# **Europe PMC Funders Group Author Manuscript** *J Psychoeduc Assess***. Author manuscript; available in PMC 2010 June 01.**

Published in final edited form as: J Psychoeduc Assess. 2009 June ; 27(3): 206–225. doi:10.1177/0734282908330583.

# **Math Anxiety and Math Ability in Early Primary School Years**

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### **Abstract**

Mathematical learning disabilities (MLDs) are often associated with math anxiety, yet until now, very little is known about the causal relations between calculation ability and math anxiety during early primary school years. The main aim of this study was to longitudinally investigate the relationship between calculation ability, self-reported evaluation of mathematics, and math anxiety in 140 primary school children between the end of first grade and the middle of third grade. Structural equation modeling revealed a strong influence of calculation ability and math anxiety on the evaluation of mathematics but no effect of math anxiety on calculation ability or vice versa —contrasting with the frequent clinical reports of math anxiety even in very young MLD children. To summarize, our study is a first step toward a better understanding of the link between math anxiety and math performance in early primary school years performance during typical and atypical courses of development.

### **Keywords**

math anxiety; development of calculation ability; mathematical learning disability

When studying learning disabilities, the emotional aspects often associated with these primarily cognitive problems are often neglected. However, particularly in the field of mathematical learning disabilities (MLDs), math anxiety may exert considerable negative effects on the academic and social life of affected children. Already in 1968, Lang stated that math anxiety is—like any other phobia—influencing individuals on three different levels. All three differential effects of math anxiety were confirmed independently: (a) Physiological reactions (i.e., sweating or high pulse rate) as frequent accompanying symptoms of math anxiety were described by Faust (1992), (b) cognitive effects of math anxiety (worrisome thoughts) were demonstrated by Richardson and Woolfolk (1980), and (c) avoiding behavior concerning number processing and calculation was first systematically analyzed by Hembree (1990).

More extensive research on the link between math ability and math anxiety began only in the 1990s (Ashcraft & Faust, 1994; Hembree, 1990; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998). According to popular behavioristic models, anxiety emerges as an obligatory

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response to an aversive stimulus (Watson & Rayner, 1920). Thus, it is plausible to speculate that frequent poor math performance or failure to understand math concepts (despite investing high efforts) leads to negative emotions such as math anxiety, which in turn is likely to provoke avoidance behavior (Miller, 1948; Mowrer, 1939, 1947). There is also evidence that the negative evaluation of failure in mathematics might be mediated by cultural influences and educational factors (i.e., parents' expectations of performance or attribution of success; Stevenson, Hofer, & Randel, 2000).

However, the association between math ability and math anxiety may not be unidirectional. Rather, it has been suggested previously that emotional factors might generally influence cognitive abilities (e.g., Easterbrook, 1959). Concerning the possible impact of math anxiety on calculation ability, some authors state that the avoidance behavior caused by math anxiety will most probably lead to a vicious cycle being characterized by less calculation practice, which will cause a lag in learning and therefore even more disappointment and emotional problems (Ashcraft, 2002; for an overview, see Dowker, 2005). The assumption that math anxiety influences math ability is strongly supported by a meta-analysis showing that successful treatment of math anxiety in adults leads to significant improvement of their calculation performance, even though math ability was not trained explicitly (Hembree, 1990). It is interesting that decreasing math anxiety also positively influenced math performance in MLD children attending Grades 4 to 7 (Kamann & Wong, 1994).

Beyond the long-term effects of avoidance behavior, worrisome thoughts are also known to have a direct negative impact on calculation performance. Worrisome thoughts are very hard to inhibit and therefore will absorb working memory and attentional resources ("deficient inhibition mechanism": Hopko et al., 1998). The assumptions of the deficient inhibition mechanism are based on two theories—namely, the inhibition theory proposed by Hasher and Zacks (1988) and the processing efficiency theory purported by Eysenck and Calvo (1992). The first theory postulates general decreases in cognitive performance during the presence of distracting stimuli (Hasher & Zacks, 1988). The second theory states that experiencing anxiety will draw on working memory capacities and therefore will compromise cognitive performance. When combining these two theories, Hopko et al. (1998) suggested that poorer calculation abilities of individuals with high math anxiety are not a direct consequence of their worrisome thoughts but rather are due to an inability to withdraw attention from these thoughts. They also reported empirical evidence for the negative impact of math anxiety and its accompanying worrisome thoughts on cognitive performance: In three groups of college students with low, medium, and high math anxiety, individuals with low anxiety were better able to inhibit distractors while reading texts compared to the other two groups. Furthermore, Ashcraft and Kirk (2001) found that relative to college students with low math anxiety, those with higher math anxiety displayed lower working memory spans for numerical tasks and moreover exhibited longer reaction times and higher error rates in addition and number transcoding tasks when asked to simultaneously solve tasks drawing on working memory capacities.

More empirical evidence for the negative effects of math anxiety on math ability during calculation performance was reported by Ashcraft and Faust (1994): Adults exhibiting high math anxiety solved calculation problems faster and less accurately compared to individuals without math anxiety. This speed–accuracy tradeoff was interpreted by the authors as being caused by the wish to terminate anxiety-eliciting situations as soon as possible.

