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Disaggregated spatial modelling for areal unit categorical data

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Summary

We consider joint spatial modelling of areal multivariate categorical data assuming a multiway contingency table for the variables, modelled by using a log-linear model, and connected across units by using spatial random effects. With no distinction regarding whether variables are response or explanatory, we do not limit inference to conditional probabilities, as in customary spatial logistic regression. With joint probabilities we can calculate arbitrary marginal and conditional probabilities without having to refit models to investigate different hypotheses. Flexible aggregation allows us to investigate subgroups of interest; flexible conditioning enables not only the study of outcomes given risk factors but also retrospective study of risk factors given outcomes. A benefit of joint spatial modelling is the opportunity to reveal disparities in health in a richer fashion, e.g. across space for any particular group of cells, across groups of cells at a particular location, and, hence, potential space—group interaction. We illustrate with an analysis of birth records for the state of North Carolina and compare with spatial logistic regression.

Keywords

Conditionally auto-regressive models; Disease mapping; Health disparities; Hierarchical models; Log-linear models; Multilevel models; Spatial random effects

1. Introduction

1.1. Overview

Our concern here is with spatial categorical data that are available at areal unit scale. A common setting where counts that are associated with such units arise is disease mapping, and we have witnessed the development of an extensive literature in this area in recent years. In these applications, patients' data are assigned to areal units because of limitations of collection or to protect subjects' confidentiality sufficiently. However, disease maps of raw, local mortality estimates in areal units have well-known deficiencies: geographically large but sparsely populated areas often visually dominate the map but provide the least reliable estimates, especially for rare diseases (Gelman and Price, 1999). Researchers have addressed these deficiencies by using methods that smooth, spatially or otherwise, raw estimates by 'borrowing strength' across areas. Bayesian methods for spatial disease mapping formally combine information on observed events in an area with information on nearby rates as well as overall rates of disease to estimate area-specific rates of disease. Such models were introduced by Clayton and Kaldor (1987) and placed in a fully Bayesian setting by Besag *et al.* (1991). They have been reviewed by Mollié (1996) and Wakefield *et al.* (2001).

Spatial smoothing in such models is typically achieved by using a *conditionally auto-regressive* prior on areal unit level random effects (Besag *et al.*, 1991), where a unit's rate estimate is smoothed on the basis of its neighbours' rates, and with 'neighbours' typically defined as the adjacent units. Such models are often used to analyse aggregated data (i.e. data that are summed over individuals and groups to achieve areal unit totals). If interest lies in subgroups, covariates can be introduced to yield area-specific rates by group. However, since this approach constrains all groups to share the same spatial random effects, it can be unsatisfying for subgroup analysis. An alternative is to fit separate models for each subgroup, but this may lead to an excessive number of parameters, sparse areal counts for some models and awkwardness if we seek to recombine groups. It would also fail to provide the type of integrated model that seems necessary to a comprehensive investigation of the aetiology of health disparities.

Instead, we begin with a joint association model. Some variables may be viewed as potential responses and others as explanatory (as, typically, in public health data), but the joint specification makes no such distinction (as is common in economics and public opinion data). In particular, we work with joint distributions of the cells in a multiway contingency table and do not specify a response variable. Hence, we do not confine ourselves to explaining a particular outcome given a particular set of risk factors (i.e. to conditional probabilities). Rather, once we have joint probabilities, we can calculate arbitrary marginal and conditional probabilities, i.e. we can aggregate or condition in any fashion, enabling a rich range of inference without having to reconsider and refit models to investigate other potential hypotheses. Flexible aggregation allows investigation of groups of interest, whereas flexible conditioning enables study of single as well as multiple outcomes given risk factors along with consideration of incidence of risk factors given outcomes, perhaps in the presence of other risk factors or outcomes. Conditioning enables sensitivity, specificity and related inference that may be appropriate for retrospective data examination. Additionally, our approach avoids potential Simpson's paradox issues that can arise under collapsing of categories (for example, see Simpson (1951) and Agresti (2002)).

Our approach assumes that observations within areal units retain individual level characteristics but without precise locations. We further assume that the characteristics are categorical (or have been made categorical) so that, for the areal unit, we can envision a contingency table and associated model that describes the cell probabilities (or, equivalently, the joint distribution). These probabilities might be driven by features of the areal unit, and, additionally, we expect spatial dependence in the sense that these joint distributions will be similar for areal units that are close to each other.

In implementing this approach, we specify spatially varying log-linear models: for each areal unit, we imagine Poisson-distributed cell counts driven by a *local* log-linear model. Some of the parameters in the log-linear model will be allowed to vary spatially, and they may also be explained by using areal unit level covariate information, i.e. contextual variables associated with these units, such as household median family income, percentage home ownership or crime rate. These contextual variables may be useful in explaining differences in cell probabilities across units. An attractive advantage of the log-linear model approach is that, with a multiway table, the number of cells in the table will typically be far greater than the number of parameters in the log-linear model. With ordinal data classifications, we can draw on suitable classes of ordinal categorical data models (Agresti, 2002) which, again, achieve dimension reduction. Hence, through log-linear modelling, we can introduce cell level spatial dependence across areal units without requiring a distinct spatial random effect for every cell at every location.

A further benefit of such modelling is its utility in addressing the complex issue of how to assess disparities in health at a meaningful geographic scale while protecting patients' confidentiality and accommodating both sparsely and densely populated areas. We can study disparities across space for any grouping of cells, in addition to being able to study disparities across groups of cells. In fact, our approach can tease out space by risk factor interactions. In the context of disparities in health, such general analysis has not been previously considered but is consistent with the US's 'Healthy people' 2010 initiative, whose two goals include the elimination of disparities in health, including those that occur by race, gender or geographic location (US Department of Health and Human Services, 2000).

