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The Importance of Number Sense to Mathematics Achievement in First and Third Grades

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Abstract

Children's symbolic number sense was examined at the beginning of first grade with a short screen of competencies related to counting, number knowledge, and arithmetic operations. Conventional mathematics achievement was then assessed at the end of both first and third grades. Controlling for age and cognitive abilities (i.e., language, spatial, and memory), number sense made a unique and meaningful contribution to the variance in mathematics achievement at both first and third grades. Furthermore, the strength of the predictions did not weaken over time. Number sense was most strongly related to the ability to solve applied mathematics problems presented in various contexts. The number sense screen taps important intermediate skills that should be considered in the development of early mathematics assessments and interventions.

The Importance of Number Sense to Mathematics Achievement in First and Third Grades

Mathematics achievement is a key educational concern in the United States. Competence in mathematics is critical to the workforce in STEM (science, technology, engineering, and mathematics) disciplines and to international leadership. Although there is an upward trend in average mathematics test scores in elementary and middle school (e.g., National Assessment of Educational Progress, 2008), U.S. students still lag behind their counterparts in many other industrialized nations (National Mathematics Advisory Panel, 2008). Moreover, within the school population, there are large individual differences in mathematics achievement associated with socioeconomic status (Lubienski, 2000), home experiences (Blevins-Knabe & Musun-Miller, 1996), culture and language (Miller & Stigler, 1987; Miura, 1987), and learning abilities (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

Although considerable attention has been devoted to mathematics achievement in elementary and secondary school, foundations for mathematics learning are established much earlier (Clements & Sarama, 2007). There is good reason to believe that the screening of mathematics achievement can be used to provide early predictors and support for interventions, before children fall seriously behind in school (Gersten, Jordan, & Flojo, 2005). In the area of reading, which has been studied more thoroughly than mathematics, reliable early screening measures with strong predictive validity have led to the development of effective support programs in kindergarten and first grade (Schatschneider, Carlson, Francis, Foorman, & Fletcher, 2002). Intermediate measures most closely allied with actual reading (e.g., knowledge of letter sounds) are more predictive of reading achievement than are more general phonological or perceptual measures (Schatschneider, Fletcher, Francis, Carlson, & Foorman, 2004). Similar to that for reading, the present study is concerned with screening key number competencies children acquire before first grade, which can serve as a ladder for learning mathematics in school.

Number Sense

Number sense that is relevant to learning mathematics takes root early in life, well before children enter school. Primary, or preverbal, number sense appears to develop without or with little verbal input or instruction, and it is present in infancy (Dehaene, 1997). The development of number sense begins with precise representation of small numbers, whereas large quantities are initially captured through approximate representations (Feigenson & Carey, 2003).

It has been argued that these primary abilities are the basis for developing secondary symbolic -- or verbal -- number competencies (Feigenson et al., 2004). When children learn the verbal count list and understand cardinal values for numbers, they learn to represent larger numbers exactly and see that each number has a unique successor (Le Corre & Carey, 2007; Sarnecka & Carey, 2008). Symbolic number sense is highly dependent on the input a child receives (Clements & Sarama, 2007) and thus is secondary to primary preverbal number sense but intermediate to the conventional mathematics that is taught in school. Key areas include counting, number knowledge and arithmetic operations. Although the relation between nonverbal and verbal number competencies is not always clear, there is general agreement that early verbal number competencies are necessary for extending knowledge with small numbers to knowledge with larger numbers and for learning school-based mathematics.

Children first map number words onto small sets (i.e., sets of 3 or less) through subitization or instant recognition of a quantity (e.g., Le Corre & Carey, 2006). For larger sets, counting usually is needed to determine the cardinal value. During preschool and kindergarten, most children learn to enumerate sets in a stable order (e.g., 1, 2, 3, 4, 5) using one-to-one correspondence and come to realize that the last number indicates the number of items in a set (Gelman & Gallistel, 1978). Comprehension of these “how to count” principles allows children to enumerate any object or entity (e.g., heterogeneous or homogeneous) in any direction (e.g., left to right or right to left and so forth).

