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Mathematical Disabilities: Reflections on Cognitive, Neuropsychological, and Genetic Components

David C. Geary
University of Missouri

Abstract

The collection of articles in this special issue and related studies over the past decade provides a fine example of the substantial progress that has been made in our understanding and remediation of mathematical learning disabilities and difficulties since 1993 (Geary, 1993). The originally proposed procedural and retrieval deficits have been supported and a number sense deficit has been identified. There is evidence for visuospatial contributions to some aspects of mathematical learning, but identification of a core visuospatial deficit underlying some forms of mathematics learning disabilities and difficulties has been elusive. The contributions of working memory to the development and expression of these deficits is more nuanced than I originally proposed as are the brain systems supporting mathematical learning. Although much has been learned about children's difficulties in learning mathematics, but there is just as much and likely more than remains to be discovered.

Cirino and Berch asked me to reflect on the articles in this special issue with respect to my 1993 review of what was then known about mathematical learning disabilities (MLD; Geary, 1993). At the time of my 1993 review, there were less than a score of cognitively-motivated studies of MLD and LA (low achievement), a somewhat richer literature in neuropsychology, and only a few behavioral genetic analyses of individual differences in mathematics achievement. The revolution in brain imaging technology was just underway and thus there were few such studies on mathematical processing and even fewer randomized control studies of cognitively-motivated interventions for MLD. In fact, there was no agreed upon criterion for diagnosing MLD. Substantial progress has been made in all of these areas, and the set of articles in this issue provides a fine illustration of how far our understanding of MLD and mathematical difficulties associated with LA has come since 1993. I reflect on some of these gains following the organization of the 1993 review; specifically, cognitive, neuropsychological, and genetic components of MLD. Unless otherwise noted, hereafter MLD refers to both MLD and LA children, as these groups were conflated in much of the earlier research.

COGNITIVE COMPONENT

With the early cognitively-motivated studies, the methods and theories used in the study of typical development were adopted to better understand the achievement deficits of children

Correspondence: Department of Psychological Sciences University of Missouri Columbia, MO 65211-2500 Phone: 573-882-6268 Fax: 573-882-7710 GearyD@Missouri.edu.

with MLD. The benefit was readymade empirical and theoretical contexts for interpreting the performance of children with MLD, but with the cost of being constrained by what was known and how this was studied. On the basis of studies available by 1993, I concluded there are three cognitive components to MLD whose expression and development were influenced in part by underlying working memory deficits. These manifested as deficits in the retrieval of arithmetic facts from long-term semantic memory, in the execution of procedures for solving arithmetic problems, and in the ability to represent and interpret visuospatial representations of mathematical information. Subsequent studies have confirmed the fact retrieval and procedural deficits (Jordan, Hanich, & Kaplan, 2003; Raghubar, Cirino, Barnes, Ewing-Cobbs, Fletcher, & Fuchs, 2009) and have extended these to a potentially more fundamental deficit in number sense. Although there were early studies of infants' and preschooler's number sense before 1993 and similar studies of adults (Mandler & Shebo, 1982; Starkey & Cooper, 1980), the corresponding methods were not typically applied to the study of individual differences in children's mathematical achievement and thus were not included in my original review.

In any case, the fundamental core of early number sense includes an implicit and potentially inherent understanding of the *exact quantity* of small collections of actions or objects and of symbols (e.g., Arabic numerals) that represent them (e.g., `3' = ■■■), and of the *approximate magnitude* of larger quantities (Butterworth & Reigosa, 2007; Dehaene, Piazza, Pinel, & Cohen, 2003; Gallistel & Gelman, 1992; Geary, 1995). This implicit knowledge is manifested in their ability to (a) apprehend the quantity of sets of 3 to 4 objects or actions without counting, that is, by subitizing (Mandler & Shebo, 1982; Starkey & Cooper, 1980; Wynn, Bloom, & Chiang, 2002); (b) use non-verbal processes or counting to quantity small sets of objects and to add and subtract small quantities to and from these sets (Levine, Jordan, & Huttenlocher, 1992; Starkey, 1992); and, (c) estimate the magnitude of sets of objects and the results of simple numerical operations (Dehaene, 1997).

