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Consequences, Characteristics, and Causes of Mathematical Learning Disabilities and Persistent Low Achievement in Mathematics

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Abstract

Objective—The goals of the review are threefold; a) to highlight the educational and employment consequences of poorly developed mathematical competencies; b) overview the characteristics of the children with persistently low achievement in mathematics; and c) provide a primer on cognitive science research that is aimed at identifying the cognitive mechanisms underlying these learning disabilities and associated cognitive interventions.

Method—Literatures on the educational and economic consequences of poor mathematics achievement were reviewed and integrated with reviews of epidemiological, behavioral genetic, and cognitive science studies of poor mathematics achievement.

Results—Poor mathematical competencies are common among adults and result in employment difficulties and difficulties in many common day-to-day activities. Among students, about 7% of children and adolescents have a mathematical learning disability (MLD) and another 10% show persistent low achievement (LA) in mathematics despite average abilities in most other areas. Children with MLD and their LA peers have deficits in understanding and representing numerical magnitude, difficulties retrieving basic arithmetic facts from long-term memory, and delays in learning mathematical procedures. These deficits and delays cannot be attributed to intelligence, but are related to working memory deficits for children with MLD, but not LA children. Interventions that target these cognitive deficits are in development and preliminary results are promising.

Conclusion—Mathematical learning disabilities and learning difficulties associated with persistent low achievement in mathematics are common and not attributable to intelligence. These individuals have identifiable number and memory delays and deficits that appear to be specific to mathematics learning. The most promising interventions are those that target these specific deficits and, in addition, for children with MLD interventions that target their low working memory capacity.

Keywords

learning disability; mathematical learning disability; poor achievement; mathematical cognition; working memory

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Few people question the importance of literacy for employment and day-to-day living in the modern world, but many under appreciate the importance of arithmetic and other basic mathematical competencies (e.g., simple algebra, measurement)[1]. In fact, the social and individual costs of poorly developed mathematical skills may be higher than those associated with poor reading skills, in part because more people have difficulty with mathematics than with reading and because of steady increases in the quantitative knowledge needed to function in many jobs today, including many blue collar jobs[1, 2, 3, 4, 5].

The consequences are detailed in the first section, followed in the second by an overview of the characteristics of children with a mathematical learning disability (MLD) and their peers who have persistently low mathematics achievement mathematics (LA), despite average abilities in most other areas. The final section provides a primer on cognitive science research, with a focus on identifying the mechanisms underlying MLD and LA and attempts to develop interventions that target them.

CONSEQUENCES

The consequences of poorly developed mathematical competencies were documented in a review of large-scale national studies of the reading and mathematical skills of children and adults in Great Britian[1]. It was not surprising that their findings revealed that poor reading skills reduced employment opportunities and wages once employed, but it was surprising that poor mathematics skills resulted in even more dire prospects, even for individuals with good reading skills[2, 4]. The gist is illustrated by the results of a large-scale longitudinal study of about 17,000 people from birth through adulthood, with reading, mathematics, and employment skills and employment history fully assessed for 10% of them at age 37[2]. The reading and mathematics assessments focused on everyday competencies. The reading test included items ranging from the ability to comprehend an advertisement to making inferences about a technical newspaper article, and the mathematics items ranged from determining the correct amount of change following a purchase to determining the relation between salary increases and cost of living increases. All of the mathematics items could be easily solved with basic arithmetic, measurement, and simple algebraic skills. To control for confounds, the focus was on individuals who did not go to college after completing high school and comparisons were across two groups, one with average reading and average mathematics skills and the other with average reading but below average mathematics skills.

For both men and women, poor mathematics skills were associated with lower rates of full-time employment, higher rates of employment in low-paying manual occupations, more frequent periods of unemployment, and a lower ability to take advantage of employer offered training and thus lower rates of promotion. Many women in this group eventually left the full time labor market and although 4 out of 5 of the men were employed full time, 50% of them had a low annual income, compared to 26% of the men in the contrast group. These finding are not limited to Great Britain, as similar relations are found in the United States[4].

These results and those for other large scale studies conducted in the United States and Canada also indicate that below average mathematical competencies at the beginning of schooling is associated with elevated risk of poor mathematical competencies at the end of schooling, above and beyond the influence of family background and the child’s social and emotional functioning and their intelligence and reading ability[1, 6]. The early identification of children who are at risk for long-term difficulties in mathematics is critical. Without intervention, these early deficits will likely compound into life-long struggles in the workplace and in dealing with the day-to-day demands of the modern world.

