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A lightweight universe?

NETA A. BAHCALL^{*} AND XIAOHUI FAN

Princeton University Observatory, Princeton, NJ 08544

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ABSTRACT How much matter is there in the universe? Does the universe have the critical density needed to stop its expansion, or is the universe underweight and destined to expand forever? We show that several independent measures, especially those utilizing the largest bound systems known clusters of galaxies—all indicate that the mass-density of the universe is insufficient to halt the expansion. A promising new method, the evolution of the number density of clusters with time, provides the most powerful indication so far that the universe has a subcritical density. We show that different techniques reveal a consistent picture of a lightweight universe with only ~20–30% of the critical density. Thus, the universe may expand forever.

Standard models of inflation-how the universe expanded in the beginning-as well as general arguments that demand no "fine tuning" of cosmological parameters, predict a flat universe with the critical density needed to just halt its expansion. The critical density, $1.9 \times 10^{-29} h^2$ g cm⁻³ (where h refers to Hubble's constant; see below), is equivalent to ~ 10 protons per cubic meter; this density provides the gravitational pull needed to slow down the universal expansion that began with the Big Bang approximately 15 billion years ago and will eventually bring it to a halt. So far, however, only a small fraction of the critical density has been detected, even when all the unseen "dark matter" in galaxy halos and clusters of galaxies is included. There is no reliable indication so far that most of the matter needed for closing the universe does in fact exist. Here we show that several independent observations of clusters of galaxies, including the mass-to-light ratio of clusters, the high baryon fraction in clusters, and the observed evolution of cluster abundance, all portray a consistent picture of a subcritical universe.

Weighing Clusters

Rich clusters of galaxies-families of hundreds of galaxies held together by the gravitational potential of the clusterare the most massive bound objects known. Cluster masses can be directly and reliably determined by using three independent methods: (i) the motion (velocity dispersion) of galaxies within clusters reflects the dynamical cluster mass, within a given radius, assuming that the clusters are in hydrostatic equilibrium (1-3); (ii) the temperature of the hot intracluster gas, like the galaxy motion, traces the cluster mass (4–6); and (iii) gravitational lensing distortions of background galaxies can be used to directly measure the intervening cluster mass that causes the distortions (7-10). All three independent methods yield consistent cluster masses (typically within radii of $\sim 1 \text{ Mpc} \sim 3 \times 10^6$ light-years), indicating that we can reliably determine cluster masses within the observed scatter $(\pm 30\%).$

Mass-to-Light Ratio of Clusters

Let us begin with the simplest argument for a low density universe. The masses of rich clusters of galaxies range from $\sim 10^{14}$ to $10^{15}h^{-1}M_{\odot}$ within $1.5h^{-1}$ Mpc radius of the cluster center (where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ denotes Hubble's constant, representing the expansion rate of the universe). When normalized by the cluster luminosity, a median mass-to-light ratio of $M/L_B \simeq 300 \pm 100h$ in solar units (M_{\odot}/L_{\odot}) is observed for rich clusters, independent of the cluster luminosity, velocity dispersion, or other parameters (3, 11). (L_B is the total luminosity of the cluster in the blue band, corrected for internal and Galactic absorption.) When integrated over the entire observed luminosity density of the universe, this mass-to-light ratio yields a mass density of $\rho_m \approx 0.4 \times 10^{-29} h^2 \text{ g cm}^{-3}$, or a mass density ratio of $\Omega_m = \rho_m / \rho_{crit} \approx 0.2 \pm 0.07$ (where ρ_{crit} is the critical density needed to close the universe). The inferred density assumes that all galaxies exhibit the same high M/L_B ratio as clusters and that mass follows light on large scales. Thus, even if all galaxies have as much mass per unit luminosity as do massive clusters, the total mass of the universe is only $\sim 20\%$ of the critical density. If one insists on esthetic grounds that the universe has a critical density ($\Omega_m = 1$), then most of the mass of the universe has to be unassociated with galaxies (i.e., with light). On large scales ($\geq 1.5h^{-1}$ Mpc) the mass has to reside in "voids" where there is no light. This would imply, for $\Omega_m = 1$, a large bias in the distribution of mass versus light, with mass distributed considerably more diffusely than light.

