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On the Mathematical Analysis of Schistosome Populations

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The concept of the stationary ecological life-table has been applied to populations of the three main species of schistosomes affecting man. In such a system, the net reproductive rate should be equal to 1.0, with reproduction and mortality balancing each other. Calculations are made of net reproduction in both snail and mammal hosts and of the probabilities of transmission from each host to the other. By far the most complete data are those for Schistosoma japonicum, *for which the net reproductive rate is calculated as 0.588. The error probably represents the inadequacy of information on rat populations. Using much less complete data, the net reproductive rate for* S. mansoni *is calculated as 1.85 and for* S. haematobium *as 2.74. The relative departure from the expected value of 1.0 thus reflects the lack of complete information on the respective parasites. Details of the method and examples of each kind of calculation are given.*

It is estimated that with adequate data from three or four areas with a range of transmission rates, a predictive model could be constructed for the epidemiology of one species of parasite, and that adequate staff could obtain the necessary data in two-and-a-half years.

> " A closer collaboration between biometrician and [parasitologist], and a better acquaintanceship of each with the methods of the other, is one of the most useful things we can work for today."
>
> HACKETT (1937) [2]

It will be acknowledged that a predictive mathematical model of the epidemiology of any disease is desirable, both from the standpoint of intellectual satisfaction and from the standpoint of the usefulness in planning measures to control the disease. It is the purpose of this communication to document the advantage of having models for the epidemiology of bilharziasis, to carry the analysis as far as possible with the data that are available, and to estimate the amount of data that would be required to write complete models. The effort required and the feasibility of obtaining sufficient information may then be weighed against the uncertainties involved in proceeding with attempts to control the disease on the basis of present knowledge. From these considerations, rational decisions may be reached as to the allocation of effort among research and control projects.

THE RATIONALE AND PURPOSE OF THE ANALYSIS

The approach taken will be that of an analysis of the parasite population (Hairston, 1962). The methods are fundamentally those of demography, and require a knowledge of the age-specific rates of survival and reproduction (Leslie & Ranson, 1940; Birch, 1948; Evans & Smith, 1952). For free-living organisms, the acquisition and handling of these data are straightforward. For example, according to the data of Pesigan et al. (1958b), the ecological life-table for *Oncomelania quadrasi*, the snail host of *Schistosoma japonicum*, is as shown in Table 1. In such a table, the proportion of female snails surviving to each successive age x is entered in the l_x column. The average number of female eggs laid per female during each successive time interval is entered in the m_x column. The proportional contribution of each age-group to the next generation of female snails is the product of the l_x and m_x entries for that age. The sum of all of these $l_x m_x$ entries is the total contribution of one generation to the next, or the net reproductive rate. Obviously, if the population is neither increasing nor decreasing, the net reproductive rate will be 1.0, and since the field data (Pesigan et al., 1958b,

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[2] The quotations throughout this paper are taken from L. W. Hackett's *Malaria in Europe*, one of the most prescient books in the history of biology.

TABLE 1
ECOLOGICAL LIFE-TABLE FOR FEMALE *ONCOMELANIA QUADRASI* IN THE FIELD

Age in days (x)	Survival of females (l_x)	Female eggs produced per female (m_x)	$l_x m_x$
0	1.000		
16	0.763		
26	0.489		
46	0.240		
66	0.138		
86	0.086		
106	0.062	0.0	0.000
126	0.051	1.5	0.077
146	0.042	4.0	0.168
166	0.034	4.0	0.136
186	0.028	4.0	0.112
206	0.023	4.0	0.092
226	0.019	4.0	0.076
246	0.016	4.0	0.064
266	0.013	4.0	0.052
286	0.011	4.0	0.044
306	0.009	4.0	0.036
326	0.007	4.0	0.028
346	0.006	4.0	0.024
366	0.005	4.0	0.020
386	0.004	4.0	0.016
406	0.003	4.0	0.012
426	0.003	4.0	0.012
446	0.002	4.0	0.008
466	0.002	4.0	0.008
486	0.001	4.0	0.004
506	0.001	4.0	0.004
526	0.001	4.0	0.004
546	0.000	4.0	0.000
			$\Sigma=0.997$ (=net reproductive rate per generation)

p. 526) indicate very stable populations over the period of observation, the calculated net reproductive rate of 0.997 indicates that the observations and assumptions made in constructing the ecological life-table must be quite accurate, although balancing errors are possible.

Obtaining the data for an ecological life-table for the parasite is considerably more complicated because reproduction takes place twice in the life-cycle and because direct observation of reproduction and mortality in the two hosts is not possible. Moreover, the survival of the parasite between the periods of reproduction must also be estimated.

Basically, the ecological life-table of the parasite would involve two tables such as Table 1 multiplied by the probabilities of survival between hosts. It can be reduced to an equation:

$$\left\{\begin{array}{l}\text{net reproductive}\\ \text{rate in snail}\end{array}\right\} \times \left\{\begin{array}{l}\text{probability}\\ \text{of infecting}\\ \text{a mammal}\end{array}\right\} \times \left\{\begin{array}{l}\text{net reproduc-}\\ \text{tive rate in}\\ \text{mammal}\end{array}\right\} \times \left\{\begin{array}{l}\text{probability}\\ \text{of infecting}\\ \text{a snail}\end{array}\right\} = \left\{\begin{array}{l}\text{net reproductive}\\ \text{rate of parasite}\end{array}\right\} \qquad (1)$$

Obtaining the data necessary for such an analysis presents formidable difficulties in effort and ingenuity. Indeed, it is legitimate to ask if the information obtained justifies the expenditure of such effort, since a mere solution of the terms in equation (1) does not yield any predictions.

The principal justification comes from the following considerations. In most areas where bilharziasis is endemic, the parasite population does not appear to change greatly unless the area is disturbed as, for example, by the introduction of irrigation systems or by immigration. This observation is based upon age-prevalence data taken at considerable intervals. Since the curves constructed from such data often remain unchanged, it follows that the parasites are not changing in abundance. Hence, the net reproductive rate of such a population of parasites must be approximately equal to 1.0.

Furthermore, it is well known that age-prevalence curves may be quite different under different conditions, as in the case of the coastal and inland areas of Palo, Leyte, Philippines (Pesigan et al., 1958a, pp. 382-386). This observation indicates that the parasite population is able to come into equilibrium at different rates of transmission, since over a range of such rates, the net reproductive rate remains constant at approximately 1.0.

This being the case, it follows that net reproduction in one or both of the hosts must be curtailed with increasing transmission and enhanced with decreasing transmission. That this actually happens is shown by the fact that *O. quadrasi*, when infected with two or more miracidia, produces no more cercariae than when infected with a single miracidium

(Pesigan et al., 1958b, pp. 565-568). Thus, the average number of progeny per successful miracidium will decrease with increasing proportions of snails infected, as superinfections are all wasted. Similar phenomena occur in the vertebrate host, probably as a result of immunity and fibrosis caused by previous infection.

Thus, there is a range of transmission rates over which compensatory mechanisms operate to keep the parasite population in equilibrium. The important point here is that there is a lower limit below which these mechanisms fail. In the case of the schistosomes, and probably the other dioecious parasites as well, the most important cause of the failure of compensatory mechanisms at low transmission rates is the increasing probability that single parasites which succeed in entering the definitive host will remain unmated (Hairston, 1962). The point at which this phenomenon becomes self-accelerating is critical for the parasite, and the ability to predict it would be extremely useful. By finding solutions for the terms in equation (1) for three or more situations involving as wide a range of transmission rates as possible, one could establish the upper limits of the net reproductive rates in the two hosts, and from this could be calculated the minimum probabilities of successful transmission that would permit the continued existence of the parasite population. Once these facts were established, a definite prediction could be made of the degree of effectiveness of any control measure that would be necessary to cause the parasite population to decline to zero. From the foregoing discussion, it is clear that no control measure need be 100% effective. However, no prediction can yet be made as to the necessary degree of effectiveness.