1.2. Motivating data set

We developed our disaggregation model for application to North Carolina birth record data. These data include information on every documented live birth in the state since 1990, including maternal age, birth weight, clinical estimate of gestation, plurality, maternal complications, congenital anomalies, maternal tobacco and alcohol use, number of living children, number of children born alive now dead and a variety of maternal and paternal demographic variables. We restricted our analysis to singleton (i.e. one fetus) live births with no congenital anomalies born to mothers aged 15–44 years who self-identified their race as either African American or white, for the period 1999–2003. These restrictions eliminate from the analysis adverse birth outcomes that are driven by the physiology of pregnancy (more than one fetus, very young mothers or advanced maternal age) or the pathophysiology of fetal conditions (congenital anomalies).

A total of 462 362 births were included in our analysis, including 52 697 (11.4%) adverse birth outcomes (i.e. infants that were low birth weight (LBW) (7.0%) or preterm (9.3%) or both (4.9%)). More precisely, we take as areal units the 100 counties in North Carolina with the following M = 5 binary classification variables: X for maternal race, African American (1) or white (0); Y for birth weight outcome, LBW (1, less than 2500 g) or not LBW (0, 2500 g or heavier); Z for infant sex male (0) or female (1); W for maternal use of tobacco during pregnancy, yes (1) or no (0); V for preterm birth (1, less than 37 weeks of gestation) or not (0, 37 weeks of gestation or longer). So we have $L = 2^5 = 32$ cells in the contingency table. Many hypotheses can be entertained involving these five variables. Any particular one would involve computing functions of cell probabilities and comparing these functions within a county or across counties. In presenting results, we focus on the putative outcome 'term LBW', which refers to women who carry their pregnancies to term (i.e. gestation at least 37 weeks) and, nevertheless, have LBW infants (i.e. under 2500 g). Most of these infants would be classified as 'small for gestational age', which is a category of particular concern to both obstetricians and paediatricians. Lastly, we note that the methodology can be extended to a richer set of variables and that we could work at other areal scales, e.g. zip codes or census units.

1.3. Format

The format of the paper is as follows. In Section 2, we provide formal details for our disaggregation modelling approach and consider marginal and conditional probabilities from the model. Assuming interest in a regression to explain outcomes, in Section 3 we compare our approach with customary logistic spatial regression models. We then illustrate our approach with an analysis of data drawn from the North Carolina birth record data in Section 4. Finally, we provide a summary and discuss possible extensions in Section 5.

2. Modelling details and properties

In this section, we formalize the disaggregation model that we employ for the balance of the paper. We also discuss marginalization and conditioning under this model.

2.1. The disaggregation model

Consider the subjects in areal unit s as cross-classified by M categorical variables with a total of L classification cells. To connect with our motivating data set let $n_l^{(s)}$ denote the population count of births in cell l in county s. The $L \times 1$ data vector for county s, $\mathbf{n}^{(s)}$, consists of entries $\{n_l^{(s)}\}$. Let $n_l^{(s)} = \sum_{l=1}^L n_l^{(s)}$ denote the total count for areal unit s. Also, define $\boldsymbol{\lambda}^{(s)} = \{\lambda_l^{(s)}\}$ as the vector of expected cell counts for county s.

We assume that, at the first stage, the data $\{n_l^{(s)}\}$ are conditionally independent such that $n_l^{(s)} \sim \operatorname{Poisson}(\lambda_l^{(s)})$. We model the expected cell counts as linear on the log-scale, i.e.

$$\log(\lambda_l^{(s)}) = \mathbf{X}_l^{\mathrm{T}} \boldsymbol{\beta}_s = \sum_{t=1}^r X_{lt} \boldsymbol{\beta}_{st}, \tag{1}$$

with \mathbf{X}_l denoting the design vector for the lth cell, $\boldsymbol{\beta}_s$ denoting the coefficients in the loglinear model for areal unit s and $r \leq L$. In practice, we do not use saturated models but rather models with fewer parameters than the number of cells, so r < L. Customarily, second-order interactions and perhaps a few three-way interactions are introduced, but rarely higher order interactions. With regard to these $\boldsymbol{\beta}s$, we shall sometimes use notation such that $\boldsymbol{\mu}$ denotes the overall (or 'grand') mean term and $\boldsymbol{\gamma}s$ denote the main effects and interaction terms. We also assume the usual constraints on the $\boldsymbol{\gamma}s$, i.e. that they sum to 0 over any index. For instance, in analysing the illustrative data set, if we used a model with all two-way interactions, we would have a 1 + 5 + 10 = 16-dimensional $\boldsymbol{\beta}$ -vector consisting of $\boldsymbol{\mu}$ and $\boldsymbol{\gamma}s$. With the assumed constraints on the $\boldsymbol{\gamma}s$, each \mathbf{X}_l -vector would have ± 1 as entries.

Collecting to a vector, we may write the first level of our model as

$$\log(\boldsymbol{\lambda}^{(s)}) = \mathbf{X}^{\mathrm{T}} \boldsymbol{\beta}_{s},\tag{2}$$

noting that the \mathbf{X}^T -matrix is $L \times r$ and does not depend on areal unit s, i.e. the design matrix is the same for each areal unit. We make this assumption to ensure that the $\boldsymbol{\beta}_s$ have the same interpretation across s, so that it is sensible to introduce spatial random effects into some components of $\boldsymbol{\beta}_s$.

We intentionally model the $\lambda_l^{(s)}$ rather than the cell probabilities $p_l^{(s)}$, to work with a product Poisson likelihood rather than a multinomial. The cell probabilities are induced by the γs ; to

extract them for each county we need to normalize by $\sum_{l=1}^L \lambda_l^{(s)}$. Hence, when fitting the hierarchical model by using Markov chain Monte Carlo techniques, at each iteration, we calculate the $\lambda_l^{(s)}$ and normalize to obtain posterior samples of the $p_l^{(s)}$.