CHARACTERISTICS

DEFINITION

The *Diagnostic and Statistical Manual of Mental Disorders* of the American Psychiatric Association defines MLD in terms of a discrepancy between performance on mathematics achievement tests and expected performance based on age, intelligence, and years of education and for adults significantly interferes with their daily activities[7]. However, it has not been established that children (or adults) with low mathematics achievement scores and with low intelligence have different forms of mathematical cognition deficit than children with low mathematics achievement and average intelligence. Some findings described in **CAUSES** suggest the mathematical deficits may in fact be the same, but the mechanisms causing these deficits may differ. At this point, the implications are that level of intelligence may not matter when identifying mathematical deficits, but different remedial approaches may be needed for children with higher or lower intelligence.

Among researchers in this field, a consensus is emerging with respect to the utility of distinguishing between children with MLD and their LA peers, with the restriction that intelligence scores are above the 15th percentile[8, 9]. Children who score at or below the 10th percentile on standardized mathematics achievement tests for at least two consecutive academic years are typically categorized as MLD in research studies, and children scoring between the 11th and the 25th percentiles, inclusive, across at least two consecutive years are categorized as LA. Response to intervention may also be used to identify and treat children with MLD and LA and is described in **COGNITIVE INTERVENTIONS**.

INCIDENCE

On the basis of several population-based prospective studies and many smaller-scale studies, about 7% of children and adolescents will be diagnosable as MLD in at least one area of mathematics before graduating from high school, and an additional 10% of children and adolescents will be identified as LA[10, 11, 12]. An analysis of more than 340,000 11 year olds between 1998 and 2007, inclusive, identified between 5% and 7% each academic year as being 3 to 4 grade levels behind in mathematics, a deficit level consistent with MLD[1].

In any one year, 10% of children will score at or below the 10th percentile in mathematics by definition, but not all of them will score in this range across multiple years and thus the 7% estimated incidence for MLD is lower than the suggested 10th percentile cutoff; the same goes for scores in the LA range. The multiple year criteria is important, because many children who score poorly in one academic year score higher in later years, and these children do not have the cognitive deficits found with children who consistently score in the MLD and LA ranges[13, 14]. Finally, the large scale studies in Great Britain indicated that about 23% of adults are functionally innumerate, that is, they do not have the mathematical competencies needed for many routine day-to-day activities.

The reason this percentage is higher than the combined MLD and LA estimates is not known, but may be related to the exclusion of lower-IQ children from MLD and LA studies, loss of basic mathematics competencies with disuse, or some combination[15]. Whatever the reasons, poorly developed mathematical competencies are common in children and adults. As we will cover in **CAUSES,** for some of these individuals their difficulties with mathematics is not due to schooling or intelligence, but rather to one or several specific cognitive delays or deficits.

ETIOLOGY

Twin and family studies reveal genetic and environmental contributions to individual differences in mathematics achievement and to MLD and LA[16, 17, 18]. A study of elementary school twins revealed genetic as well as shared (between the pair of twins) and unique environmental contributions to individual differences in mathematics achievement and to MLD, with the latter defined by cut-offs at the 5th and 15th percentiles on a mathematics achievement test. Depending on the grade and mathematics test used, from 50% to 67% of the individual differences in mathematics achievement were attributable to genetic variation and the remainder to shared and unique experiences[16].

The same genetic influences that contributed to MLD contributed to individual differences at all levels of performance[16, 19]. There are not specific MLD genes, but rather the genetic influences on MLD are the same as those that influence mathematics achievement across the entire range of scores. Roughly 33% of the genetic influences on mathematics achievement overlapped the genetic influences contributing to variation in intelligence, 33% overlapped the genetic influences contributing to variation in reading ability independent of intelligence, and 33% were unique to mathematics.

Moderate genetic influences on MLD should not be equated with constraint on the potential to remediate these deficits, because changes in the individual's environment may alter the relative extent of these genetic and environmental influences. In any case, the genetic studies also indicate important environmental influences on mathematics learning and MLD. Schooling influences mathematics achievement in general and the nascent interventions for MLD (**COGNITIVE INTERVENTIONS**) improve the mathematics achievement of these children above and beyond the influence of general education, even if they do not eliminate variation in mathematical outcomes[20].

