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Do Word-Problem Features Differentially Affect Problem Difficulty as a Function of Students' Mathematics Difficulty With and Without Reading Difficulty?

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Abstract

This study examined whether and, if so, how word-problem features differentially affect problem difficulty as a function of mathematics difficulty (MD) status: no MD (n = 109), MD only (n = 109), or MD in combination with reading difficulties (MDRD; n = 109). The problem features were problem type (total, difference, or change) and position of missing information in the number sentence representing the word problem (first, second, or third position). Students were assessed on 14 word problems near the beginning of third grade. Consistent with the hypothesis that mathematical cognition differs as a function of MD subtype, problem type affected problem difficulty differentially for MDRD versus MD-only students; however, the position of missing information in word problems did not. Implications for MD subtyping and for instruction are discussed.

Keywords

problem solving; mathematics; quantitative research method; discalculia

Some students perform poorly in mathematics yet relatively well in reading, whereas others perform poorly in mathematics and have concurrent reading difficulties (Gross-Tsur, Manor, & Shalev, 1996). This distinction may represent a productive scheme for subtyping mathematics difficulty (MD; Geary, 1995; Robinson, Menchetti, & Torgesen, 2002; Rourke & Finlayson, 1978). Students may struggle concurrently with reading and math due to weak phonological processing skills (Hecht, Torgesen, Wagner, & Rashotte, 2001; Robinson et al., 2002), whereas math difficulties that occur without concurrent reading difficulties may be due to poor number sense (Robinson et al., 2002). To better understand mathematical cognition as a function of this MD subtyping framework and to assess the tenability of the subtyping scheme, researchers often compare two groups of students: those with concurrent difficulty with math and reading (MDRD) and those with difficulty only in math (MD only; e.g., Andersson & Lyxell, 2007; Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Swanson & Jerman, 2006).

In these studies, the procedures used to identify students for MD-only and MDRD groups vary. Common criteria include performance 1, 1.5, or 2 standard deviations below the mean of a national normative framework (e.g., Fuchs & Fuchs, 2002), discrepancies of one or two between achievement and IQ (Parmar, Cawley, & Frazita, 1996), or nationally norm-referenced percentile cutoff points (e.g., Jordan & Montani, 1997), with cutoffs including the 10th, 25th, 31st, 35th, or 45th percentiles (Mazzocco, 2005). So a student identified as having math difficulties in one study might fail to meet the inclusion criteria in another. Despite these differences in identification criteria, some reliable differences emerge in the mathematical cognition of MDRD versus MD-only students. In this introduction, we synthesize prior work describing these differences. Next, we summarize studies examining differences specifically on word problems. Then, we consider the potential role of word-problem features in determining difficulty. Finally, we describe the purpose of the present study.

MDRD and MD-Only Mathematical Cognition

Research on mathematical differences between MDRD and MD-only students suggests distinctive mathematical profiles. Geary, Hoard, and Hamson (1999) assessed 25 MDRD and 15 MD-only students in the fall and spring of first grade on a global mathematics test, assessing performance across several dimensions of mathematical cognition. MDRD students scored below the 20th percentile on math and reading, whereas MD-only students scored below the 20th percentile in math but had average reading scores. In terms of number comprehension, MDRD students performed significantly below MD-only students and average-performing peers, and many MDRD students were unfamiliar with proper representations of number. In terms of counting, MD-only students performed better than MDRD students when items were out of order but performed worse than MDRD students on tasks where items were counted twice. In terms of arithmetic, both groups demonstrated deficient addition skill; however, MDRD students made more procedural and retrieval errors and less frequently used the more sophisticated min strategy (starting with the bigger number and counting up; see Groen & Parkman, 1972).

In a subsequent study, Geary, Hamson, and Hoard (2000) followed students longitudinally, testing 16 MDRD and 12 MD-only students in the fall and spring of first and second grade. On a global test of mathematics, MDRD students scored below the 20th percentile on math and reading, whereas MD-only students scored below the 20th percentile in math but above the 35th percentile in reading. Similar to the Geary et al. (1999) study, MDRD students had difficulty determining values of adjacent numbers on number-comprehension tasks. By contrast, MD-only students did not demonstrate number-comprehension difficulties and performed similarly to students without math difficulties. Both MDRD and MD-only students demonstrated understanding of basic counting principles, but both groups made assumptions that counting must be carried out in a sequential manner. Again, on arithmetic problems, MD-only students used more efficient counting procedures. As MD-only students entered second grade, they used the min procedure more frequently and caught up to average-performing peers. No MDRD students used the min procedure in first grade, and this group of students did not catch up to peers during second grade. Also looking across grades, Andersson and Lyxell (2007) analyzed differences between MDRD and MD-only students in the second, third, and fourth grades on mathematical working-memory tasks. Students were classified into difficulty groups by performance on math and reading tests, although the authors did not report cutoffs. On arithmetic, MD-only students answered as many number combinations correctly as did average-performing peers but only when given more than 3 seconds to respond. MDRD students did not solve as many problems correctly regardless of the time provided. Other researchers (e.g., Jordan & Hanich, 2003) have also documented distinctions in the number concepts and computational skills according to an MDRD versus MD-only subtyping scheme.