Counting facility extends numerical understanding in important ways (Baroody, 1987). It helps children see that numbers later in the count list have larger quantities than earlier ones (e.g., n; $n + 1$; $(n + 1) +1$, etc.) (Sarnecka & Carey, 2008) and manipulate sets through addition and subtraction, with and without object representations (Levine, Jordan, & Huttenlocher, 1992). Learning difficulties in mathematics have been traced to weaknesses in intermediate number competencies related to counting, number comparisons, and set transformations (Geary, 1990; Mazzocco & Thompson 2005). These number abilities are highly sensitive to socioeconomic status, suggesting the importance of early input and instruction (Jordan, Huttenlocher, & Levine, 1992). For example, low-income kindergartners perform worse than their middle-income counterparts on oral number combinations and story problems involving addition and subtraction (Jordan, Levine, & Huttenlocher, 1994); they also use counting strategies less adaptively (e.g., they do not use their fingers to count on from addends; Jordan, Kaplan, Ramineni, & Locuniak, 2008).

Measuring number sense

Key number competencies can be reliably measured in kindergarten and early elementary school. Jordan and colleagues (Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan, Kaplan, Locuniak, & Ramineni, 2007) developed a "core" number-sense battery for screening children in kindergarten and first grade. To assess *counting*, children are asked recite the count sequence, to count sets of different sizes, to recognize correct, incorrect (e.g., counting the first object twice), and correct but unusual counts (e.g., counting from right to left). To assess *number knowledge*, they are asked to make numerical magnitude judgments (e.g., indicating which of 2 numbers is bigger or smaller, what number comes one and two after another number). Children also are asked to perform simple addition and subtraction calculations presented in three contexts. On *nonverbal problems*, children are shown a set of chips, which is then covered. Chips are either added to or taken away from the cover. The child must indicate how many chips are under the cover after the addition or subtraction transformation. *Story problems*, which refer to objects, are orally phrased as "Sue has *m* pennies. Bill gives her *n* more pennies. How many pennies does Sue have now?" and "Sue has *m* pennies. Bill takes away *n* of her pennies. How many pennies does Sue have now?" *Number combinations* were orally phrased as "How much is *n* and *m*?" and "How much is *n* take away *m*?". Developmental studies show that children can reliably solve simple nonverbal calculations (e.g., 2 + 1) as early as three years of age, while the ability to solve comparable story problems and number combinations develops later, starting around four years of age (Levine, Jordan, & Huttenlocher, 1992).

Longitudinal assessment over multiple time points in kindergarten showed three empirically separate growth trajectories in overall number sense as well as in number subareas (Jordan et al., 2006; 2007): (a) children who started with low number competence and stayed low; (b) children who started with high number competence and remained there; and (c) those who started with low number competence but made relatively good growth. Low-income kindergartners were much more likely to be in the low-flat growth class than were middle-income kindergartners, especially with respect to addition and subtraction story problems. Children's overall performance on the number sense battery and their growth rate between kindergarten and first grade predicted overall performance and the growth rate in general mathematics achievement between first through third grades (Jordan, et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009) Although all subareas were significantly related to each other and to achievement outcomes, early facility with addition and subtraction number combinations was most predictive of later achievement (Jordan et al., 2007).

Although our number sense battery has good reliability and predictive validity, it has a relatively long administration time and thus may be of limited practical value to classroom teachers. To address this issue, Jordan, Glutting, and Ramineni (2008) developed a reliable but abbreviated screen (referred to as the *Number Sense Brief* or NSB) through Rasch item analyses as well as a more subjective review of issues related to item bias. Internal reliability for the screen was at least .80 in kindergarten and first grade. Although the number sense brief screen is positively correlated with mathematics achievement measures, its predictive validity has not been established.