No matter if one plans to study math anxiety as a dependent or an independent variable, there is a need for a standardized measurement instrument. The first published math anxiety questionnaire was the Mathematics Anxiety Rating Scale (MARS; Richardson & Suinn, 1972), which, like the Abbreviated Math Anxiety Scale (Hopko, Mahadevan, Bare, & Hunt,

2003), is a self-reporting questionnaire assessing math anxiety in adults. However, the MARS is not suitable for exploring math anxiety in children because questions and answers are phrased in a quite complex way and some of these questions deal with calculation in contexts that are not yet relevant for primary school children.

The first and to our knowledge still only self-reporting scale suitable for studying math anxiety in primary school children was constructed by Thomas and Dowker (2000) and is called the Math Anxiety Questionnaire (MAQ). The need for this instrument arose in a clinical context, as a considerable proportion of MLD children not only showed the above described triad of avoidance behavior, physiological reactions, and math anxiety but likewise exhibited associated psychosomatic symptoms (e.g., stomach ache) or psychiatric disorders (e.g., depression). Thus, there is a clear need for a standardized instrument exploring emotional factors associated with math performance in primary school children.

A first pilot study using the MAQ (for a detailed description of the questionnaire, see the "Method" section) showed significant positive correlations between self-perceived performance, attitudes, and calculation ability but no correlations between poor performance unhappiness as well as math anxiety and calculation ability in 6- and 9-year-old children (Thomas & Dowker, 2000). When considering the consistently found negative correlations between math ability and math anxiety in adults (for an overview, see Hembree, 1990), Thomas and Dowker (2000) hypothesized that the link between math ability and math anxiety is age dependent (i.e., may become stronger with age and/or schooling). In a previous study using a German translation of the MAQ ("Fragebogen für Rechenangst") in a large sample of primary school children (Krinzinger et al., 2007, published in German), we reported significant correlations in the range of .25 to .45 between self-perceived performance/attitudes with calculation ability as well. Though the results concerning the association between calculation ability and math anxiety differed somewhat across age groups, they did not lend support to a developmental increase as suggested by Thomas and Dowker (2000).

Overall, there is accumulating evidence for a link between math ability and math anxiety in adults. Nonetheless, the question of the direction of causality between these two factors remains as yet unresolved. Longitudinal developmental studies are strongly needed to further disentangle the impact of math anxiety on math ability and vice versa. Taking into account that a better understanding of the link between math anxiety and math ability could have important implications for designing and improving remediation programs for children diagnosed with MLDs and mathematics curricula alike, it is quite surprising that so far no such investigations have been carried out.

To study the possible causal relations between math anxiety and math ability after the beginning of formal schooling, we administered the MAQ as well as timed calculation tasks to a group of primary school children four times at an interval of about 6 month each. Using this longitudinal design, we aimed at investigating (a) whether math anxiety predicts future calculation ability, (b) whether calculation ability predicts future math anxiety, and (c) whether such associations are mediated by the individual's evaluation of mathematics.

# **Method**

### **Participants**

One hundred and forty-nine children participated in this study. All children attended mainstream classes and were recruited from five different schools supporting mostly middleincome families in Aachen, Germany. Written and informed consent was given by all parents, teachers, and headmasters involved. Seven children of the initial sample had to

repeat a school year because of severe learning problems (which is a quite frequent procedure in Germany) and were therefore excluded from the sample. Another two children were excluded for withdrawn consent (once by the parents and one child refused to participate after the second testing session), thus resulting in a final sample of 140 children. Of these 140 children, 80 (57.1%) were female and 60 (42.9%) were male. Mean age at the end of first grade when children first participated in our study was 7 years and 6 months (SD  $=$  4 months, range  $=$  6.8 to 8.5 years). Five children moved away during the overall testing period of 1 and a half years. As none of them had shown any learning problems, their missing values were treated like missing values caused by illness (replaced by the means of all other children at the level of task raw scores).

### **Materials and Procedure**

The same children were tested at four different time points, with approximately half a year between the testing sessions (T1: end of first grade; T2: middle of second grade; T3: end of second grade; T4: middle of third grade). Individual testing sessions were carried out by the first author of this article or a graduate psychology student, and group testing sessions were always carried out by the first author.

The calculation tasks were presented in four blocks, for which the overall processing time was measured in seconds each. The four blocks included small and large additions as well as small and large subtractions. In individual testing sessions, calculation problems were presented one by one on a sheet of paper, and children were asked to orally solve the problems as fast as possible. Small additions and subtractions composed all possible combinations of single-digit numbers from 2 to 9, with a result smaller than 10 (28 items each). Large calculations included single-digit plus single-digit number items (numbers also from 2 to 9), with a result larger than 10 for complex additions, and teen numbers minus single-digit numbers, with half the results smaller and half larger than 10 for complex subtractions (20 items each). To make performance comparable across development, we calculated performance measures (correctly solved problems per minute) for each of the four calculation types for each time point.

The math anxiety questionnaire used in this study was the German translation of the MAQ (Thomas & Dowker, 2000). In a standardization study published in German, the internal consistency (Cronbach's alpha) was reported to vary between .83 and .91 for the whole questionnaire depending on the age group (Krinzinger et al., 2007).

The MAQ requires children to answer four different types of questions ("How good are you at … ?" "How much do you like … ?" "How happy or unhappy are you if you have problems with … ?" and "How worried are you if you have problems with … ?") on one training situation (writing; e.g., "How good are you at writing?") and on seven subsequent math-related situations each (math in general, written calculations, mental calculations, easy calculations, difficult calculations, math homework, and listening and understanding during math lessons; e.g., "How much do you like math in general?"). Children were asked to mark their respective answers on a 5-point scale using different pictures for each type of question (see Figure 1; Krinzinger et al., 2007). The ratings varied from 0 for the most negative possible answer to 4 for the most positive possible answer, thus resulting in an overall minimum score of 0 (most negative) and an overall maximum score of 28 (most positive).

