

Melting and crystallization of colloidal hard-sphere suspensions under shear

Yu Ling Wu^a, Didi Derks^{a,b}, Alfons van Blaaderen^a, and Arnout Imhof^{a,1}

^aSoft Condensed Matter, Debye Institute for Nanomaterials Science, Utrecht University, Princetonplein 5, 3584 CC, Utrecht, The Netherlands; and ^bLaboratoire de Physique Statistique, Ecole Normale Supérieure, 24 Rue Lhomond, 75231 Paris Cedex 05, France

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Shear-induced melting and crystallization were investigated by confocal microscopy in concentrated colloidal suspensions of hard-sphere-like particles. Both silica and polymethylmethacrylate suspensions were sheared with a constant rate in either a counter-rotating parallel plate shear cell or a counterrotating cone-plate shear cell. These instruments make it possible to track particles undergoing shear for extended periods of time in a plane of zero velocity. Although on large scales, the flow profile deviated from linearity, the crystal flowed in an aligned sliding layer structure at low shear rates. Higher shear rates caused the crystal to shear melt, but, contrary to expectations, the transition was not sudden. Instead, although the overall order decreased with shear rate, this was due to an increase in the nucleation of localized domains that temporarily lost and regained their ordered structure. Even at shear rates that were considered to have melted the crystal as a whole, ordered regions kept showing up at times, giving rise to very large fluctuations in 2D bond-orientational order parameters. Low shear rates induced initially disordered suspensions to crystallize. This time, the order parameter increased gradually in time without large fluctuations, indicating that shear-induced crystallization of hard spheres does not proceed via a nucleation and growth mechanism. We conclude that the dynamics of melting and crystallization under shear differ dramatically from their counterparts in quiescent suspensions.

The majority of complex fluids are non-Newtonian liquids. That is, when subjected to a shear flow, they exhibit shear thinning or shear thickening behavior. It has long been established that the macroscopic properties of a material are coupled to its microstructure. To understand the macroscopic behavior of flowing complex fluids, much research has been done on revealing the microstructure under shear.

Apart from shear thinning and shear thickening, also shear banding, which is characterized by a discontinuous jump in the flow profile, is observed for many complex fluids (1). For worm-like micellar systems this has been extensively studied (2–5). Shear banding has also been observed in rod-like colloidal suspensions (6) and in crystallizing suspensions of spherical colloids (7–10), but for the latter, the number of studies is limited. Zukoski and coworkers (11–14) found the origin of shear banding with rheometry and small angle neutron scattering. They found that colloidal crystals shear thin discontinuously and that this is associated with a transition from a polycrystalline structure to a sliding layer structure. For not too soft potentials, these sliding layers have a hexagonal symmetry (15, 16). By using microscopy, it has been shown that this transition also leads to a shear banded flow (7).

Shear has a dual influence on the order in colloidal suspensions. At high shear rates, it can disorder, or melt, a colloidal crystal (17–19), but at low shear rates or at low-amplitude oscillatory shear, it can induce order (20–22). Shear changes the growth kinetics, i.e., the induction time, density of nuclei, and crystal growth rate (23). This has been studied on soft spheres both with experiments (24, 25) and computer simulations (26–28). One of the characteristics reported for crystallization under flow is that the crystals align in 1 direction. Crystals with a

different orientation are destroyed, after which they recrystallize in the preferred direction. Third, the nucleation rate depends on the stability of the crystal and liquid under shear. For soft spheres, the melting line shifts when a shear flow is applied (25, 26). For hard spheres, it is unknown whether or how the melting phase transition changes by shear, but a disordering transition is clearly evident from scattering experiments (16, 17).