The present study examined predictive validity of the NSB screening measure. Children were given the screening measure at the beginning of first grade and mathematics outcomes were obtained at end of both first and third grades. Outcomes included overall mathematics achievement, as well as subareas of written computation and applied problem solving. It was hypothesized that number sense proficiency may be more relevant to applied problem solving than written computation, which may be more dependent on learned algorithms. To examine the unique contribution of number sense (as measured by the number sense brief) to these later mathematics outcomes, we also added the common predictors of age, verbal and spatial abilities, and working memory skills in our analyses.

Method

Participants

Participants were drawn from a multi-year longitudinal investigation of children's mathematics development. They all attended the same public school district in northern Delaware. Background characteristics of children in first grade (n = 279) and in third grade (n = 175) are presented in Table 1. The first graders included children who completed all measures in first grade and the third graders were children who completed all measures in first and third grade. In the first grade sample, 55% of the children were boys, 52% were minority, and 28% came from low-income families. In the third grade sample, 54% of the children were boys, 42% were minority, and 22% came from low-income families. Income status was determined by participation in the free or reduced-price lunch program in school. Moreover, most of the low-income children lived in urban, low-income neighborhoods. The differential attrition from first to third grade by minority and income status may limit the generalizability of findings and should be kept in mind when interpreting the current results. Participant attrition was due primarily to children moving out of the school district, rather than withdrawal from the study or absence on the day of testing. Although we were not able to determine why children left the school district, attrition may reflect lower family stability. All children were taught mathematics with the same curricular content and approach in first through third grade.

Procedure

The measures were given to children individually in school by one of several trained research assistants. The NSB items were given in October of first grade, the cognitive measures (Vocabulary, Matrix Reasoning, and Digit Span tests) in January of first grade and the math achievement measures in April of first grade and again in April of third grade.

Materials

Number Sense Brief Screen—The NSB is a shortened version of a longer number battery given to children (e.g., Jordan et al., 2006; Jordan et al., 2007; Locuniak & Jordan, 2008; Jordan et al., 2009). The NSB has 33 items, which are presented in the Appendix. The items assess counting knowledge and principles, number recognition, number comparisons, nonverbal calculation, story problems and number combinations. The measure is reliable, with a coefficient alpha of .84 at the beginning of first grade (Jordan, Glutting, & Ramineni, 2008).

Cognitive tasks—The Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999) was used to assess oral vocabulary and spatial reasoning. At age 7, internal reliability is .86 for the Vocabulary subtest and .94 for the Matrix Reasoning subtest. The correlation between the Vocabulary and overall verbal IQ is .93 and the correlation between Matrix Reasoning and overall performance (nonverbal) IQ is .87.

A digit span test (Wechsler, 2003) was used to measure short-term and working memory. A series of single digits of varying lengths were read orally to each child. Children were asked to repeat digits verbatim on a Digit Span Forward section, and then, to repeat digits in reverse order on a Digit Span Backward section. Digit Span Forward is a measure of short-term recall and Digit Span Backward a measure of working memory or active recall (Reynolds, 1997). At age 7, internal-consistency reliability is .79 for Digit Span Forward and .69 for Digit Span Backward.

Mathematics Achievement—Math achievement was assessed with the Woodcock-Johnson III (WJ-III; McGrew, Schrank, & Woodcock, 2007), which is normed through adulthood. The composite achievement score (Math Overall) was the combined raw scores for subtests assessing written calculation (written calculations in a using a paper and pencil format; Math Calculation) and applied problem solving (orally presented problems in various contexts; Math Applications). Internal-consistency reliability is above .80 between first and third grades for each subtest. The correlation between Math Calculation and Math Applications is .68 for ages 6 − 8.

Results

Raw scores from the NSB were used for all analyses. Mean raw scores and standard deviations on all tasks are presented in Table 2 for both first- and third-grade samples. Bivariate correlations are presented in Table 3 between the NSB raw scores and raw scores on the cognitive measures at first- and third-grades, as well as between the NSB and age at the beginning of kindergarten. All of the correlations were positive and statistically significant (i.e., all p values ≤ .05), with the two lowest correlations being kindergarten start age (.19) and Digit Span Forward (.34) and the highest correlations being Math Applications in first and third grades (.73 and .74, respectively).