For each different type of situation, children were asked to mark their respective answers to the four related questions in a different color stated by the experimenter. The MAQ was administered in groups of 15 to 20 children.

In the previously mentioned standardization study (Krinzinger et al., 2007), we employed Facetted Smallest Space-Analysis (Shye, 1985; Shye & Elizur, 1994) to examine the empirical structure underlying the four different types of questions. This nonmetric multidimensional similarity scaling method represents pairwise similarities of objects in a lowdimensional space in a way that objects (items) with high similarity are close to each other in space. If items are assessing similar aspects, the respective items should cluster together in space, and items containing different aspects should be spatially separable. The similarity measure used is the so-called monotonicity coefficient—a measure of monotone relationship between items. We found that items belonging to the first two types of questions ("How good are you at … ?" and "How much do you like … ?") as well as those tapping the second two types of questions ("How happy or unhappy are you if you have problems with … ?" and "How worried are you if you have problems with … ?") were located close together and well separated from each other, respectively. We interpret the first aspect as general mathrelated attitudes (evaluation of mathematics) and the second factor as negative emotions and anxiety concerning mathematics (math anxiety).

Structural Equation Modeling (using AMOS 7.0) was applied to examine potential influences between the three variables of calculation ability, evaluation of mathematics, and math anxiety. This analytical method allows for testing the adequacy of a set of postulated multivariate regression (single-headed arrows) or correlation (double-headed arrows) equations between so-called latent variables or constructs based on the correlation matrix of observed variables using the maximum-likelihood method (Hoyle,  $1995$ ).<sup>1</sup> The observed variables measuring the three constructs were (a) correctly solved calculation problems per minute for each of the four operations (small and large additions, small and large subtractions) per time point for calculation ability (CALC1 to CALC4), (b) the sums of the item scores belonging to the first and the second question type of the MAQ per time point for evaluation of mathematics (EVAL1 to EVAL4), and (c) the sums of the third and the fourth type of question of the MAQ per time point for math anxiety (ANX1 to ANX4). Our model was specified according to the regular developmental sequence (see Figure 2). Regressions (paths) were only allowed from one latent variable at one time point to the other latent variables at the next time point half a year later to make sure they capture causal and not only correlational relations. To our knowledge, no empirical evidence has been presented so far concerning the relations between calculation ability, evaluation of mathematics, and math anxiety at the beginning of formal schooling. Therefore, we allowed only for correlations between these three factors at T1 (end of first grade).

Error covariances—as suggested by modification indices in the process of the evaluation of model fit—were only allowed between observed variables measuring the same latent variable at the same time point or between the same observed variables at different time points.

To evaluate the goodness of fit of our model to the sample data, we used the chi-square test statistic of model fit  $(X^2$ ; Jöreskog, 1969) and its relation to the degrees of freedom, the Adjusted Goodness of Fit Index (AGFI; Jöreskog & Sörbom, 1984), the Comparative Fit Index (CFI; Bentler, 1990), the Root Mean Square Error of Approximation (RMSEA; Browne & Cudeck, 1993), and the Akaike Information Criterion (AIC; Akaike, 1987). For a good model, the relation between  $X^2$  and its associated degrees of freedom should be less than 2, CFI and AGFI larger than 0.90, RMSEA smaller than 0.05, and AIC smaller compared to the saturated model (all paths free) and the independence model (all paths set to zero).

<sup>&</sup>lt;sup>1</sup>The correlation coefficients among the observed variables can be obtained from the authors.

J Psychoeduc Assess. Author manuscript; available in PMC 2010 June 01.

## **Results**

#### **Descriptive Statistics**

Descriptive statistics for the observed variables at the four time points are reported in Table 1. To examine whether there was a (linear) trend over time, we computed the regression slopes of the means of the observed variables for each latent variable over the four time points (cf., Figure 3) for each individual child. Then, we tested whether the mean slope was significantly different from zero by means of one-sample t tests (as proposed by Lorch  $\&$ Myers, 1990). We could show (a) a highly significant linear increase of calculation ability with time (mean slope = 3.39),  $t(139) = 33.44$ ,  $p < .001$ , with all 140 children presenting a positive slope; (b) a significant but small linear decrease in the evaluation of mathematics over time (slope =  $-.41$ ),  $t(139) = -3.13$ ,  $p = .002$ , with 58 children presenting with a positive slope, 79 children with a negative slope, and 3 children with a slope equal to zero; and (c) significantly increasing math anxiety with age (slope = 1.32),  $t(139) = -8.97$ ,  $p <$ . 001, with 109 children presenting with a negative, 29 with a positive, and 2 with no developmental change.

#### **Longitudinal Model**

The fit statistics of our model can be found in Table 2. Overall, the fit indices can be considered as satisfying. The standardized solution for the structural model with at least marginally significant paths ( $p < 0.1$ ) can be seen in Figure 4.

In general, the high regression weights from the latent variables at one time point to the same latent variables half a year later (all  $p < .001$ ) can be interpreted as quite stable development concerning all three constructs of interest. This developmental stability is most pronounced for calculation ability (regression weights between .79 and .91), showing that children who are good calculators compared to their peers at T1 are also superior at all other time points.

