COMMENTARY

How static is static friction?

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Physics 101 wisdom suggests that we need to overcome the static friction force, F_s, to initiate the lateral motion of one solid body over another one and that a force greater than the kinetic friction, F_k, is required to maintain the gliding nature of the contact. To drag a solid through a fluid, no such threshold forces exist. Many scientists are therefore tempted to conclude that a smaller force is required to push a large cruise ship through still waters than to slide a salt shaker over the dining table, provided that time is not an issue. The potential pitfall in this reasoning stems from the difficulty of clearly distinguishing a pinned from a sliding state, which makes it difficult to determine a precise value for static friction. Extremely small sliding velocities may remain unnoticed, because the distance that a supposedly pinned solid displaces within a day or even a year is below the detection limit of the experimental apparatus. A problem with static friction is that it may be conceptually ill-defined. First, F_s is not single-valued even if the materials in contact, the load, and a potentially present lubricant are well specified. Instead static friction is known to depend on the age of the contact (the increase is logarithmic in time over a broad range of contact ages) and the rate with which the shear stress is increased. Second, static friction may not even be static. Transient creep-like motion, difficult to detect at the macroscopic scale, can take place before the rapid slip event (1). To probe the fundamental laws of static friction, one therefore needs to study extremely small sliding velocities v_s. Going down to v_s slightly <1 μm/s for a paper-on-paper system, Baumberger and coworkers (2, 3) showed that creep occurs in those systems during the stick phase, although the lateral forces were well below F_s. In this issue of PNAS, Yang, Zhang, and Marder (4) push the envelope even more slowly and manage to resolve sliding velocities down to 10^{-5} μm/s. Their analysis of this experimental data in terms of a rate and state model for friction suggests that slip precedes static friction and furthermore confirms the expectation that creep takes place at shear forces much below the static friction.

Rate and State Models: From Micrometers to Tectonic Scales

Rate and state models for friction were originally developed in the geophysical community to describe the slow time evolution of mechanical contacts, in particular those of rock (5–7). These theories are based on the observation that intimate mechanical contact tends to happen only at a miniscule fraction of the apparent contact area, which is caused by the rough, self-affine nature of typical surface topographies (8, 9). At the onset of sliding, previously existing microscopic contact points break and new ones are formed, leading to a complicated dynamics of aging and rejuvenation. The key idea of rate and state theories is to cast the history dependence into one or several (phenomenological) state variables Θ, whose physical interpretation differs. In the simplest case, Θ is associated with the (average) age of contact points, but alternative interpretations are possible. In addition to the logarithmic dependence of friction on the waiting time (or state variable), friction increases logarithmically with sliding velocity. To reflect these observations (at sufficiently large values of Θ and v_s), Yang, Zhang, and Marder (4) chose the following expression for the friction coefficient μ (ratio of normal and lateral force):

$$\mu = \mu_0 + A \ln\left(\frac{v}{v^*} + 1\right) + B \ln\left(\frac{\Theta}{\Theta^*} + 1\right). \qquad [1]$$

where μ_0 can be associated with the static friction coefficient ($v_s = 0$) of a nascent contact ($\Theta = 0$). A, B, v^*, and Θ^* are phenomenological, system-dependent coefficients in addition to μ_0. The advantage of Eq. **1** over the more regularly used expression $\mu = \mu_0 + A\ln(v/v^*) + B\ln(\Theta/\Theta^*)$ is that no singularities arise in the zero limits of v_s and Θ.

To complete the theory, an expression for the time evolution of the state variable needs to be found. In systems that show a lot of aging at long times, e.g., when the materials yield plastically at the points of contact, Θ tends to be associated with the age of the contact. However, on time scales between 30 min and 2 weeks, Yang, Zhang, and Marder (4) found no evidence for aging in their systems (steel rubbed against either silicon or quartz). This finding motivated their choice of an expression originally suggested by Ruina (6), in which Θ is a function of past sliding velocities. Considering a limit of Ruina's formulation, in which aging does not play a role, the following expression for the time derivative of Θ was found:

$$\frac{d\Theta}{dt} = \frac{v}{D_c}\left(\frac{1}{1 + v/v_\Theta} - \Theta\right), \qquad [2]$$

where D_c is a characteristic linear dimension of a single contact point and v_Θ is a system-dependent term. As the physical nature of the variable is unspecified, it may be difficult to fully rationalize this result. However, one can ascertain that in the limit of large v, the rate with which the contact "rejuvinates" is v/D_c, whereas the contact becomes "stiffer" when the sliding velocity is small. At zero sliding, no stiffening (roughly similar to aging) occurs.

The theoretical framework was applied to two types of experiments. In the first, a steel frame, into which the samples (quartz or silicon) were clamped, was extended by 0.5 μm for 16 s and then it was not further elongated for another 284 s. This 5-min protocol was repeated several times. In the second sort of experiment, the frame was stretched continuously as a function of time with loads varying from 120 to 1,500 N. The rate-state model captured the experimental features quantitatively. However, it was necessary to set the coefficient for static friction of a nascent contact, μ_0, to zero. The implication would be that surfaces need to slip on a submicron scale before they can lock together. Of course, this does not mean that the static friction itself is zero. An (upper-bound) estimate for static friction is $F_s = B\ln(1 + 1/\Theta^*)$; Θ cannot exceed unity because of Eq. **2**.

Microscopic Origin of Interlocking

It may be worth placing the interpretation resulting from the rate state treatment into Coulomb's microscopic picture for solid friction (10): ". . . ou bien il faut supposer que les molécules des surfaces des deux plans en contact contracte, par leur proximetée, une cohérence qu'il faut vaincre pour conduire le movement" (. . . or one has to assume

Author contributions: M.H.M. wrote the paper.

The author declares no conflict of interest.

See companion article on page 13264.

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that the surface molecules of the two opposing planes contract because of their proximity into a coherence, which needs to be overcome to produce motion). The rate-state model suggests that one needs a little bit of slip on a microscopic scale before this "contraction" takes place. This idea seems provocative given that there are quite a few static friction mechanisms not requiring submicron slips. Some of these mechanisms are interlocking because of adsorbed layers, pinning via (chemical) defects, material mixing and/or cold welding, or local, elastic and plastic instabilities (11). One can speculate that the concept of contact stiffness resolves the issue: solid–solid interfaces have been found to behave as if they had a shear stiffness κ: for forces well below F_s, linear, elastic displacements between the two solids have been identified (12). Interestingly, κ does not appear to correlate with the elastic properties of the contacting solids, but it can rather be described as the ratio, $\kappa = L/\lambda$, formed by the normal force or load L pushing the solids together and a length λ in the order of 1–2 μm, which like D_c, corresponds to typical linear dimensions of single-asperity contacts. However, although the concept of contact stiffness shows the correct trends, the values of $1/\kappa$ appear to be too small to account for the magnitude of the measured effects.

Creep takes place at shear forces much below the static friction.

The observations by Yang, Zhang, and Marder (4) certainly further solidify the idea that static friction is not truly static and it is intriguing that their observations can be cast into a simple and thus elegant rate-state model, which would require interfaces to slip before they stick. Still, it will not be necessary to rewrite all aspects on solid friction in physics 101 textbooks. Velocities of 10^{-5} μm/s can be considered negligible for most practical purposes and it will remain easier to push a cruise ship through still waters than to slide a salt shaker over the dining table at such small velocities.

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