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Order Parameter Dynamics of Body-scaled Hysteresis and Mode Transitions in Grasping Behavior

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Abstract Several experimental studies have shown that human grasping behavior exhibits a transition from one-handed to two-handed grasping when to-be-grasped objects become larger and larger. The transition point depends on the relative size of objects measured in terms of human body-scales. Most strikingly, the transitions between the two different behavioral 'modes' of grasping exhibit hysteresis. That is, one-to-two hand transitions and two-to-one hand transitions occur at different relative object sizes when objects are scaled up or down in size. In our study we approach body-scaled hysteresis and mode transitions in grasping by exploiting the notion that human behavior in general results from self-organization and satisfies appropriately-defined order parameter equations. To this end, grasping transitions and grasping hysteresis are discussed from a theoretical perspective in analogy to cognitive processes defined by Haken's neural network model for pattern recognition. In doing so, issues such as the exclusivity of grasping modes, biomechanical constraints, mode-mode interactions, single subject behavior and population behavior are explored.

Keywords Order parameters · Hysteresis · Grasping

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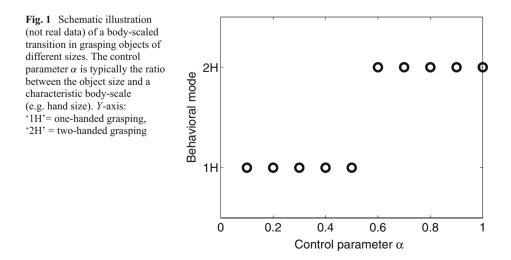
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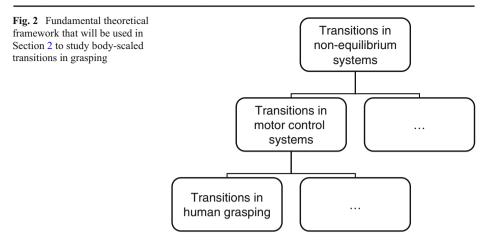
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1 Introduction

Many objects in every-day situations (books, pencils, bags and so on) can be grasped with one or two hands. However, not all large objects can easily be grasped with one hand, and objects that exceed certain critical limit sizes are usually grasped with two hands. That is, the ratio between characteristic object- and body-scales determines whether objects are grasped with one hand or with two hands. In general, if objects are large with respect to characteristic body scales (e.g. the hand size), humans tend to use two hands for grasping. In contrast, if objects are relatively small, humans show the tendency to handle objects with one hand only, as is schematically illustrated in Fig. 1. Consequently, at critical object-body scale ratios, transitions between one-handed and two-handed grasping occur. Several studies have been devoted to examining these so-called body-scaled transitions. For example, body-scaled transitions in grasping have been studied in the field of motor development [1–6]. Body-scaled transitions have also been studied in the context of cognitive processes [7] and social behavior [7, 8]. Studies on body-scaled transitions in general [1–4] have addressed issues regarding the ecological approach to perception and action [9] and in particular [5–7] the actualization of perceived action possibilities, often referred to as affordences [9–12].

The transitions between different behavioral modes of grasping are examples of the behavioral transitions that have been extensively studied in motor control systems [13–29]. Behavioral transitions in motor control systems in turn are examples of transitions that naturally occur in complex non-equilibrium systems [30], as is shown in Fig. 2. Such transitions can be understood using the principles of dynamical systems theory and self-organization [30]. A key ingredient of self-organizing systems are control parameters. Control parameters are certain parameters that induce transitions when they are scaled across critical values, but do not specify the spatio-temporal patterns or behavioral modes that emerge due to these transitions. For self-organized motor-control systems, the control parameters of particular relevance capture the informational variables that specify the information that is available to that human actor in a given environment and the information that is available to that human actor in that environment. In some instances, this relationship may assume the form of a mapping [31, 34, 35]. The information variables



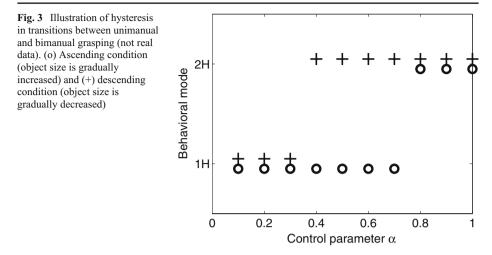


is mapped to different types of behavioral modes. For behavioral transitions that involve hysteresis (such as grasping transitions, as discussed below), however, such a map must involve a dynamic element (see Section 3) and the relationship between behavior and information can be described in terms of the relationship between behavioral modes and control parameters of dynamical self-organizing systems.

Due to the omnipresence of grasping behavior in every-day activities, understanding the dynamics and self-organization of such behavior is important for an understanding of motor control systems in general. However, a dynamical systems approach that studies explicit forms of evolution equations to describe the phenomenon of body-scaled transitions in human grasping has not yet been developed. The aim of the current paper is to provide some initial steps in this direction. To this end, we will present a dynamic system that accounts for three key properties of grasping transitions: hysteresis, exclusivity of behavioral modes, and the distinction between biomechanically induced transitions and transitions induced by interacting behavioral modes.

In several experimental studies it has been shown that grasping behavior shows hysteresis [1, 5, 7]. If object size is increased gradually (ascending condition), the transition from one-handed grasping to two-handed grasping occurs at a particular critical value of a control parameter α representing an object-size to body-scale ratio. If object size is decreased gradually (descending condition), the transition from two-handed grasping to one-handed grasping typically occurs at a different critical value of the control parameter. That is, grasping transitions typically show hysteresis as illustrated schematically in Fig. 3. Typically, the critical control parameter of the one-to-two hand transition in the ascending condition is larger than the critical parameter of the two-to-one hand transition in the descending condition (for exceptions, see, for example, [7, 36]). The question arises as to what can be inferred from the observation of hysteresis? To what extent does the phenomenon of hysteresis provide us with insights into the structure of the dynamical systems underlying grasping transitions?

Grasping transitions involve two mutually exclusive behavioral modes: grasping with one hand and grasping with two hands. That is, there is not a continuous spectrum of grasping modes that is bounded on either side by the one-handed and two-handed grasps, respectively. For example, there does not exist a 1.5-handed grasping behavior. Grasping



transitions confront us with the question of how to approach the exclusivity of grasping modes from a dynamical systems perspective.