Concerning correlational relations between the three constucts at T1 (end of first grade), significant correlations were found between calculation ability and evaluation of mathematics ( $p < .001$ ) as well as between evaluation of mathematics and math anxiety ( $p$ )  $\langle 0.001 \rangle$  but not between calculation ability and math anxiety ( $p > 0.1$ ).

Both regression weights between calculation ability and evaluation of mathematics and vice versa were significant from T1 to T2 (CALC1  $\rightarrow$  EVAL2:  $p = .015$ ; EVAL1  $\rightarrow$  CALC2:  $p$ < .001), but only the regressions from calculation ability to evaluation of mathematics were significant from T2 onward (CALC2  $\rightarrow$  EVAL3:  $p = .085$ ; CALC3  $\rightarrow$  EVAL4:  $p = .009$ ).

No regression weights between math anxiety and evaluation of mathematics were significant from one time point to the next, but modification indices suggested to include regressions from anxiety to evaluation at T2 (ANX2  $\rightarrow$  EVAL2:  $p < .001$ ) as well as at T3 (ANX3  $\rightarrow$ EVAL3:  $p = .032$ ), indicating an immediate rather than a delayed causal relation between math anxiety and evaluation of mathematics.

No significant causal relations between calculation ability and math anxiety could be observed at all. The negative regression weight from math anxiety at T1 to calculation ability at T2 ( $p = .01$ ) should be interpreted as the "neutralization" of a doubling of indirect effects: (a) via the correlation between math anxiety and evaluation of mathematics at T1 and the regression from evaluation at T1 to calculation ability at T2 on one hand and (b) the correlation between evaluation and calculation at T2 and the regression from calculation at T1 to T2 on the other hand. In other words, the two paths of indirect effects from math anxiety at T1 to calculation ability at T2 sum up to a higher effect than can actually be found

in the data, and this is compensated for by the respective direct negative regression weight  $(ANX1 \rightarrow CALC2)$ . As indicated by the AMOS output for standardized indirect effects, the actual indirect effect from math anxiety at T1 to calculation ability at T2 is less than .001.

# **Discussion of the Longitudinal Model**

To summarize the results of our longitudinal model, we found very stable developmental trajectories for calculation ability and moderate developmental stability for the two emotional latent variables evaluation of mathematics and math anxiety. It is a well-known finding that individual differences in simple arithmetic such as fact retrieval are already present at the beginning of primary school and persist until much later in development (e.g., Geary, Brown, & Samaranayake, 1991; Ostad 1998). In contrast, our understanding of the developmental trajectories of math anxiety and other emotional factors related to mathematics in early primary school years is very limited. To the best of our knowledge, this study was the first to investigate possible causal relations between math anxiety, evaluation of mathematics, and math ability using a longitudinal design in primary school children.

What we found was that calculation ability and evaluation of mathematics were correlated at the end of first grade and influenced each other mutually until the middle of second grade. From the middle of second grade onward, only calculation ability was predicted evaluation of mathematics, with the highest influence from the second last to the last time point. This may mean that only for very young children, who have not received a lot of external feedback concerning their calculation ability, their self-perceived performance and attitudes might influence their calculation performance via motivational factors. Furthermore, results suggest that later during formal education, when children have received more feedback and thus have become more experienced in comparing their own abilities to those of their peers, results suggest that only actual calculation ability is a predictor for future evaluation of mathematics but not the other way around.

Evaluation of mathematics was also influenced by math anxiety. Yet this impact was found not from one time point to the next but between both emotional factors at the same time point. This may be interpreted as reflecting a simultaneous and direct impact of unhappiness and worrisome thoughts on children's evaluation of mathematics.

Concerning the most important research question of our study—the possible causal relations between math anxiety and math ability—our findings were not suggestive of any direct influence of one on the other. Thus, our results replicate the null result reported in the pilot study by Thomas and Dowker (2000) using the MAQ in a small sample of 6- and 9-yearolds and contradict the regular finding of a negative correlation between math anxiety and math ability in adults (for an overview, see Hembree, 1990). In our study, this null result cannot be explained by small sample size. Alternatively, it may be speculated that both math anxiety and poor calculation ability exhibit negative effects on each other only if they are extremely pronounced, which could be attenuated in our model by the much larger proportion of typically developing children (i.e., children without diagnosed MLDs).

To examine this possibility, we conducted further analyses using specific subgroups of children. If only very low calculation performance has an influence on math anxiety, we would expect group differences either in the math anxiety scores (higher for very poor calculators) or more specifically in the developmental slope of math anxiety (steeper for very poor calculators). On the other hand, if very high math anxiety has an impact on calculation ability, we would expect either lower calculation scores or a less shallow slope of calculation development for children with very high math anxiety.