Is there a strong bias in the universe, with most of the dark matter residing on large scales, well beyond galaxies and clusters? A recent analysis of the mass-to-light ratio of galaxies, groups, and clusters by Bahcall, Lubin, and Dorman (11) suggests that there is not a large bias. The study shows that the M/L_B ratio of galaxies increases with scale up to radii of $R \sim 0.2 h^{-1}$ Mpc, due to very large dark halos around galaxies (see also refs. 12 and 13). The M/L ratio, however, appears to flatten and remain approximately constant for groups and rich clusters from scales of ~ 0.2 to at least $1.5h^{-1}$ Mpc and possibly even beyond (Fig. 1). The flattening occurs at $M/L_B \simeq$ 200–300h, corresponding to $\Omega_m \simeq 0.2$. (An M/L_B ~ 1350 h is needed for a critical density universe, $\Omega_m = 1$.) This observation contradicts the classical belief that the relative amount of dark matter increases continuously with scale, possibly reaching $\Omega_m = 1$ on large scales. The available data suggest that most of the dark matter may be associated with very large

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Abbreviations: Mpc, megaparsec $\simeq 3 \times 10^6$ light-years; h, Hubble's constant in units of 100 km s⁻¹Mpc⁻¹; Ω_m , cosmological mass density parameter of the universe in units of the critical density; M/L, mass-to-light ratio; Ω_b , baryon mass density in the universe in units of the critical density.

^{*}To whom reprint requests should be addressed. e-mail: neta@astro. princeton.edu.



FIG. 1. The dependence of mass-to-light ratio, M/L_B , on scale, R, for average spiral galaxies (blue symbols), elliptical galaxies (green), and groups and clusters (red). [Adapted from Bahcall, Lubin, and Dorman (11)]. The large scale point at ~15h⁻¹ Mpc represents Virgo cluster infall motion results (11). The location of $\Omega_m = 1$ and $\Omega_m = 0.3$ are indicated by the horizontal lines. A flattening of M/L_B is suggested at $\Omega_m \simeq 0.2 \pm 0.1$.

dark halos of galaxies and that clusters do not contain a substantial amount of additional dark matter, other than that associated with (or torn-off from) the galaxy halos, plus the hot intracluster gas. This flattening of M/L with scale, if confirmed by further larger-scale observations, suggests that the relative amount of dark matter does not increase significantly with scale above $\sim 0.2h^{-1}$ Mpc. In that case, the mass density of the universe is low, $\Omega_m \sim 0.2$ –0.3, with no significant bias (i.e., mass approximately following light on large scales).

Baryons in Clusters

Clusters contain many baryons, observed as gas and stars. Within $1.5h^{-1}$ Mpc of a rich cluster, the x-ray emitting gas contributes $\sim 6h^{-1.5}\%$ of the cluster virial mass (14–16). Stars contribute another $\gtrsim 4\%$. The baryon fraction observed in clusters is thus:

$$\Omega_b / \Omega_m \gtrsim 0.06 h^{-1.5} + 0.04.$$
 [1]

Standard Big Bang nucleosynthesis limits the baryon density of the universe to (17, 18):

$$\Omega_h \simeq 0.017 h^{-2}.$$
 [2]

These facts suggest that the baryon fraction observed in rich clusters (Eq. 1) exceeds that of an $\Omega_m = 1$ universe $(\Omega_b/(\Omega_m = 1) \approx 0.017h^{-2}$; Eq. 2) by a factor of ≥ 3 (for $h \geq 0.5$). Because detailed hydrodynamic simulations (14, 16) show that baryons do not segregate into rich clusters, the above results imply that either the mean density of the universe is lower than the critical density by a factor of ≥ 3 , or that the baryon density is much larger than predicted by

nucleosynthesis. The observed high baryonic mass fraction in clusters (Eq. 1), combined with the nucleosynthesis limit (Eq. 2), suggest (for $h \approx 0.5 - 1$)

$$\Omega_m \simeq 0.2 \pm 0.1.$$
 [3]