Word-Problem Solving in MDRD and MD-Only Students

The studies just discussed provide support for an MD subtyping scheme that differentiates between students with math difficulties with and without concurrent reading problems. Another dimension of mathematical performance that has been less well studied is word problems. This lack of attention is unfortunate because solving word problems is an especially difficult task for many students with learning difficulties. Word problems may be challenging due to the variety of skills needed to solve these problems (Parmar et al., 1996). That is, to solve a word problem, students must use text to identify missing information, construct a number sentence, and set up a calculation problem for finding the missing information (Fuchs et al., 2008). In a related way, because language and reading are closely linked, we might expect students with MDRD to have greater difficulty with word problems. The literature is consistent with this hypothesis. Even when simple calculations are required to solve word problems (Pellegrino & Goldman, 1987), both MDRD and MD-only students struggle with problem solving compared with average-performing peers, but MD-only students generally outperform MDRD students.

Jordan and colleagues conducted a series of studies to examine word-problem performance as a function of MD subtype and to describe the calculation strategies MD students use to derive answers. Jordan and Hanich (2000) administered 14 arithmetic word problems (i.e., Ashley has 7 pennies. Jason has 4 pennies fewer than Ashley. How many pennies does Jason have?) to 10 MDRD and 10 MD-only second-grade students. MDRD students had scored below the 30th percentile on global tests of reading and math; MD-only students scored below the 30th percentile in math but above the 40th percentile in reading. MD-only students answered significantly more simple word problems correctly than did MDRD students. Although both groups used a counting strategy the majority of the time to derive answers, MD-only students calculated answers correctly, whereas MDRD students' answers tended to be incorrect. Jordan and Montani (1997) extended this work to third graders. MDRD students scored below the 30th percentile in reading and math, and MD-only students scored below the 30th percentile in math but above the 30th percentile in reading. Students (MDRD, $n = 12$; MD only, $n = 12$) were administered 18 simple word problems under timed and untimed conditions, as done in Jordan and Hanich. Under timed conditions, MD-only students and MDRD students performed similarly and scored significantly worse than an average-performing group of students. Under untimed conditions, however, MD-only students performed significantly better than MDRD students. In fact, the MD-only students performed similarly to control students in the untimed condition. Again, both MDRD and MD-only students relied on counting strategies, but MD-only students had better accuracy in using these strategies.

In a subsequent study, Hanich, Jordan, Kaplan, and Dick (2001) expanded their focus beyond differences in strategies for calculating answers to consider the operations students used when solving the word problems. Hanich et al. also considered the proximity of errors to the correct answers. Ten simple word problems were presented orally without time constraints. Second-grade MDRD ($n = 52$) students scored below the 35th percentile in math and reading; MD-only students ($n = 53$) scored below the 35th percentile in math but above the 40th percentile in reading. MD-only students answered more word problems correctly than did MDRD students. MDRD students more frequently added to solve every word problem even though 7 of the 10 problems required subtraction. In addition, when MDRD and MD-only students answered incorrectly, MDRD students' answers were significantly farther from the correct response.

Fuchs and Fuchs (2002) moved beyond simple word problems to evaluate the performance of MDRD and MD-only students on simple, complex, and real-world problems. At fourth grade, MDRD students ($n = 22$) performed 1.5 standard deviations below the national mean on math and reading fluency tests, whereas MD-only ($n = 18$) students performed 1.5 standard deviations below the mean in math but within the average range in reading. Answers were scored in terms of the accuracy of math operations and the accuracy of problem-solving strategies. For simple word problems, MD-only students performed significantly better than MDRD students. Accuracy on math operations averaged 80% for MD-only students but only

59% for MDRD students. For problem-solving strategies, accuracy for MD-only and MDRD students was 71% and 52%, respectively. By contrast, on complex word problems and real-world word problems, where performance was low (10%–15% correct for MD-only students and 5%–11% correct for MDRD students), there were no significant differences between MD-only and MDRD students in terms of operations or problem-solving accuracy. This reveals the challenges of examining MD subtype differences on complex word problems, where floor effects may be problematic, and raises the possibility that response to intervention may be another productive methodological approach for examining differences between MD-only and MDRD students on more complex word problems.

The only intervention study examining differences in the response of MD-only versus MDRD students was conducted by Fuchs, Fuchs, and Prentice (2004). MDRD students scored below the 25th percentile in reading and mathematics, whereas MD-only students scored below the 25th percentile in math but above the 40th percentile in reading. MD-only ($n = 8$) and MDRD ($n = 12$) students participated in 32 sessions of validated problem-solving instruction conducted in whole-class sessions with classroom peers over 16 weeks. Another group of 20 MDRD and 5 MD-only students, who did not receive validated problem-solving instruction, served as a comparison group. Students were pretested and posttested on a complex word-problem measure. Overall, MDRD and MD-only students showed significantly better improvement over comparison students, but MDRD students improved less with validated instruction than did MD-only students. Effect sizes (ESs) comparing MDRD students who did and did not receive validated instruction were 1.40, 0.90, and 1.35 for understanding, computation, and labeling, respectively. Corresponding figures for MD-only students were 2.15, 1.06, and 1.85.