Grasping objects with one hand becomes impossible for objects that are simply too large to be picked up with one hand. Consequently, grasping transitions from one-handed grasps to two-handed grasps may be induced by biomechanical limitations. The argument of biomechanical limitations, however, does not necessarily apply to grasping transitions from two-handed to one-handed grasps as observed under the aforementioned descending object presentation condition. Relatively small objects such as pencils can be picked up with two hands although adult participants in an experimental study might feel uncomfortable doing so. The two-to-one hand transitions observed in experimental studies suggest that there is a second process that triggers grasping transitions. We will argue in Sections 2 and 3 that this second process reflects interactions between unimanual and bimanual grasping modes. In short, when discussing grasping transitions from a theoretical perspective, we need to take two different types of processes into consideration. First, one of the two grasping modes may become unavailable due to biomechanical constraints. In this context, we would deal with a transition due to biomechanical limitations. Second, both behavioral modes are available and interact with each other. In this context, we would deal with transitions involving a mode selection. How can these two processes be rephrased in the language of dynamical systems theory? How can these two types of transitions be related to attractors and repellers of a dynamical system underlying grasping transitions?

In the subsequent sections, we will study grasping transitions from a dynamical systems perspective by adopting a model proposed by [37] (see also [38–40]). Haken's model is tailored to describe pattern recognition processes (e.g. the recognition of faces and scenes). In particular, it has been applied to study oscillatory perceptual processes centered on ambiguous patterns [41]. The model proposed by Haken has often been interpreted as a neural network model (see, for example [42–44]). In this context, states of neural activity are determined by attractors and repellers of the neural system of humans and animals. In our context, we need to take a slightly different perspective. That is, the dynamical system involved in grasping transitions must incorporate both the individual and the environment. More specifically, the dynamical system involved in grasping transitions must incorporate the complementary relation that exists between the relevant properties of the

to-be-grasped objects (i.e. object-length or size) and the relevant properties of the individual's action system (i.e. hand size). It is this animal-environment relation (more precisely, the information that specifies this animal-environment relation) that constrains and guides the grasping behavior of an individual [9, 10, 32]. The attractors and the repellers that we will discuss in our study do not exist exclusively within the body or brain of an individual; rather, they exist in the animal-environment system composed of the individual and its environment [45, 46].

With Haken's model at hand, we will address in detail the three key issues mentioned above. In Section 2.1 we will show that the observation of hysteresis suggests that the dynamical system involved in grasping transitions exhibits a bistable parameter domain. We will show that the exclusivity of behavioral modes naturally results from a winner-takes-all dynamics as described by Haken's model for pattern recognition. We will study fixed point attractors that vanish entirely and discuss attractors that become repellers when control parameters are varied. We will show to what extent these two types of bifurcations are related to the two above-mentioned processes that may result in grasping transitions. While Section 2.1 is concerned with individual behavior, in Section 2.2, predictions for participant populations will be derived. In particular, we will demonstrate how to derive, from our dynamical systems approach, the hysteresis curves for participant populations that have been reported in several studies [4, 5, 7].

2 Haken's model of pattern recognition applied to grasping transitions

In our study, we will restrict our analysis to the order parameter dynamics of Haken's model for pattern recognition [37].

2.1 Individual participant behavior

Let ξ_1 and ξ_2 denote the generalized amplitudes of one-handed and two-handed grasping movements, respectively. Consequently, if ξ_1 is finite and ξ_2 equals zero, then one-handed grasps are performed. In contrast, if we have $\xi_2 > 0$ and $\xi_1 = 0$ then two-handed grasps are performed. In general, ξ_1 and ξ_2 are so-called order parameters that emerge close to bifurcation points and evolve with time. Accordingly, grasping behavior is determined by the evolution of the order parameters ξ_1 and ξ_2 . In line with Haken's model for pattern recognition, we assume that the order parameter dynamics satisfies the coupled set of nonlinear first-order dynamical equations [37, Section 5.2, Eq. 5.33]:

$$\frac{d}{dt}\xi_1(t) = \lambda_1\xi_1 - B\xi_2^2\xi_1 - C\left(\xi_1^2 + \xi_2^2\right)\xi_1,$$

$$\frac{d}{dt}\xi_2(t) = \lambda_2\xi_2 - B\xi_1^2\xi_2 - C\left(\xi_1^2 + \xi_2^2\right)\xi_2.$$
(1)

Note that in what follows we will consider only positive order parameters $\xi_1, \xi_2 \ge 0$. The linear terms involve the parameters λ_1, λ_2 that can assume both positive and negative values. λ_1 and λ_2 correspond to the growth rates of the amplitudes ξ_1 and ξ_2 , if ξ_1 and ξ_2 are close to zero. Consequently, the order parameters can only grow from a state $\xi_{1,2} \approx 0$ to $\xi_1 > 0$ or $\xi_2 > 0$ if at least one of the λ parameters is positive. In the context of Haken's model for pattern recognition the parameters λ_1 and λ_2 have been called attention parameters because a pattern can be recognized only if its corresponding λ parameter is positive [37,

Section 5.2]. Accordingly, $\lambda > 0$ means that attention is directed to a pattern, whereas $\lambda < 0$ means that no attention is paid to the pattern. In our study, we will refer to λ_1 and λ_2 as attention parameters as well, in order to be consistent with the literature. However, we will not necessarily interpret them as parameters that express a degree of attention (see Section 2.1.8). In line with the study by [41] on oscillatory perceptual instabilities, we assume that in general the attention parameters can differ from each other. The nonlinear terms involve two parameters *B*, *C* > 0. Equation 1 can equivalently be expressed in terms of the potential dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\xi_{1}(t) = -\frac{\partial}{\partial\xi_{1}}V(\xi_{1},\xi_{2}),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\xi_{2}(t) = -\frac{\partial}{\partial\xi_{2}}V(\xi_{1},\xi_{2})$$
(2)

involving the two-dimensional potential

$$V(\xi_1,\xi_2) = -\frac{1}{2} \left(\lambda_1 \xi_1^2 + \lambda_2 \xi_2^2 \right) + \frac{B}{2} \xi_1^2 \xi_2^2 + \frac{C}{4} (\xi_1^2 + \xi_2^2)^2 .$$
(3)

Note that in our study we will not evaluate the magnitudes of ξ_1 and ξ_2 . For example, the states $(\xi_1, \xi_2) = (1, 0)$ and $(\xi_1, \xi_2) = (2, 0)$ both describe one-handed grasping. We will focus on the issue of whether a behavioral mode is present (e.g. $\xi_1 > 0$) or absent (e.g. $\xi_1 = 0$). We will briefly return to the relevance of the magnitudes of the order parameters ξ_1 and ξ_2 in Section 3.