Most studies of complex fluids under shear have been done with light, X-ray, or neutron scattering. Scattering techniques have the disadvantage that the data are averages over the sampled volume; information about local processes cannot be obtained. Real-space experiments, using microscopy, have been undertaken to study behavior under shear in more detail. Two-dimensional colloidal crystals at a liquid interface were observed in real-space (29, 30) showing the motion of strings of particles past each other when sheared. Shear-induced crystallization has been observed in 3 dimensions in hard sphere suspensions under low-frequency oscillatory shear by taking snapshots at the extremes of the oscillation cycle (31). When crystals were confined to a narrow gap by applying oscillatory shear, several unusual crystal structures were found (32). Finally, Palberg et al. (33) used a special microscopy method in which positional correlations of particles could be directly imaged but not individual particles themselves. For confined, very soft colloids, these authors observed a zig-zag motion of sliding hexagonal layers as well as a disordering transition at higher shear rates (34, 35). Despite these efforts, a unified picture of the structural transitions of colloidal crystals under shear is still far from complete. Shear alignment of colloids has already been used to align large colloidal crystals for photonic applications (36) even on a semiindustrial scale (37–39), and it has also been reported in the related process of spin-coating (40–43).

In previous work we examined sheared colloidal crystals in 3D using confocal microscopy and a counter rotating cone-plate shear cell (44, 45). The zig-zag motion of sliding hexagonal layers could be clearly observed and the local flow profile could be determined. As the shear rate was increased displacements of particles from their lattice positions grew larger, and particle diffusivity increased. The instrument used enables one to determine structure on a single-particle level. The counterrotation principle creates a plane of zero velocity in which a collection of particles can be tracked for an extended time. In the present work, we also used a counter translating parallel plate shear cell with a small enough plate separation that we could observe individual particles throughout the gap with a confocal microscope (46). This enabled us to obtain local information about the complete system. As we will show, local information is valuable in characterizing the sliding-layer structure and the observation of shear banding. Moreover, it reveals interesting differences

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¹To whom correspondence should be addressed. E-mail: a.imhof@uu.nl.

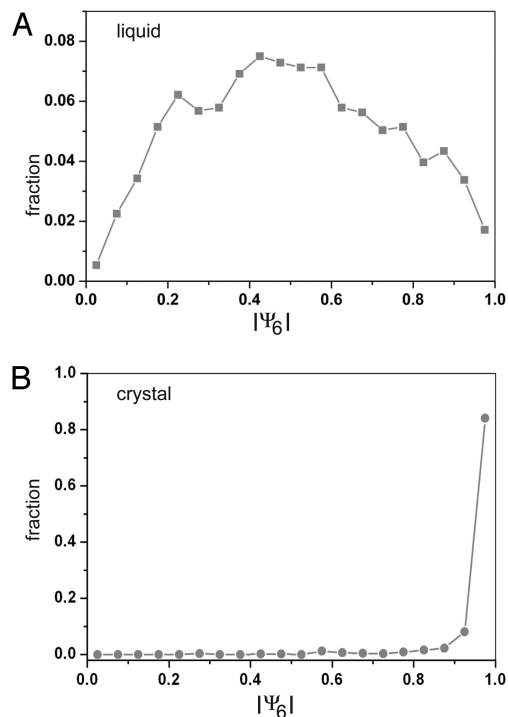


Fig. 3. Histograms of ψ_6 values of particles in an unsheared system before (A) and after (B) crystallization had set in. The data were collected on a suspension at the same volume fraction as in the shear melting and crystallization experiments.

The order parameter ψ_6 of particle i is a summation over all its n_c nearest neighbors. $\theta(r_{ij})$ is the angle between the vector connecting particle i to j and an arbitrary fixed reference axis. In a perfect hexagonal layer, the angles between the 6 nearest neighbors are multiples of 60° and $|\psi_6| = 1$. Fig. 3 shows histograms of the $|\psi_6|$ values of the particles in an unsheared sample before it had crystallized (Fig. 3A) and after complete crystallization (Fig. 3B). Before crystallization, the histogram was broad and had a maximum of approximately $|\psi_6| = 0.5$. After crystallization, almost all particles had a $|\psi_6| > 0.8$.