A primary purpose of the study was to determine the unique contribution of the NSB in predicting criterion mathematics performance. Specifically, the study examined the extent to which the NSB predicted mathematics performance *above-and-beyond* the contribution of the control (nuisance) variables of age and general cognition related to language (Vocabulary), spatial ability (Matrix Reasoning) and memory (Digit Span Forward and Digit Span Backward). To accomplish these goals, students' scores on the NSB were regressed on a series of established mathematics achievement outcomes (Math Overall, Math Calculation, Math Applications) using the two-stage model recommended by Keith (2006). This methodology is sometimes referred to as a variance partitioning analysis (Pedhazur, 1997) and/or a sequential variance decomposition analysis (Darlington, 1990). At step one (model 1), the control (nuisance) variables entered simultaneously into an analysis. Step 2 (model 2) comprised entry of the NSB. The analyses were used to predict mathematics achievement in first grade and then in third grade. The independent contributions of predictors were evaluated through the interpretation of squared partial coefficients (Meyers, Gamst, & Guarina, 2006; Tabacnick & Fidell, 2007). Effect sizes were estimated for the predictors using Cohen's (1988) f^2, where values of .02 equal a small effect, values of .15 equal a medium effect, and values of .35 a large effect.

Mathematics Overall

Table 4 presents the results for predicting criterion performance on the Woodcock-Johnson III mathematics composite score (Math Overall). Model 1 (age and general cognitive measures) accounted for 47% of the variance in math in first grade ($p < .01$) (with Vocabulary, Matrix Reasoning, and Digit Span Backward reaching significance) and 45% of the variance in third grade ($p < .01$) (with Vocabulary, Matrix Reasoning, Digit Span Forward, and Digit Span Backward reaching significance). Results showed that the NSB made statistically significant, unique contributions to the prediction at first grade ($p < .01$) and third grade ($p < .01$) outcomes in Math Overall. In each instance, the NSB accounted for about 12% more criterion variance than the control variables. More importantly, Cohen's (1988) f^2 represented a medium-to-large effect sizes for both first- and third-grade criterion performance (respectively, .29, .21).

Mathematics Calculation

Table 5 presents the results for predicting Mathematics Calculation. Model 1 (age and general cognitive measures) accounted for 35% of the variance in Math Calculation in first grade ($p < .01$) with Vocabulary, Matrix Reasoning, and Digit Span Backward reaching significance, and 33% of the variance in third ($p < .01$), with Vocabulary and Matrix Reasoning reaching significance. Model 2 accounted for 41% of the variance in first grade and indicating that the NSB measure accounted for 6% more variance than the control variables. Cohen's (1988) f^2 value for the NSB was .10, which represented a small-to-medium effect size. Results for third grade were more impressive. The NSB accounted for a 14% more variance of Math Calculation than the control variables and Cohen's (1988) f^2 (.26) represented a medium-to-large effect size.

Mathematics Applications

Table 6 presents the results for Mathematics Applications where the results were most impressive. Model 1 accounted for 44% of the variance in Math Applications in first grade (p <.01) with Vocabulary, Matrix Reasoning, and Digit Span Backward reaching significance, and 45% of the variance in third (p grade <.01), with Vocabulary and Matrix Reasoning reaching significance. Not only did the NSB make significant, unique contributions that accounted for 14% to 17% of the criterion's variance, Cohen's (1988) f^2 represented a *large effect siz*e in predicting first-grade NSB performance (.44) and third-grade NSB performance (.45).