2.1.1 Fixed points, attractors and repellers

Equation 1 exhibits, in general, up to four fixed points $(\xi_{1,st}, \xi_{2,st})$ that will be denoted by *I*, *II*, *III*, and *IV* (here 'st' refers to 'stationary'). Fixed point *I* is given by

$$\xi_{1,\text{st}} = \sqrt{\frac{\lambda_1}{C}} , \ \xi_{2,\text{st}} = 0$$
 (4)

and describes one-handed grasping activity. Fixed point II assumes the form

$$\xi_{1,\text{st}} = 0, \ \xi_{2,\text{st}} = \sqrt{\frac{\lambda_2}{C}},$$
(5)

and describes two-handed grasps. Fixed point III satisfies

$$\xi_{1,st} > 0 , \ \xi_{2,st} > 0 .$$
 (6)

This fixed point describes the co-existence of the one- and two-handed grasping modes. Since the grasping modes are mutually exclusive, such a co-existence is forbidden. We will show below to what extent the dynamical system (1) indeed does not exhibit such states of co-existence, although, under particular parameter conditions, fixed point *III* is a solution of Eq. 1. Fixed point *IV* is the origin, given by

$$\xi_{1,st} = 0, \ \xi_{2,st} = 0.$$
 (7)

This fixed point describes the absence of grasping activity. Let us discuss next the existence and stability of the fixed points I - IV. To this end, we will proceed along the lines of the discussion carried out in [37].

2.1.2 *Case* $\lambda_1 > 0, \lambda_2 \leq 0$

In this case fixed points *I* and *IV* exist. Fixed points *II* and *III* do not exist. A linear stability analysis shows that fixed point *I* represents a stable node. That is, there is an attractor of the one-handed grasping mode. The attractor corresponds to a minimum of the potential *V* as illustrated in Fig. 4. Using linear stability analysis it can be shown that the origin (fixed point *IV*) corresponds to an unstable node. That is, the no-action-at-all mode corresponds to a repeller. As can be seen from Fig. 4 this repeller corresponds to a maximum of the potential *V*. Consequently, for any initial condition $\xi_1(0) > 0$, $\xi_2(0) > 0$ the dynamical system (1) converges to the fixed point *I*, indicating that under the condition $\lambda_1 > 0$, $\lambda_2 \leq 0$ only one-handed grasps are performed.

2.1.3 *Case* $\lambda_1 \leq 0, \lambda_2 > 0$

Under these conditions, fixed points *II* and *IV* exist, whereas fixed points *I* and *III* do not exist. By means of linear stability analysis we can show that fixed point *II* is a stable node. Fixed point *IV* corresponds to an unstable node. The dynamical system (1) converges for any initial condition $\xi_1(0) > 0$ and $\xi_2(0) > 0$ to the fixed point *II*. Consequently, only two-handed grasps are performed.

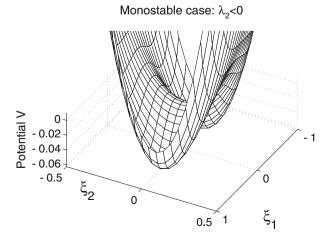
2.1.4 *Case* $\lambda_1 > 0$, $\lambda_2 > 0$

The fixed points *I*, *II* and *IV* exist for any pair (λ_1, λ_2) with $\lambda_1 > 0$ and $\lambda_2 > 0$. The existence of fixed point *III* will be discussed separately below. Linear stability analysis shows that fixed point *IV* is an unstable node. In order to examine fixed point *I* by means of linear stability analysis we put $\xi_1 = \xi_{1,st} + \epsilon_1, \xi_2 = \epsilon_2$, where ϵ_1 and ϵ_2 correspond to small perturbations. Substituting these relations into Eq. 1, we see that the perturbations evolve as

$$\frac{\mathrm{d}}{\mathrm{d}t}\epsilon_{1}(t) = -2\lambda_{1}\epsilon_{1}(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\epsilon_{2}(t) = -\left[\lambda_{1}\left(1 + \frac{B}{C}\right) - \lambda_{2}\right]\epsilon_{2}(t).$$
(8)

Fig. 4 Case $\lambda_1 > 0, \lambda_2 \leq 0$. The potential (3) exhibits a minimum at the fixed point I defined by Eq. 4. The amplitude (order parameter) of the one-handed grasping mode is finite, whereas the amplitude (order parameter) of the two-handed grasping mode vanishes. Note that in this figure and in all subsequent figures we will present the potential V in the full two-dimensional plane spanned by ξ_1 and ξ_2 which improves the overall visibility of the potential function. In our study the focus is on the part of this plane given by the sub-domain $\xi_1 \ge 0$ and $\xi_2 \ge 0$



At this stage we introduce the parameter

$$g = 1 + \frac{B}{C} > 1$$
, (9)

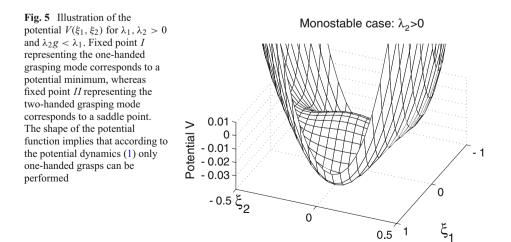
that will be used frequently in subsequent steps of our analysis. From Eqs. 8 and 9 it follows that for $\lambda_1 g > \lambda_2$ we are dealing with a stable node *I*, whereas for $\lambda_1 g < \lambda_2$ fixed point *I* is unstable and corresponds to a saddle point. Likewise, let us examine the stability of fixed point *II* by introducing the perturbations ϵ_1 and ϵ_2 defined by $\xi_1 = \epsilon_1$ and $\xi_2 = \xi_{2,st} + \epsilon_2$. From Eq. 1 we obtain

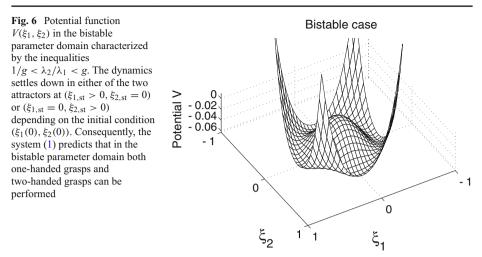
$$\frac{\mathrm{d}}{\mathrm{d}t}\epsilon_1(t) = -\left[\lambda_2\left(1+\frac{B}{C}\right) - \lambda_1\right]\epsilon_1(t) ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\epsilon_2(t) = -2\lambda_2\epsilon_2(t) .$$
(10)

Consequently, for $\lambda_2 g > \lambda_1$ we find that fixed point *II* is a stable node, whereas for $\lambda_2 g < \lambda_1$ fixed point *II* corresponds to a saddle point. Figure 5 illustrates this latter case: λ_2 is larger than zero, but the dynamical system (1) is still monostable because we have $\lambda_2 g < \lambda_1$.