#### **Subgroup Analyses**

To investigate whether only very poor calculation ability or very high math anxiety might have an influence on the development of the other construct, we first calculated the mean of the observed variables loading on calculation ability or math anxiety for each time point. Next, children scoring below the 15th percentile for each of the two factors at any time point were selected. To reduce the number of different possible developments, three performance groups for each of the two factors were established: children never scoring below the 15th percentile, children scoring below the 15th percentile one or two times out of four, and children scoring below the 15th percentile three or four times out of four. For calculation ability, the numbers of children scoring one, two, three, or four times below the 15th percentile were 16, 5, 7, and 6, respectively. For math anxiety, the respective numbers of children were 27, 11, 5, and 3. Therefore, the respective group sizes for falling never, one or two times, or three or four times below the 15th percentile were 96, 21, and 13 for calculation ability and 94, 38, and 8 for math anxiety. Hence, the group of poor calculators (scoring one or two times below the 15th percentile in calculation) contained 15% of all children, and the group of very poor calculators (scoring three or four times below the 15th percentile) composed 9.2% of all children. The group of children with high math anxiety (scoring one or two times below the 15th percentile for math anxiety) included 27.1% and the group with very high math anxiety (scoring below the 15th percentile for math anxiety three or four times) 5.7% of all children.

The developmental slopes for the three groups of calculation performance and math anxiety for both factors can be seen in Figures 5 through 8.

Descriptively, the score levels and the slopes of math anxiety development were comparable for the different calculation performance groups. Likewise, no significant group differences emerged with respect to level of math anxiety in the score levels or in the slopes of calculation ability.

These observations were statistically confirmed: In a MANOVA using calculation performance group (three levels) and math anxiety group (three levels) as between-subject factors and calculation ability slopes and math anxiety as well as the respective scores for all four time points as dependent variables, neither significant effects of math anxiety group on calculation performance measures, all  $F(2, 139) < 0.76$ , all  $p > .47$ , nor significant effects of calculation performance group on math anxiety measures, all  $R_2$ , 139) < 2.25, all  $p > .10$ , were found. Similarly, no significant interactions of math anxiety group and calculation performance group on any of the dependent variables were found, all  $R(2, 139) < 1.94$ , all p  $> .12.$ 

As regard differences in development of math anxiety by math anxiety group, significant group differences in math anxiety scores for all four time points, all  $R(2, 139) > 7.5$ , all  $p <$ . 001, emerged, which is according to expectation. Post hoc multiple comparisons showed that these group differences between math anxiety groups for math anxiety scores were significant for all comparisons at all time points (all  $p < .03$ , Bonferroni-corrected), except for the comparison between the very high and the high anxiety group at  $T2$  ( $p = .077$ , Bonferroni-corrected). Yet we could not find differences in the developmental slope for math anxiety between the three math anxiety groups,  $F(2, 139) = 0.22$ ,  $p = .80$ .

Similar differences were found for the development of calculation ability by calculation group for the respective scores at the four different time points, all  $R(2, 139) > 14.84$ , all p  $<$  0.001. These differences were confirmed for all group differences at all time points (all  $p$ ) < .023, Bonferroni-corrected), except for the comparison between very poor and poor calculators at T1 ( $p = .23$ , Bonferroni-corrected), applying post hoc multiple comparisons.

Contrary to the similar slopes for math anxiety development in the different math anxiety groups, results reflected differential slopes regarding calculation development for the three calculation performance groups,  $F2$ ,  $139$ ) = 7.7,  $p < .001$ . Post hoc multiple comparisons revealed that the slopes of the children never scoring below the 15th percentile in calculation were significantly steeper compared to the poor calculators (scoring below the 15th percentile one or two times out of four;  $p = .001$ , Bonferroni-corrected) and the very poor calculators (scoring below the 15th percentile three or four times out of four;  $p < .001$ , Bonferroni-corrected). Nonetheless, the slopes did not differ significantly between the poor and the very poor calculators ( $p = .25$ , Bonferroni-corrected).

These results clearly showed that in our sample not even the poorest calculators display differential effects concerning math anxiety development. Moreover, our findings were not suggestive of a significant influence of very high math anxiety on the development of calculation performance. However, it is interesting that our results revealed a parallel development of math anxiety in the three different math anxiety groups but a steeper developmental slope for calculation development in children who never scored below the 15th percentile in calculation compared to poor and very poor calculators. This indicates that the performance lag of poor calculators seems to increase between the end of first grade and the middle of third grade, whereas the difference in math anxiety level in the three math anxiety groups seems to remain stable during this developmental period.

# **General Discussion**

It has been known for a long time that in adults math anxiety is highly relevant for affected individuals (Lang, 1968) and related to math ability (Hembree, 1990). Previous findings support a possible causal role both of math ability on math anxiety (Dowker, 2005) as well as for math anxiety on math ability (Ashcraft & Faust, 1994; Ashcraft & Kirk, 2001; Hembree, 1990; Hopko et al., 1998; Kamann & Wong, 1994). As frequently observed by clinicians, math anxiety and related problems such as depression or psychosomatic disorders exert negative effects on children diagnosed with MLDs (for an overview, see Dowker, 2005). Yet almost nothing is known about the causal relations between math anxiety and math ability during development, particularly at the beginning of primary school, when children are introduced to formal calculation, receive feedback on their performance, and start to compare their skills with those of their peers.

To address this question and to investigate a possible mediating role of individual children's evaluation of mathematics on the possible influences of math anxiety on math ability and vice versa, a large sample of primary school children was assessed longitudinally over a period of 1 and a half years with respect to their calculation ability, their math anxiety, and their evaluation of mathematics.

Applying Structural Equation Modeling to analyze our longitudinal data set, we found that evaluation of mathematics is influenced both by past calculation performance as well as by concomitant math anxiety level. Therefore, evaluation of mathematics is unlikely to be a mediator between math anxiety and math ability for early primary school children.