THE METHOD OF ANALYSIS

The various terms indicated in equation (1) have been estimated only for *Schistosoma japonicum* in the Philippines (Hairston, 1962) and even in this case there are gaps in information sufficient to cause uneasiness.

Net reproductive rate in the snail

> " Now, while most of the books repeat these original calculations of Ross with some gusto, since we all relish the pleasant excitement of such large numbers, a little investigation shows that nothing like this actually happens."

The reproduction of the parasite in the snail is the most directly obtainable portion of the ecological life-table. It is necessary to keep experimentally infected snails in the laboratory, obtaining counts or accurate estimates of the daily production of cercariae for as long as these are produced. This will provide the data for the m_x column. In order to obtain data for the l_x column, the survival of infected snails in the field must be estimated. For *O. quadrasi* infected with *S. japonicum*, the best estimate was obtained by, *first*, comparing the survival of infected and uninfected snails in the laboratory; *second*, obtaining an estimate of the survival of largely uninfected snails in the field; and, *third*, assuming that the proportional survival observed in the laboratory was true of the field (Hairston, 1962). The mortality rates of male and female snails differ. It is therefore necessary to compute the net reproductive rate of the parasite separately for the two sexes of snails, as has been done in Table 2. The m_x data are taken from Pesigan et al. (1958b, p. 566). For *Schistosoma haematobium* and *S. mansoni*, only one table for each would be necessary, as the snails are hermaphroditic. Obtaining the m_x data, however, would be more tedious because of the much greater number of cercariae produced daily.

In these two species, the detailed data necessary to produce an accurate life-table of flukes in snails are still lacking, but crude estimates are possible. For example, approximate rates of the production of cercariae per day are widely quoted, although it is inevitably the maximum rather than the average figure that is given. These estimates indicate a daily output of 800-1000 cercariae for *S. haematobium* and 3500 for *S. mansoni*. The death-rates of infected snails in the field can be estimated from successive field samplings. Such data are given in an unpublished quarterly report of the Egypt-49 project.[1] Death-rates may be calculated from the decline in either the proportion positive or the absolute numbers of infected snails collected. This method assumes that no new infection appears during the interval, and the estimated death-rates are therefore minimal. The formula for estimating such death-rates is:

$$\text{(Proportion remaining after } x \text{ days)} = e^{-ax}$$

where a is the daily death-rate and e is the base of natural logarithms.

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[1] Joint United Arab Republic/WHO/UNICEF Bilharziasis Control Pilot Project and Training Centre.

TABLE 2

ECOLOGICAL LIFE-TABLES FOR *SCHISTOSOMA JAPONICUM* IN THE SNAIL HOST[a]

Age of infection in days (x)	Survival of infected female snails (l_x)	Cercariae produced per infected snail (m_x)	l_xm_x (females)	Survival of male snails (l_x)	l_xm_x (males)
0	1.000	0		1.000	
5	0.907	0		0.875	
15	0.746	0		0.668	
25	0.614	0		0.510	
35	0.505	0		0.390	
45	0.416	12.98	5.400	0.298	3.867
55	0.342	55.50	18.981	0.228	12.654
65	0.282	54.09	15.254	0.174	9.412
75	0.232	37.84	8.779	0.133	5.033
85	0.191	30.77	5.877	0.102	3.139
95	0.157	15.80	2.481	0.078	1.232
105	0.129	11.34	1.463	0.059	0.669
115	0.106	22.27	2.361	0.045	1.002
125	0.087	7.43	0.646	0.035	0.260
135	0.072	3.77	0.271	0.026	0.098
145	0.059	1.27	0.075	0.020	0.025
155	0.049	3.80	0.186	0.015	0.057
165	0.040	9.39	0.376	0.012	0.113
175	0.033	17.29	0.571	0.009	0.156
185	0.027	15.93	0.430	0.006	0.095
195	0.022	7.74	0.170	0.004	0.031
205	0.018	0	0	0.002	0
215	0.015	0	0	0.002	0
225	0.012	0	0	0.001	0
235	0.010	0	0	0.001	0
			$\Sigma = 63.321$		$\Sigma = 37.843$

[a] Differential survival of male and female snails requires separate life-tables for the parasite.

For infected *Bulinus truncatus*, the estimates range from 0.0340 to 0.0676, with a mean of 0.0548; for infected *Biomphalaria alexandrina*, the range is from 0.0327 to 0.077, with a mean of 0.0563. These figures are somewhat higher than the estimated death-rates of field snails generally (unpublished Egypt-49 project quarterly report), as might be expected.

If the death-rates are constant, the mean length of life can be calculated as:

$$\frac{1}{1 - e^{-a}}$$

This yields values of 18.82 days for infected *Bulinus truncatus* and 18.27 days for infected *Biomphalaria alexandrina*. Thus, the expected average output of

S. *haematobium* cercariae would be 18.82 × 800 or 15 056 per snail after the start of cercarial shedding. For *S. mansoni* cercariae, the average output of cercariae per snail would be 18.27 × 3500 or 63 945. These figures, however, do not take into account the death of infected snails before the infection becomes mature. If the estimated death-rates hold from the time the miracidium enters the snail, then only 0.147 of infected *Bulinus truncatus* will live to produce cercariae, and the average number of progeny of a successful *S. haematobium* miracidium would be only 2213. For *Biomphalaria* infected with *S. mansoni*, only 0.139 would survive the 35 days of incubation, and the average progeny of a successful miracidium would be 8888. Of course, the death-rate during incubation may not be as high as it is afterwards, but it would hardly be lower than that for snails in the field generally. If there is no difference between the death-rates of uninfected snails and infected snails during incubation, the net reproductive rate for *S. haematobium* in the snail would be 3163, and that of *S. mansoni* 11 252.

The probability of survival of cercariae

> " The difficulty is that the mosquito and the plasmodium do not seem to have any random habits. They do not behave like the black and white balls in the statistician's bag. They are like the little boy in Arkansas who would ruther do what he'd ruther."

The number of cercariae present. If it were possible to select a restricted area and estimate the number of cercariae released into the water daily, and to estimate the number of flukes that are acquired daily by the definitive hosts, it would be a simple matter to calculate the survival of the parasite during this phase of its existence. Both estimates are difficult to obtain. Accurate quantitative samples of the snail populations in all habitats within the area are necessary before snail infection rates and the daily emission of cercariae are at all helpful. There has been a marked reluctance among bilharziasis workers to expend the effort necessary to obtain quantitative samples of the snail populations, and even when this is done, the habitats sampled are nearly always widely separated and hence of relatively little value in yielding the kinds of data that are sought for the present analysis. The best data are from the village of Malirong, Palo, Leyte, (Hairston, 1962). Over a two-year period, the average number of infected snails in the area was 280 309 females and 157 674 males. Since the average daily output of cercariae was 2.214 for females, and 2.261 for males, it can be estimated that there were 977 105 cercariae of *S. japonicum* in the waters of this village on the average day, or 3.57 × 10^8 per year.