We may summarize the results that we have derived so far by assuming that $\lambda_1 > 0$ is fixed and that λ_2 is increased gradually. Then, we see that the dynamical system (1) defined on $\xi_1 \ge 0$ and $\xi_2 \ge 0$ is monostable for $\lambda_2/\lambda_1 < 1/g$ and exhibits a unique stable attractor given by fixed point *I*. The system (1) is bistable for $1/g < \lambda_2/\lambda_1 < g$ and exhibits two attractors related to the fixed points *I* and *II*. The dynamic system becomes monostable again for $\lambda_2/\lambda_1 > g$ and exhibits a unique stable attractor given by the fixed point *II*. In the monostable parameter domains the system converges to its uniquely defined stable fixed point for arbitrary initial conditions $\xi_1(0), \xi_2(0) > 0$. This implies that only one of the two grasping modes can be performed. In the bistable parameter domain the system converges to either of the two attractors. That is, the system may converge to the fixed point *I* reflecting the one-handed grasping mode or to the fixed point *II* reflecting the two-handed grasping mode. Both grasping modes are available. Which mode is selected





depends on the initial conditions. Figure 6 illustrates the potential $V(\xi_1, \xi_2)$ for the bistable case. The attractors at $(\xi_{1,st} > 0, \xi_{2,st} = 0)$ (fixed point *I*) and $(\xi_{1,st} = 0, \xi_{2,st} > 0)$ (fixed point *II*) correspond to potential minima of *V*.

2.1.5 Co-existence in the bistable parameter domain?

In the preceding analysis we neglected the fixed point *III*. From Eq. 1 it follows that fixed point *III* is defined by

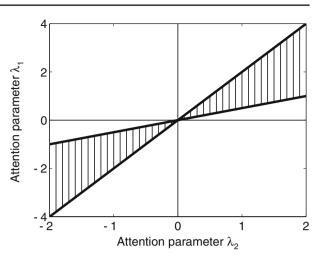
$$\xi_1 = \sqrt{\frac{C}{B}(\lambda_1 g - \lambda_2)},$$

$$\xi_2 = \sqrt{\frac{C}{B}(\lambda_2 g - \lambda_1)}.$$
(11)

Equation 11 reveals that fixed point *III* exits only in the bistable parameter domain. In the parameter space ξ_1 - ξ_2 this bistable domain is given by the hatched area shown in Fig. 7.

In order to determine the character of the fixed point *III* we first note that within the bistable parameter domain the number of fixed points does not change. Likewise, the stability of the fixed points I, II, and IV does not change. From these two properties and from the fact that the potential V depends in a continuous way on the attention parameters λ_1 and λ_2 , we conclude that the stability of the fixed point III cannot change within the bistable parameter domain. In other words, the topology of the potential V does not depend on the parameters λ_1 and λ_2 as long as they are selected from the bistable parameter domain (i.e. as long as they satisfy the inequality $1/g < \lambda_2/\lambda_1 < g$). From this argument it follows that the fixed point III for the special case $\lambda_1 = \lambda_2$ exhibits the same stability as for all other attention parameters satisfying $1/g < \lambda_2/\lambda_1 < g$. For the special case $\lambda_1 = \lambda_2$ it has been shown that the fixed point *III* corresponds to a saddle point [37]. Consequently, if fixed point *III* exists, then it is an unstable fixed point that corresponds to a saddle point of the potential $V(\xi_1, \xi_2)$. Fixed point *III* corresponds to the saddle between the potential minima at $(\xi_1 > 0, \xi_2 = 0)$ and $(\xi_1 = 0, \xi_2 > 0)$ as illustrated in Fig. 6. With this result at hand, we can conclude our discussion on the existence and stability of the fixed points of Eq. 1. Table 1 summarizes our results.

Fig. 7 Illustration of the bistable domain (hatched area) defined by $1/g < \lambda_2/\lambda_1 < g$ in the two-dimensional parameter space ξ_1 - ξ_2 . Note that the size of the area depends on the parameter *g* defined by Eq. 9, which in turn depends on the ratio B/C. For $B/C \rightarrow 0$ the hatched area vanishes and reduces to the diagonal. In contrast, for $B/C \gg 1$ the hatched area would basically correspond to the entire first and third quadrants



2.1.6 Bistability and hysteresis

Since Eq. 1 exhibits a bistable parameter domain and satisfies a potential dynamics involving the potential function $V(\xi_1, \xi_2)$ we conclude that Eq. 1 features hysteresis. If we gradually increase the ratio λ_2/λ_1 , then from Table 1 we can read off that for $\lambda_2/\lambda_1 < g$ the dynamical system (1) will settle down to the fixed point *I* irrespective of its initial condition. Accordingly, participants perform one-handed grasping movements for $\lambda_2/\lambda_1 < g$. If the ratio λ_2/λ_1 is increased further such that the system operates in its bistable parameter domain, then the system dynamics will not leave the attractor I. Only if λ_2/λ_1 is increased beyond the threshold $\lambda_2/\lambda_1 = g$, the system will settle down to the attractor of fixed point II. Accordingly, the one-handed grasping mode vanishes and the two-handed grasping mode emerges. The transition from one- to two-handed grasps occurs at $\lambda_2/\lambda_1 = g$. We can identify this transition with the grasping transition in the ascending condition (see Fig. 3). In contrast, if we gradually decrease the ratio λ_2/λ_1 starting with a value $\lambda_2/\lambda_1 > g$, then the system will settle down initially to the attractor of fixed point II and will remain there until the critical threshold $\lambda_2/\lambda_1 = 1/g$ is reached. That is, Eq. 1 predicts that the twohanded grasping mode is active during the descending condition as long as $\lambda_2/\lambda_1 > 1/g$. At $\lambda_2/\lambda_1 = 1/g$ the two-handed grasping mode vanishes and the one-handed grasping mode emerges (see descending condition in Fig. 3). In short, Eq. 1 predicts that the one-to-two hand grasping transition occurs at $\lambda_2/\lambda_1 = g$ in the ascending condition, whereas the twoto-one hand grasping transition occurs at $\lambda_2/\lambda_1 = 1/g$ in the descending condition.

	$\lambda_2/\lambda_1 < 1/g$	$1/g < \lambda_2/\lambda_1 < g$	$\lambda_2/\lambda_1 > g$	
Ι	stable	stable	unstable	1H grasping mode
II	unstable	stable	stable	2H grasping mode
III	-	unstable	-	
IV	unstable	unstable	unstable	no action

Table 1 Summary of the stability analysis of the fixed points Eqs. 4, ..., 7 of the system (1)

2.1.7 Winner-takes-all dynamics and exclusivity of grasping modes

From our previous discussion it follows that the dynamical system (1) does not exhibit a stable fixed point *III*. That is, Eq. 1 does not exhibit a state in which the amplitudes ξ_1 and ξ_2 of the two grasping modes are both finite. In other words, one mode dominates the other mode. We may refer to the dominant mode as the 'winner' mode. The winner mode takes all activity and 'inhibits' the other mode completely. Consequently, the winner-takes-all dynamics of Eq. 1 reflects the exclusivity of grasping modes.