In sum, on simple word problems, MD-only students outperform MDRD students and rely on more efficient counting strategies to derive answers (Hanich et al., 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997). MDRD students' understanding of word problems also appears inferior to that of MD-only students, as revealed in (a) MDRD students' tendency to add across problems even when the problem requires subtraction (Hanich et al.), (b) the greater distance of their answers from correct responses (Hanich et al.), and (c) their less sophisticated problem-solving strategies (Fuchs & Fuchs, 2002). In addition, on complex word problems, MDRD students' response to validated instruction is weaker than that of MD-only students (Fuchs et al., 2004).

Problem Type and Position of Missing Information in Word Problems

Although MD-only students appear to outperform MDRD students on word problems, we identified no prior studies that asked whether word-problem features differentially impact student performance as a function of MD subtype. This is unfortunate because this question represents another strategy for extending conceptual understanding about MD and gaining insight into whether concurrent reading difficulty represents a promising framework for MD subtyping. Findings may also carry implications for intervention.

Separate from the MD-subtyping issue, some prior work does suggest that two word-problem features may contribute to problem difficulty. The first potentially important word-problem feature is problem type. Previous research has focused on change, combine, compare, or equalize problem types (Garcia, Jiménez, & Hess, 2006; Jordan & Hanich, 2000; Riley & Greeno, 1988) or on problem types involving joining, separating, part-partwhole, comparison, and equalize (Carpenter, Hiebert, & Moser, 1981). The second potentially important word-problem feature is the position of the missing variable that needs to be found to solve problems (Carpenter & Moser, 1983; DeCorte & Verschaffel, 1987). In the word problem, "Sam has 4 pencils. Maria has 5 pencils. How many pencils do they have altogether?" the equation $4 + 5 = x$ represents the problem, with the missing information in the third position. For the problem, "Abby had some pencils. Then, Ben gave her 7 more pencils. Now, Abby has 10 pencils. How many pencils did Abby start with?" the missing information is in the first position: $x + 7 = 10$.

To examine the effect of problem type and position of missing information on word-problem performance, Garcia et al. (2006) assessed 104 low-performing math students and 44 average-performing math students in Spain. Low-performing math students performed below the 25th percentile on a global math test. All students were administered 40 simple word problems representing change, combine, compare, and equalize problem types. Although problem type alone did not determine the difficulty of the problem, a combination of problem type and the position of the missing information did determine problem difficulty level, with missing-information position having a greater influence than problem type. Students had the greatest difficulty when the missing information was in the first position; second-position problems were easier than first-position problems, and third-position problems were the easiest. Garcia et al. did not, however, examine the effects of problem type or position of the missing variable as a function of MD subtype.

Again not focusing on MDRD and MD-only differences, Schatschneider, Fuchs, Fuchs, and Compton (2006) used item-response theory to determine the difficulty of simple word problems based on problem type, position of missing information, and operation in a representative sample of 958 third-grade students. As with Garcia et al. (2006), problems with missing information in the third position were easier to solve than those with missing information in the second or first positions, but missing information in the first position presented the greatest challenge. Problem difficulty was not influenced by the problem type or the operation needed to solve the problem.

Together, Garcia et al. (2006) and Schatschneider et al. (2006) produced similar findings. Problem type did not affect (Schatschneider et al., 2006) or did not completely determine (Garcia et al., 2006) problem difficulty. Both studies concluded that the position of the missing information influences word-problem difficulty more than does problem type. Neither study, however, examined whether word-problem features differentially affect problem difficulty as a function of MD status.

Purpose of the Study

Prior work suggests that MD-only students perform better on word problems as compared with MDRD students (Hanich et al., 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997). Only a handful of studies, however, have examined how word-problem features affect difficulty, and neither of the pertinent studies (Garcia et al., 2006; Schatschneider et al., 2006) examined whether word-problem features differentially affect problem difficulty as a function of MD status. This was the purpose of the present study. We recruited MDRD and MD-only students using a relatively stringent cutoff for MD and focused on two potentially important problem features: problem type (total, difference, and change) and the position of the missing information in problems (first, second, and third). In addition, for comparison purposes, we incorporated a sample of students without math difficulty (no MD). Based on previous research, we anticipated, in line with the hypothesis that mathematical cognition differs as a function of MD subtype, that problem features would differentially affect problem difficulty for MDRD versus MD-only students.

Method

Participants

The data on the MD-only and MDRD students described in this article were collected as part of a series of three related studies assessing the effects of computation and word-problem instruction. Across the three studies, participants were sampled from 134 third-grade classrooms in 24 Title I and 8 non–Title I schools (1–8 teachers per school) in a southeastern metropolitan school district and one non–Title I parochial school (1 teacher) in the same urban area as the metropolitan school district. Students were first assessed on the *Wide-Range Achievement Test–Arithmetic* (WRAT-Arithmetic; Wilkinson, 1993) in September of third grade in whole-class administrations. (As reported by Wilkinson, median reliability for students aged 5–12 years is 0.94 on WRAT-Arithmetic and 0.94 on WRAT-Reading.) Then, students participated in an individual testing session in which WRAT-Reading, Wechsler Abbreviated Intelligence Scale–Vocabulary (WASI-Vocabulary; The Psychological Corporation, 1999), and WASI–Matrix Reasoning subtests were administered. (According to the test manual, median reliability for students aged 6–16 exceeds 0.92. From 1,801 students with consent, 218 students qualified to participate.) All 218 students scored at or below the 25th percentile on WRAT-Arithmetic and had at least one WASI *T*-score above 30. Of the 218, 109 also scored at or below the 25th percentile on WRAT-Reading. These students were designated as having MDRD. The other 109 students, scoring at or above the 40th percentile on WRAT-Reading, were designated as having MD only. There were no significant interactions between study cohort and MDRD versus MD only on WRAT-Arithmetic or WRAT-Reading.