For *S. haematobium* and *S. mansoni*, truly comparable data do not exist, but if we are willing to extrapolate sufficiently, the necessary calculations can be made. The Egypt-49 project data for the Kom Ishu and Kom El Birka sections indicate that at the end of May 1963, 208.980 km of canals and drains were infested with *Bulinus truncatus* and 321.015 km were infested with *Biomphalaria alexandrina*. Quantitative sampling was done in some of these streams, and if the samples come from representative snail habitats, the total number of snails can be estimated, since 360 samples were taken, and each sample represents 1/8 linear metre of watercourse. The mean density of *Bulinus truncatus* was found to be 8.444 per metre, and of *Biomphalaria alexandrina*, 27.622 per metre. The latter figure has very wide confidence limits because the snail occurs in high densities in restricted localities. Going ahead with the calculations, one can estimate that in these two sections there were, at the end of May, 1.76 × 10^6 *Bulinus truncatus* and 8.87 × 10^6 *Biomphalaria alexandria* present. Infection rates were 0.00156 and 0.00655 respectively, and hence it can be estimated that there were 2.75 × 10^3 infected *Bulinus truncatus* and 5.8 × 10^4 infected *Biomphalaria alexandrina* in the two divisions at the time of the survey. At the quoted rates of cercarial production, there would be 800 × 2.75 × 10^3 or 2.20 × 10^6 *S. haematobium* cercariae and 3.5 × 10^3 × 5.8 × 10^4 or 2.03 × 10^8 *S. mansoni* cercariae in the waters of Kom Ishu and Kom El Birka each day.

These estimates are for the period of maximum snail abundance, and near the maximum snail infection rates. Thus, from the annual cycle of snail abundance and infection, it can be calculated that the production during the 18.82 days during which one could expect these numbers of cercariae to be present represents 0.19 of all the cercarial production of the six months of most intense transmission, and 0.179 of the cercariae produced during the entire year. This means that the total annual production of cercariae of *S. haematobium* is 2.31 × 10^8. Similar considerations lead to the calculation that there are 1.69 × 10^{10} cercariae of *S. mansoni* shed annually into the waters of Kom Ishu and Kom

El Birka. It should be pointed out that these figures are far less reliable than the comparable figures for *S. japonicum*, primarily because the sampling data are much less complete, but also because of the instability of the snail populations.

The number of cercariae that are successful. The number of cercariae that are able to infect the definitive host must be estimated by indirect methods, at least as far as the human host is concerned. For the non-human hosts of *S. japonicum* the method can be somewhat more direct, since the numbers of worms present can be counted in a sample of the population. If estimates can be made of the average age of these animals, the rate of acquisition of flukes can be determined (Hairston, 1962). Using the death-rate estimated by Spencer & Davis (1950) for rats in Hawaii, it was calculated that the mean length of life was 60.6 days. Since the average worm load was 20.43 per rat, the daily acquisition could be estimated as 0.337 cercariae per rat. These were field rats (probably *Rattus rattus mindanensis*) and only a crude guess was possible as to the population but the final estimate was of a population of 3075 in the vicinity of Malirong. Thus, there would have been 1036 successful cercariae per day, or a success rate of 1.04×10^{-3} for parasites entering rats. The probability of success in infecting dogs (1.65×10^{-6}) or pigs (9.82×10^{-6}) was much lower.

The only source of information on the success of cercariae in infecting humans is the rate at which previously negative persons become positive. This may be done directly or indirectly. Direct observations have been made in Leyte, Philippines (Pesigan et al., 1958b) and in Egypt (unpublished). In the coastal and inland areas of Leyte, the annual rates were 0.11 and 0.54 respectively. For the Kom Ishu and Kom El Birka sections of the Egypt-49 project area the rates vary with age; for the first five years of life, the annual incidence is 0.155 for *S. haematobium* and 0.04 for *S. mansoni*, and for children aged 5-7 years, the rates are 0.333 for *S. haematobium* and 0.135 for *S. mansoni*. These rates may be estimated in an independent manner from the age-prevalence curves, using the two-stage catalytic model of Muench (1959). For the Egyptian data, the agreement between the two approaches is excellent as regards *S. haematobium* and satisfactory for *S. mansoni* (Hairston, 1965).

From the proportion of people becoming positive each year, an estimate of the average number of worms acquired per person can be made, if we can permit the assumption that worms are distributed randomly among the population. This assumption is unlikely to be correct, but minor perturbations of randomness do not have a serious influence on the results of the calculation, and major possible sources of non-randomness have either been eliminated, as in the stratification between urban and rural areas, or shown to be unimportant, as in differences between the sexes.

The rationale of the approach is not difficult to follow. If we let p be the proportion of people acquiring female worms and p' be the proportion acquiring male worms, and q and q' be the proportions not acquiring females and males respectively, then:

$$\begin{aligned} &(p + q) = 1 \\ &(p' + q') = 1 \\ &(p + q)(p' + q') = 1 \\ &pp' + pq' + p'q + qq' = 1 \end{aligned} \tag{2}$$

In the final equation, pp' is the proportion acquiring both sexes and hence is the proportion positive. In the Philippines the sex ratio was 1:1 among worms recovered from mice exposed to natural waters, and thus p is equal to p'. Hence, $pp' = p^2$ and the proportion of the people acquiring female worms is the square root of the proportion becoming positive. There is some evidence that the sex ratio is not equal among the cercariae of *S. mansoni*, the recoveries from field-infected mice being 189 males and 164 females. Even with this difference, the calculations are not seriously upset, as shown in the following calculations

For people older than 5 years, the annual incidence is 0.135.

If $p = p', p^2 = 0.135$,
$p = 0.367$ (fraction with females),
$q = 0.633$ (fraction without females).

Under the assumption of a random distribution of worms among people,

$$q = e^{-m}$$

where m is the mean number of worms per person and e is the base of natural logarithms.

Solving, $m = 0.455$ females acquired per person per year and there are $2m$ or 0.91 worms acquired per person per year.

If we use the observed, $p' = 1.15\,p$,

$1.15p \times p = 0.135$

$$p^2 = \frac{0.135}{1.15} = 0.1174$$

$$p = 0.343$$
$$p' = 0.394$$
$$q = 0.657$$
$$q' = 0.606$$

$m = 0.42$

$m' = 0.50$

sum = 0.92 worms acquired per person per year

The difference between the two estimates is thus negligible.

Completing the analysis for *S. mansoni*, the annual rate of becoming positive is 0.04 for children under 5 years of age. Using the same mathematical manipulations, it is estimated that the average acquisition of worms is 0.44 per year for each child in this age-group.

For persons older than 5 years, a correction is necessary because some of the negatives at the start of the observation were probably carrying unmated female worms, and this fraction should not be counted in the annual rate of acquisition of worms. Children in the age-groups 4, 5 and 6 years showed a prevalence of 0.117, indicating that a total of 0.342 were carrying female worms. Thus, 0.342-0.117 or 0.225 of them were carrying unmated females. This represents $\frac{0.225}{1 - 0.117}$ or 0.255 of the apparent negatives. In the next year, 0.394 of these would have been positive. Since 0.394 × 0.255 equals 0.10 of the persons acquiring worms, the estimated 0.92 worms per year must be reduced by this fraction, leaving a corrected estimate of 0.828 worms acquired per year per person over 5 years old.

The age structure of the population indicates that 0.165 are under 5 years old. This amounts to 2540 of the 15 396 people in Kom Ishu and Kom El Birka. Hence, 0.44 × 2540 or 1118 flukes are acquired per this age-group. The rest of the population, 12 856, are estimated to acquire 12 856 × 0.828 or 10 645 worms per year. The total successful *S. mansoni* cercariae, then, is 11 763 out of the 1.69×10^{10} cercariae that are present. The probability of success of *S. mansoni* cercariae, then, is

$$\frac{1.18 \times 10^{4}}{1.69 \times 10^{10}} \text{ or } 6.98 \times 10^{-7}.$$

For *S. haematobium*, no data are available on the sex ratio of cercariae in the field, and the assumption is made that the sexes are equally abundant. For children less than 5 years old, the rate of becoming positive is 0.155 per year. This would mean that 0.394 acquire female flukes, for an average per person of 0.5 females per year. Thus, the 2540 children acquire an average of one fluke per person per year. The remainder of the population becomes positive at a rate of 0.333 per year, which means that they acquire, on the average, 1.46 flukes each, after correcting for the unmated females that contributed to the rate of acquisition. Thus, there are 1 × 2540 plus 1.46 × 12 856 or 21 310 successful cercariae of *Schistosoma haematobium* in Kom Ishu and Kom El Birka each year. This represents a probability of success of

$$\frac{2.13 \times 10^{4}}{2.31 \times 10^{8}} \text{ or } 9.22 \times 10^{-5}.$$

S. japonicum has the possibility of infecting almost any mammal. The author has calculated that in Malirong four species of mammals contribute significantly to its maintenance. The probabilities of success of cercariae in infecting non-human mammals have been given above. In the case of man, the method of calculation is very similar to that for *S. haematobium*. It is estimated that an average of 4.2 cercariae succeed in infecting people in Malirong daily. Hence, the probability of success for cercariae is

$$\frac{4.2}{9.7 \times 10^{5}} \text{ or } 4.3 \times 10^{-6}.$$

This may be compared with 1.04×10^{-3} for infecting rats, 1.69×10^{-6} for infecting dogs, and 9.82×10^{-6} for infecting pigs.