Fig. 4 shows the average value of $|\psi_6|$ of all ≈ 100 particles in a frame as a function of time for the silica and the PMMA systems, respectively. For easy comparison, data taken at various shear rates are shown in the same graph with an offset in time. The silica system was always ordered at the start of an experiment, whereas the PMMA system had already been shearing at a constant rate for several minutes. When an initially crystalline suspension was sheared, the order showed a nearly instant decrease, after which $\langle |\psi_6| \rangle$ kept fluctuating wildly around a certain average value. The same fluctuations in order parameter were observed in suspensions that had already been shearing for some time. The snapshots in Fig. 2 are accompanied by their $|\psi_6|$ to illustrate the large fluctuations in order. At higher shear rates, this time-averaged value decreased, but at the same time, the fluctuations became larger, making it difficult to say at which shear rate a suspension had completely melted. Even at the highest shear rates, ordered domains regularly nucleated and melted again.

We further analyzed the distribution of $|\psi_6|$ values by determining $|\psi_6|$ for each individual particle over the whole time span and making histograms of these values (Fig. 5). The histograms of systems sheared at low shear rates ($\dot{\gamma} < 2 \text{ s}^{-1}$ for the silica system and $\dot{\gamma}$ up to 2.3 s^{-1} for the PMMA system) show that the majority of particles had an order above $|\psi_6| = 0.9$, similar to the histogram of a pure crystal (Fig. 3B) but that this majority

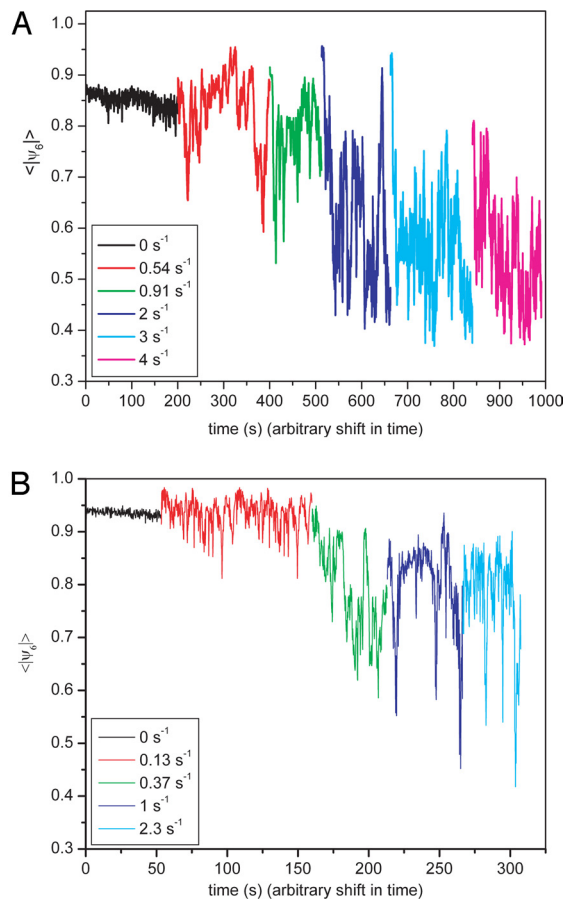


Fig. 4. The average order, quantified by ψ_6 , of initially crystalline suspensions sheared at increasing shear rates. (A) The results of a system of $1.2\text{-}\mu\text{m}$ silica particles in EPTA. (B) The results for PMMA particles in CHB/decaline. The graphs at the different shear rates are shifted horizontally for clarity.

shrinks with increasing shear rate. The overall features were the same for the 2 systems, except that the PMMA system was even more ordered than the silica system, probably owing to a higher volume fraction. When the shear rate was so high that the crystal had largely melted, a clear signature of ordered domains remained present, which is shown by the almost bimodal distribution. Otherwise, the broadness of the peak of $|\psi_6|$ values of the disordered particles is very similar to that of the particles in a pure unsheared liquid (Fig. 3A).