In addition to the 218 MDRD and MD-only students, a group of 109 no-MD third-grade students was included for comparison purposes. These students came from 29 third-grade classrooms in six Title I and two non–Title I schools in the same southeastern metropolitan school district. The 109 no-MD students were sampled from 313 students participating in a randomized-field experiment testing the efficacy of whole-class, word-problem-solving instruction. The pool of 313 students had been selected using parallel methods as used to identify the MD-only and MDRD students. That is, students were first assessed on WRAT-Arithmetic in September of third grade in whole-class administrations. Then, students participated in one individual testing session in which WRAT-Reading, WASI-Vocabulary, and WASI–Matrix Reasoning were administered. No-MD students were randomly selected from the subset of 171 students who met the following criteria: (a) scoring above the 39th percentile on WRAT-Arithmetic and WRAT-Reading, (b) having at least one WASI *T*-score above 30, and (c) not being served in special education.

On WRAT-Arithmetic, no-MD students' raw score average was 25.35 ($SD = 2.35$), MD-only students' average was 21.68 ($SD = 1.61$), and MDRD students' average was 21.37 ($SD = 1.97$). As expected, one-way analysis of variance (ANOVA) revealed a significant difference as a function of MD status, $F(2, 324) = 133.24$, $p < .001$. Follow-ups indicated that no-MD students scored higher than MD-only students ($p < .001$) and higher than MDRD students ($p < .001$); MD-only and MDRD students scored comparably ($p = .216$).

On WRAT-Reading, no-MD students' raw score average was 31.67 ($SD = 3.92$), MD-only students' average was 32.42 ($SD = 2.97$), and MDRD students' average was 24.04 ($SD = 3.63$). As expected, ANOVA revealed a significant difference as a function of MD status, $F(2, 324) = 179.54$, $p < .001$. Follow-ups indicated that no-MD students scored comparably to MD-only students ($p = .111$), but no-MD students and MD-only students scored higher than MDRD students (both $p < .001$).

As indexed on the 2-subset WASI, no-MD students' IQ averaged 102.58 ($SD = 12.97$); MD-only students, 92.85 ($SD = 12.37$); and MDRD students, 85.25 ($SD = 9.46$). ANOVA revealed a significant relation with MD status, $F(2, 324) = 60.08$, $p < .001$, with no-MD students higher than MD-only students and higher than MDRD students and with MD-only students higher than MDRD (all *p*s < .001). Inspection of each of the two WASI subsets revealed patterns consistent with cognitive characteristics associated with MD-only versus MDRD students.

That is, on WASI-Vocabulary, the mean scores for no-MD, MD-only, and MDRD students, respectively, were 27.87 (*SD* = 6.72), 27.05 (*SD* = 6.81), and 24.32 (*SD* = 6.32). ANOVA was significant, $F(2, 324) = 8.49$, $p < .001$, with no-MD and MD-only students performing comparably ($p = .368$) but with no-MD and MD-only students scoring higher than MDRD students ($p < .001$ and $p = .003$, respectively). The pattern differed on WASI–Matrix Reasoning, where scores for the three respective groups were 18.62 (*SD* = 5.44), 13.37 (*SD* = 6.63), and 12.12 (*SD* = 5.51). ANOVA was significant, $F(2, 324) = 37.52$, $p < .001$, but no-MD students scored higher than MD-only ($p < .001$) and MDRD ($p < .001$) students, with the two MD subtypes scoring comparably ($p = .132$). Because the pattern of the IQ subtest scores is consistent with characteristics associated with MD-only versus MDRD students, these differences strengthen the external validity of the study. (Given the overall significant IQ differences among groups, all analyses were test run with IQ as a covariate. The pattern of findings was the same with and without the IQ covariate; so, for simplicity and reader ease, IQ was not used as a covariate.)