Net reproductive rate in the definitive host

> " That this [reproduction] is not impossible can easily be demonstrated by a simple mathematical calculation in geometrical progression, the one so often used in poultry catalogues to show how prodigiously living things multiply if given a free rein . . . The trouble is, no such numbers can be demonstrated . . ."

The only direct observations available for calculating reproduction in the human host are those of the daily output of eggs in faeces or urine. Before these data can be used in the construction of an ecological life-table, however, it is necessary to know how many female flukes are present and their mortality rate. Essentially, the mortality rate is the only unknown, since the rate of acquisition of female flukes has already been estimated above. Several cases have been reported in the literature of people with bilharziasis passing viable eggs 20 years or more after leaving an endemic area, but these can scarcely represent the average conditions, since all three species show a marked decline in prevalence after a peak between the ages of 12 and 30 years. This decline, which is particularly marked in *S. haematobium*, must mean that a large fraction of the female worms die or cease laying eggs after a much shorter time than 20 years. The alternative explanation, that the flukes are alive and laying eggs which never reach the urine or faeces, can be treated as effective death, since such eggs would contribute nothing to the life-table.

In principle, the death-rate of flukes should be obtainable from experimental infections terminated at variable times. In practice, this method yields erratic results which usually indicate excessive death-rates, probably because excessive numbers of cercariae are used initially. Also, this practice results in low recoveries. In one case where this practice was not followed and only one to three pairs of cercariae were used per mouse, recovery amounted to 72.6% (Pesigan et al., 1958a, p. 447).

Two methods remain, one of which has been used by the author (Hairston, 1962). This derives from changes in the sex ratio and is based upon the biology of the parasites. It is estimated that the female moves into the venules of the submucosa 12 times daily to lay eggs (Pesigan et al., 1958a). This results in stretching and irritating the surrounding tissues and she runs the risk of being injured in the process. The male runs no such risk, and it is not surprising that necropsies nearly always reveal an excess of males. If males have a negligible death-rate, the sex ratio of flukes in infections of known age should give the death-rate of females. These ratios were 1.043 males per female in rats with an average life of 60.6 days, and 1.718 males per female in dogs estimated to have been infected for two years. In these two cases, the calculated death-rates of females agreed very well: 0.253 per year for the rats and 0.27 per year for the dogs. These rates, of course, apply specifically only to *S. japonicum.*

A second method of estimating the death-rate of females comes from the use of the calculated rate of loss of active infections, which can be done from sufficiently detailed age-prevalence data (Muench, 1959). In situations or among age-groups where the mean number of female worms per active case is very low, approaching 1.0, the rate of loss of infections will be a minimal estimate of the death-rate of female worms. Where the mean number of worms per active case is higher, the rate of loss of infections will be a power of the death-rate of females. For example, if the death-rate of females is 0.5 per year, this can be considered a probability and if two worms are present, the probability of loss of infection will be 0.5 for each, or 0.25. The maximum calculated loss of infection, then, will approach the death-rate of female worms. For a detailed consideration of the use of Muench's catalytic models in the epidemiology of bilharziasis, the reader is referred to the original book and to the present author's communication on the subject (Hairston, 1965). It will suffice here to state that for *S. haematobium*, the calculated rate of loss of infections amounts to 0.30 per year for children under 5 years of age and 0.09 per year for older persons. Thus, the minimal death-rate of female flukes can be stated as 0.3 per year. *S. mansoni* analysed in the same way give loss rates of infections ranging from 0.10 to 0.336.

The death-rates of female flukes must be combined with host death-rates in order to obtain entries for the l_x column. Entries in the m_x column have been shown to vary, in the case of *S. japonicum*, with the mean density of female flukes, and also with the age of the human host at the time the parasite enters. At least two factors appear to affect the output of eggs in faeces: the physiological state of the host, and previous experience with the parasite. Children 10-15 years of age give much higher egg-counts than any other group, and the increase per female worm present may be associated with the onset of puberty. In older people, immunity and/or fibrosis reduces the output of eggs per female. It is difficult to distinguish these factors from a crowding effect, but the problem does not affect the arithmetic, since a decrease in egg output represents the same loss to the parasite population, regardless of whether the cercaria was successful in reaching maturity or died in the skin.

Detailed counts of the number of eggs passed in the urine or faeces daily are available only for *S. japonicum.* For Malirong (in the inland division) and the coastal division of Palo, Philippines, these counts are given for people in successive age-groups in Table 3. Since the number of flukes acquired per year has been estimated, as well as the death-rate of females, it is possible to estimate the number of worms likely to be present in the average person of any given age. This is calculated in a fairly simple but somewhat tedious manner. Under the assumptions of random distribution of flukes (of each sex) among

TABLE 3

METHOD OF CALCULATING EFFECTIVE EGG OUTPUT PER FEMALE WORM PER DAY FOR HUMAN HOSTS IN MALIRONG AND COASTAL DIVISION OF PALO, LEYTE, PHILIPPINES

Age-group of host (years)	Total female eggs in faeces per day	Proportion hatchable	Hatchable female eggs per day	Estimated average number of female flukes per stool-positive person	Hatchable female eggs per female fluke per day
			Malirong		
5-9	742	0.400	297	3.051	97
10-14	2 357	0.484	1 141	4 617	247
15-19	578	0.367	212	5.054	42
20-29	525	0.367	193	5.200	37
≥30	297	0.400	119	5.200	23
			Coastal division		
5-9	459	0.400	184	1.379	133
10-14	1 965	0.484	951	1.998	476
15-19	578	0.367	212	2.150	63
20-29	525	0.367	193	2.163	56
≥30	297	0.400	119	2.163	34

people, and the likelihood that it is only the mated females that have the measurable death-rate of 0.25 per year, the proportion of females that are mated can be calculated, their death-rate taken into account, and the average number of female worms per stool-positive person estimated.