Discussion

Number sense, as assessed by our screening measure, is a powerful predictor of later mathematics outcomes – both at the end of first grade and the end of third grade. In terms of overall mathematics achievement, number sense made a significant and unique contribution to our regression models, over and above both age and cognitive factors. Its predictability was as strong in third grade as it was in first grade, contributing an additional 12% of the variance in mathematics achievement at both grades. Our findings are in keeping with those of other investigations suggesting that weaknesses in intermediate symbolic number sense, or number competencies related to counting, number relationships, and basic operations, underlie most mathematics learning difficulties (e.g., Gersten. Jordan & Flojo, 2005; Geary, Hoard, Byrd-Craven et al., 2007; Landerl, Bevan, & Butterworth, 2004).

Analysis of mathematics achievement outcomes by the subareas of calculation and applied problem solving was additionally revealing. Calculation, a paper and pencil task, assessed conventional operations and procedures, whereas applied problems required children to solve novel problems in everyday contexts. Although number sense was a unique predictor of both mathematics achievement subareas, it was more predictive of applied problem solving. Noticeably, the effect of number sense as a predictor was large and significant for both first and third grade. With general predictors included in our model, number sense contributed an additional 14% of the variance in first grade and an additional 17% of the variance in third grade. Most surprising was the sustained and even stronger relationship between earlier number sense and applied problem solving over time. That is, we expected number sense at the beginning of first grade to predict mathematics problem solving at the end of first grade, since the content of the two measures are closely allied during this period (Jordan et al., 2009). Mathematics problem solving becomes more complex by third grade, requiring children to solve novel problems involving a range of numerosities and operations. Likewise, the effect of number sense as a predictor of mathematics calculation became greater between first and third grades. Our findings support the notion that children who bring foundational knowledge of numbers to first grade are more likely to benefit from mathematical experiences throughout the elementary grades than those who do not have this knowledge (Baroody, Lai, & Mix, 2006) and that the effect of weak number sense may be cumulative. Knowledge of number concepts and skill with mathematics procedures appear to be mutually supportive, each facilitating the development of the other area (Baroody & Ginsburg, 1986; Rittle-Johnson, Siegler, & Alibali, 2001)

Previous findings, as well as the ones reported in the present investigation, establish that general verbal and spatial abilities are related to mathematics achievement (e.g., Donlan, Cowan, Newton, & Lloyd, 2007; Shea, Lubinksi, & Benbow, 2001). Studies of children with disabilities show that language impairments compromise the acquisition of spoken number sequences (Donlan et al., 2007) while spatial impairments inhibit understanding cardinality concepts (Ansari, Donlan, Thomas, Ewing, Peen, & Karmiloff-Smith, 2003). Moreover, weaknesses in working memory capacity are a characteristic of young children with math difficulties (Geary, Brown, & Samaranayake, 1991; Koontz & Berch, 1996; Wilson & Swanson, 2001; Swanson & Beebe-Frankenberger, 2004). For example, working memory weaknesses make it difficult for a child to hold one term of the problem in memory while counting on the number in the other term to solve an addition problem (Lefevre, Destefano, Coleman, & Shanahan, 2005). Despite the influence of general cognitive factors, however, the present findings show that number sense is uniquely and meaningfully related mathematical development. This observation supports the suggestion that number concepts and principles develop independently of other abilities and might represent a relatively separate cognitive system (Donlan, et al. 2007; Landerl et al., 2004).

Our relatively brief number sense screen is a valid and powerful measure that can be used to predict which children at the beginning of school are likely to have trouble learning mathematics. In reading, similar screening measures have been devised to help schools provide additional support and interventions (Gersten, Jordan, & Flojo, 2005). Not surprisingly, early literacy skills related to letter-sound knowledge are more predictive of subsequent reading achievement than are more general cognitive factors (Schatschneider, Fletcher, Francis, Carlson, & Foorman, 2004). It has been suggested that number sense is an intermediate ability that is achievable through early instruction (Ginsburg, Lee, & Boyd, 2008). Previous studies have shown that poor mathematics outcomes for low SES children are mediated by weak number sense (Jordan et al., 2009). Many disadvantaged, low-income children come to school with fewer number experiences than their middle-income peers (Clements & Sarama, 2008). It is likely that the lack of such experiences results in deficient symbolic number sense upon entry to elementary school. Number sense, which involves interrelated concepts of counting, number knowledge, and operations, has promise for guiding the development of early intervention programs. Future work should also consider children's strategy use on number tasks, especially on addition and subtraction problems. Understanding whether children can use efficient techniques such as counting on from a cardinal value might add to achievement predictability as well as inform instruction. Although the present findings suggest considerable stability in mathematical knowledge between kindergarten and third grade, there is good reason to believe that this knowledge can be improved with targeted interventions (e.g., Baroody, Eiland, & Thompson, 2009; Ramani & Siegler, 2008).