Table 1 shows gender, race, subsidized-lunch status, special-education status, and English-language-learner status of MD students. Chi-square values testing the relation between demographic variables and MD status were run. There was no significant relation between MD status and gender, $\chi^2(2, N = 327) = 0.32$, $p = .853$, race, $\chi^2(6, N = 327) = 11.02$, $p = .088$, or English-language-learner status $\chi^2(2, N = 327) = 5.10$, $p < .078$. There was a significant relation between MD status and subsidized-lunch status, $\chi^2(2, N = 327) = 22.50$, $p < .001$. Using the Benjamini and Hochberg (1995) method to control for experimenter-wise error, we found that status was comparable for no-MD and MD-only students ($p = .050$)— however, MDRD students were disproportionately on subsidized lunch compared with no-MD ($p < .001$) and MD-only students ($p = .004$). There was also a significant relation between MD status and special-education status, $\chi^2(2, N = 327) = 22.50$, $p < .001$. Again, using the Benjamini and Hochberg (1995) procedure, we found that status was comparable for no-MD and MD-only students ($p = .024$) but not for no-MD and MDRD students ($p < .001$) or for MD-only and MDRD students ($p < .001$). Disproportionate representation in subsidized-lunch status for MDRD versus MD-only students has been reported previously (Hanich et al., 2001), and special-education differences reflect MDRD students' reading and language problems (Jordan, Hanich, & Kaplan, 2003); so, these differences add to the external validity of the study. All data analyses were run including and excluding the 17 English-language learners. There were no differences between the two runs, so English-language learners taught in English-language classrooms were retained in the study to enhance sample size.

Word-Problem Measure

Word Problems (Jordan & Hanich, 2000) comprises 14 simple word problems involving sums or minuends of 9 or fewer. Originally, Jordan and Hanich (2000) categorized these problems as change, combine, compare, and equalize problem types. We reclassified the problems into total, difference, and change problem types to mirror the psychometric framework provided by Schatschneider et al. (2006) and due to ambiguities in the equalize category (see Note [1]). (We ran analyses using our three problem-type categorization [total, difference, change] and using Jordan and Hanich's, 2000, four problem-type categorization [combine, change, compare, equalize]. The pattern of findings was similar between runs.) The measure is printed on two pages. The first seven problems are on the first page, the second seven problems on the next page. Below each word problem is blank space for students to write their work, and

[1]For example, in Problem 1 in Table 2, the amount that Alex has to give away to end with the same number of pennies as Chris can be conceptualized as (a) the difference between Alex's and Chris's amounts (i.e., a difference problem) or (b) how much change (decrease) in Alex's amount is needed (i.e., a change problem). In a similar way, in Problem 7 in Table 2, the quantity that Claire needs to obtain to have as many pennies as Ben can be conceptualized as (a) the difference between Claire's and Ben's amounts (i.e., a different problem) or (b) how much change (increase) in Claire's amount is needed (i.e., a change problem).

students write their answers on a line to the left of each written problem. The score is the number of correct answers. All tests were double scored with agreement of 100%. Coefficient alpha on this sample was 0.85.

This 14-item *Word Problems* test (or a briefer 10-item version) has been used frequently across Grades 1 to 3 to assess student performance on simple word problems (Fuchs et al., 2005; Fuchs & Fuchs, 2002; Hanich et al., 2001; Jordan et al., 2003; Jordan & Hanich, 2003). On standardized tests and in textbooks, third-grade students are asked to solve a variety of word problems ranging from simple problems, similar to those included in *Word Problems*, to complex two-step problems with important information provided in charts or graphs (Hoover, Hieronymous, Dunbar, & Frisbie, 1993). Although *Word Problems* does not assess all word-problem types in the third-grade curriculum, it is important to note that (a) this study focused on the word-problem-solving skill of students struggling with math; (b) we anticipated a floor effect in the MD population at the beginning of third grade with more complex problems, as demonstrated by Fuchs and Fuchs (2002); and (c) we did not experience a ceiling effect in the present study (on average, no-MD third graders answered 79% of problems correctly; MD-only, 68%; and MDRD, 49%).

As noted, for this study, problems were categorized into three problem types based on the work of Schatschneider et al. (2006). With total problems, two amounts sum to a total. With difference problems, bigger and smaller quantities are compared. With change problems, an amount becomes bigger or smaller. Of the 14 word problems, 2 problems are total, 6 are difference, and 6 are change. Coefficient alpha by problem type on this sample was 0.23, 0.76, and 0.70, respectively. We also considered the location of the missing information within the word problem. The missing variable was in the first position in three problems (i.e., $x + 3 = 7$), second position in four problems (i.e., $9 - x = 2$), and third position in seven problems (i.e., $4 + 5 = x$). Coefficient alpha by position in this sample was 0.45 for the first position, 0.73 for the second position, and 0.79 for the third position.

Table 2 lists each problem according to problem type, position of missing information, and Jordan and Hanich's (2000) original problem type. *Word Problems* does not sample problem types or position of the missing information equally. We nevertheless used *Word Problems* because it has been widely used, making connections between the present and many prior studies transparent. Future studies should develop other measures that represent problem type and position of missing information comparably.

Procedure

Across no-MD, MD-only, and MDRD students, *Word Problems* was administered in large groups by trained examiners, each of whom demonstrated 100% accuracy during a mock administration. The examiner reads each problem aloud while students follow along silently; students have 30 seconds to respond and can ask for rereading(s) as needed before the examiner reads the next problem.

Data Analysis and Results

An average percentage correct score on total, difference, and change problems as well as on the position of missing information was calculated for each student. Table 3 provides means and standard deviations by MD status and across MD status for each problem type and for each missing-information position.