An example of the method will be given. It has been noted that the rate of acquisition of flukes in Malirong has been estimated as 2.3 per person per year, or 1.15 of each sex. A downward correction for children below 5 years was made, indicating that those between 3 and 5 years old acquire 0.63 worm of each sex annually. Children 4 years old, then, would have 1.26 worms. If worms of each sex are distributed randomly among people then the proportion of female worms that are mated can be calculated as $1-e^{-m}$, where m is the mean number of male worms per person. In the present case, m is 0.63 and hence 0.467 of the female worms will be mated, and the mean number of mated female worms in people 4 years old is 0.294 per person, including, of course, those having no female worms at all and those having unmated females. The fraction of persons with one or more worms among a population having a mean load of 0.294 can be calculated as $1-e^{-0.294}$. Thus, 0.2543 of the children of this age have all of the mated females. Hence, the average number of mated females per stool-positive individuals is

$$\frac{0.294}{0.2543} \text{ or } 1.156.$$

During the next year, 0.63 female per person will be added, but there will be a death-rate among the 0.294 female per person, amounting to e^{-a}, where a is the instantaneous death-rate per year, estimated as 0.25. Hence, 0.2211 of the 0.294, or an average of 0.065 per person, will die during the year. This must be subtracted from the $0.63+0.63$ or 1.26 that has been acquired during the two years from 3 to 5 years of age. This leaves an average of 1.155 females per person, while the male flukes would simply accumulate to 1.26 per person. In this year, $1-e^{-1.26}$ or 0.716 of the females would be mated, for a mean of 0.856 per person. The mean number of female worms per stool-positive person would be

$$\frac{0.865}{1-e^{-0.856}} \text{ or } 1.484.$$

During the next year, the rate of addition of worms was estimated to increase to 1.15 of each sex per person. The calculations carried out in the same way indicate that 6-year-olds in Malirong have an average of 2.284 female flukes per stool-positive individual. Under the assumptions already given, virtually all female worms are mated in persons over 10 years of age since the population of male flukes has increased steadily to a mean of 8.16 per person, leaving less than 0.001 of these children with no males.

The death-rate of females will eventually balance the number acquired, and in persons over 20 years of age, the mean number of females present remains constant at 5.2 per person.

Table 4 gives the estimated mean number of female worms per stool-positive person at each age from 4 years up for the coastal and inland divisions of Palo. The faecal egg-counts are averages for five-year groups and it is necessary to obtain the average density of egg-laying female worms per person over these age-groups. From these figures, the number of eggs escaping in the faeces per female fluke can be estimated.

TABLE 4

THE ESTIMATED AVERAGE NUMBER OF FEMALE *S. JAPONICUM* PER STOOL-POSITIVE PERSON OF EACH AGE IN TWO DIVISIONS OF PALO

Age (years)	Average number of female flukes per positive person	
	Inland	Coastal
4	1.156	—
5	1.484	1.005
6	2.284	1.094
7	2.973	1.265
8	3.493	1.463
9	3.883	1.647
10	4.183	1.800
11	4.417	1.916
12	4.597	2.000
13	4.734	2.058
14	4.842	2.098
15	4.926	2.123
16	4.996	2.143
17	5.050	2.150
18	5.088	2.156
19	5.120	2.162
20	5.145	2.163
21	5.165	2.163
22	5.180	2.163
23	5.192	2.163
24	5.200	2.163
25	5.200	2.163
26	—	2.163

The relevant figures, however, include only the female eggs, assumed to be one-half of the total. Pesigan et al (1958a) have also shown that less than half of the eggs passed in human faeces hatch. The proportions vary with age, as shown in Table 3, which gives estimates for the hatchable female eggs per female fluke per day for the significant parts of the human population in Malirong and the coastal division.

From these figures, it is clear that some factor prevents a simple addition of expected eggs for each female fluke added to the host, since the average number of eggs produced per infected person is not proportional to the average number of female worms estimated to be present. The most likely reasons for this are the immunity produced by previous exposure to the parasites and fibrosis of the gut wall caused by egg depositions of female worms arriving early in life. Whatever the cause, the effect is as though the worms, in some way, responded negatively to each other's presence, as far as effective reproduction is concerned. This factor must, of course, be taken into account in calculating the life-table of flukes in humans. Inasmuch as rate functions are involved, the relationships are best expressed in logarithms. The data in Table 3 lead to the following equations:

For 5- 9-year-olds: $\log h = 2.18 - 0.40 \log f$
For 10-14-year-olds: $\log h = 2.90 - 0.78 \log f$
For 15-19-year-olds: $\log h = 1.96 - 0.474 \log f$
For 20-29-year-olds: $\log h = 1.91 - 0.474 \log f$
For 30 years and over: $\log h = 1.68 - 0.474 \log f$

where h is hatchable female eggs per female per day, and f is the mean number of female worms per positive person.

This represents a correction of Hairston (1962), in which the number of flukes acquired annually in the coastal zone was incorrectly stated to be 0.4 per person. This is the rate of acquisition of female flukes.

A second important feature of the data is the much larger average reproductive success of female flukes in persons 10-15 years old—larger than flukes in persons either older or younger. This can only be explained as some kind of response connected with the physiological conditions of the human host at this age and the onset of puberty suggests itself as the most likely explanation.

In calculating the ecological life-table for flukes in people, it is necessary to take into account three factors: the decrease in effective egg production with increased numbers of female worms; the change in effective egg production with age of host; and the increase in proportion of females mated with increased infection rates.

As an example of the exact method of calculating the ecological life-table of *S. japonicum* in man, female worms entering 5-year-old children in Malirong will be used. The l_x column is calculated as the combination of survival of female flukes and the survival of people. As noted above, if the death-rate is known, the proportion surviving to any age can be calculated. Taking the mid-points of successive years (x) from 5 onwards, the survival of females will be $l_{0.5x} = e^{-0.5a}$, $l_{1.5x} = e^{-1.5a}$, etc., where a is the estimated death-rate of 0.25 per year. In Table 5, the successive values in the column headed "l_x flukes" were calculated in this way. The crude death-rate of people in Leyte is given by Pesigan et al. (1958a) as 0.012. This figure has been used to construct a column headed " l_x people ". The effective survival of the parasite to any given age is the product of the survival of the parasite and that of the host. The column headed " l_x, flukes in people " is thus the column used in calculating the net reproductive rate. In order to obtain m_x entries, the equations given above for successive age-groups are used to obtain the number of hatchable female eggs per female fluke per day. The number of females per stool-positive individual is obtained by interpolating between the entries in Table 4 for people 5.5, 6.5 years old, etc. When the equation is solved for any age, the solution must be multiplied by 365 to obtain m_x, the number of hatchable eggs passed per female per year. The net reproductive rate is the sum of the products of l_x and m_x entries, 174 504. This calculation assumes that all female worms are mated, which is untrue, especially when small mean numbers are present. The calculation of the success of cercariae, however, included all worms infecting humans. In order to make the net reproductive rate usable in equation (1), the unmated female worms must be taken into account. This is done by multiplying the $l_x m_x$ entries in Table 5 by the proportion of females mated, as has been done in the last two columns of the table. This reduces the net reproductive rate to 165 535. The reduction caused by unmated worms is much greater for areas where the rate of acquisition of parasites is lower than in Malirong.

A detailed examination of Table 5 shows that the net reproductive rate depends upon the age of the human host when the parasite enters. This is due to the great difference in egg-passing by people of different ages. The details of calculation will not be repeated, but the results at 5-year intervals are given in Table 6. Because of the variation in both net reproductive rate and worm load with age of host, the net reproduction must be apportioned among the worms in the population. Table 6 gives the method. The average fluke in Malirong thus has a net reproductive rate of 90 471 while in the human host. The same series of calculations leads to the estimated net reproductive rate of 147 421 for worms in people in the coastal area of Leyte. This is much higher than in any of the other three mammal hosts. Calculations of these terms is relatively simple, since no effect of the age of the host is known. The net reproductive rates, as calculated by the author (Hairston, 1962), are: in rats, 63; in dogs, 5015; in pigs, 640.

No such data on daily passage of eggs in stools or urine have been recorded for *S. haematobium* or *S. mansoni*, although studies are being carried out both at the Ross Institute Bilharziasis Research Unit at Mwanza, Tanzania, and at the joint United Arab Republic/WHO/UNICEF project near Alexandria, Egypt.