We next add a second stage to our model, in the spirit of the multilevel models of Raudenbush and Bryk (2002). We set

$$\beta_{st} = \eta_t^{\mathrm{T}} \mathbf{w}_s + \overset{\sim}{\varphi}_t^{(s)} = \sum_{u=1}^q \eta_{tu} w_{su} + \overset{\sim}{\varphi}_t^{(s)}, \tag{3}$$

where η_{tu} is the parameter that is associated with the areal unit covariate w_{su} . In our illustrative setting, w_{s1} might be median family income, w_{s2} might be percentage home ownership, w_{s3} might be crime rate, etc. The random effects $\varphi_t^{(s)}$, t=1, 2, ..., r, are assumed to be spatially structured. To do so, we introduce an overall intercept for each t, φ_t^0 , and write $\varphi_t^{(s)} = \varphi_t^0 + \varphi_t^{(s)}$ so that the $\varphi_t^{(s)}$ are centred at 0. (Implicitly, this means that \mathbf{w}_s does not include an intercept.) We address below the question of whether or not to include a φ for each t as well as the modelling for the φ s that we do include. However, for now, employing notation that includes all φ s, we can write β_s as

$$\beta_s = \boldsymbol{\varphi}^0 + \eta \mathbf{W}_s + \boldsymbol{\varphi}^{(s)},\tag{4}$$

where φ^0 is the $r \times 1$ vector of intercepts, η is an $r \times q$ matrix of second-level model parameters, \mathbf{w}_s is a $q \times 1$ column vector of second-level covariates and $\varphi^{(s)}$ is an $r \times 1$ vector of spatial random effects.

Inserting equation (4) into equation (2) and expanding provides

$$\log(\boldsymbol{\lambda}^{(s)}) = \mathbf{X}^{\mathrm{T}} \boldsymbol{\varphi}^{0} + \mathbf{X}^{\mathrm{T}} \boldsymbol{\eta} \mathbf{w}_{s} + \mathbf{X}^{\mathrm{T}} \boldsymbol{\varphi}^{(s)}.$$
 (5)

The interpretation of equation (5) is attractive. We explain the λs through the individual level characteristics, through the local contextual variables (recall that the first entry in every **X**-vector is a 1), through interactions between individual characteristics and local covariates, with spatial adjustment to the intercept (again, since the first entry in every **X**-vector is a 1), and also spatial adjustment to the coefficients that are associated with individual characteristics. Thus, we achieve an extremely flexible explanatory model.

We note that we could develop an analogue of equation (5) without introducing a log-linear model. An advantage of the log-linear approach is that we must explain only $r \beta$ s rather than $L \lambda$ s where r will usually be much smaller than L. Hence, we must introduce only at most r sets of random effects.

The usual modelling intent using equation (5) is to incorporate spatial dependence between cell probabilities and to effect spatial smoothing of these probabilities, so we model the $\varphi_t^{(s)}$ as purely spatial effects. (Heterogeneous random effects can be envisioned also, as in many disease mapping applications (for example, see Banerjee *et al.* (2004) and references therein). However, they are difficult to separate from the spatial random effects; the prior specifications essentially drive the inference.) Hence, as priors for the $\varphi_t^{(s)}$ we use customary independent intrinsic conditionally auto-regressive models (Besag, 1974; Besag *et al.*, 1991, Banerjee *et al.*, 2004), though dependence in these random effects could be achieved with

multivariate conditionally auto-regressive priors, such as in Gelfand and Vounatsou (2003) or Jin et al. (2005). We do not use multivariate conditionally auto-regressive priors here, expecting that introducing such dependence will not gain much with regard to inference about desired probabilities since these probabilities arise as complicated non-linear functions of these effects. Details of the model implementation are provided in Appendix A.

We also remark that a slightly more general form is possible: we need not require the same set of covariates to explain each β_{st} . This generalized form is given in Raudenbush and Bryk (2002). Thus, in equation (3), q would change to q_t and we could change the above notation accordingly. Of course, we can always work with the full set of covariates of interest and let the data tell us which η_{tu} s are significant.

Finally, we comment that ordinal classifications can be accommodated. Using typical models for such classifications (see Agresti(2002)), the associated X_{lt} s would introduce appropriate scores.

2.2. Marginal and conditional probabilities

It is straightforward to write down general forms for any marginal and conditional probabilities under the above modelling. In general, the marginal probabilities that are associated with the collection of cells $L' \subset L$ are

$$\sum_{l \in L'} p_l^{(s)} = \sum_{l \in L'} \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\beta}_s) / \sum_{l \in L} \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\beta}_s).$$

Conditional probabilities arise as

$$P(l \in L^{''}|l \in L^{'}) = \sum_{l \in L^{''}} \exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\beta}_{s}) / \sum_{l \in L^{'}} \exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\beta}_{s}).$$

Relative risks and odds ratios can be computed similarly.

For instance, in the illustrative example that was introduced in Section 1.2 and is described in detail below in Section 4, the probability of an LBW infant born to an African American mother in areal unit s is $\sum_k \sum_m \sum_n p_{11kmn}^{(s)}$. The conditional probability in areal unit s of LBW given an African American mother is

$$\sum_{k} \sum_{m} \sum_{n} p_{11kmn}^{(s)} / \sum_{j} \sum_{k} \sum_{m} \sum_{n} p_{1jkmn}^{(s)}.$$

Aggregation and conditioning could occur over s as well, in addition to that over i, j, k, m and n.

In considering marginal and conditional probabilities, we might ask under what spatial random-effects specifications will they be spatially varying? As in the examples above, marginalizations of interest will typically be over a set of the *M* categorical variables in the *M*-way table. Conditional events of interest will also typically involve probabilities in both numerator and denominator that marginalize over categories.

In this regard, we turn to the question of which $\varphi_t^{(s)}$ should be chosen to be spatially varying. Suppose, for the moment, that we have no contextual variables so $\beta_{st} = \varphi_t^0 + \varphi_t^{(s)}$. It is clear that, if $\beta_{s1} = \mu^{(s)}$ is the grand mean term, for any probability, $\varphi_1^{(s)}$ will cancel out in the ratio. If we introduce spatial effects only on this overall 'intercept', we shall not achieve spatially varying probabilities. In other words, an intercept random effect for each areal unit will not work and, in fact, there is no reason to introduce them. However, if we introduce $\varphi_t^{(s)}$ for every other t, we ensure that any conditional probability will be spatially varying. However, r-dimensional spatial random-effects vectors might be more than the data can explain and might introduce problems of identifiability and unstable computation. Below, we illustrate, by specifying spatial effects for the intercept, all main effects and all two-way interactions. (The reader can verify that this guarantees spatially varying relative risks and odds ratios.)

