

Effective communication of standard errors and confidence intervals

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1. INTRODUCTION

When reporting estimates and associated standard errors (ses) or confidence intervals (CIs), the standard formats, “estimate (se)” and “estimate (95% CI: [lower, upper]),” can be confusing in text; in tables they hinder comparisons. Furthermore, some readers can misinterpret the CI format as indicating equal support for all reported values. To remedy these deficits, we recommend formats that (1) improve clarity in text and tables and (2) emphasize that an estimate and its associated uncertainty should be “connected at the hip” as a single unit.

2. RECOMMENDED DISPLAYS

We recommend displaying an estimate with its se using $\text{est}_{(se)}$ and displaying an estimate with its CI using the triple of percentiles, $_{2.5}50_{97.5}$. The first author encouraged these formats in articles published in the *Journal of the American Statistical Association*, Applications and Case Studies articles. In their discussion of estimated deaths in Iraq, Zeger and Johnson (2007) extended the CI format to the 5-number summary, $_{2.5}25_{50}75_{97.5}$. This display is reminiscent of the 5-number summary introduced by Tukey (1977). In both CI formats, the decreasing point size communicates decreasing likelihood. Indeed, the 5-number summary graphic is reminiscent of a likelihood function or posterior distribution.

3. EXAMPLES

In text: Compare the clarity and message of “the estimate is 1.48 ($se = 0.09$)” to “the estimate is $1.48_{(0.09)}$ ” and the clarity and message of “the estimate of excess deaths is 654 (95% CI : 393 to 943)” to that of “the estimate of excess deaths is $_{393}654_{943}$.” Furthermore, note both the clarity and the information content of the 5-number summary, $_{393}560_{654}748_{943}$. The recommended formats are easier to read and reinforce the message that uncertainty measures are an integral part of an estimate.

In tables: Tabulations using the new methods pay big dividends in clarity. Note the ease of making row and column comparisons in Table 1. Similar clarity is conferred by tabulating $\text{est}_{(se)}$ rather than using the standard format (see Hoeting *and others*, 2003).

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Table 1. *Table 6 from Barnard and others (2003) (converted to recommended format): “ITT Effect of Private School Attendance on Test Scores.”*

| Grade at application | Applicant’s school: Low | | | Applicant’s school: High | | |
|----------------------|-------------------------|---------------------|--------------------------|--------------------------|---------------------|--------------------------|
| | Reading | | Math | Reading | | Math |
| 1 | -2.0 | 3.4 _{8.7} | 3.0 7.7 _{12.4} | -7.3 | 1.9 _{10.3} | 0.2 7.4 _{14.6} |
| 2 | -3.7 | 0.7 _{5.0} | -2.4 1.9 _{6.2} | -9.4 | -0.9 _{7.3} | -6.2 1.5 _{9.3} |
| 3 | -4.1 | 1.0 _{6.1} | -0.8 5.0 _{10.7} | -9.5 | -0.8 _{7.7} | -4.9 4.0 _{12.5} |
| 4 | -1.5 | 4.2 _{10.1} | -1.6 4.3 _{10.1} | -6.3 | 2.7 _{11.3} | -4.7 3.5 _{11.9} |
| Overall | -0.9 | 2.2 _{5.3} | 1.4 4.7 _{7.9} | -7.1 | 0.6 _{7.7} | -2.6 4.2 _{10.9} |

ACKNOWLEDGMENTS

Conflict of Interest: None declared.

FUNDING

US National Institutes of Health (RO1 DK061662, RO1 ES0154-02, and 5P30ES03819-17).

APPENDIX: LATEX CODE

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\newcommand{\estse}[2]{\{#1\}_\{(#2)\}}
\newcommand{\cithree}[3]{_\{#1\}\;\!\{#2\}_\{\;\{#3\}\}}
\newcommand{\cifive}[5]{_\{_\{#1\},\}\{#2\}\;\!\{#3\}_\{\,\{#4\}_\{,\{#5\}\}\}}
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[Received February 25, 2008; revised March 14, 2008; accepted for publication April 4, 2008]