Jordan, Glutting, and Ramineni's (this issue) screening battery provides a much needed means to assess these and related emerging mathematical competencies. They have shown the core number sense competencies assessed in their battery are predictive of later mathematics achievement, above and beyond the influence of IQ and working memory (see also Fuchs, Geary, Compton, Fuchs, & Hamlett, in press; Geary, Bailey, & Hoard, 2009; Jordan, Kaplan, Olah, & Locuniak, 2006; Locuniak & Jordan, 2008). The availability of the battery is especially important, because research since 1993 indicates children with MLD and to lesser degree LA children have deficits or developmental delays in both the exact quantity and approximate magnitude systems (Butterworth, 2005; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Halberda, Mazzocco, & Feigenson, 2008; Koontz & Berch, 1996; Landerl, Bevan, & Butterworth, 2004). The relation between these number sense deficits and the memory retrieval and procedural deficits identified in Geary (1993) is unclear.

One possibility is an early number sense deficit impedes children's transition to formal mathematics in school (e.g., learning Arabic numerals and mapping these to representations of quantity) and through this results in difficulties learning arithmetic facts and procedures, among other potential problems. This is not likely to be the whole story, however. As Ansari (this issue) aptly reviews, there are multiple brain systems engaged during children's mathematical learning and a developmental delay or deficit in any one of them could result in similar functional deficits. In other words, it is not likely that MLD and LA is due to a single underlying deficit in a manner analogous to phonemic awareness and early reading disability (RD). In as yet unpublished data from the Missouri Longitudinal Study (Geary, in press), we have identified two groups of LA children, both of whom have moderate deficits on two number sense tasks relative to typically achieving (TA) children. Both groups have an average IQ and average scores on measures of the central executive, phonological loop, and visual spatial sketch pad components of working memory competencies, as well as normal procedural competencies, at least for solving simple addition problems.

However, one group of LA children has a severe fact retrieval deficit. Indeed, from 2nd to 4th grade, they show little growth in ability to correctly remember addition facts. Across these two groups and in comparison to TA children, we have dissociations between number sense deficits (found in both groups), procedural deficits (found in neither group), and fact retrieval deficits (found in one group). A potentially important feature of the retrieval deficit of this group of LA children is a very high frequency (21% in 2nd, 3rd, and 4th grade) of counting-string intrusions (e.g., retrieving `5' to the problem, `3+4') during the retrieval process. The intrusions are consistent with Engle, Tuholski, Laughlin, and Conway's (1999) inhibitory control model of working memory, and with Passolunghi and Siegel's (2004) finding that LA children have more intrusions of irrelevant information into working memory, but our sample of LA children, as noted, had average scores on measures of three core working memory systems.

These findings are understandable in the context of Raghubar, Barnes, and Hecht's (this issue) review of working memory and mathematics achievement. These authors highlight that there are multiple subcomponents of working memory above and beyond or subsumed under the three commonly studied core systems, and that we do not fully understand individual differences in strategic approaches to working memory tasks (Berch, 2008). Mazzocco and Murphy (this issue) make the parallel point that tasks commonly thought to assess fluency and speed of processing may in fact engage working memory resources for some children. To further complicate matters, Meyer, Salimpoor, Wu, Geary, and Menon (this issue) show that the contributions of one component of working memory or another to individual differences in achievement may vary across development or with change in mathematical content being assessed in one grade or the next. These findings are consistent with Ansari's (this issue) caution that brain-mathematics relations found in the developed brain may be very different from those found in the developing brain.

Returning to my three original cognitive deficits, support for the predicted visuospatial deficit has been mixed. Visuospatial competencies do contribute to some aspects of mathematics learning, as found by Meyer et al. (this issue), and some children with MLD have difficulties in the visuospatial organization of mathematical information (Raghubar et al., 2009), but these difficulties do not appear to be as common as the fact retrieval, procedural, and number sense deficits. As Raghubar et al. (this issue) point out, visual working memory and visual representations are not the same as spatial ones. We may need more precise measures of individual differences in the ability to form spatial representations to fully assess the potential relation to MLD. There is, for instance, evidence that the brain regions that support spatial representations for distance and magnitude (e.g., area) might be situated near the same regions that support the approximate representational system and the learning of some aspects of mathematics, such as the number line (Zorzi, Priftis, & Umiltá, 2002; Geary et al., 2008). There is some evidence for delayed maturation of this system, as related to learning the mathematical number line, in children with MLD, but any such deficit is not detected by standard measures of the visuospatial sketch pad.