More important, no significant association was found between calculation ability and math anxiety, which contradicts the consistently reported negative correlations between math anxiety and math ability in adults (Hembree, 1990). Employing subgroup analyses, we could rule out that this null result might be due to the fact that only very pronounced calculation difficulties or extremely high math anxiety might influence the development of the other factor.

So which alternative explanations could be raised to explain the absence of any causal influence between math ability and math anxiety during early primary school years in our study?

First, it might be that for primary school children the association between math anxiety and calculation performance is weak to nonexistent, for example because math anxiety may be more related to personality aspects such as general anxiety or because it might be very strongly mediated by teachers' or parents' attitudes (Stevenson et al., 2000). Yet clinicians working with MLD children frequently report that already during second and third grade, math anxiety and related psychosomatic or even psychiatric problems such as depression are frequently diagnosed in children with MLD (Dowker, 2005). As it is very unlikely that MLD is caused by anxiety as a personality trait (for a recent overview about possible causes of MLD, see Wilson & Dehaene, 2007), we believe that the frequent occurrence of math anxiety in MLD children even in second and third grade underlines the possibility that math anxiety and math ability are closely interlinked in elementary school children with severe math problems or high math anxiety. Rather, our inability to find such influences may be due to methodological issues such as the choice of study measures.

We believe that the number of correctly solved simple addition and subtraction problems per minute is well suited to capture the development of calculation performance, because even for solving more complex problems such as multidigit calculations (which children are expected to learn during the second half of second grade in Germany), it is important to solve the intermediate steps fast and accurately. The adequateness of this measure is further supported by our finding of its very high developmental stability.

Concerning math anxiety, we now believe that asking children about their experienced unhappiness and worry caused by problems in calculation might not be the best measure if one is interested in the possible mutual influences between math anxiety and math ability in early primary school children. First, it could be that the cognitive level of anxiety ("worrisome thoughts"; Lang, 1968) may not be the most suitable aspect of math anxiety to be examined in children as young as 7 to 9 years old. Instead, physiological reactions such as high pulse or avoiding behavior concerning calculations might be more reliable and valid measures for math anxiety at that age. Second, it might also be that the exact phrasing of the MAQ questions poses difficulties for young children. Asking directly for anxiety in mathrelated situations (and not for unhappiness and worry in situations where you have problems in calculation) might be a more direct way to investigate worrisome thoughts.

To conclude, the findings of this longitudinal study revealed a close relationship between math anxiety and math ability on evaluation of mathematics in primary school children. On the other hand, in our sample, math anxiety did not exert direct and/or developmentally stable effects on math ability (and vice versa). By emphasizing the negative impact of math anxiety and poor math ability on children's evaluation of mathematics, the present findings are relevant for future research endeavors targeted at investigating the development of math anxiety and its relation to math performance. Furthermore, our results stress the need for standardized and developmentally appropriate instruments to assess math anxiety during early primary school years, the time when math anxiety most probably first emerges.

### **Acknowledgments**

We would like to thank the participating children for their cooperation and time dedicated to our study. Moreover, we are grateful to their caregivers and teachers as well as headmasters, who gave their permission to participate and who supported this longitudinal project. Furthermore, we want to thank Katharina Wopp, Jan Lonnemann, and especially Frederike Feldmann for help with testing the children. The first author was supported by the European

Union Marie-Curie RTN "NUMBRA" Proposal 504927; Liane Kaufmann was supported by the Austrian Science Fund (T286-B05) and the Tyrolean Science Fund (UNI-0404/523).

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### **Figure 1.**

MAQ Pictorial Rating Scales for the Four Types of Questions Source: Krinzinger et al. (2007), copyright by courtesy of Hans Huber Verlag. Note: The four types of questions are as follows: "How good are you at … ?" (very good to very bad) "How much do you like ... ?" (not at all to very much) "How happy or unhappy are you if you have problems with ... ?" (very happy to very unhappy) "How worried are you if you have problems with ... ?" (very worried to very relaxed). The check marks and crosses are for very good to very bad self-perceived performance, wasps and candies are for very negative to very positive attitudes, happy and unhappy faces are for poor performance unhappiness, and worried and relaxed faces are for anxiety.



#### **Figure 2.**

Model Specification (Structural Part Only) for a Longitudinal Developmental Model Linking Calculation Performance (CALC; 4 Observed Variables Each), Evaluation of Mathematics (EVAL; 2 Observed Variables Each), and Math Anxiety (ANX; 2 Observed Variables Each) for Primary School Children From the End of Grade 1 (T1) to the Middle of Grade 3 (T4) With Paths Allowed Between All Three Latent Variables at T1 (Correlations) and From Each Latent Variable from one time point to All Three Constructs From One Time Point to the Next Half a Year Later



### **Figure 3.**

Development of Mean Raw Scores ( $n = 140$ ) of Calculation Ability (Correctly Solved Items Per Minute), Evaluation of Mathematics (Sum Score With 0 as Most Negative and 28 as Most Positive Possible Outcome), and Math Anxiety (Sum Score With 0 as Most Negative and 28 as Most Positive Possible Outcome) From T1 (End of Grade 1) to T4 (Middle of Grade 3)

Note: Error bars represent standard deviations of the mean.