Evolution of Cluster Abundance

In a recent study by Bahcall, Fan, and Cen (19, 20), we show that the evolution of the number density of clusters as a function of cosmic time (or redshift) provides a powerful constraint on Ω_m (19–22). The growth of high-mass clusters from initial Gaussian fluctuations depends strongly on the cosmological parameters Ω_m and σ_8 (where σ_8 is the root-mean-square mass fluctuation on $8h^{-1}$ Mpc scale; refs. 23–27). In low-density models, density fluctuations evolve and freeze out at early times, thus producing only relatively little evolution at recent times (z < 1). In an $\Omega_m = 1$ universe, the fluctuations start growing more recently, thereby producing strong evolution in recent times; a large increase in the abundance of massive clusters is expected from $z \sim 1$ to $z \sim 0$. The evolution is so strong in $\Omega_m = 1$ models that finding even a few Coma-like clusters at z > 0.5 over $\sim 10^3 \text{ deg}^2$ of sky contradicts an $\Omega_m = 1$ model where only $\sim 10^{-2}$ such clusters would be expected (when normalized to the observed presentday cluster abundance). The evolution of the number density of Coma-like clusters was recently determined from observations and compared with cosmological simulations (19-21). The data show only a slow evolution of the cluster abundance to $z \sim 0.5$, with $\sim 10^2$ times more clusters observed at these redshifts than expected for $\Omega_m = 1$. The results yield $\Omega_m\simeq 0.3\pm 0.1.$

The evolutionary effects increase with cluster mass and with redshift. The existence of the three most massive clusters observed so far at $z \sim 0.5$ –0.9 places the strongest constraint yet on Ω_m and σ_8 . These clusters (MS0016+16 at z = 0.55, MS0451 - 03 at z = 0.54, and MS1054 - 03 at z = 0.83, from the Extended Medium Sensitivity Survey, EMSS; ref. 28) are nearly twice as massive as the Coma cluster and have reliably measured masses (including gravitational lensing masses, temperatures, and velocity dispersions; refs. 3, 9, 29-32). These clusters posses the highest masses ($\geq 8 \times 10^{14} h^{-1} M_{\odot}$ within 1.5h⁻¹ comoving Mpc radius), the highest velocity dispersions $(\geq 1200 \text{ km s}^{-1})$, and the highest temperatures $(\geq 8 \text{ kev})$ in the z > 0.5 EMSS survey. The existence of these three massive distant clusters, even just the existence of the single observed cluster at z = 0.83, rules out Gaussian $\Omega_m = 1$ models for which only $\sim 10^{-5}$ $z \sim 0.8$ clusters are expected instead of the 1 cluster observed (or $\sim 10^{-3} z > 0.5$ clusters expected instead of the 3 observed). [See Bahcall and Fan (29).]

In Fig. 2, we compare the observed versus expected evolution of the number density of such massive clusters. The expected evolution is based on the Press–Schechter (23) formalism that describes the growth of structure in a hierarchical universe with standard initial Gaussian density fluctuations; this formalism agrees well with direct numerical cosmological simulations (20, 26). The expected evolution is shown for different Ω_m values (each with the appropriate normalization σ_8 that satisfies the observed present-day cluster abundance, $\sigma_8 \approx 0.5 \Omega_m^{-0.5}$; refs. 26 and 33). The model curves range from $\Omega_m = 0.1$ ($\sigma_8 \approx 1.7$) at the top of the figure (flattest, nearly no evolution) to $\Omega_m = 1$ ($\sigma_8 \approx 0.5$) at the bottom (steepest, strongest evolution). The difference between high and low



FIG. 2. Evolution of the number density of massive clusters as a function of redshift: observed versus expected (for clusters with mass $\geq 8 \times 10^{14} h^{-1} M_{\odot}$ within a comoving radius of $1.5 h^{-1}$ Mpc). [Adapted from Bahcall and Fan (29)]. The expected evolution is presented for different Ω_m values by the different curves. The observational data points (see text) show only a slow evolution in the cluster abundance, consistent with $\Omega_m \simeq 0.2^{+0.15}_{-0.1}$. Models with $\Omega_m = 1$ predict $\sim 10^5$ fewer clusters than observed at $z \sim 0.8$, and $\sim 10^3$ fewer clusters than observed at $z \sim 0.6$.