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Appendix

Items (N = 33) in the Number Sense Brief Screener (Jordan, Glutting & Ramineni, 2008)

Give the child a picture with 5 stars in a line. Say: “Here are some stars. I want you to count each star. Touch each star as you count.” When the child is finished counting, ask, “**How many stars are on the paper?”**

1 Enumerated 5

2 Indicated there were 5 stars were on the paper

Say: “I want you to count as high as you can. But I bet you're a very good counter, so I'll stop you after you've counted high enough, OK?” Allow children to count up to 20. If

3 Counted to at least 10 without error.

Show the child a line of 5 alternating blue and yellow dots printed on a paper. Say: **“Here are some yellow and blue dots. This is Dino** (show a finger puppet), and he would like you to help him play a game. Dino is going to count the dots on the paper, but he is just learning how to count and sometimes he makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes if it was OK to count the way he did or not OK. So, remember you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong. Do you have any questions?”

4 Counted Left to Right (correct)

5 Counted Right to Left (correct)

6 Counted Yellow then Blue (correct)

7 Counted first Dot twice (incorrect)

[For items 8 through 11, point to each number that is printed on a separate card and say: **“What number is this?”]**

8 13

9 37

10 82

11 124

12 What number comes right after 7?

13 What number comes two numbers after 7?

14 Which is bigger: 5 or 4?

15 Which is bigger: 7 or 9?

16 Which is smaller: 8 or 6?

17 Which is smaller: 5 or 7?

18 Which number is closer to 5: 6 or 2?

Say: “We are going to play a game with these chips. Watch carefully.” Place two chips on your mat. **“See these, there are 2 chips.”** Cover the chips and put out another chip**.**

“Here is one more chip.” Before the transformation say, “Watch what I do. Now make yours just like mine or just tell me how many chips are hiding under the box.” Add/remove chips one at a time.

19 2 + 1

20 4 + 3

21 3 + 2

22 3 − 1

Say: “I'm going to read you some number questions and you can do anything you want to help you find the answer. Some questions might be easy for you and others might be hard. Don't worry if you don't get them all right. Listen carefully to the question before you answer.”

23 Jill has 2 pennies. Jim gives her 1 more penny. How many pennies does Jill have now?

24 Sally has 4 crayons. Stan gives her 3 more crayons. How many crayons does Sally have now?

25 Jose has 3 cookies. Sarah gives him 2 more cookies. How many cookies does Jose have now?

26 Kisha has 6 pennies. Peter takes away 4 of her pennnies. How many pennies does Kisha have now?

27 Paul has 5 oranges. Maria takes away 2 of his oranges. How many oranges does Paul have now?

28 How much is 2 and 1?

29 How much is 3 and 2?

30 How much is 4 and 3?

31 How much is 2 and 4?

32 How much is 7 take away 3?

33 How much is 6 take away 4?

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Table 1

Demographic Information for Participants at the End of First Grade (n=279) and the End of Third Grade (n=175)

Variable	End of First Grade	End of Third Grade
Gender		
Male	55%	54%
Female	45%	46%
Race		
Minority[a]	52%	42%
Non-minority	48%	58%
Income		
Low income	28%	22%
Middle income	72%	78%
Mean kindergarten start age (SD)	5yr-6mo (4mo)	5yr-6mo(4mo)

[a]Minority refers to African-American (29%, *n* = 81), Asian (6%, *n* = 17), and Hispanic (17%, *n* = 47) at the end of First grade; and African-American (25%, *n* = 44), Asian (6%, *n* = 11), and Hispanic (11%, *n* = 19) at the end of Third grade.