3. Connection with customary logistic spatial regression

In many categorical settings interest focuses on modelling an outcome variable given a set of explanatory variables. In this regard, if we have a log-linear model with, say, m = 3 and we identify the binary outcome variable X as the response with Y and Z as covariates, then $\log (p_{1jk}/p_{0jk})$ is the logistic regression that is associated with the conditional event that X = 1 given Y = j and Z = k. In fitting this model, we look at the counts n_{1jk} as the number of successes in $n_{:jk}$ Bernoulli trials at level Y = j, Z = k, each with conditional probability $p_{1|jk} = p_{1jk}/(p_{0jk} + p_{1jk})$. In situations where Z = 1 denotes occurrence of disease, the $p_{1|jk}$ are usually small and the $n_{:jk}$ large so a Poisson approximation for the distribution of n_{1jk} is adopted with parameter $\lambda_{jk} = n_{:jk}p_{1|jk}$, i.e. λ_{jk} is the expected number of cases of disease given Y = j and Z = k. With the definition, $\lambda_{ijk} = n_{:jk}p_{ijk}$, we have $\lambda_{jk} = \lambda_{1jk}/(\lambda_{0jk} + \lambda_{1jk})$. The familiar disease mapping setting indexes the λ by location and produces spatial modelling for the resulting $\lambda^{(s)}$.

3.1. The usual disease mapping model

Arguably the most widely used model to explain areal unit counts arises in the setting of disease mapping using the log-relative-risk in a Poisson regression with random effects, i.e.

$$O^{(s)}|\theta^{(s)} \stackrel{\text{ind}}{\sim} \text{Poisson}(\lambda^{(s)} = E^{(s)} \times \theta^{(s)}),$$
 (6)

$$\log(\theta^{(s)}) = \alpha + \mathbf{w}_s^{\mathsf{T}} \boldsymbol{\xi} + U_s. \tag{7}$$

(See, for example, Besag *et al.* (1991), Banerjee *et al.* (2004) or Waller and Gotway (2004).) In the context of the example in Section 1.2, we would identify an outcome, such as LBW among term births (again, 'term LBW' birth), leading to $O^{(s)}$ as the observed number of term LBW births in county *s*. Then, $E^{(s)}$ is the expected number of term LBW births in county *s*, $\theta^{(s)}$ is the relative risk of a term LBW in county *s* and \mathbf{w}_s are county level covariates (analogous to our earlier \mathbf{w}_s , and with corresponding coefficients $\boldsymbol{\xi}$).

To model this conditional outcome with the usual disease mapping model, the analyst must restrict the data to only term births, i.e. the binary birth weight outcome would be conditioned on the set of term births; all preterm births would be discarded. By contrast, the joint approach uses all of the data to fit the model; hence all of the data bear on the conditional inference and, in fact, on any inference of interest. Continuing, the U_s are

county-specific random effects that are spatially structured via a conditionally autoregressive prior and result in spatial smoothing towards a local mean rate. The $O^{(s)}$ are regarded as random and conditionally independent given the $\theta^{(s)}$ and $E^{(s)}$. The $E^{(s)}$ are regarded as fixed and known. When possible the $E^{(s)}$ are obtained externally to the data set but in many applications in the literature are calculated via internal standardization (Fleiss *et al.*, 2003). When this is so, the data are used on both the 'left-hand side' and the 'right-hand side' of the model. By contrast, in our modelling, we work with λ s rather than λ s and the full vector of counts $\mathbf{n}^{(s)}$, eliminating the need for standardization.

3.2. Modelling multiple groups

In practice, overall rates as well as rates for specific subgroups are of interest. How would the model in equations (6) and (7) be used to infer about rates for subgroups within areal units? There appear to be two approaches:

- **a.** introduce an analysis-of-variance style of model incorporating suitable main and interaction effects in the model for the $\log(\theta)$ s in equation (7) to capture the subgroups of interest; or
- **b.** fit a separate model for each subgroup.

The former would introduce one set of spatial effects as in equation (7), arguably too restrictive though frequently used (e.g. in Waller *et al.* (1997)). The latter would introduce a set of spatial random effects for each subgroup.

Option (b) raises numerous questions and concerns, primarily due to modelling $\tilde{\lambda}$ rather than

 λ . For example, should the $\tilde{\lambda}_l^{(s)}$ be independent across l? In this regard, our modelling, by introducing multiple spatial random effects for each cell count, as discussed in Section 2.2, automatically introduces dependence to the $\tilde{\lambda}$. Should we adopt a distinct ξ for each subgroup or a common ξ (to connect the groups)? How do we make comparisons across groups, e.g. rates of risk or odds ratios? Further, to investigate aggregated rates across groups we would have to fit a new model for every aggregation. With a large number of cells, we would fit models to data sets having many cells with sparse counts, leading to potentially unstable fitting.

Whether we create the specification under (a) or (b), the models would provide conditional probabilities of outcome given subgroup. These are the only probabilities that can be studied. Retrospective analyses for risk factors given presence (or absence) of outcome are precluded. In addition, by converting log-linear models to (essentially) logistic regression models, some effects will disappear; we cannot assess their explanatory contribution. Finally, one must specify, *a priori*, which subgroups to study and which variable(s) or functions of variables to regard as an 'outcome' (as is the case in Section 3.1 with the outcome term LBW birth). These questions and concerns seem to us either not to arise or to be more easily accommodated under the disaggregated model proposed. Finally, in using all of the available categorical information, the disaggregated model produces improved inference for the λ s relative to a model of the form given in equations (6) and (7), as we demonstrate in Section 4.