Problem Type by MD Status

A two-way ANOVA, with MD status as a between-subjects factor and with problem type as a within-subjects factor, was applied to the data. Follow-up tests were conducted using the Benjamini and Hochberg (1995) procedure to control for experimenter-wise error. ESs involving between-subject differences were calculated by subtracting the means and dividing by the pooled *SD* (Hedges & Olkin, 1985). ESs involving within-subject differences were calculated by subtracting the means and dividing by the following: the pooled *SD* divided by the square root of $2(1-r)$, where r is the correlation between the two measures (Glass, McGaw, & Smith, 1981).

The main effect of MD status was significant, $F(2, 324) = 42.05$, $p < .001$, as was the interaction between MD status and problem type, $F(4, 648) = 3.19$, $p = .013$. (The main effect for problem type was not, $F(2, 648) = 49.21$, $p = .058$.) To follow up the interaction, which supersedes the MD-status main effect, we examined the two-way interaction for each pair of MD-status groups (i.e., problem type as a function of no MD vs. MD only, problem type as a function of no MD vs. MDRD, and problem type as a function of MD only vs. MDRD). The purpose of these follow-up tests was to pinpoint the pair(s) of MD-status groups in which the interaction resided. Results showed that problem type differentially affected only the performance of MD-only versus MDRD students. So we followed up the interaction between problem type and MD-only versus MDRD students by contrasting performance on total versus difference problems, performance on difference versus change problems, and performance on total versus change problems for MD-only versus MDRD students. To control for Type I error in this follow-up strategy, which involved nine tests, we used the Benjamini and Hochberg (1995) procedure.

The specific results were as follows. When the follow-up, two-way ANOVA was conducted using no MD and MD only as the two levels of MD status, the interaction between problem type and MD status did not meet the critical p value, $F(2, 432) = 3.42$, $p = .034$. (Only the main effect for MD status was significant, $F(1, 216) = 15.74$, $p < .001$; across problem types, no-MD students scored higher than those with MD only, with an ES of 0.46.) When the follow-up, two-way ANOVA was conducted using no MD and MDRD as the two levels of MD status, the interaction between problem type and MD status also was not significant, $F(2, 432) = 1.20$, $p = .302$. (Both main effects were significant. For MD status, $F(1, 216) = 43.71$, $p < .001$; across problem types, no-MD students performed stronger than MDRD students, with an ES of 1.36. None of the follow-ups to the problem type main effect, $F(2, 432) = 5.02$, $p = .007$, met the critical p value.)

When the follow-up, two-way ANOVA was conducted using MD-only and MDRD as the two levels of MD status, the interaction was significant, $F(2, 432) = 4.73$, $p = .009$. Among the MD-only students, total and difference problems were comparably difficult ($p = .814$), but change problems were easier than difference problems ($p = .019$; ES = 0.16), and change problems were easier than total problems ($p = .018$; ES of 0.18). By contrast, among the MDRD students, total problems were easier than difference problems ($p = .005$; ES = 0.28), but change and difference problems were comparably difficult ($p = .220$), as were change and total problems ($p = .065$).

Position of Missing Information by MD Status

A two-way ANOVA, with MD status as a between-subjects factor and with position of missing information as a within-subjects factor, was applied to the data. Follow-up tests were conducted using the Benjamini and Hochberg (1995) procedure, and ESs were calculated as already described. The interaction between MD status and position of missing information did not meet the critical p value, $F(4, 648) = 2.20$, $p = .067$. The main effect of MD status was significant, $F(2, 324) = 44.69$, $p < .001$, as was the main effect for position of missing information, F(2,

648) = 106.65, $p < .001$. In follow-up to the MD-status main effect, we found that no-MD students outperformed those with MD only ($p < .001$; ES = 0.46), no-MD students outperformed those with MDRD ($p < .001$; ES = 1.36), and MD-only students outperformed those with MDRD ($p < .001$; ES = 0.76). In following up the position of missing-information main effect, we found that the third position was easier than the first ($p < .001$; ES = 2.69) or the second position ($p < .001$; ES = 1.36) and that the second position was easier than the first position ($p < .001$; ES = 0.69).

Discussion

The purpose of the present study was to examine whether word-problem features differentially affect problem difficulty as a function of MD status. In this way, we sought to extend prior work suggesting that MD-only students outperform MDRD students on simple word problems (Hanich et al., 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997). Although these differences are documented in the literature, we identified no prior study that asked whether word-problem features differentially affect student performance as a function of MD subtype. This is unfortunate because this question represents another strategy for extending conceptual understanding about MD and gaining insight into whether concurrent reading difficulty represents a promising framework for MD subtyping. Findings may also carry implications for intervention. Focusing on the differential impact of word-problem features on student performance as a function of MD subtype represents a unique contribution of the present study.

For problem type, results supported the tenability of the MD subtyping framework that distinguishes students with MD based on whether the students have concurrent reading difficulty. Follow-up tests to the significant interaction between problem type and MD status showed that the interaction resided within the MD-only versus MDRD contrast (not the contrasts involving no-MD students). Among MD-only students, total and difference problems were comparably difficult, but change problems were easier than difference problems (ES = 0.16), and change problems were easier than total problems (ES = 0.18). By contrast, for MDRD students, total problems were easier than difference problems (ES = 0.28), but change and difference problems were comparably difficult, as were change and total problems.