2.1.8 Sign of attention parameters and two types of grasping transitions

Figures 4 and 5 illustrate that there are two types of monostable parameter domains. There are two monostable parameter domains (defined by $\lambda_1 \leq 0$ or $\lambda_2 \leq 0$) for which the dynamical system (1) exhibits only one fixed point different from the origin, as exemplified by Fig. 4. In addition, there are two monostable parameter domains (defined by $\lambda_1, \lambda_2 > 0$ and either $\lambda_1 > g\lambda_2$ or $\lambda_2 > g\lambda_1$) for which the system (1) exhibits two fixed points different from the origin, but only one of the two corresponds to an attractor (see, for example, Fig. 5). This implies that we can distinguish between two cases. In the first case there is only one grasping mode available at all. In the second case there are two grasping modes available. However, only one amplitude of the two mode amplitudes has an attractor. The other mode amplitude has a saddle point. We can identify the first case with experimental conditions under which a particular grasping mode cannot be performed at all. As mentioned in the introduction, there are instances in which larger objects simply can not be grasped with one hand due to biomechanical limitations. Monostability in the second case arises from an interaction between the one- and two-handed grasping modes. In terms of the attention parameters λ_1 and λ_2 , we may say that if a grasping mode is physically forbidden (i.e. can not be performed due to biomechanical constraints), then the mode exhibits a negative attention parameter. For example, if we continously increase the size of an object then the attention parameter λ_1 of the one-handed grasping mode as a function of the object size will become negative at a certain critical object size. However, if potentially both grasping modes are available (if they are consistent with biomechanical constraints) but only one grasping mode is selected for all possible initial conditions, then both attention parameters are positive and one attention parameter is critically larger than the other (i.e. we have either $\lambda_1 > g\lambda_2$ or $\lambda_2 > g\lambda_1$) such that the underlying dynamics (1) is in a monostable parameter domain. Consequently, the two processes mentioned in the introduction that can induce transitions between grasping modes can be classified in terms of attention parameters. A transition from one grasping mode to another mode due to biomechanical limitations is related to an attention parameter that becomes negative. A transition from one grasping mode to another mode due to mode-mode interactions occurs when one attention parameter becomes critically larger than the other in the sense that one of the inequalities $\lambda_1 > g\lambda_2$ and $\lambda_2 > g\lambda_1$ is satisfied.

2.1.9 Object-body ratio (control parameter) and attention parameters

In order to relate Haken's model (1) closer to experimental findings, we need to introduce the control parameter α typically used in experimental studies on grasping transitions. As mentioned earlier, the control parameter α reflects typically the ratio between the size of an object and a characteristic body-scale. For example, α was defined as the ratio of the size of the to-be-grasped object and the hand size of a participant (see e.g. [4, 7]). In general, all parameters λ_1 , λ_2 , *B*, *C* occurring in Eq. 1 may depend on this control parameter α . In this section, we will assume that the parameters *B* and *C* describing the magnitudes of the system nonlinearities do not depend on α (we will briefly address the more general case in the discussion, see Section 3). We are left to discuss the functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$. From the empirically observed transition in the ascending and descending conditions, we conclude that the attention parameter λ_1 of the one-handed grasping mode decreases with α , whereas the attention parameter λ_2 of the two-handed mode increases with α . The most parsimonious approach is to linearize the functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$ such that we obtain

$$\lambda_1 = \lambda_{1,0} - \beta \alpha ,$$

$$\lambda_2 = \lambda_{2,0} + \beta \alpha$$
(12)

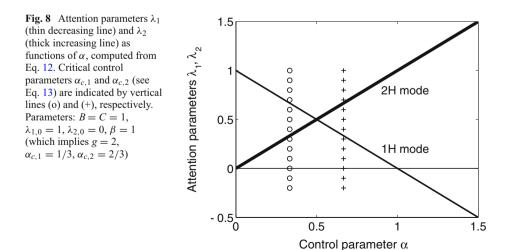
with $\beta > 0$ and $\lambda_{1,0} > \lambda_{2,0}$. From Eq. 12 and the critical thresholds $\lambda_2/\lambda_1 = 1/g$ and $\lambda_2/\lambda_1 = g$, we can derive the critical control parameters:

$$\alpha_{c,1} = \frac{1}{\beta(1+g)} \left(\lambda_{1,0} - g \lambda_{2,0} \right) ,$$

$$\alpha_{c,2} = \frac{1}{\beta(1+g)} \left(g \lambda_{1,0} - \lambda_{2,0} \right) .$$
(13)

The critical control parameter $\alpha_{c,1}$ corresponds to the case $\lambda_2/\lambda_1 = 1/g$, whereas $\alpha_{c,2}$ reflects the case $\lambda_2/\lambda_1 = g$. It can be shown that $\alpha_{c,2} > \alpha_{c,1}$. Figure 8 illustrates the linear functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$ as well as the critical control parameters $\alpha_{c,1}$ and $\alpha_{c,2}$.

Using the linear functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$ defined by Eq. 12 in combination with Eq. 1, we can derive the stationary order parameters $\xi_{1,st}$ and $\xi_{2,st}$ as functions of α for the ascending and descending conditions. Figure 9 shows the functions $\xi_{1,st}(\alpha)$ and $\xi_{2,st}(\alpha)$ as obtained by solving Eqs. 1 and 12 numerically. In line with our analytical discussion, we see that in the ascending condition the amplitude $\xi_{1,st}$ of the one-handed grasping mode is finite and the amplitude $\xi_{2,st}$ of the two-handed grasping mode vanishes for $\alpha < \alpha_{c,2}$, whereas for $\alpha > \alpha_{c,2}$ we have $\xi_{1,st} = 0$ and $\xi_{2,st} > 0$. That is, the one-to-two hand transition occurs at $\alpha_{c,2}$ (see left panel in Fig. 9). In contrast, in the descending condition the order parameter $\xi_{1,st}$ of the two-handed grasping mode is finite, and the order parameter $\xi_{1,st}$ of