An additional benefit deserves comment. A primary reason for investigating many public health databases is to illuminate disparities. Within the disaggregation framework, we can straightforwardly study disparities. Let L' denote a particular subgroup of the L cells with

probabilities $P_{L'}^{(s)}$. Then we can directly study spatial disparities by examining this set across the areal units. Next, suppose that we seek to compare subgroups. Let L'' be a second sub-

group with probabilities $P_{l''}^{(s)}$. We can compare these with the $P_{l'}^{(s)}$ locally (at each unit) and make comparison across space (to detect group–space interaction). Such interactions would suggest that the spatial random effects are serving as surrogates for an environmental variable with spatial structure not included in our model (i.e. not in the \mathbf{w}_s) but interacting with some aspect that defines the subgroup. By averaging, in a suitable fashion, we can make partial spatial or non-spatial comparisons. Again, such detailed disparity analysis is precluded by customary modelling approaches.

4. Analysis of the motivating data set

4.1. Applying the disaggregation approach

As a point of departure, we began our analysis by ignoring the role of county and fitting a non-spatial model for the entire state of North Carolina. For our resulting (M=) 5-way table, we first fit a saturated log-linear model with 32 parameters. With L=32 cells, many cell probabilities will be small (i.e. non-zero) and there is little reason for concern with more than 460 000 births. With smaller data sets, we might have to confine ourselves to fewer variables. In any event, no cell will have 0 counts across all counties. If a particular outcome were that rare, we would collapse over it. Moreover, in this regard, the benefit of Bayesian hierarchical modelling emerges: our model will borrow strength across counties to infer about cell probabilities for any particular county. In any case, we could reduce our model by eliminating non-significant parameters to (XYWV, XZV, YZV), in which all 22 terms were either significant or compelled by the hierarchy principle. With such a large data set, not surprisingly, this baseline model emerged from both classical and Bayesian fitting. The number of parameters was reduced by a third; more pronounced reduction would be expected with a larger M.

We used this baseline log-linear model (*XYWV*, *XZV*, *YZV*) as the starting point for our spatial analysis. In addition, we considered county level household median income data from the census for inclusion (as \mathbf{w}_s with an associated $\boldsymbol{\eta}$). We constructed four variants from this model:

- **a.** a baseline model with neither spatial random effects nor county level covariates (model 0);
- **b.** a model without spatial random effects but with county level covariates (model A);
- c. a model with spatial random effects but without county level covariates (model B);
- **d.** a model with both spatial random effects and county level covariates (model AB).

For spatial random effects, we placed spatial structure on terms up to two-way interaction terms (i.e. the three-way and four-way interaction terms did not receive spatial structure). In total, 15 of the 22 terms

$$(\gamma_{i}^{X(s)}, \gamma_{j}^{Y(s)}, \gamma_{k}^{Z(s)}, \gamma_{m}^{W(s)}, \gamma_{n}^{V(s)}, \gamma_{ij}^{XY(s)}, \gamma_{ik}^{XZ(s)}, \gamma_{im}^{XW(s)}, \gamma_{in}^{XV(s)}, \gamma_{jk}^{YZ(s)}, \gamma_{jm}^{YW(s)}, \gamma_{jm}^{YV(s)}, \gamma_{km}^{ZW(s)}, \gamma_{km}^{ZV(s)}, \gamma_{km$$

and $\gamma_{mn}^{WV(s)}$) received spatial random effects in the form of independent conditionally autoregressive prior distributions, whereas the grand mean terms ($\mu^{(s)}$) received independent vague, normal priors. We completed the model specification in accord with the details that are provided in Appendix A, and we fitted all four models in WinBUGS (http://www.mrc-bsu.cam.ac.uk/bugs/welcome.shtml). (Run times are slow, between 2 and 7 days for the spatial models, owing to the size of the data set and the very large number of spatial random effects. Much faster run times could be achieved by using C code.)

To inform model selection, we used four simple measures, with the only stipulation being that they operate in the space of the *ps* (since these are the objects of inferential concern).

The measures that are presented are as follows: MSE is the mean over all LS = 3200 cells of the squared errors, based on the observed probability $\widehat{P}_{ijkmn}^{(s)}$ and the modelled value $P_{ijkmn}^{(s)}$; $\log(p)$ is the sum over all cells of $\{\log(\widehat{P}_{ijkmn}^{(s)}) - \log(p_{ijkmn}^{(s)})\}^2$; χ^2 is the sum over all cells of $(O_{ijkmn}^{(s)} - E_{ijkmn}^{(s)})^2/E_{ijkmn}^{(s)}$ where O denotes observed and E expected counts; and KL is the sum over all cells of $\widehat{P}_{ijkmn}^{(s)}\log(\widehat{P}_{ijkmn}^{(s)}/P_{ijkmn}^{(s)})$, the Kullback–Leibler distance (Kullback and Leibler, 1951) of fitted against observed, all with standard adjustment for observed 0 cell counts. Table 1 also provides median values and corresponding 95% credible intervals (CIs) based on samples from the posterior. We found that models B and AB, the two with spatial structure, performed substantially better than the models without spatial structure (models 0 and A) by using these measures and we would expect this to be so by using any other appropriate criterion. Even with penalization for model complexity, models 0 and A would not be competitive.

Table 2 provides main effect parameter estimates from the two spatial models (B and AB) and shows the substantial similarity of their results. We note that all main effect parameters obtain statistical significance, on the basis of 95% CIs, in the models. For illustration of our methodology, we employ the more parsimonious model B in what follows.