From the meagre data available, some calculations may be made, if sufficient assumptions are permitted. The first derives from what is known of the actual egg-laying rates of *S. mansoni* and *S. japonicum*. Moore & Sandground (1956) estimated that the actual number of eggs produced daily by a female *S. mansoni* was 1/10 of the number produced by *S. japonicum*. Since Pesigan et al. (1958a) determined experimentally that *S. japonicum* produced 140 eggs per day, this would mean 140 eggs laid daily by *S. mansoni*. The average length of life of *S. japonicum* is estimated as $\frac{1}{1 - e^{-a}}$ or 4.52 years, where a is the exponential rate of death per year. This would mean that the average female *S. japonicum* would lay 2.31×10^6 eggs in its life. If half of these are female, and 0.42 are potentially hatchable (Pesigan et al., 1958a, p. 445), then the potential net reproductive rate would be 4.85×10^5, counting eggs that remain in the body of the host. Based on

TABLE 5

ECOLOGICAL LIFE-TABLE FOR FEMALE FLUKES ENTERING THE HUMAN HOST WHEN THE LATTER IS 5 YEARS OLD, BASED ON DATA FROM MALIRONG

Age of flukes in years (x)	l_x for flukes	l_x for people	l_x, flukes in people	Hatchable female eggs per female fluke per day	m_x	l_xm_x	Proportion mated	l_xm_x for all female flukes entering people
0	1.000	1.000	1.000					
0.5	0.882	0.994	0.877	118	43 070	37 772	0.813	30 709
1.5	0.687	0.982	0.675	102	37 230	25 130	0.941	23 647
2.5	0.535	0.970	0.519	95	34 675	17 996	0.982	17 672
3.5	0.417	0.958	0.399	90	32 850	13 107	0.994	13 028
4.5	0.325	0.947	0.308	87	31 755	9 781	0.998	9 761
5.5	0.253	0.936	0.237	255	93 075	22 059	1.000	22 059
6.5	0.196	0.925	0.181	246	89 790	16 252	1.000	16 252
7.5	0.153	0.914	0.140	239	87 235	12 213	1.000	12 213,
8.5	0.119	0.902	0.107	234	85 410	9 139	1.000	9 139
9.5	0.093	0.891	0.083	231	84 315	6 998	1.000	6 998
10.5	0.072	0.881	0.063	43	15 695	989	1.000	989
11.5	0.056	0.871	0.048	42	15 330	736	1.000	736
12.5	0.044	0.861	0.038	42	15 330	583	1.000	583
13.5	0.034	0.850	0.029	42	15 330	445	1.000	445
14.5	0.027	0.840	0.023	42	15 330	353	1.000	353
15.5	0.020	0.830	0.017	37	13 505	230	1.000	230
16.5	0.016	0.820	0.013	37	13 505	176	1.000	176
17.5	0.013	0.811	0.011	37	13 505	149	1.000	149
18.5	0.010	0.801	0.008	37	13 505	108	1.000	108
19.5	0.007	0.791	0.006	37	13 505	81	1.000	81
20.5	0.006	0.781	0.005	37	13 505	68	1.000	68
21.5	0.005	0.772	0.004	37	13 505	54	1.000	54
22.5	0.003	0.763	0.002	37	13 505	27	1.000	27
23.5	0.003	0.754	0.002	37	13 505	27	1.000	27
24.5	0.002	0.745	0.001	37	13 505	13	1.000	13
25.5	0.002	0.736	0.001	22	8 030	8	1.000	8
26.5	0.001	0.727	0.001	22	8 030	8	1.000	8
27.5	0.001	0.719	0.001	22	8 030	8	1.000	8
28.5	0.001	0.710	0.001	22	8 030	8	1.000	8
						Σ=174 518		Σ=165 549

the eggs that appear in the faeces, the net reproductive rate of the average female *S. japonicum* in Malirong is 90 471, indicating that $\frac{9.05 \times 10^4}{4.85 \times 10^5}$, or 0.187, of the eggs escape the body. The effect of immunity is probably responsible for the low figure. In the coastal area, where transmission is lower, the fraction estimated to escape in the faeces is 0.303.

S. mansoni would lay 140 × 365, or 51 100 per year, and, applying the minimum estimated death-

TABLE 6

CALCULATION OF AVERAGE NET REPRODUCTIVE RATE OF FEMALE *S. JAPONICUM* IN MALIRONG

Age-group of host (years)	Net reproductive rate of flukes	Proportion of positive people in age-group	Relative contribution per female present in people of age-group	Average number of female flukes per positive person	Relative contribution of all worms in age-group	Relative number of worms in age-group
5-9	165 549	0.152	25 161	3.051	76 766	0.464
10-14	262 700	0.185	48 600	4.617	224 386	0.854
15-19	56 301	0.082	4 617	5.054	23 334	0.444
20-24	49 811	0.093	4 632	5.200	24 086	3.021
25-29	45 716	0.086	3 932	5.200	20 446	
≥30	30 474	0.402	12 251	5.200	63 705	
Total					432 723	4.783 [a]

[a] Average contribution per female fluke $= \frac{432\,723}{4.783} = 90\,471$.

rate, 0.336 per year, the total egg production would be 51 100 × 3.5, or 1.79×10^5. If the assumption is permitted that the proportion escaping from the body is the same as for *S. japonicum* in an area of comparable transmission rates (coastal division, Palo) each female *S. mansoni* would produce 54 213 eggs that would appear in the faeces of its host. Applying the sex ratio (1 : 1) and hatchability (0.424) figures from *S. japonicum* data, this would give a net reproductive rate of 11 493 for female *S. mansoni* in the human host.

For *S. haematobium*, no such conversion factors are available. Data on egg counts that have been reported are either for a restricted age-group or for persons otherwise selected. If the Egypt-49 report of 23.17 eggs per 150 ml of urine is used as an average figure, it can be estimated that for an average output of 1.4 litres of urine per day there would be 248 eggs passed per person. Although *S. haematobium* eggs show a very high peak count in early afternoon, the figures cited by Bradley (1963) indicate that approximately the same fraction of eggs is passed between 8 a.m. and noon (when the urines were mostly collected) as the fraction of urine passed during the same period. Thus, no correction of the estimate of 248 eggs is necessary on account of the non-uniform rate of egg passage. It is likely, however, that it is a minimal figure, since the report implies that some specimens consist of less than 150 ml and that no correction is made for this.

It is possible to calculate the average number of female flukes per positive person, since data on the number acquired annually and the death-rate are both known (see pages 52 and 53 above). This is done in Table 7. Using unpublished data from the Egypt-49 quarterly report for April 1963 to apportion positive cases among age-groups, it is estimated that the average worm load is 2.757 females per person. This would mean that for each female present, 90 eggs are passed in the urine daily. From the death-rate of 0.30 per year, the average length of life is estimated as 3.84 years, which would mean that each female is responsible for 126 144 eggs in the urine of its host. This would give a production of 63 072 female eggs. There are no hatchability data for *S. haematobium* eggs, but it is certainly less than 100%. If it is the same as for *S. japonicum* the net reproductive rate is 26 743.

The probability of survival of miracidia

> ". . . the number of carriers and the number of vectors are not unrelated variables, but the one depends on the other, so that if the vectors decrease, the chances that they will meet carriers go down even more rapidly."