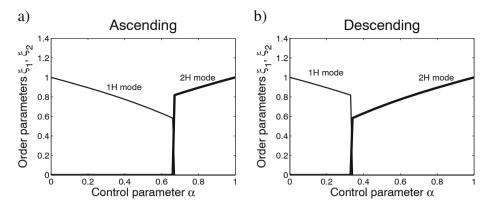


Fig. 9 Stationary order parameters ξ_1 (thin line) and ξ_2 (thick line) versus α obtained by solving numerically the system (1) for the linearized attention parameter functions (12). In the ascending case (panel a) α was increased in steps $\Delta \alpha$. The order parameter ξ_1 of the one-handed mode vanished and the order parameter ξ_2 of the two-handed mode emerged at the theoretically predicted value $\alpha_{c,2}$. In the descending case (panel b) α was decreased in steps $\Delta \alpha$ and the two-to-one hand transition was observed at $\alpha_{c,1}$. Parameters are defined as in Fig. 8 and $\Delta \alpha = 0.01$. Equation 1 was solved using an forward Euler method with $\Delta t = 0.1$. Total number of iterations for every α : 10000

the one-handed grasping mode vanishes for $\alpha > \alpha_{c,1}$. For $\alpha < \alpha_{c,1}$ we have $\xi_{1,st} > 0$ and $\xi_{2,st} = 0$. That is, the two-to-one hand transition occurs at $\alpha_{c,1}$ (see right panel in Fig. 9).

2.2 Participant populations and mean performance of individuals

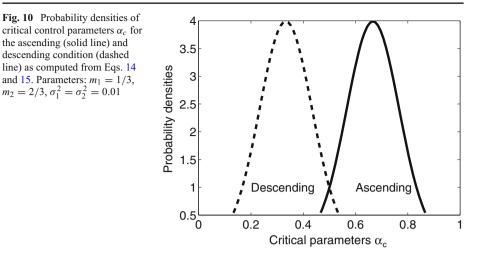
2.2.1 Participant populations

Individual participants show transitions between grasping modes at different critical control parameters (see, for example, [5]). In order to account for this kind of individuality, we assume that in general the parameters $B, C, \beta, \lambda_{1,0}$, and $\lambda_{2,0}$ of Eqs. 1 and 12 differ slightly among participants. Consequently, in a population of participants the parameters $\alpha_{c,1}$ and $\alpha_{c,2}$ defined by Eq. 13 will be distributed according to a certain law. For sake of simplicity, we assume that $\alpha_{c,1}$ and $\alpha_{c,2}$ are Gaussian distributed around some mean values m_1 and m_2 , respectively. That is, we consider probability densities $P_1(\alpha_{1,c})$ and $P_2(\alpha_{2,c})$ of the critical control parameters $\alpha_{c,1}$ and $\alpha_{c,2}$ defined by

$$P_1(\alpha_{1,c}) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left\{-\frac{(\alpha_{c,1} - m_1)^2}{2\sigma_1^2}\right\},$$
(14)

$$P_2(\alpha_{2,c}) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left\{-\frac{(\alpha_{c,2} - m_2)^2}{2\sigma_2^2}\right\},$$
(15)

where σ_1^2 and σ_2^2 denote the variances of the parameters $\alpha_{c,1}$ and $\alpha_{c,2}$, as shown in Fig. 10. In order to compute the percentage of participants that perform two-handed grasps given a certain control parameter α in the ascending condition, we need to compute the probability



 $p(\alpha_{c,2} < \alpha)$ to find critical control parameters $\alpha_{c,2}$ smaller than α . The probability $p(\alpha_{c,2} < \alpha)$ can be computed from

$$p(\alpha_{c,2} < \alpha) = \int_{-\infty}^{\alpha} P_2(\alpha_{c,2}) \,\mathrm{d}\alpha_{c,2} \,. \tag{16}$$

Using $y = (\alpha_{c,2} - m_2)/\sqrt{2\sigma_2^2}$, we obtain

$$p(\alpha_{c,2} < \alpha) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-y^2} \, \mathrm{d}y + \frac{1}{\sqrt{\pi}} \int_{0}^{(\alpha - m_2)/\sqrt{2\sigma_2^2}} e^{-y^2} \, \mathrm{d}y$$
$$= \frac{1}{2} \left(1 + \operatorname{erf}\left\{\frac{\alpha - m_2}{\sqrt{2\sigma_2^2}}\right\} \right), \tag{17}$$

where the error-function $\operatorname{erf}(z)$ is defined by $\operatorname{erf}(z) = 2\pi^{-1/2} \int_0^z \exp\{-y^2\} dy$. Likewise, we can compute the probability $p(\alpha_{c,1} < \alpha)$ to find participants that perform two-handed grasp in the descending condition given a certain control parameter α :

$$p(\alpha_{c,1} < \alpha) = \int_{-\infty}^{\alpha} P_1(\alpha_{c,1}) \, \mathrm{d}\alpha_{c,1}$$
$$= \frac{1}{2} \left(1 + \mathrm{erf}\left\{\frac{\alpha - m_1}{\sqrt{2\sigma_1^2}}\right\} \right) \,. \tag{18}$$

As argued earlier, $p(\alpha_{c,2} < \alpha)$ describes the probability of finding participants that perform two-handed grasping movements in the ascending condition. Consequently, $p_{2H,ascend}(\alpha) =$ $100p(\alpha_{c,2} < \alpha)$ is the percentage of participants that perform two-handed grasps in the ascending condition. Likewise, $p_{2H,descend}(\alpha) = 100p(\alpha_{c,1} < \alpha)$ is the percentage of participants that perform two-handed grasps in the descending condition. We have

$$p_{2H,ascend}(\alpha) = \frac{100}{2} \left(1 + \operatorname{erf}\left\{\frac{\alpha - m_2}{\sqrt{2\sigma_2^2}}\right\} \right) , \qquad (19)$$

$$p_{2H,descend}(\alpha) = \frac{100}{2} \left(1 + \operatorname{erf}\left\{\frac{\alpha - m_1}{\sqrt{2\sigma_1^2}}\right\} \right) .$$
 (20)

Let $p_{1H,ascend}(\alpha)$ and $p_{1H,descend}(\alpha)$ denote the percentages of participants performing unimanual grasps in the ascending and descending condition, respectively. From $p_{1H,ascend}(\alpha) + p_{2H,ascend}(\alpha) = 100$ and $p_{1H,descend}(\alpha) + p_{2H,descend}(\alpha) = 100$ it follows that

$$p_{1H,ascend}(\alpha) = \frac{100}{2} \left(1 - \operatorname{erf}\left\{\frac{\alpha - m_2}{\sqrt{2\sigma_2^2}}\right\} \right) , \qquad (21)$$

$$p_{1H,descend}(\alpha) = \frac{100}{2} \left(1 - \operatorname{erf}\left\{\frac{\alpha - m_1}{\sqrt{2\sigma_1^2}}\right\} \right) \,. \tag{22}$$