Table2

Raw Score Means (SD) for the Measures in First and Third Grade

Measure	First Grade Mean (SD) *n* =279	Third Grade Mean (SD) *n* =175
Number Sense Brief	21.83 (5.85)	23.26 (4.83)
Math Composite	32.55 (6.40)	48.85 (8.19)
Math Applications	24.41 (4.02)	33.75 (5.30)
Math Calculation	8.14 (2.92)	15.09 (3.47)
Vocabulary	20.91 (7.18)	22.58 (6.42)
Matrix Reasoning	10.13 (5.92)	11.13 (5.75)
Digit Span Forward	6.37 (1.99)	6.56 (1.80)
Digit Span Backward	4.51 (1.91)	4.85 (1.41)

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Table 3

Correlations Between First Grade Number Sense Brief and Control Variables

Variable	Number Sense Brief
Math Composite (End of First Grade)	0.72
Math Applications (End of First Grade)	0.73
Math Calculation (End of First Grade)	0.58
Math Composite (End of Third Grade)	0.70
Math Applications (End of Third Grade)	0.74
Math Calculation (End of Third Grade)	0.66
Kindergarten Start Age	0.19
Vocabulary	0.56
Matrix Reasoning	0.53
Digit Span Forward	0.34
Digit Span Backward	0.50

Note. All correlations are significant, $p < 0.01$

Table 4

Results of Block Entry Regression for the End of First Grade Math Overall and the End of Third Grade Math Overall: Regression Coefficients and Variance Explained by Each Block of Variables

		First Grade Math Overall					Third Grade Math Overall				
Model		***B***	**β**	***t* value**	***p* value**	***Effect size******	***B***	**β**	***t* value**	***p* value**	***Effect size******
One											
Age		0.11	0.07	1.50	0.14		−0.07	−0.03	−0.55	0.59	
Vocabulary		0.27	0.30	5.58	0.00		0.30	0.24	3.62	0.00	
Matrix Reasoning		0.34	0.31	6.16	0.00		0.56	0.39	6.26	0.00	
Digit Span Forward		0.06	0.02	0.33	0.74		0.68	0.15	2.24	0.03	
Digit Span Backward		0.78	0.23	4.38	0.00		0.99	0.17	2.77	0.01	
Two											
Age		0.05	0.03	0.79	0.43	-	−0.09	−0.04	−0.82	0.42	-
Vocabulary		0.12	0.14	2.71	0.01	0.03	0.14	0.11	1.78	0.08	-
Matrix reasoning		0.18	0.17	3.62	0.00	0.05	0.36	0.25	4.27	0.00	0.08
Digit Span Forward		0.11	0.03	0.71	0.48	-	0.39	0.09	1.43	0.16	-
Digit Span Backward		0.38	0.11	2.31	0.02	0.02	0.41	0.07	1.26	0.21	-
Number Sense Brief		0.53	0.48	8.97	0.00	0.29	0.78	0.46	6.83	0.00	0.21
Model	R Square	R Square Change	F Change	df 1	df 2		R Square	R Square Change	F Change	df 1	df 2
One	0.47		47 99**	5	273		0.45		28.03**	5	169
Two	0.59	0.12	80.43**	1	272		0.57	0.12	46.70**	1	168

*Cohen's (1988) f^2 statistic where 0.02 is a small effect size, 0.15 is a medium effect size, and 0.35 is a large effect size.