#### **Figure 4.**

Longitudinal Model (Standardized Solution, Structural Part Only) for Linking Calculation Performance (CALC; 4 Observed Variables Each), Evaluation of Mathematics (EVAL; 2 Observed Variables Each), and Math Anxiety (ANX; 2 Observed Variables Each) for Primary School Children From the End of Grade 1 (T1) to the Middle of Grade 3 (T4) Note: All paths  $p < .10$ ; fit statistics:  $X^2/df = 1.29$ ; Adjusted goodness of Fit Index = .771; Comparative Fit Index = .968; Root Mean Square Error Approximation = .046.



## **Figure 5.**

Development of Mean Raw Calculation Ability Scores (Correctly Solved Items per Minute) for Each Math Anxiety Group From T1 (End of Grade 1) to T4 (Middle of Grade 3) Note: Black line is the anxiety score 15th percentile at all time points; long dashed line is the anxiety score < 15th percentile at one or two time points; short dashed line is the anxiety score < 15th percentile at three or four time points. Error bars represent standard deviations of the mean.



### **Figure 6.**

Development of Mean Raw Math Anxiety Scores (Sum Score With 0 as Most Negative and 28 as Most Positive Possible Outcome) for Each Calculation Performance Group From T1 (End of Grade 1) to T4 (Middle of Grade 3)

Note: Black line is the calculation score 15th percentile at all time points; long dashed line is the calculation score < 15th percentile at one or two time points; short dashed line is the calculation score < 15th percentile at three or four time points. Error bars represent standard deviations of the mean.



#### **Figure 7.**

Development of Mean Raw Math Anxiety Scores (Sum Score With 0 as Most Negative and 28 as Most Positive Possible Outcome) for Each Math Anxiety Group From T1 (End of Grade 1) to T4 (Middle of Grade 3)

Note: Black line is the anxiety score 15th percentile at all time points; long dashed line is the anxiety score < 15th percentile at one or two time points; short dashed line is the anxiety score < 15th percentile at three or four time points. Error bars represent standard deviations of the mean.



#### **Figure 8.**

Development of Mean Raw Calculation Ability Scores (Correctly Solved Items per Minute) for Each Calculation Performance Group From T1 (End of Grade 1) to T4 (Middle of Grade 3)

Note: Black line is the calculation score 15th percentile at all time points; long dashed line is the calculation score < 15th percentile at one or two time points; short dashed line is the calculation score < 15th percentile at three or four time points. Error bars represent standard deviations of the mean.

**Table 1**

T2 140 5.60 48.00 21.69 0.59 7.03 0.59a 0.78 T3 140 8.97 41.54 22.80 0.59 6.93 0.40 −0.13 T4 140 14.74 60.00 30.56 0.64 7.56 0.41a 0.85a

 $-0.13$  $0.85^{\it a}$  $1.01<sup>4</sup>$ 

 $0.40$ 

6.93

22.80

0.78

 $0.59^{d}$ 

 $7.03$ 

T2 140 2.07 32.57 11.85 0.40 4.69 0.90a 1.97a T3 140 3.69 24.78 13.23 0.41 4.81 0.51a −0.38 T4 140 4.67 38.00 17.77 0.47 5.62 0.65a 1.13a

11.85 13.23

 $\mathbb{R}$  $\Gamma$  $\overline{\Gamma}$ 

 $0.40$ 

 $-0.38$ 

4.81

 $0.41$ 

 $1.13<sup>a</sup>$ 

 $0.65<sup>a</sup>$ 

5.62

 $0.47$ 

17.77

38.00

0.66 0.66

 $0.84<sup>4</sup>$ 

4.18

0.35

9.42

23.66 37.33

 $1.97<sup>2</sup>$ 

 $0.90<sup>4</sup>$  $0.51<sup>2</sup>$ 

 $0.91<sup>4</sup>$ 

3.90 4.69

0.33

9.06

2.47 2.07 3.69 4.67 2.13 2.82 3.09

 $\overline{\Box}$ 

 $0.41<sup>2</sup>$ 

7.56

0.64

30.56

 $60.00$ 22.35 32.57 24.78

14.74

Descriptive Statistics of Observed Variables Descriptive Statistics of Observed Variables

**Observed Variable Time Point** *n* **Min. Max.** *M SE SD* **Skewness Kurtosis**  $L_{\text{M}}$   $=$   $\frac{1}{0.91}$   $=$   $\frac{0.73}{0.91}$   $=$   $\frac{0.73}{0$  $L_{\rm M}$ ge subtractions: correct per minute T1 140 140 140 140 140 1194 1.19a 1.19a 1.19a 1.19a 1.19a 1.19a 1.19a 1.19a 1.19a 1.10 1.19a 1.10 1.19a 1 MAQ Question 1: sum T1 140 11 28 21.83 0.36 4.22 −0.35 −0.69 MAQ Question 2: sum T1 5.64 7 20.47 20.47 20.47 0.48 5.64 −0.61 −0.61  $S_{\rm 25}$   $S_{\rm 34}$   $S_{\rm 41}$   $S_{\rm 35}$   $S_{\rm 26}$   $S_{\rm 27}$   $S_{\rm 28}$   $S_{\rm 29}$   $S_{\rm 21}$   $S_{\rm 21}$   $S_{\rm 24}$   $S_{\rm 24}$   $S_{\rm 24}$   $S_{\rm 24}$   $S_{\rm 24}$   $S_{\rm 24}$   $S_{\rm 25}$   $S_{\rm 24}$   $S_{\rm 25}$   $S_{\rm 26}$   $\frac{S}{S}$   $\frac{S}{S}$  **Time Point** Large subtractions: correct per minute Small subtractions: correct per minute Small additions: correct per minute Large additions: correct per minute MAQ Question 1: sum MAQ Question 2: sum Observed Variable