Where does this leave us? The predicted delayed procedural development for children with MLD and to a lesser extend LA children is an accurate reflection of at least one source of their poor mathematics achievement. Fuchs, Powell, Seethaler, Cirino, Fletcher, Fuchs, and Hamlett (this issue) demonstrate how fine-grain studies of this procedural delay can be used to develop effective interventions for addressing them. Their study demonstrates the effectiveness of using explicit instruction, combined with deliberate practice, to teach children how to use the most developmentally mature counting procedure to solve simple addition and subtraction problems. Their approach is one that should be followed in future intervention research on MLD and LA.

The predicted fact retrieval deficit has also been confirmed, but the sources of the deficit are more complex than I originally suggested. Although the retrieval deficits of children with MLD often co-occur with reading disability, my proposal that the common underlying source of these deficits was the phonological loop and the semantic memory systems that support word retrieval is not a good characterization of these retrieval deficits, at least for many children; some children have arithmetic retrieval deficits and no reading difficulties. The learning of arithmetic facts and the process of retrieving them is more complicated than I originally believed, especially when placed in the context of the developing brain, as done by Ansari (see also, Cho, Kondos, Geary, & Menon, 2009). As noted, there is also much to be learned about my proposed visuospatial deficit. A better characterization might be spatial rather than visual per se, and the mixed evidence might be more due to the types of mathematics studied rather than the absence of this form of deficit.

In 1993 I did not identify a number sense deficit as contributing to MLD and LA, but subsequent work, including some of our own, has revealed such a deficit. The deficit appears to involve the exact, small quantity and approximate representational systems, and may extend to children's implicit understanding of the effects of addition and subtraction on quantity. Finally, working memory deficits do appear to influence the expression of MLD, as argued in 1993, but not always in straightforward ways, as illustrated by Meyer et al. (this issue), Mazzocco and Murphy (this issue), and Raghubar et al. (this issue), and in some cases not at all. Many LA children do not have working memory deficits on standard measures of the central executive, phonological loop, or visuospatial sketch pad, but a subset of them may have specific deficits on the inhibitory control subcomponent of the central executive.

NEUROPSYCHOLOGICAL COMPONENT

In 1993, I suggested posterior left- and right-hemispheric involvement, with some subcortical contributions, to differing degrees as underlying the proposed MLD subtypes. These conclusions were based almost entirely on studies of adult and developmental dyscalculia, that is, deficits associated with known or inferred brain injury. Since that time and with the development of brain imaging technologies, there have been substantive gains in our understanding of the multiple brain regions that contribute to mathematical learning and that likely contribute to the development and on-line expression of MLD and LA. I agree with Ansari's (this issue) main points: 1. The brain systems that support mathematical processing in the adult brain are not likely to be the same as those that support the learning of these processes in the developing brain; 2. The study of MLD and LA have to be placed in the context of our emerging understanding of brain development; and 3. The same behavioral outcomes can be achieved with different brain systems.

Further complicating the study of the brain systems underlying MLD and LA is the effects of schooling and practice on the development of the system of brain regions that support mathematical learning. We know that practice in adults and as implied in cross-sectional studies results in changes in the brain regions underlying mathematical performance (Delazer, Ischeback, Domahs, Zamarian, Koppelstätter, Siedentopf, Kaufman et al., 2005; Rivera, Reiss, Eckert, & Menon, 2005), but as Ansari (this issue) argues we do not know how practice affects the developing brain. We need to understand if the brain at different points in development responds differently to the same instructional practices: Does the mapping of magnitude representations of Arabic numerals onto the approximate representational system – presumably supported in part by the intraparietal sulcus (Dehaene et al., 2003) – occur differently in the three year old brain than in the six year old brain? Does it matter for latter learning?