 Ω_m models is dramatic for these high mass clusters: $\Omega_m = 1$ models predict $\sim 10^5$ times fewer clusters at $z \sim 0.8$ than do $\Omega_m \sim 0.2$ models. The large magnitude of the effect is due to the fact that these are very massive clusters, on the exponential tail of the cluster mass function; they are rare events and the evolution of their number density depends exponentially on their "rarity," i.e., depends exponentially on $\sigma_8^{-2} \propto \Omega_m$ (20, 23, 29). The number of clusters observed at $z \sim 0.8$ is consistent with $\Omega_m \sim 0.2$ and is highly inconsistent with the $\sim 10^{-5}$ clusters expected if $\Omega_m = 1$. The data exhibits only a slow, relatively flat evolution; this is expected only in low Ω_m models. $\Omega_m = 1$ models have a $\sim 10^{-5}$ probability of producing the one observed cluster at $z \sim 0.8$, and, independently, a $\sim 10^{-6}$ probability of producing the two observed clusters at $z \sim 0.55$. These results rule out $\hat{\Omega}_m = 1$ Gaussian models at a very high confidence level. The results are similar for models with or without a cosmological constant. The data provide powerful constraints on Ω_m and σ_8 : $\Omega_m = 0.2^{+0.15}_{-0.1}$ and $\sigma_8 = 1.2 \pm 0.3$ (68% confidence level) (29). The high σ_8 value for the mean mass fluctuations indicates a nearly unbiased universe, with mass approximately tracing light on large scales [because the galaxy fluctuations, which represent the light, exhibit a similar value of $\sigma_{\rm g}({\rm galaxy}) \simeq 1$]. This conclusion is consistent with the suggested flattening of the observed M/L ratio on large scales (Fig. 1).

In Fig. 3 we summarize the four independent Ω_m (σ_8) constraints obtained from the cluster results discussed above: (*i*) the present-day cluster abundance constraint (26, 33) $\Omega_m^{0.5} \approx 0.5/\sigma_8$; (*ii*) the high-redshift ($z \sim 0.5$ –0.9) cluster abundance constraint (29) [the overlap of the $z \sim 0$ and $z \sim 0.5$ –0.9 abundance constraints of (*i*) and (*ii*) yields the cluster evolution constraint discussed above]; (*iii*) the Ω_m derived from the high baryon fraction in clusters; and (*iv*) the Ω_m obtained from cluster masses. The results are all consistent with each other for $\Omega_m = 0.2 \pm 0.1$ and $\sigma_8 = 1.2 \pm 0.2$ (1σ level). $\Omega_m = 1$ models are highly incompatible with these results ($\leq 10^{-6}$ probability).

Summary

We have shown that several independent observations of clusters of galaxies all indicate that the mass-density of the universe is subcritical: $\Omega_m \approx 0.2 \pm 0.1$. A summary of the results, presented in Fig. 3, is highlighted below.

1. The mass-to-light ratio of clusters of galaxies and the suggested flattening of the mass-to-light ratio on large scales suggest $\Omega_m \simeq 0.2 \pm 0.1$.

2. The high baryon fraction observed in clusters of galaxies suggests $\Omega_m \simeq 0.2 \pm 0.1$.

3. The weak evolution of the observed cluster abundance to $z \sim 1$ provides a robust estimate of $\Omega_m \simeq 0.2^{+0.15}_{-0.1}$, valid for any Gaussian models. An $\Omega_m = 1$ Gaussian universe is ruled out as a $\leq 10^{-6}$ probability by the cluster evolution results (Figs. 2 and 3).

4. All the above-described independent measures are consistent with each other and indicate a low-density universe with $\Omega_m \simeq 0.2 \pm 0.1$ (Fig. 3). $\Omega_m = 1$ models are ruled out by the data. While non-Gaussian initial fluctuations, if they exist, will affect the cluster evolution results, they will not affect arguments 1 and 2 above. Gaussian low-density models (with or without a cosmological constant) can consistently explain all the independent observations presented here. These independent cluster observations indicate that we live in a lightweight universe with only ~20–30% of the critical density. Thus, the universe may expand forever.

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FIG. 3. Constraining the mass-density parameter, Ω_m , and the mass fluctuations on $8h^{-1}$ Mpc scale, σ_8 , from several independent observations of clusters: cluster dynamics (blue band); baryon fraction in clusters (pink); present-day cluster abundance ($z \sim 0$; green); and cluster abundance at redshift $z \sim 0.7$ (yellow). (The latter two abundances yield the cluster evolution constraints shown in Fig. 2; see text.) All of these model-independent observations converge at the allowed range of $\Omega_m = 0.2 \pm 0.1$ and $\sigma_8 = 1.2 \pm 0.2$ (68% confidence level; red). The dotted lines illustrate the mean microwave fluctuations constraints, based on the COBE satellite results, for a Cold-Dark-Matter model with h = 0.7 (with and without a cosmological constant, denoted as LCDM and OCDM respectively; both models are consistent, within their uncertainties, with the best-fit $\Omega_m - \sigma_8$ regime of the cluster observations.)

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