Shear-Induced Crystallization. In a different set of experiments, we started with a disordered suspension, which would have spontaneously crystallized had it been given sufficient time. Upon applying a low shear rate, the particles started to order much more rapidly. For the silica system, we first applied a very high shear rate to a suspension with a volume fraction in the fluid–crystal coexistence region to completely melt the system. Immediately after that, a low continuous shear rate of $\dot{\gamma} = 0.45 \text{ s}^{-1}$ was applied. We then monitored the local ordering of the particles in the zero-velocity plane. Snapshots of a time series are shown in Fig. 6. It can be seen that the order increased gradually until a hexagonal layer had been formed, of which the order did not change significantly in time. The orientation of the hexagonal layer was such that a close-packed line was always parallel to the flow direction. Crystallization of the PMMA system at a shear rate of 0.50 s^{-1} occurred in much the same way. In neither of the systems was the formation of locally ordered nuclei observed. Rather, the suspensions gradually ordered as a whole, i.e.,

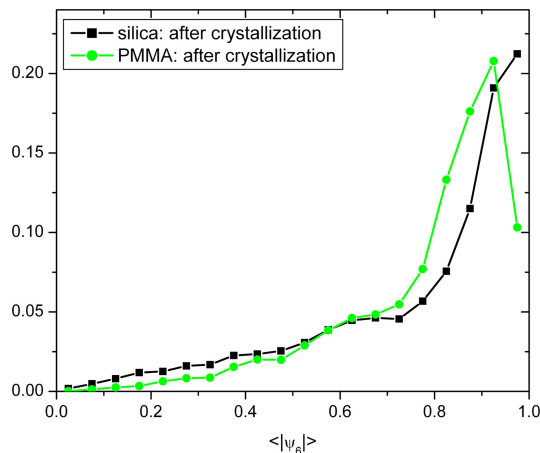


Fig. 8. Histogram of the individual ψ_6 values of particles during shear-induced crystallization after the transition into a crystal.

with the system spending more time in the liquid state at higher drive (51, 52). This result is reminiscent of the melting scenario that we find, but it remains to be seen whether the fluctuations survive in bulk systems and when hydrodynamic interactions (HI) are included.

The melting scenario that we describe suggests that for hard spheres, it is difficult, if not impossible, to pinpoint a shear melting transition, such as has been described for soft-sphere suspensions on the basis of nonequilibrium Brownian dynamics simulations (26, 27, 48, 49) and scattering experiments (25). Why this should be so different is unclear. It is possible that the behavior is sensitive to the balance between electrostatic forces and hydrodynamic interactions. The nonequilibrium Brownian dynamics simulations do not include HI, but Stokesian dynamics simulations show that for hard spheres, HI strongly influence shear-induced structure (23, 53). It would be very interesting, therefore, to do particle-tracking experiments similar to ours on soft spheres.

Shear-induced crystallization of initially disordered suspensions followed a different scenario than shear melting. Here, the order parameter increased much more gradually in time with only small fluctuations. Localized ordered domains growing in a surrounding fluid, as would be expected in a nucleation and growth-type of mechanism, were not observed. This is again different from the picture painted by Brownian dynamics simulations on soft spheres (26, 28).

It is remarkable that the scenarios for shear melting and shear crystallization appear to be so different. Shear-induced melting is apparently a local process in which small ordered or disordered domains appear and disappear. Conversely, in shear-induced crystallization, the particles ordered collectively, without much local or temporal fluctuation. Strikingly, the scenario of melting and crystallization induced by shear described here are approximately opposite to that without shear. In the latter case, which is well established, it is crystallization that takes place through nucleation and growth, whereas melting proceeds uniformly from the surface of a crystal inward.