We next turn to other quantities of interest that can be built from the disaggregated probabilities. Fig. 1 shows estimated incidence of term LBW in its centre map, flanked by analogous maps for the eight race \times infant sex \times tobacco use subgroups. In Fig. 1 internally derived quintiles determine the cut points of the categories for each map, so, despite being on very different scales, the darkest shading denotes, in each map, the 20 counties with highest incidence, permitting us to make a particular type of relative comparison. Fig. 2 displays the same estimates of term LBW incidence, but with 'global' quintiles based on the rates over all eight of the race \times infant sex \times tobacco use subgroups, permitting absolute comparison across maps. Figs 1 and 2 together help to illuminate the story. Fig. 1 is most effective for seeing within-map variation whereas Fig. 2 reveals across-map variation. Though the flanking maps in Fig. 2 do not individually reveal much variation, a comparison between them is striking.

There are several striking features in Figs 1 and 2 that demonstrate the utility of our disaggregation model. In Fig. 1, the large central map of overall term LBW incidence shows that counties in eastern North Carolina generally suffer higher overall incidence. However, the eight flanking maps of the race × infant sex × tobacco use subgroups tell a different story. Counties in north-western North Carolina tend to have higher incidence rates *for each subgroup*. The overall incidence map is subject to confounding with respect to county level composition of the eight subgroups, i.e., in the overall map, a county's rate is a weighted average of the eight subgroup rates, but the precise weighting differs from county to county in North Carolina, leading to the Simpson's paradox situation (Agresti, 2002) that emerges in the ensemble of maps.

The notable visual impression from Fig. 2 is the lack of overlap among the eight flanking subgroup maps. For example, the counties in the upper leftmost map (for female infants born to African American mothers who used tobacco during pregnancy) are entirely in the upper quintile, whereas nearly every county in the lower rightmost map (for male infants born to white mothers who did not use tobacco during pregnancy) belongs to the lowest quintile. In this vein, the centre map in Fig. 2 shows that the variability in overall county term LBW birth incidence is rather small compared with the variability across the eight subgroup maps, and that within-map variability is small compared with across-map variability.

In addition, Fig. 2 recapitulates the parameter estimates from Table 2, particularly for the interaction of maternal race and LBW (MR \times LBW). Table 2 shows that the magnitude of this effect (0.1819 in model B) is similar to the magnitude of the effect for the interaction of LBW and maternal tobacco use (LBW \times MTU; 0.1844 in model B). Comparing the range of values in

- a. the leftmost map in the lower row with the map second from the left in the upper row and
- **b.** the map second from the right in the lower row with the rightmost map in the upper

demonstrates what these parameter estimates also show: the effect of maternal race on LBW incidence is about the same as the effect of maternal tobacco use. Such conclusions might be difficult to obtain with other modelling approaches.

Although Figs 1 and 2 suggest differences across racial groups for adverse birth outcome incidence, our modelling framework is sufficiently flexible to support direct measures of racial disparity easily. For this, Fig. 3 depicts county level odds ratios OR of a term LBW

birth for African Americans *versus* whites, $OR^{(s)} = p_{11..0}^{(s)} p_{00..0}^{(s)} / p_{10..0}^{(s)} p_{01..0}^{(s)}$. For these ORs a value of 1.0 would reflect no disparity, and Fig. 3 therefore documents the large (estimated county ORs ranged from 1.8 to 3.0, with median 2.2) and geographically pervasive burden in term LBW incidence borne by African Americans *versus* whites. In particular, areas in central and eastern North Carolina (mostly) suffer from greater-than-average levels of disparity. Other measures of disparity, such as those advocated by the Centers for Disease Control and Prevention (Keppel *et al.*, 2005), may also be built from the model output.

By viewing Fig. 3 in conjunction with Fig. 1 and/or Fig. 2, we see that

- **a.** disparity can be high even where incidence of term LBW is relatively low for both African American and white mothers,
- **b.** disparity can be low even where term LBW incidence is relatively high for both African American and white mothers and
- **c.** disparity can (of course) be high where African American mothers have relatively high term LBW incidence while white mothers have relatively low incidence.

For (a) and (b), policy makers might have competing interests and trade-offs to consider in crafting intervention strategies, whereas for (c) intervention strategies might be focused on a subset of the county population. Such intervention strategies based on Figs 1–3 could consider whether to target the geographic areas with the worst overall outcomes, the subpopulations with the worst outcomes, areas with the worst racial disparity or some combination of these.

4.2. Comparing the conditional and disaggregation approaches

Finally, we compare our approach with the conditional model given in Section 3.1. We consider two different outcomes for this comparison:

- a. term LBW, as in Section 4.1, and
- b. LBW.

For fits to the North Carolina birth record data, the county level 95% CIs from the conditional model are wider than the corresponding 95% CIs from the disaggregation model for both outcomes. For term LBW, the county level CIs are on average about 30% wider whereas for LBW the CIs are on average about 25% wider, and in many cases more than

40% wider (for both outcomes). Further, for the four largest counties (each with over 20 000 births), where we would expect both models to perform well, the CIs are still about 15% wider for the usual model for term LBW and over 25% wider for LBW. Finally, we recall that, for term LBW, fitting the conditional model excluded preterm births from the analysis, whereas the disaggregation approach allows us to use all the data. This may explain the even poorer performance of this model for this outcome.

To support further the advantages of the disaggregation model, we investigated a simulated data set. To place the simulation on more familiar terrain for the usual model, we did not include preterm birth as an outcome, focusing only on the other four dichotomous variables (maternal race, birth weight outcome, infant sex and maternal tobacco use). Simulated data were generated from a known model similar to the 5 years of North Carolina data and with spatially varying cell probabilities, i.e. cell counts for the $100 \times 2^4 = 1600$ categories were generated. The disaggregation model was fitted to these cell counts whereas the usual model was fitted to totals marginalized over the demographic variables. Model performance was based on estimation of the 100 known county level probabilities of LBW. As expected, both models achieved roughly 95% empirical coverage: 93/100 for the disaggregation model and 97/100 for the conditional model. However, the CIs were roughly 50% wider for the usual model compared with the disaggregation model. Also, the disaggregation model had an MSE that was less than half of the MSE from the usual model. In other words, the interval estimates for the usual model were too long and not well centred. Lastly, turning to the 100 \times 16 = 1600 underlying cell probabilities which the disaggregation model estimates, satisfyingly, we found 1528 of 1600 CIs (95.5%) contained the true cell probability.