Figure 11 depicts the functions (19, ..., 22) for particular values of m_1, m_2, σ_1^2 and σ_2^2 . The parameters m_1 and m_2 determine 50 percent levels under ascending and descending conditions. These 50 percent levels have been interpreted as critical transition parameters for participant populations [7]. The variances σ_1^2 and σ_2^2 determine the slope of the functions $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ at $\alpha = m_1$ and $\alpha = m_2$, respectively. For example, for large values the functions $p_{2H,ascend}(\alpha)$ and $p_{2H,descend}(\alpha)$ increase slowly, whereas for very small values the functions $p_{2H,ascend}(\alpha)$ and $p_{2H,descend}(\alpha)$ jump from 0 to 100 at $\alpha = m_1$ and $\alpha = m_2$. In Fig. 11 the case $\sigma_1^2 = \sigma_2^2$ and $m_2 > m_1$ is shown. Population

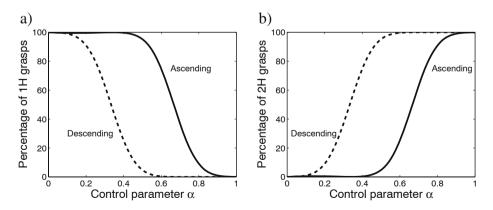


Fig. 11 Percentages of one-handed grasps (panel a) and two-handed grasps (panel b) as functions of the control parameter α for the ascending (solid lines) and descending condition (dashed lines) as computed from Eqs. 19, ..., 22. Parameters as used in Fig. 10

results $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ have been reported in several empirical studies [4, 5, 7]. The empirical graphs presented in these studies correspond qualitatively to the functions $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ defined by Eqs. 19, ..., 22.

2.2.2 Mean performance of individuals

In Section 2.2.1, we derived the functions $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ under the assumption that the parameters of our system (1) and (12) vary between participants. The technical arguments made in Section 2.2.1 remain valid if we assume alternatively that the system parameters vary between trials performed by a single individual. In this case, we can re-interpret the function $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ as describing the mean grasping behavior of individuals. In fact, in [1] sigmoidal functions for individual participants have been reported that correspond qualitatively to the functions $p_{1H/2H,ascend}(\alpha)$ and $p_{1H/2H,descend}(\alpha)$ defined by Eqs. 19, ..., 22.

3 Discussion

We studied body-scaled transitions in human grasping from a theoretical perspective using a dynamical system defined by Eq. 1 that was originally proposed in [37]. Equation 1 describes the evolution of the amplitudes (or order parameters) ξ_1 and ξ_2 of unimanual and bimanual grasping modes. If an order parameter becomes finite, then the corresponding grasping mode emerges. If an order parameter vanishes, then the corresponding grasping mode vanishes as well. Our analysis showed that the system (1) accounts for several experimental findings: the hysteresis in grasping modes, and the exclusivity of grasping modes, the interaction between grasping modes, and the existence of biomechanical constraints that make the performance of certain grasping modes (such as the unimanual mode) impossible.

Our analysis suggests that hysteresis of grasping transitions results from the bistability of the underlying order parameter dynamics. That is, although the dynamics described in Eq. 1 should be regarded as an approximation of the actual order parameter dynamics of grasping, our analysis suggests that the actual order parameter dynamics will also involve a bistable parameter domain that gives rise to the hysteresis patterns observed in experimental studies.

By means of Eq. 1 we demonstrated that a winner-takes-all dynamics is tailored to describe transitions between mutually exclusive grasping modes. Again, we may generalize this result by stating that human-environmental systems that feature mutually exclusive behavioral patterns are likely to feature an underlying winner-takes-all dynamics. The winner-takes-all dynamics may be approximately described by Eq. 1 or by another winner-takes-all model.

We exploited the attention parameters λ_1 and λ_2 of Eq. 1 in order to distinguish between biomechanical limitations and the mode-mode interactions that may result in grasping transitions. If an attention parameter of a grasping mode becomes negative, then a stationary finite order parameter of that mode no longer exists because the fixed point of the mode entirely vanishes. In this sense, a mode with a negative attention parameter is no longer part of the repertoire of available behavioral patterns. For this reason we interpreted in Section 2.1.8 negative attention parameters as counterparts to biomechanical constraints that make the performance of behavioral patterns impossible. For example, if we ask a participant to grasp a cube with index finger and thumb, then the participant simply can

not perform this task with one hand if the size of the cube exceeds the span between index finger and thumb. This experimental condition would correspond to a negative attention parameter λ_1 for the unimanual grasping mode. In contrast, if an attention parameter is positive but critically smaller than the other attention parameter, then the corresponding grasping mode is available but unstable. In this case due to mode-mode interactions a mode cannot be performed although it is part of the repertoire of available behavioral patterns. For example, in the descending condition a transition from bimanual to unimanual grasping occurs as illustrated schematically in Fig. 3. However, if we would instruct participants in the descending condition to grasp objects always with two hands even if the objects become relatively small, the transition would not occur or would be shifted to very small object sizes. In view of Eq. 1 the instruction to use bimanual grasps would imply that the order parameter ξ_1 is fixed to zero. Under the condition $\xi_1 = 0$, Eq. 1 reduces to $d\xi_2/dt = \lambda_2\xi_2 - C\xi_2^3$ and the two-to-one hand transition for $\lambda_2 > 0$ shown in Fig. 9 (panel b) would disappear. This illustrates that if ξ_1 is *not* fixed to zero, the two-to-one hand transition at $\lambda_2 = \lambda_1/g > 0$ shown in Fig. 9 (panel b) is due to a mode-mode interaction. Since human behavior is frequently subjected to biomechanical constraints, dynamical systems approaches to human behavior in general should account for the aforementioned two types of transitions. Equation 1 helps us to illustrate explicitly in the context of grasping transitions how this can be done.

We linked the attention parameters λ_1 , λ_2 to the experimentally accessible object-body scale ratio α . To this end, in Section 2.1.9 we wrote the attention parameters λ_1 and λ_2 as functions of α : $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$. Taking a dynamical systems perspective, α corresponds to a control parameter. In Section 2 we showed that α captures the unimanual and bimanual grasping mode in a dynamical framework. Equations 1 and 12 predict, for any time point *t*, for any given pair $\xi_1(t)$ and $\xi_2(t)$, and for any given parameter α , which grasping mode will be selected. By solving Eqs. 1 and 12 numerically (see simulations in Fig. 9) and analytically (see the fixed point analysis in Sections 2.1.1,...,2.1.5) from Eqs. 1 and 12 a mapping

$$\{\xi_1(t), \xi_2(t), \alpha\} \to \{\xi_{1,st}, \xi_{2,st}\}$$
 (23)

can be derived. This mapping involves a dynamical aspect because the time point t on the left hand side of Eq. 23 does not equal the time point that occurs on the right hand side of Eq. 23. Dynamic perception as opposed to static or passive perception accounts for the fact that human-environmental systems evolve in time and that the emergence of behavioral patterns is also a process that involves time. Thus, Eqs. 1 and 12 illustrate how the issues of dynamic perception and action can be addressed explicitly by recourse to dynamical systems theory.