The principles used in calculating the numbers of eggs likely to survive after leaving the body are the same as those used for the numbers of cercariae likely to survive. It is necessary to know the number

TABLE 7

CALCULATION OF AVERAGE NUMBER OF *S. HAEMATOBIUM* PER PERSON AND PER POSITIVE PERSON IN THE CONTROL DIVISION OF THE EGYPT-49 PROJECT AREA[a]

Age of host (years)	Average number of flukes per person		Proportion of female worms that are mated	Average mated females per person	Average female worms per positive person
	Male worms	Female worms			
1	0.50	0.50	0.394	0.197	1.107
2	1.00	0.95	0.632	0.600	1.330
3	1.50	1.29	0.777	1.002	1.583
4	2.00	1.53	0.865	1.323	1.802
5	2.50	1.69	0.918	1.551	1.967
6	3.23	2.02	0.961	1.941	2.239
7	3.96	2.25	0.981	2.207	2.480
8	4.69	2.41	0.991	2.388	2.630
9	5.42	2.52	0.996	2.510	2.731
10	6.15	2.60	0.998	2.595	2.805
11	6.88	2.66	0.999	2.657	2.856
12	7.61	2.70	1.000	2.700	2.894
13	8.34	2.73	1.000	2.730	2.920
14	9.07	2.75	1.000	2.750	2.938
15	9.80	2.76	1.000	2.760	2.946
16	10.53	2.77	1.000	2.770	2.956
17	11.26	2.78	1.000	2.780	2.964
18	11.99	2.79	1.000	2.790	2.971
19	12.72	2.79	1.000	2.790	2.971
≥20	13.45	2.79	1.000	2.790	2.971

[a] Calculations based on rates of becoming positive of 0.17 per year below 5 years of age and 0.333 per year above 5. Death-rate of mated females estimated at 0.30 per year.

of eggs passed per unit of time within a defined area, and the number of snails becoming infected in the same area in the same unit of time.

For *S. japonicum* in Malirong, the egg output by people has been estimated by Hairston (1962) as 327 273 hatchable eggs per day. In arriving at this figure, the different rates of egg passage by different groups (see Table 3) and the number of stool-positive persons in each age-group have been used. The total population of the village was 768. For this species, other mammal hosts are important in maintaining the parasite. Dogs and pigs contributed 55 992 and 12 240 hatchable eggs respectively each day. These estimates were easily obtained, but a comparable one for rats proved very difficult, both because data were lacking on absolute abundance of rats and because of inconsistencies in egg-counts. A final estimate was made from laboratory infections in native rats and from what was no more than a guess as to their real abundance. Using a figure of 436 896 hatchable eggs passed daily by rats, the total hatchable eggs available for infecting snails amounted to 832 401 per day. As has been stated above, the area supported an average standing crop of 280 309 demonstrably positive females and 157 674 demonstrably positive male snails. With mean lengths of life of 51.3 days and 37.6 days respectively, the number becoming positive daily was 5465 females and 4153 males. The time from becoming infected to becoming demonstrably positive was estimated

TABLE 8

CALCULATION OF AVERAGE NUMBER OF *S. MANSONI* PER PERSON AND PER POSITIVE PERSON IN THE CONTROL DIVISION OF THE EGYPT-49 PROJECT AREA[a]

Age of host (years)	Average number of flukes per person		Proportion of female worms that are mated	Average mated females per person	Average female worms per positive person
	Male worms	Female worms			
1	0.22	0.22	0.197	0.043	1.012
2	0.44	0.43	0.356	0.153	1.077
3	0.66	0.63	0.483	0.304	1.160
4	0.88	0.76	0.585	0.445	1.240
5	1.10	0.85	0.667	0.567	1.309
6	1.51	1.10	0.779	0.857	1.488
7	1.92	1.27	0.853	1.083	1.638
8	2.33	1.37	0.903	1.237	1.742
9	2.74	1.43	0.935	1.337	1.814
10	3.15	1.46	0.957	1.383	1.846
11	3.56	1.48	0.972	1.439	1.886
12	3.97	1.48	0.981	1.452	1.896
13	4.38	1.48	0.987	1.461	1.902
14	4.79	1.47	0.992	1.458	1.902
≥15	5.20	1.46	0.996	1.460	1.902

[a] Calculations based on rates of becoming positive of 0.13 per year up to 7.5 years of age and 0.17 per year thereafter. Death-rate of mated females estimated as 0.336 per year.

as 30 days, by which time only 0.557 of the infected females and 0.446 of the infected males were left alive. Thus, it was estimated that 9810 females and 9401 males became infected daily, for a total of 19 211. The probability that a hatchable egg would succeed in infecting a snail was thus

$$\frac{19\,211}{832\,401} \text{ or } 0.0231.$$

The calculations for *S. mansoni* start from the net reproductive rate in humans, estimated above as 11 493. This would mean that twice as many hatchable eggs are produced per female, since only female eggs are involved in the net reproductive rate. The average female, then, would be expected to place 22 986 hatchable eggs into the faeces of its host during the 3.5 years or 1278 days of its existence. Calculations based upon the rate of acquisition of worms and their death-rate indicate an average of 1.81 female worms per stool-positive individual (Table 8). The average positive individual, then, would be expected to pass 32.55 hatchable eggs per day. At the stated prevalence of 40.2%, there would be 6189 egg-passing people in Kom Ishu and Kom El Birka (in the Egypt-49 project area), who would pass a total of 2.01×10^5 hatchable eggs daily.

It has been estimated above that at the end of May, there were 5.8×10^4 demonstrably positive *Biomphalaria alexandrina* present. From repeated samplings of the snail population, it can be estimated that this represents 0.22 of the annual number of positive snails. Thus, there would be 2.64×10^5 becoming positive each year. The death-rates of infected snails indicate that only 0.139 of those infected live long enough to become demonstrably positive. From this it is concluded that 1.9×10^6 become infected each year for a daily average of 5.2×10^3. The probability of success of a hatchable egg of *S. mansoni*, then, would be

$$\frac{5.2 \times 10^3}{2.0 \times 10^5} \text{ or } 2.59 \times 10^{-2}.$$

Applying the same kind of calculations to *S. haematobium*, it is estimated that each female parasite is responsible for 53 486 hatchable eggs in the urine of the host. This represents the reproduction during 3.86 years, or 1408 days. Thus, each female worm places 38 hatchable eggs per day into the urine of the host, which from Table 7 contains 2.757 worms on the average. This would mean that the average positive individual passes 105 hatchable eggs per day. With a prevalence of 35.4%, there are 5450 positive people in Kom Ishu and Kom El Birka, and they pass 5.71×10^5 hatchable eggs daily.

It has been estimated in a foregoing section that there are 2.75×10^3 positive *Bulinus truncatus* in the area at the end of May. Since this has been estimated to represent 0.179 of the annual infections, there would be 1.54×10^4 snails becoming positive annually. Of the snails that become infected, only 0.147 survive to become demonstrably positive. This means that 1.05×10^5 become infected each year, or 2.88×10^2 per day. This represents a success rate of

$$\frac{2.88 \times 10^2}{5.71 \times 10^5} \text{ or } 5.04 \times 10^{-4}$$

for hatchable eggs for *S. haematobium.*

Ecological life-tables for the schistosomes

It is now possible to solve equation (1) for all three species of parasites, since all of the terms have been obtained for each species. Under the initial assumption of a stable parasite population, the solutions should equal 1.0 in each case, and the degree of approximation to this value should reflect the amount and accuracy of what is known about the reproduction and survival of the species concerned.

The solutions are:

$$\left.\begin{array}{l}\text{net reproduc-}\\\text{tive rate in}\\\text{snail}\end{array}\right\} \times \left\{\begin{array}{l}\text{probability}\\\text{of infecting}\\\text{a mammal}\end{array}\right\} \times \left\{\begin{array}{l}\text{net reproduc-}\\\text{tive rate in}\\\text{mammal}\end{array}\right\} \times$$

$$\times \left\{\begin{array}{l}\text{probability}\\\text{of infecting}\\\text{a snail}\end{array}\right\} = \left\{\begin{array}{l}\text{net reproduc-}\\\text{tive rate of}\\\text{parasite.}\end{array}\right.$$

$$54.15 \times \Sigma \begin{pmatrix} 4.3 & \times & 10^{-6} & \times & 9.05 & \times & 10^4 \\ 1.04 & \times & 10^{-3} & \times & 63 & & \\ 1.69 & \times & 10^{-6} & \times & 5.02 & \times & 10^3 \\ 9.82 & \times & 10^{-6} & \times & 6.4 & \times & 10^2 \end{pmatrix} \times$$

$$\times\ 2.31 \times 10^{-2} = 0.588 \text{ for } \textit{S. japonicum.}$$

The summation is for the four significant mammal hosts.