T2 140 2.82 37.33 14.34 0.51 5.99 0.69a 0.66 T3 140 3.09 39.07 17.46 0.53 6.22 0.45a 0.46 T4 140 7.81 39.07 21.92 0.58 6.82 0.20 −0.20

14.34 17.46 21.92

 $\mathbb{R}$  $\Gamma$  $\rm{F}$ 

 $\overline{\Gamma}$ 

 $0.69<sup>d</sup>$ 

5.99 6.22 6.82

 $0.51$ 

 $T2$  140 0.00 22.35 8.07 0.37 4.32 0.85<sup>a</sup> 0.64 T3 140 1.13 24.00 9.64 0.40 4.72 0.67a 0.12 T4 140 2.39 35.63 12.85 0.46 5.42 0.79a 1.62<sup>a</sup>

0.37

 $0.40$  $0.46$ 

> 12.85 21.83 21.76 20.36 21.58

35.63  $28$  $28$  $28\,$  $28$  $28$  $28$  $28$  $28$  $28$  $28$ 

 $1.24<sup>a</sup>$ 

 $1.19^{d}$  $0.85^{\it a}$  $0.67<sup>4</sup>$  $0.79<sup>d</sup>$  $-0.35$  $-0.65$ 

 $3.10$ 4.32 4.72 5.42 4.22 4.76 4.89 4.08

5.20 8.07 9.64

 $0.00$  $0.00$  $1.13$ 2.39

 $\overline{40}$  $\overline{40}$  $\frac{40}{5}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$ 

 $\overline{\phantom{m}}$ 

 $\Gamma$  $\Gamma$  $F_{4}$ 

 $0.64\,$ 

 $0.46$  $-0.28$ 

 $0.45<sup>a</sup>$ 

0.53

39.07

 $0.20$ 

0.58  $0.26$ 

39.07  $16.52$ 22.35  $24.00$ 

7.81

 $1.62^{\it a}$  $-0.69$  $-0.08$ 

 $0.12$ 

T2 140 9 28 28 0.40 4.76 −0.65 −0.65 T3 140 6 28 20.36 0.41 4.89 −0.43 0.01 T4 140 8 28 21.58 0.34 4.08 −0.66 0.58

 $0.40$ 

0.36

 $\equiv$  $\circ$  $\epsilon$  $\infty$ 

 $\overline{\Box}$ 

 $\Gamma$  $\mathbb{E}$  $F_{4}$  T2 140 5 28 28 5.88 5.88 −0.50 140 5.66 T3 140 0 28 17.91 0.51 6.08 −0.25 −0.37 T4 140 4 28 19.14 0.52 6.12 −0.32 −0.79

 $-0.56$  $-0.37$  $-0.79$  $-0.09$  $1.92^{\it a}$ 

 $-0.41$ 

 $-0.25$  $-0.32$  $0.14$ 

 $6.08$ 6.12 5.43 4.70

 $0.51$ 

17.91

0.52

19.14

 $0.46$  $0.40$ 

14.46

 $-0.27$ 

 $-0.61$ 

5.64 5.88

20.47 20.00

 $\mathbf{v}$  $\mathbf{v}$  $\circ$  $\overline{4}$ 

 $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$ 

 $F R$  $\mathbb{E}$  $F_{\!\!\!4}$ 

 $\overline{40}$ 

0.58

 $-0.66$ 

0.34  $0.48$  $0.50$ 

 $0.01$ 

 $-0.43$ 

 $0.41$ 

MAQ Question 3: Sum T1 140 1 28 14.46 0.46 5.43 0.14 −0.09

 $F R$ 

MAQ Question 3: Sum

 $\circ$ 

T2  $140$  0 28  $12.37$  0.40  $4.70$  0.74  $1.92^a$ 

12.37

 $0.74$ 

J Psychoeduc Assess. Author manuscript; available in PMC 2010 June 01.

**Kurtosis** 

**Skewness**  $0.61^{\it a}$ 

 $S\!D$ 

 $SE$ 

 $\boldsymbol{M}$ 

Max.

Min. 6.32 5.60 8.97

 $\boldsymbol{z}$ 

 $0.01\,$ 

5.42

 $0.46$  $0.59$  $0.59$ 

16.65 21.69

32.94 48.00 41.54

 $140$ 140  $\overline{40}$  $\overline{a}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{40}$  $\overline{a}$ 

 $\overline{\Box}$ 

 $\Gamma$  $\mathbb{E}$  $\mathbb{F}$ 





Note: T1 = Time 1; T2 = Time 2; T3 = Time 3; T4 = Time 4; MAQ = Math Anxiety Questionnaire. Note:  $T1 = T$ ime 1;  $T2 = T$ ime 2;  $T3 = T$ ime 3;  $T4 = T$ ime 4;  $MAQ = M$ ath Anxiety Questionnaire.

 ${}^{\rm 2}{\rm This}$  parameter deviates substantially from a standard normal distribution. This parameter deviates substantially from a standard normal distribution.

#### **Table 2**

### Fit Statistics of the Longitudinal Model Selected



Note: AGFI = Adjusted Goodness of Fit Index; CFI = Comparative Fit Index; RMSEA = Root Mean Square Error Approximation; AIC = Akaike Information Criterion.