5. Summary and extensions

We have argued that, in the case where we have observations assigned to areal units but which retain individual characteristics, there are numerous advantages to modelling at the individual level through the use of log-linear models incorporating spatial random effects. In particular, we can spatially investigate arbitrary marginal and conditional probabilities. In the context of public health databases, this enables us to illuminate disparities in a richer fashion than previously available.

In this analysis, we studied term LBW in North Carolina with such a model. We observed patterns that might have been difficult or impossible to study with other techniques, such as the higher overall rates in eastern North Carolina yet higher rates for each subgroup in western North Carolina. This same model was used to provide a picture of racial disparity in term LBW, and to classify areas of high disparity into subcategories that might call for different intervention strategies.

In future work, we shall apply this modelling approach to contingency tables of much higher dimension and to tables involving ordinal classifications. In addition, there will be interest in how selected probabilities and disparities change not only across space but also evolve in time, for instance, under an annual model for the birth record data from Section 1.2. This will lead us to dynamic versions of our approach, perhaps through space—time autoregressive random-effects specifications.

Finally, there are data sets where we do have the geocoded locations that are associated with the individuals. This would encourage us to work at the point level rather than the areal unit level. Conceptually, we would envision a spatial process of log-linear models. Practically,

we would have one multinomial trial at each of $n = \sum_{s=1}^{S} n_{\cdot}^{(s)}$ locations rather than n multinomial trials ignoring location or using only areal unit information. Structured spatial

dependence enables us to handle the former case, and investigation of this approach is currently work in progress.

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Appendix A: Model fitting

We build our fully Bayesian model as follows. For prior distributions, we use $\varphi_t^0 \sim N(0, 100^2)$ for t = 1, ..., r; $\eta_{tu} \sim N(0, 100^2)$ for t = 1, ..., r and u = 1, ..., q, and $\{\varphi_t^{(1)}, \varphi_t^{(2)}, ..., \varphi_t^{(S)}\}$ $\sim \text{CAR}(\tau_t)$ for certain t, as given in Section 1.2. For the t without spatial structure, $\varphi_t^{(1)} = \varphi_t^{(2)} = ... = \varphi_t^{(S)} = 0$, with the exception of the grand mean term. For the grand mean term, each parameter received an $N(0, 100^2)$ prior. The hyperpriors for the τ_t s are $\tau_t \sim \text{gamma}(0.5, 0.005)$, which is discussed in Wakefield et al. (2001). The joint posterior is

$$\pi(\boldsymbol{\varphi}^0, \boldsymbol{\varphi}^{(s)}, \eta, \boldsymbol{\tau} | \mathbf{n}) \propto \prod_{s} \prod_{l} \pi(n_l^{(s)} | \boldsymbol{\varphi}^0, \boldsymbol{\varphi}^{(s)}, \eta, \boldsymbol{\tau}) \pi(\boldsymbol{\varphi}^0) \pi(\boldsymbol{\varphi}^{(s)} | \boldsymbol{\tau}) \pi(\boldsymbol{\tau}) \pi(\eta),$$

writing $(\varphi^0, \varphi^{(s)}, \eta)$ for λ . After simplification, the likelihood term is

$$\prod_{s} \prod_{l} \frac{\exp\{-\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\varphi}^{(0)})\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\eta}\mathbf{w}_{s})\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\varphi}^{(s)})\}\{\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\varphi}^{(0)})\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\eta}\mathbf{w}_{s})\exp(\mathbf{X}_{l}^{\mathsf{T}}\boldsymbol{\varphi}^{(s)})\}^{n_{l}^{(s)}}}{n_{l}^{(s)}!}$$

The resulting full conditional distributions are as follows:

a.
$$P(\varphi_t^0|\cdot) \propto \prod_s \prod_l \exp\{n_l^{(s)} \mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^{(0)} - \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^{(0)}) \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\eta} \mathbf{w}_s) \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^{(s)}) - (\varphi_t^0)^2/(2 \times 100^2)\}$$
;

b.
$$P(\varphi_t^{(s)}|\cdot) \propto \prod_l \exp\{n_l^{(s)} \mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^{(s)} - \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^0) \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\eta} \mathbf{w}_s) \exp(\mathbf{X}_l^{\mathsf{T}} \boldsymbol{\varphi}^{(s)}) - \tau_t \omega_{s.} (\varphi_t^{(s)} - \overline{\varphi}_t^{(s)})^2 / 2 \}$$
, where δ_s is the set of neighbours of s , defined by adjacency so that $\omega_{sv} = 1$ if $v \in \delta_s$ (and otherwise $\omega_{sv} = 0$) with the convention that $\omega_{ss} = 0$, where $\omega_{s.} = \sum_{v=1}^{s} \omega_{sv}$, and where $\overline{\varphi}_t^{(s)} = (1/\omega_{s.}) \sum_{v=1}^{s} \omega_{sv} \varphi_t^{(v)}$;

c.
$$P(\eta_{tu}|\cdot) \propto \prod_{s} \prod_{l} \exp\{n_{l}^{(s)} \mathbf{X}_{l}^{T} \eta \mathbf{w}_{s} - \exp(\mathbf{X}_{l}^{T} \boldsymbol{\varphi}^{(0)}) \exp(\mathbf{X}_{l}^{T} \eta \mathbf{w}_{s}) \exp(\mathbf{X}_{l}^{T} \boldsymbol{\varphi}^{(s)}) - \eta_{tu}^{2}/(2 \times 100^{2})\}$$
;

d.
$$P(\tau_t|\cdot) \sim \text{gamma}\{0.5+S/2, 0.0005+\frac{1}{2}\sum_{s=1}^{S}\sum_{v < s}\omega_{sv}(\varphi_t^{(s)}-\varphi_t^{(v)})^2\}$$
, based on Mollié (1996).