** $p < 0.01$

Table 5

Results of Block Entry Regression for the End of First Grade Math Calculation and the End of Third Grade Math Calculation: Regression Coefficients and Variance Explained by Each Block of Variables

		First Grade Math Calculation					Third Grade Math Calculation				
Model		***B***	**β**	***t* value**	***p* value**	***Effect size****	***B***	**β**	***t* value**	***p* value**	***Effect size****
One											
Age		0.07	0.02	0.47	0.64		0.16	0.04	0.69	0.49	
Vocabulary		0.10	0.24	3.96	0.00		0.15	0.28	3.72	0.00	
Matrix Reasoning		0.14	0.29	5.19	0.00		0.21	0.34	4.87	0.00	
Digit Span forward		−0.07	−0.05	−0.77	0.44		−0.08	−0.04	−0.57	0.57	
Digit Span backward		0.40	0.26	4.38	0.00		0.25	0.13	1.73	0.09	
Two											
Age		−0.01	0.00	−0.03	0.97	-	0.03	0.01	0.16	0.88	-
Vocabulary		0.05	0.12	2.02	0.05	0.02	0.06	0.12	1.69	0.09	-
Matrix Reasoning		0.10	0.19	3.40	0.00	0.05	0.11	0.18	2.75	0.01	0.04
Digit Span Forward		−0.05	−0.04	−0.61	0.54	-	−0.04	−0.02	−0.30	0.77	-
Digit Span Backward		0.27	0.18	2.97	0.00	0.03	0.07	0.03	0.51	0.61	-
Number Sense Brief		0.17	0.33	5.19	0.00	0.10	0.33	0.48	6.86	0.00	0.26
Model	R Square	R Square Change	F Change	df 1	df 2		R Square	R Square Change	F Change	df 1	df 2
One	0.35		29.46**	5	273		0.33		18.77**	5	187
Two	0.41	0.06	26.89**	1	272		0.47	0.14	47.12**	1	186

*Cohen's (1988) f^2 statistic where 0.02 is a small effect size, 0.15 is a medium effect size, and 0.35 is a large effect size.

** $p < 0.01$

Table 6

Results of Block Entry Regression for the End of First Grade Math Applications and the End of Third Grade Math Applications: Regression Coefficients and Variance Explained by Each Block of Variables

		First Grade Math Applications					Third Grade Math Applications				
Model		***B***	**β**	***t* value**	***p* value**	***Effect size****	***B***	**β**	***t* value**	***p* value**	***Effect size****
One											
Age		0.37	0.09	1.95	0.05		0.01	0.00	0.02	0.98	
Vocabulary		0.17	0.31	5.56	0.00		0.22	0.28	4.08	0.00	
Matrix Reasoning		0.19	0.29	5.49	0.00		0.38	0.41	6.51	0.00	
Digit Span Forward		0.12	0.06	1.11	0.27		0.07	0.03	0.38	0.71	
Digit Span Backward		0.39	0.19	3.37	0.00		0.34	0.12	1.74	0.08	
Two											
Age		0.21	0.05	1.27	0.21	-	−0.21	−0.04	−0.78	0.44	-
Vocabulary		0.07	0.13	2.54	0.01	0.03	0.08	0.10	1.67	0.10	-
Matrix Reasoning		0.09	0.13	2.79	0.01	0.03	0.22	0.24	4.20	0.00	0.11
Digit Span Forward		0.16	0.08	1.65	0.10	-	0.14	0.05	0.89	0.37	-
Digit Span Backward		0.11	0.05	1.07	0.29	-	0.04	0.01	0.21	0.83	-
Number Sense Brief		0.36	0.52	9.69	0.00	0.44	0.55	0.54	9.11	0.00	0.45
Model	R Square	R Square Change	F Change	df 1	df 2		R Square	R Square Change	F Change	df 1	df 2
One	0.44		43.01**	5	273		0.45		30.34**	5	187
Two	0.58	0.14	93.89**	1	272		0.62	0.17	82.97**	1	186

* Cohen's (1988) f^2 statistic where 0.02 is a small effect size, 0.15 is a medium effect size, and 0.35 is a large effect size.

** $p < 0.01$