuration counts or proportions (e.g., the number of triangles or the mean clustering coefficient) as model covariates. It may seem natural to represent a disproportionately high number of triangles in a network by a triangle term in the model. But because a single edge can complete a large number of triangles, the dyadic dependence effects amplify quickly, so a model with a positive coefficient on a triangle term will almost always lead to degenerate behavior. This is a form of “collapse” or threshold behavior well known in complex systems.

The goal is to develop substantively meaningful models in which the dependent process is properly constrained: to represent the additional heterogeneity in the process of tie formation that limits the range of dependence. One plausible mechanism is strong attribute mixing, which would reduce the impact of dependence beyond the preferred group of nodes: a triad created within the group is more likely than a triad created between groups. Another mechanism is a nonlinear impact of the dependent configuration, e.g., a tie completing two triangles is not twice as likely as a tie completing one triangle, but something less than that. This is similar to the classic declining marginal returns models in economics. The **ergm** package provides a flexible framework for representing such processes with “curved exponential family models” (Hunter and Handcock 2006). These allow one to specify a single **ergm** term that is a parametric summary of an entire distribution of statistics, such as degree statistics, instead of a nonparametric specification that uses one term for each value in the distribution.

In some cases, model re-specification will simply require selecting different **ergm** terms from the list included in the **ergm** package (Handcock, Hunter, Butts, Goodreau, and Morris 2003a). In other cases, it may require coding a new **ergm** term using the methods provided for user-coded terms. As discussed in Morris *et al.* (2008), the terms in an ERGM are network statistics that must be calculated for the observed network, and for each step of the MCMC sequence. So every term requires its own algorithm. This is another way in which network ERGMs differ from traditional statistical models. In a traditional linear model, for example, the covariates are measured prior to estimation, and while they might be transformed as a part of model specification, most transformations are simple arithmetic operators on one or more vectors, and most interactions are simple products of vectors. Network statistics, by contrast, may require complex algorithms for their construction (e.g., a cycle census over the whole network). **statnet** includes code for many of the most common network statistics used in the social network field, but the terms relevant for any particular network, or for networks outside the social sciences, may be different. Re-specifying an ERGM may not just be a simple matter of adding another term to the model; it may require writing a piece of program code to represent that term.

**statnet** provides a range of diagnostic tools to help identify when a model is degenerate, discussed in Goodreau *et al.* (2008a) and (Hunter *et al.* 2008b). Since statistical methods for simulating stochastic random networks have only recently been developed, the models and parameter values relevant to real networks are only beginning to be understood. A good discussion of model specification for social networks can be found in Snijders, Pattison, Robins, and Handcock (2006).

#### 4. Other capabilities

**statnet** is a package that is undergoing long term development, and the **statnet** development team is continually adding functionality. We welcome your comments and even suggestions, though naturally we can not promise to respond to all requests! We also welcome inquiries about becoming a member of the **statnet** development team. We sponsor a **statnet** mail list for questions and discussions; sign-up is through the **statnet** Web page at <http://statnetproject.org/>. Further information about **statnet** is also available at the Web site.

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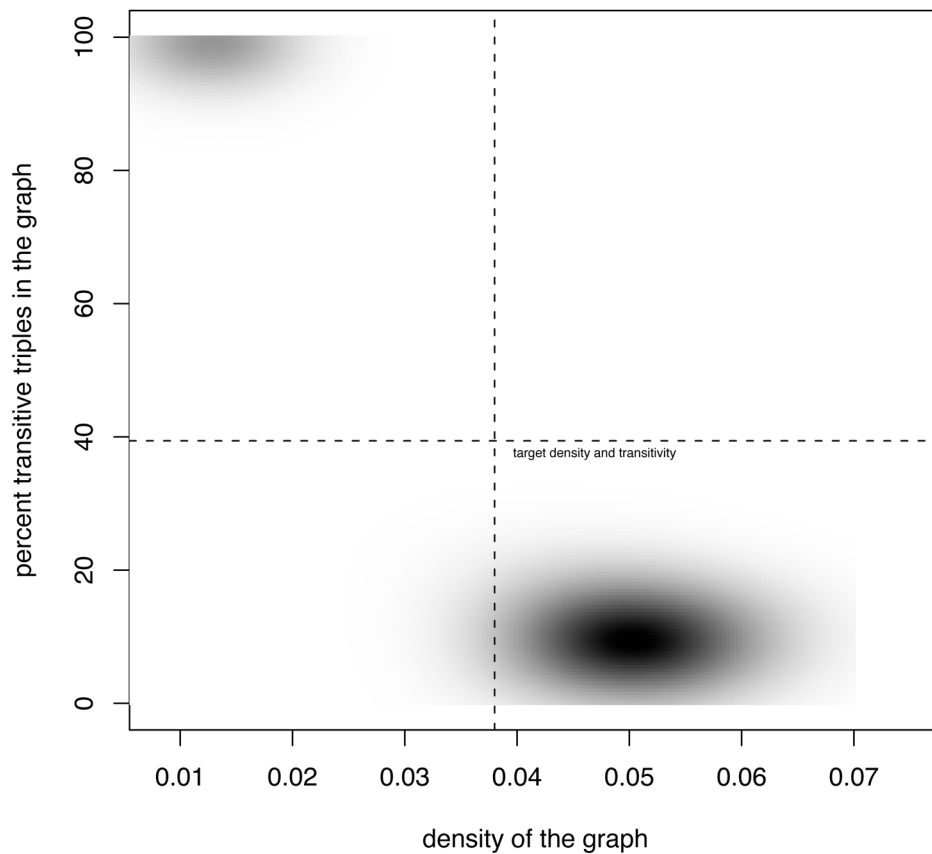
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### Distribution of Graphs from this model



**Figure 1.**

Darker gray indicates higher probability density in this plot showing the true distribution of networks according to a particular two-statistic ERGM containing edge density and mean clustering coefficient. The population mean vector, specified by a particular choice of the model parameters, is shown at the intersection of the two dotted lines. The fact that there is very little probability mass near this mean is emblematic of degeneracy.