Materials and Methods

Colloidal Suspensions. We studied 2 types of colloidal suspensions: a system with silica spheres and one with polymethylmethacrylate (PMMA) spheres. The silica system consisted of 1.2- μm diameter silica particles dispersed in

ethoxylated trimethylolpropane triacrylate (EPTA) [MW 428, viscosity 0.072 Pa·s (measured with a TA Instruments AR1000-N rheometer), density = $1.1 \times 10^3 \text{ kg/m}^3$]; Aldrich]. This polar solvent matches the refractive index of the particles. In this system, the interparticle distance in a crystal formed by sedimentation was 1.1 times the particle diameter. The particle volume fraction was $\phi = 0.50$, which is in the fluid–crystal coexistence region. A 0.4- μm diameter core of the particles had been labeled with FITC by the method described in ref. 54. This made it possible to image the particles with a confocal microscope after excitation of the dye with the 488-nm line of the Ar laser of the confocal microscope.

The PMMA system consisted of 1.67- μm diameter rhodamine labeled PMMA particles that were made by dispersion polymerization and sterically stabilized by a layer of poly(12-hydroxystearic acid) (PHS) (55, 56). They were dispersed in a 3:1 wt/wt mixture of cyclohexylbromide and *cis*-decalin, saturated with tetrabutylammoniumbromide (TBAB). In this mixture, the particles were nearly density- and refractive index-matched, and they behaved hard-sphere like (44). The particle volume fraction was $\phi = 0.57$, which is in the crystal part of the phase diagram.

Shear Cells. The experiments with the silica particles were performed in the parallel plate shear cell described and characterized in ref. 46. The PMMA system was sheared with the cone-plate shear cell described in detail in ref. 44. These shear cells could be mounted on top of an inverted confocal microscope (TCS-SP2; Leica).

For the parallel plate shear cell, the dimensions of the top glass plate were 30 mm in the *x* (velocity)-direction and 15 mm in the *y* (vorticity)-direction. The bottom glass plate was a standard no. 1 (thickness 0.13–0.16 mm) microscopy glass slide of size 50×24 mm. Because the working distance of the objective lens was 100 μm , when imaging through a no. 1 glass slide, the gap width was set $< 100 \mu\text{m}$. This allowed us to image particles over the complete gap width of the cell. A parallel plate geometry is used in an oscillatory mode by necessity, but the large plates and small gap allowed a constant shear to be sustained unidirectionally for several minutes in the present experiments. To prevent unwanted capillary forces that might disturb the flow of the suspension, we overfilled the cell and used as much as 100 μL of dispersion for each experiment. The dispersion was placed on the bottom glass plate after which the top glass plate was lowered onto the droplet.

The cone of the cone-plate shear cell had an angle of 1° or 4° . The bottom plate was a no. 1 glass plate with a diameter of 6.5 cm. At the position of imaging, the gap width was 0.4 or 1.7 mm depending on the cone used.

In both shear cells, the bottom plate and top plate (cone) translated (rotated) in opposite directions. This established a plane of zero velocity with respect to the microscope. Particles in this plane had no net velocity and could therefore be imaged over a long period. In most experiments, the zero-velocity plane was a few layers above the bottom plate. This distance was such that wall effects were negligible.

Imaging. Imaging was done with an inverted Leica TCS-SP2 confocal scanning laser microscope. We used a $100\times$ Leica immersion objective with N.A. 1.4. The 488-nm laser line of an Argon laser was used for imaging the FITC-labeled silica particles and the 543-nm laser HeNe line for the rhodamine-PMMA particles. Scanning along the *z*-axis was done by using a piezo focusing drive (Physik Instrumente). This direction coincided with the gradient direction of the shear flow field. The positions of the particles were determined by using algorithms similar to those of Crocker and Grier (57).

There are various methods to determine the flow profile. We used the method introduced by Derks et al. (44). Images were taken in the velocity-gradient (*xz*)-plane at a slow scan rate. Such an image is built up by sequential scanning of horizontal lines along the *x* (velocity)-direction. While a particle is being scanned, it translates because of the shear and therefore appears deformed in the image. The local velocity is then calculated from the deformation by cross-correlating consecutive image lines, assigning the average particle displacement to the maximum correlation.

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