So it is interesting to consider the conceptual underpinnings and challenges associated with these problem types in connection with problem difficulty for MD-only versus MDRD students. For total problems, narratives describe the combining of two parts to create a total, the addends can be combined in either order, and addition is always used as the operation in the number sentence representing the problem structure. For difference problems, narratives describe a difference between or compare two quantities, the order in which the subtrahend and minuend are arranged in the number sentence is critical, and subtraction is always used as the operation in the number sentence representing the problem structure. For change problems, narratives describe a starting amount that changes to become a new quantity, the order in which the operators are arranged in the number sentence is critical, and addition or subtraction can be used as the operation in the number sentence representing the problem structure. In considering these aspects of the three problem types, we note that Garcia et al. (2006) found that word problems requiring addition are easier than those involving subtraction (total problems always involve addition, whereas difference problems always involve subtraction, and change problems can involve addition or subtraction). Moreover, many difference problems contain relational statements (e.g., *more* or *less*) that have been shown to be conceptually confusing, thereby increasing word-problem difficulty (Lewis & Mayer, 1987).

For these reasons, it is not surprising that total problems were reliably easier than difference problems for MDRD students. In addition, there was some suggestion in the data that with larger samples, total problems may emerge as easier than change problems. This seems possible because change problems require students to distinguish between addition and subtraction as the operation represented in the story narrative and because, in change problems, the order of the operators in the number sentence representing the narrative is essential for successful problem solving. In fact, given MDRD students' language difficulties (Robinson et al., 2002; Hecht et al., 2001), it is possible that higher performance on total problems may be due simply to MDRD students' tendency to add in the face of word problems, regardless of the structure of those narratives (Hanich et al., 2001). What is more surprising is that MD-only students performed comparably on total and difference problem and found change problems to be reliably easier than total and difference problems. Superior performance on change problems suggests that MD-only students are attending to problem narratives either with greater attention or with greater understanding than their MDRD peers. With greater attention or understanding, the problem structure underlying change problems may translate more transparently to the mathematical concepts taught in school. For example, in the early grades, addition and subtraction concepts are typically taught in a manner aligned with change word problems, with students frequently generating stories to represent pictures that depict a starting quantity that increases (for addition) or decreases (for subtraction). In any case, finding that problem type affects the performance of MD-only students in a different manner than it does MDRD students provides additional support for the MD subtyping framework. It also suggests that different intervention strategies may be required to remediate word-problem deficits in these subpopulations with MD.

At the same time, the interaction between MD status and the position of missing information in the number sentences associated with the word problems was not significant. Clearly, additional research to explore whether and, if so, how problem features differentially affect problem difficulty as a function of MD subtype is needed. Moreover, additional work for assessing the tenability of this MD subtyping framework involving co-occurring reading difficulty is warranted. It is nonetheless interesting to consider the main effects documented in the interaction between MD status and the position of missing information.

In terms of MD status, regardless of the position of the missing information in problems, no-MD students performed higher than each of the MD subgroups. The ESs were moderate when compared with MD-only students (0.46) and large in comparison to MDRD students (1.36). In a related way, MD-only students outperformed MDRD students (ES = 0.76), a finding that echoes previous work (Hanich et al., 2001; Jordan & Hanich, 2000); Jordan & Montani, 1997). So although MD-only students perform below no-MD students, the gap between no-MD and MDRD students is even more sizeable. This finding reemphasizes the seriousness of the word-problem difficulties that MDRD students experience and the need to focus research on strategies for effective prevention and remediation of word-problem difficulties for students with comorbid reading and math learning disabilities.

In terms of the position of missing information, large differences existed, which overrode any possible interacting effect with MD subtypes. Clearly, problem structures that describe missing information in the last position of the equation are easier for students than problem structures describing missing information in the first or second position (with an ES of 1.36 compared with problems with the missing information in the second position and an ES of 2.69 compared with problems with the missing information in the first position). This finding corroborates prior work that found strong evidence that problems with missing information in the first and second positions are more difficult in a representative of students (Schatschneider et al., 2006) and in students with and without MD (Garcia et al., 2006). It is, therefore, unfortunate that teachers typically focus instruction on problems with the lowest conceptual demand: those with missing information in the last position. Results suggest the need to rethink the curriculum to emphasize more linguistically challenging problems in which missing information occurs in the first and second positions.

In sum, results demonstrate that word-problem structure (i.e., problem type), which reflects varying conceptual and calculation demands, interacts with MD subtype in determining problem difficulty. That is, MDRD students experience total problems as easier than difference problems, whereas MD-only students score better on change problems compared with total or difference problems. This finding extends prior work showing that students with MDRD have poorer word-problem deficits relative to MD-only students by elucidating how problem types affect difficulty for which subtypes. Results lend support to the hypothesis that concurrent reading difficulty may provide a productive scheme for subtyping MD and provide potential directions for differentiating intervention for the two MD subgroups. At the same time, the position of missing information carried such a strong effect as to render an interaction between MD status and the position of missing information untenable. Given the difficulty that students with and without MD experience when missing information is in the first or second position of equations representing the structure of word problems, greater attention on these conceptually demanding word problems is warranted in schools.