The angular gyrus might support fact retrieval in adults, but it is much more complicated with children, as noted by Ansari (this issue). By using a combination of behavioral, analytic, and brain imaging techniques, Cho et al. (2009) identified subgroups of 2^{nd} and 3^{rd} graders who largely used retrieval or counting to solve simple addition problems, and demonstrated that children in these subgroup engaged different brain regions during problem solving. As contrasted with the use of counting procedures, the process of retrieval engaged the bilateral hippocampus and areas of the fusiform gyrus, as well as the right intraparietal sulcus and left lateral prefrontal cortex, among other regions. The retrieval deficits associated with MLD and LA could result from developmental delays or deficits in one or several of these brain regions or in the interactions between them. The implication is that there may be multiple forms of retrieval deficit. There are other brain regions and mathematical process and learning that remain to be explored (Dehaene et al., 2003). For instance, does the precuneus - involved in mental imagery, among other forms of cognition (Cavanna & Trimble, 2006) - support aspects of the spatial imagery that is likely to be necessary for some forms of geometry, topography, and other areas of mathematics? Is this a potential contributor to visuospatial deficits associated with some forms of MLD?

GENETIC COMPONENT

Following an early study by Gillis and DeFries (1991) and related research, I concluded in 1993 that there may be common genetic influences on MLD and RD, and that it was unclear whether MLD represented a distinct genetic disorder or the lower end of the normal distribution. Hart, Petrill, and Thompson (this issue) confirm the earlier studies suggesting shared genetic influences on MLD and RD. Their findings also strengthen the recent conclusion that there are independent genetic influences on mathematical achievement and that genetic components of MLD are likely to be the same as those underlying individual differences in mathematics achievement (Kovas, Haworth, Dale, & Plomin, 2007); in other words, most cases of MLD are not likely the result of a distinct genetic disorder. Mazzocco and Murphy's (this issue) finding that the numerical deficits of girls with Turner syndrome are selective, not specific to mathematics and may result from more general deficits in the achievement of fluency and automaticity supports the latter conclusion. The deficits of girls with Turner syndrome may be more of a performance deficit than an underlying numerical deficit. The performance deficit results from difficulty in achieving the automaticity for otherwise routine tasks that in turn makes these tasks more dependent on working memory resources than often assumed.

Hart et al.'s (this issue) finding that the genetic influences on reading fluency are independent of those on arithmetical fluency are inconsistent with my working hypothesis that the comorbidity of MLD and RD is due to a shared semantic memory deficit; specifically, difficulty in word retrieval and arithmetic fact retrieval which would promote fluency in both reading and mathematics. As noted above, the fact retrieval deficit may be more strongly related to poor inhibition of irrelevant associations than the systems that support phonetic memory and word retrieval. At the same time, there may be multiple forms of retrieval deficit, some of which are almost certainly independent of reading ability and RD but others which show more overlap.

CONCLUSION

The collection of studies and reviews in this special issue illustrate how far we have come in our understanding of children's mathematical development and the supporting brain and cognitive systems, and in our understanding of and how to eventually remediate the underlying sources of MLD and LA. As originally proposed, there is good evidence for delayed procedural development for children with MLD and to a lesser degree their LA peers. The sources of these procedural delays remain to be fully determined; a working memory deficit contributes to some of the procedural errors committed by these children, as proposed, but this is not the whole story. The originally proposed fact retrieval deficit has been confirmed, but the underlying source (or sources) of the deficit are more complicated and nuanced than I originally suspected. Support for the proposed visuospatial deficit has been mixed, whether this is because it is less common than the other deficits or because the mathematical tasks used to assess MLD and LA do not require these competencies also remains to be determined.

Although working memory does contribute to some of the deficits associated with MLD, as originally proposed, this again is not the whole story; relations between working memory and the mathematical deficits of children with MLD is also more nuanced than I had suspected. Finally, I did not anticipate a core number sense deficit contributing to MLD and LA, but the evidence strongly supports its existence. The relation between delayed development of or deficit in the number sense systems and the three originally proposed deficits is yet to be determined. In all, many steps have been made since 1993, but we are a long way from completing this journey.

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