In general, order parameters of a system can be obtained by first identifying the entire set of evolution equations that describe the system and second by diagonalizing the linearized parts of these evolution equations [30]. In the context of body-scaled transitions, we are dealing with humans acting and perceiving in environments. Consequently, in order to derive Eq. 1 from a structural (bottom-up) approach we would need to take microscopic evolution equations into account that involve both environmental components and components of human motor control systems. Therefore, in our context, the order parameter dynamics (1), and the potential function $V(\xi_1, \xi_2)$, as well as the attractors and repellers that can be associated with the dynamics (1), reflect both environmental components and structural components of human actors. In particular, we argued that the parameters λ_1 , λ_2 , B, and C depend on a control parameter α that represents an object-size to body-scale ratio. However, other properties of human-environmental systems such as the age and gender of actors, the mass of objects, and the availability of tools that assist grasping of larger objects may affect these parameters as well. In fact, such human-environment properties have been addressed in experimental studies [1–7]. Future efforts may be devoted to explore the impacts of such properties on body-scaled transitions from a dynamical systems perspective centered on Eq. 1.

In our study we did not focus on the magnitudes of the order parameters ξ_1 and ξ_2 . The reason for this is that according to our approach the deterministic grasping behavior is completely determined by two types of fixed points. Fixed point *I* has coordinates $\xi_1 > 0, \xi_2 = 0$ and reflects unimanual grasps. Fixed point *II* has coordinates $\xi_1 = 0, \xi_2 > 0$ and reflects bimanual grasping patterns. That is, all states with $\xi_1 > 0, \xi_2 = 0$ are mapped to one-handed grasping and all states with $\xi_1 = 0, \xi_2 > 0$ are considered as states that represent two-handed grasping. However, when stochastic aspects of grasping transitions are studied, the magnitudes of the order parameters have to be examined carefully. Close to critical points in the parameter space of Eq. 1 at which grasping mode attractors of the potential $V(\xi_1, \xi_2)$ vanish, fluctuations may induce transitions between attractors of unimanual and bimanual grasping. Such transitions depend on the magnitudes of the order parameters ξ_1 and ξ_2 and particularly on the height of the potential barriers that separate minima of $V(\xi_1, \xi_2)$. A stochastic account in this direction, however, is beyond the scope of the present study.

The order parameter dynamics (1) was proposed by Haken (see [37]) in the context of pattern recognition processes. Haken not only addressed two order parameters but an arbitrarily large set of order parameters. Accordingly, the recognition of a particular face or scene implies that the order parameter of that face or scene becomes finite. In Section 2 we interpreted the order parameters ξ_1 and ξ_2 of Eq. 1 as amplitudes of two behavioral modes in order to make contact with experimental studies on transitions between unimanual and bimanual grasping [1–7]. In line with Haken's original model, we may interpret ξ_1 and ξ_2 as amplitudes of different perceptual judgments or cognitive modes. In grasping experiments there are two fundamental perceptual judgments. Participants judge an object as something that they would grasp with one hand (1H cognitive mode) or participants judge an object as something that they would grasp with two hands (2H cognitive mode). In fact, in [7] grasping was studied this way. Participants were asked to judge whether or not they would grasp an object with one or two hands. In these judgment trials grasping acts themselves were not performed. Transitions were observed between the 1H and 2H cognitive or perceptual judgment modes. Thus, when interpreting ξ_1 and ξ_2 in this latter way, Eq. 1 in principle also applies to the judgment findings reported in [7].

In [7] grasping was also studied from a social perspective. Participants were grouped into pairs. These pairs of participants were asked to transport planks of different sizes from one position to another. The participants could carry these planks alone or as couples. In [7] one-person grasping as well as two-person grasping was observed. Transitions between the one-person and two-person grasping mode were observed as well as hysteresis. The results found in the study on social interaction can also be addressed from the dynamical systems perspective developed in Section 2 by re-interpreting the order parameters ξ_1 and ξ_2 as order parameters of modes of social behavior.

The winner-takes-all model (1) allows for the study of individual behavior (see Section 2.1) and the behavior of a population (see Section 2.2). In fact, experimental studies have revealed qualitative differences in individual performance [1, 7]. Future research may be directed at studying individual performance differences from the perspective of Eq. 1. In particular, questions regarding the extent to which performance differs qualitatively

between individuals (or pairs of individuals) can be related to the parameters λ_1 , λ_2 , *B*, and *C* of Eq. 1.

Finally, in Section 2.1.9 the attention parameters λ_1 and λ_2 were assumed to depend linearly on the control parameters. For the sake of simplicity we assumed that λ_1 and λ_2 increase and decrease by the same amount for a varying control parameter α . That is, in Eq. 12 we used the same β -factor for the functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$. A natural generalization of Eq. 12 involves different β -factors such that we have

$$\lambda_1 = \lambda_{1,0} - \beta_1 \alpha ,$$

$$\lambda_2 = \lambda_{2,0} + \beta_2 \alpha$$
(24)

with β_1 , $\beta_2 > 0$. In general, the functions $\lambda_1(\alpha)$ and $\lambda_2(\alpha)$ may depend in a nonlinear fashion on the control parameter α . Generalized relationships such as given by Eq. 24 may be useful in order to relate experimental data to the order parameter dynamics (1). As mentioned in Section 2.1.9, in the most general case, the nonlinearity parameters *B* and *C* will depend on α as well. This would imply that the factor *g* changes with α . In this case the transition between one-handed and two-handed grasps in the ascending condition would occur at the critical control parameters α_c satisfying

$$\lambda_2(\alpha_c) = g(\alpha_c)\lambda_1(\alpha_c) \tag{25}$$

whereas the transition between two-handed and one-handed grasps in the descending condition would occur at

$$\lambda_1(\alpha_c) = g(\alpha_c)\lambda_2(\alpha_c) . \tag{26}$$

A challenge for future experimental and theoretical work, therefore, is to design experiments and to develop data analysis methods such that the functions $\lambda_1(\alpha)$, $\lambda_2(\alpha)$, and $g(\alpha)$ can be extracted from experimental data. Identifying the information that specifies the different behavioral modes as well as manipulating different biomechanical, cognitive, and social variables will be essential to such experimentation.

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