Before closing, we review some key limitations to the present study and note directions for future work. First, as already discussed in the measure section, it is unfortunate that our word-problem measure did not completely balance problem type with position of the missing information. We adopted the Jordan and Hanich (2000) measure in the present study to facilitate connections between present and earlier findings. Nevertheless, future work should consider balanced measures. Second, the present study did not address the range of word problems found at third grade. We avoided those complex word problems because prior work has shown that a floor effect may occur at the beginning of third grade for MD students (Fuchs & Fuchs, 2002). This raises the importance of adopting a responsiveness-to-intervention framework for studying MD subtype differences on more complex problems, as Fuchs et al. (2004) did in a retrospective study. Additional work conducted prospectively is warranted. Finally, the present study findings, wherein different problem types were differentially difficult for MD-only versus MDRD students, suggest potential directions for differentiating intervention for the two MD subgroups. Future research should investigate these possibilities.

Biography

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Table 1

Student Demographics by MD Status

Variable	MDRD (n = 109)		MD Only (n = 109)		No MD (n = 109)	
	n	%	n	%	n	%
Males	58	53.2	59	54.1	55	50.5
Race						
African American	67	61.4	60	55.0	47	43.1
White	21	19.3	30	27.5	33	30.3
Hispanic	16	14.7	16	14.7	19	17.4
Other	5	4.6	3	2.8	10	9.2
Reduced/free lunch	93	85.3	75	68.8	61	56.0
School-identified disability	26	23.9	5	4.6	0	0.0
English-language learners	13	11.9	4	3.7	9	8.3

Note: MD = mathematics difficulty; MDRD = mathematics difficulty and reading difficulty.

Table 2

Word Problems by Problem Type, Position of Missing Information, and Original Problem Type

Word Problem	Problem Type	Position of Missing Information	Original Problem Type[a]
Alex has 8 pennies. Kris has 6 pennies. How many pennies does Alex need to give away to have as many as Kris?	Difference	Third $8 - 6 = x$	Equalize
Sue had 5 pennies. Then Mike gave her 2 more pennies. How many pennies does Sue have now?	Change	Third $5 + 2 = x$	Change
Chelsea has 6 pennies. Max has 4 pennies. How many pennies does Max have less than Chelsea?	Difference	Third $6 - 4 = x$	Compare
Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?	Change	Third $9 - 3 = x$	Change
Janet has 3 pennies. Andy has 5 more pennies than Janet. How many pennies does Andy have?	Difference	First $x - 3 = 5$	Compare
Carol had 4 pennies. Then Nick gave her some more pennies. Now Carol has 6 pennies. How many pennies did Nick give her?	Change	Second $4 + x = 6$	Change
Claire has 4 pennies. Ben has 9 pennies. How many more pennies does Claire need to have as many as Ben?	Difference	Third $9 - 4 = x$	Equalize
Jen had 7 pennies. Then she gave some pennies to Joe. Now Jen has 2 pennies. How many pennies did she give to Joe?	Change	Second $7 - x = 2$	Change
Emily has 3 pennies. John has 6 pennies. How many pennies do they have altogether?	Total	Third $3 + 6 = x$	Combine
Maria and Kevin have 8 pennies together. Maria has 3 pennies. How many pennies does Kevin have?	Total	Second $3 + x = 8$	Combine
Ashley has 7 pennies. Jason has 4 pennies less than Ashley. How many pennies does Jason have?	Difference	Second $7 - x = 4$	Compare
Dennis has 7 pennies. Molly has 5 pennies. How many pennies does Dennis have more than Molly?	Difference	Third $7 - 5 = x$	Compare
Karen had some pennies. Then Matt gave her 4 more pennies. Now Karen has 6 pennies. How many pennies did she have to start with?	Change	First $x + 4 = 6$	Change
Lisa had some pennies. Then she gave 3 pennies to Bill. Now Lisa has 5 pennies. How many pennies did Lisa have to start with?	Change	First $x - 3 = 5$	Change

[a] Source: Jordan and Hanich (2000).

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Table 3

Performance by MD Status, Problem Type, and Position of Missing Information

	MDRD (*n* = 109)		**MD Only (*n* = 109)**		**Across MDRD and MD Only (*n* = 218)**		**No MD (*n* = 109)**		**Across All Students (*n* = 327)**	
Variable	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Problem type										
Total	0.55	0.27	0.65	0.33	0.60	0.31	0.82	0.28	0.67	0.31
Difference	0.47	0.30	0.65	0.31	0.56	0.32	0.79	0.25	0.64	0.32
Change	0.50	0.27	0.71	0.27	0.60	0.29	0.78	0.23	0.66	0.29
Across problem types	0.49	0.24	0.68	0.26	0.58	0.27	0.79	0.21	0.65	0.27
Missing-information position										
First	0.33	0.29	0.54	0.34	0.51	0.34	0.65	0.32	0.51	0.35
Second	0.38	0.34	0.63	0.33	0.58	0.36	0.74	0.33	0.58	0.36
Third	0.63	0.29	0.76	0.29	0.69	0.30	0.88	0.20	0.76	0.28
Across positions	0.49	0.24	0.68	0.26	0.58	0.27	0.79	0.21	0.65	0.27

Note: MD = mathematics difficulty; MDRD = mathematics difficulty and reading difficulty.