We implemented our fully Bayesian model by using Markov chain Monte Carlo methods (Gilks *et al.*, 1996). We ran three overdispersed, random, parallel chains of 15 000 Markov chain Monte Carlo simulations, discarding the first 5 000 iterations from each chain as burnin and retaining every 10th iteration after burn-in (to reduce potential problems with autocorrelation), for a total of $3 \times 10~000 = 30~000$ samples for posterior inference. There were no problems with convergence on the basis of Gelman–Rubin statistics and visual inspection of the mixing of the chains (Gelman and Rubin, 1992).

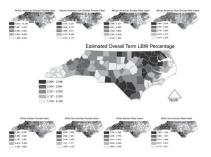


Fig. 1. Overall term LBW incidence (centre) flanked by the eight race \times infant sex \times tobacco use combinations: in these maps, internally derived quintiles determine the cut points of the categories for each map

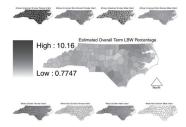


Fig. 2. Overall term LBW incidence (centre) flanked by the eight race \times infant sex \times tobacco use combinations: in these maps, a continuous colour scale corresponds to the overall range of values

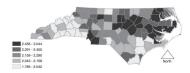


Fig. 3.Odds ratio OR of LBW for African Americans *versus* whites: this OR may be regarded as a measure of racial disparity in term LBW incidence, with the null value of 1.0 reflecting no disparity and values greater than 1.0 demonstrating the excess burden borne by African Americans

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Table 1

Model fit metrics for four models

Metric			Results for the following models:	lowing models:	
		Model 0	Model A	Model B	Model AB
MSE	$Median \times 10^{-4}$	261.3	243.6	5.10	5.14
	$95\%~CI\times10^{-4}$	(260.9, 261.8)	(243.0, 244.1)	(4.49, 5.89)	(4.52, 5.97)
$\log(p)$	Median	8376	\$998	2956	2964
	95% CI	(8326, 8429)	(8593, 8741)	(2867, 3053)	(2877, 3062)
2	Median	71650	64310	3205	3226
	95% CI	(71300, 72010)	(63960, 64670)	(3092, 3331)	(3113, 3351)
內	Median	14.92	14.38	1.098	1.108
	95% CI	(14.89, 14.95)	(14.36, 14.41)	(1.062, 1.139) (1.072, 1.150)	(1.072, 1.150)

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Table 2

First-level effect estimates from two models

Effect [†]	,	Results for the following models:	
		Model B	Model AB
Grand mean	Median	-3.463	-3.812
	95% CI	(-4.015, -3.255)	(-3.931, -3.521)
MR	Median	-0.7312	-0.7377
	95% CI	(-0.7541, -0.7100)	(-0.7993, -0.6712)
LBW	Median	-0.7652	-0.7686
	95% CI	(-0.7789, -0.7526)	(-0.7831, -0.7548)
IS	Median	-0.01868	-0.01873
	95% CI	(-0.02726, -0.01031)	(-0.02791, -0.00969)
MTU	Median	-0.6741	-0.7329
	95% CI	(-0.6900, -0.6591)	(-0.7649, -0.6982)
PT	Median	-0.5493	-0.5512
	95% CI	(-0.5633, -0.5351)	(-0.5702, -0.533)
$MR \times LBW \\$	Median	0.1819	0.1837
	95% CI	(0.1700, 0.1935)	(0.1716, 0.1961)
$MR \times IS \\$	Median	-0.01263	-0.01264
	95% CI	(-0.01875, -0.00658)	(-0.01884, -0.00623)
$MR \times MTU \\$	Median	-0.1924	-0.1579
	95% CI	(-0.2076, -0.1785)	(-0.1788, -0.137)
$MR \times PT$	Median	0.008638	0.01439
	95% CI	(-0.00458, 0.02165)	(0.001109, 0.02778)
$LBW \times IS \\$	Median	-0.08882	-0.08856
	95% CI	(-0.09646, -0.08107)	(-0.09666, -0.08044)
$LBW \times MTU \\$	Median	0.1844	0.1886
	95% CI	(0.1729, 0.1956)	(0.1777, 0.2000)
$LBW \times PT \\$	Median	0.9077	0.9161
	95% CI	(0.8952, 0.9208)	(0.9025, 0.9298)
$IS \times MTU \\$	Median	0.004782	0.00427
	95% CI	$(5.49 \times 10^{-6}, 0.009607)$	$(-6.73 \times 10^{-4}, 0.009278)$
$\text{IS} \times \text{PT}$	Median	0.07691	0.07704
	95% CI	(0.06922, 0.08475)	(0.06903, 0.08492)
$MTU \times PT \\$	Median	-0.04761	-0.0391
	95% CI	(-0.05984, -0.03526)	(-0.05055, -0.02775)
$MR \times LBW \times MTU$	Median	-0.05651	-0.05609
	95% CI	(-0.06663, -0.04676)	(-0.06584, -0.04614)
$MR \times LBW \times PT$	Median	-0.02172	-0.0208
	95% CI	(-0.03464, -0.00935)	(-0.03100, -0.01065)
$MR \times IS \times PT$	Median	-0.01193	-0.01199
	95% CI	(-0.01747, -0.00628)	(-0.01749, -0.00645)

Effect †		Results for the following models:	
		Model B	Model AB
$MR \times MTU \times PT$	Median	0.02444	0.0245
	95% CI	(0.013, 0.03611)	(0.0143, 0.03482)
$LBW \times IS \times PT$	Median	0.03118	0.03121
	95% CI	(0.02373, 0.03869)	(0.02393, 0.03868)
$LBW \times MTU \times PT$	Median	-0.06719	-0.06723
	95% CI	(-0.0799, -0.05373)	(-0.07728, -0.05704)
$MR \times LBW \times MTU \times PT$	Median	0.01125	0.01104
	95% CI	$(8.48 \times 10^{-4}, 0.02155)$	(0.001786, 0.02126)

 $^{^{\}dagger} \rm{MR},$ maternal race; IS, infant's sex; MTU, maternal to bacco use; PT, preterm.