It is done because the successful cercariae are apportioned among the hosts very differently, and the reproduction in the hosts is vastly different. It is quite possible that success is different for eggs from the different mammals, but since nothing is known of this, it is necessary to calculate a single term.

For *S. mansoni* the solution is:

$$8.89 \times 10^3 \times 6.98 \times 10^{-7} \times 1.15 \times 10^4 \times 2.59 \times 10^{-2} = 1.85$$

and for *S. haematobium*

$$2.21 \times 10^3 \times 9.77 \times 10^{-5} \times 2.67 \times 10^4 \times 5.04 \times 10^{-4} = 2.75$$

Thus, it is clear that for the two species about which the information is very fragmentary, the final solution of the equation is much less satisfactory than it is for *S. japonicum*, for which detailed data exist on all phases of the problem, except for the rat population and the reproduction of worms in the rats.

DISCUSSION

> " But the biometricians, besides establishing some essential rules of evidence, have told us one very important thing: that there is probably a critical *per capita* density of anopheline vectors below which malaria will tend to disappear altogether, instead of merely dropping to a lower level of intensity."

The principal accomplishment that can be claimed to date for this approach to the epidemiology of bilharziasis is in the detailed working out of the exact method of analysis. It is true that some of the assumptions involved might be improved. The most obvious of these is the assumption of a random distribution of worms among people—an assumption that is unlikely to be true. The only data bearing on this point are those of Pesigan et al. (1958a, p. 436), which give the distribution of egg-counts among children. These data suggest a slightly non-random distribution fitting a negative binomial with a value of about 4 for *k*. This constant varies from 0 at maximal clumping to infinity for a random distribution. The indicated value does not represent strong clumping, and such a distribution would not alter the calculations in any serious way.

Other parasites are much more strongly clumped; for example, Hairston & Jachowski (in preparation) estimate the distribution of adult female *Wuchereria bancrofti* among Samoans to fit a negative binomial with values of 0.4–0.7 for *k*.

It is concluded that the details of the method are reasonably satisfactory, and that what is primarily lacking is adequate data. For *S. japonicum*, the most important gaps in information are data on rat populations as explained above and by Hairston (1962). For the other two species, the only really satisfactory data are those concerning prevalence in the different ages of humans. Under these conditions detailed comparisons of the values obtained are not really justified.

A really good understanding of the population dynamics would require repetition of the studies in three or more areas with a wide variation in transmission rates• If the studies were complete enough, the equations should balance in all cases, and from them could be deduced the critical levels at which the parasites can no longer maintain themselves.

It seems probable that the critical level would be expressible as a function of any part of the ecological life-table, and hence as a function of any combination of values in the ecological life-table.

For example, if snail control were being attempted, and the critical level of transmission were known in terms of miracidial survival, it would be a simple matter to estimate the number of snails necessary to maintain this critical amount of success of miracidia. Any significantly lower number would then be known to represent satisfactory snail control, with the dynamics of the parasite population operating in favour of the control effort.

As pointed out in the introductory paragraphs, the aspect of population dynamics that is most likely to operate against the parasite at low rates of transmission is the decreasing probability that worms of opposite sexes will occur in the same host. These unmated worms are wasted, as far as the parasite population is concerned. Even though such worms have been shown experimentally to be capable of reproducing after long periods as single-sex infections, the time involved before they become mated is an important source of lost opportunity. The combinations must in principle become self-reinforcing if transmission rates are brought to a sufficiently low level and held there.

The desirability of being able to predict the degree of perfection to which control must be brought is obvious. It may be worth while to estimate the amount of effort that would be required to obtain the necessary data. The data needed have been listed by the author (Hairston, 1962) but will be repeated here for the sake of completeness:

Vertebrate host(s)

Population of each species in the area
Mortality rate of each species
Population fluctuations of each species

Parasites in vertebrates

Prevalence for each species of host
Prevalence for each age-group of long-lived hosts
Rate at which hosts become infected
Mortality rate in vertebrate hosts
Total potential miracidia produced by each prevalence category

Molluscan host

Total population in the area
Mortality rate of infected snails
Population fluctuations

Parasites in molluscs

Prevalence:
  by location
  by season
Cercarial output:
  by location
  by season
  by other factors (weather, time of day).

It is clear that other factors besides those listed affect transmission. These revolve especially around the habits of people. Since such factors are difficult to quantify, it would be important to select the areas of differing endemicity carefully, so that the social factors can be considered constant.

An undertaking of this magnitude is certainly not to be attempted in every country where bilharziasis control is planned. Therefore, careful selection of areas is indicated, so that the conclusions drawn would apply to as many other places as possible. If one may judge from the experience of the Philippines, it should be possible for a well-staffed and properly equipped unit to obtain the necessary data from at least three areas in two years after the complete assembly of staff and equipment, and after sufficient preliminary data had been obtained to provide for intelligent choice of the areas. Allowing six months for this, a minimum of two-and-a-half years would be required. Staffing would undoubtedly be the most difficult problem, as 15 first-rate technicians would be needed, besides a population ecologist, a parasitologist, an engineer and a biometrician, in addition to supporting staff, such as an administrator, secretaries, drivers and animal keepers. If such a unit were expected to accomplish anything more than acquiring the necessary data, the staff or the time should be expanded accordingly.

From this discussion, it can be seen that the cost of obtaining the predictive models is high, especially as the observations would need to be made in at least four parts of the world. Against this cost must be weighed the cost of control programmes that have failed completely or have been only partly successful.

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RÉSUMÉ

Ce travail reprend en détail un modèle mathématique décrit précédemment pour l'étude de l'épidémiologie de *Schistosoma japonicum*; il donne des exemples de presque tous les calculs qui n'ont pas encore été publiés. Les données fondamentales concernant l'écologie des populations permettent d'établir ce modèle en considérant la population parasite. Une équation est formulée pour les zones où les parasites ne sont en nombre ni croissant ni décroissant. Si toutes les données sont exactes, la solution de cette équation doit être 1,0, représentant le taux net de reproduction de toute population stationnaire. La solution exige quatre termes:

> Taux net de reproduction du parasite chez le mollusque × Probabilité d'infecter un mammifère × Taux net de reproduction du parasite chez le mammifère × Probabilité d'infecter un mollusque = Taux net de reproduction du parasite.

Chaque terme représente une combinaison de facteurs et des méthodes sont décrites pour calculer chacun d'eux à partir de renseignements que l'on peut obtenir en principe.

Les données les plus complètes ont été obtenues pour une population de *S. japonicum*, dans un village de Leyte, Philippines. Dans ce cas, la solution de l'équation est 0,588; il semble que la principale cause de divergence d'avec la solution théorique 1,0 soit le manque d'informations concernant la population murine, qui représente un important hôte-réservoir.

Pour *S. mansoni* et *S. haematobium*, les renseignements sont fournis par les rapports du Projet Pilote de Lutte contre la Bilharziose en Egypte. Les informations sur les populations de mollusques et l'excrétion des œufs étaient très fragmentaires au moment de la rédaction de cet article, ce qui se traduit par la médiocrité des solutions obtenues: 1,85 pour *S. mansoni* et 2,74 pour *S. haematobium*. L'auteur estime que des solutions correctes de l'équation dans au moins trois zones d'endémicité très différente devraient permettre de prédire les densités critiques de mollusques et de mammifères nécessaires au maintien de la population parasite.

Une conclusion est très importante: en principe, une lutte radicale n'est pas nécessaire; à des taux très faibles de transmission, la probabilité que les parasites pénétrant chez l'homme restent sans s'accoupler augmente fortement. Dans ces conditions, la population parasite tendra finalement vers zéro, même si une certaine transmission persiste quelque temps.

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