COMORBID DISORDERS

The genetic factors that influence achievement across academic domains may explain why many children with MLD have reading disability (RD) or other difficulties that interfere with learning in school, such as attention deficit hyperactivity disorder (ADHD)[10, 12, 21]. Barbaresi et al. found between 57% and 64% of students with MLD also had RD, depending on the diagnostic criteria used to define MLD. The earlier noted large-scale study of 11 year olds in Great Britain found that 6% of these children showed achievement levels consistent with MLD, and 2 out of 3 of them were also poor readers[1].

Research on children identified in their schools with a specific learning disability reveal these children often have an array of social deficits and that children referred for evaluation due to severe emotional or behavioral problems in school are often classified as learning disabled[22, 23]. Although most of these studies have focused on RD, they may still be relevant to MLD. One meta-analysis indicated that, as a group and in comparison to typically achieving (TA) children, children who are classified as learning disabled (in reading, mathematics, or both) experience more social rejection, have poor social problem solving skills, and are reported by others as being aggressive and immature, among other issues.

At the same time, multiple large-scale prospective studies that have tracked the relation between family background, social-emotional factors, attentional control, intelligence, and academic skills at school entry (i.e., five to six years of age) and long-term achievement in mathematics and reading do not find a relation between social-emotional problems and poor mathematics outcomes[6]. The best predictor of mathematics achievement throughout schooling was entery level mathematics skills. Early attentional skills also predicted later achievement, but the magnitude of this effect was less than 25% of the magnitude of the effect for entry-level mathematics skills. Internalizing (e.g., anxiety) and externalizing (e.g., aggression) problems at school entry were not related to later achievement nor were more general measures of social skills. This analysis suggests that the early social and behavioral profile of children is not related to their long-term mathematics achievement.

The discrepant results indicate there is much work to be done is this area, and especially with respect to children with MLD. At this point, a preliminary conclusion is that social-emotional functioning does not causally affect children's mathematics learning but that children with MLD may show a host of comorbid social and behavioral problems.

CAUSES

Cognitive scientists and neuropsychologists have conducted detailed studies of the number, counting, and arithmetic competencies of children with MLD and LA children, as well as children and adults with acquired (following traumatic brain injury) mathematical difficulties, in attempts to identify the source or sources of their poor mathematics achievement [24, 25, 26, 27, 28, 29]. Many of these studies also include assessments of general abilities – intelligence, working memory, and processing speed – that influence learning across academic domains. The goal is to determine whether there are cognitive deficits specific to learning mathematics, and whether these deficits are independent of or interact with the domain general abilities during mathematical learning or performance. The behavioral genetic studies suggest both overlap in the genetic and environmental mechanisms contributing to mathematics and other forms of learning in school, as well as mechanisms that are unique to mathematics. The results of the cognitive studies overviewed here are consistent with these findings.

Before moving to this discussion, it is important to illustrate the severity of the mathematical achievement deficits of children with MLD and their LA peers. Figure 1 shows these deficits as contrasted with TA children and a group of children with intelligence scores below the 10th percentile (Low-IQ, Mean IQ = 78). These data are from the Missouri Longitudinal Study of Mathematical Development and Disability and show the first to fifth grade, inclusive, achievement trajectories of the children in these groups[30, 31]. The MLD group included children who scored in the bottom 10 percent of the sample on a mathematics achievement test from second to fifth grade, inclusive, whereas the LA group included children scoring between the 11th and 25th percentile, inclusive. The children in the MLD group had low-average IQ scores (M = 91) and the LA (M = 101) and TA (M = 103) children average scores.

The most striking findings are that the mathematics achievement of the MLD children lags behind that of the Low-IQ children after third grade and the achievement of the LA and Low-IQ groups overlap, despite a 23 point difference in mean IQ (the mean intelligence scores of the Low-IQ and LA groups were at the 7th and 53rd percentiles, respectively). Across grades, the advantage of the TA children widens in mathematics, but the reading gap closes. Clearly, the poor mathematics achievement of the MLD and LA groups cannot be attributed to low intelligence or reading ability.

The focus of the cognitive and neuropsychological studies is on the identification of the deficits that are underlying these achievement patterns. The consistent findings suggest deficits in the ability to form representations of numerical magnitude, form memory representations of basic arithmetic facts or retrieve these facts once memories are formed, and developmental delays in the learning of arithmetical procedures.

NUMBER

There is evidence for a core system of inter-related quantitative competencies that may contribute to children's learning of formal mathematics in school.[32] Human infants, preschoolers, as well as individuals from many other species are able to discriminate smaller from larger amounts (e.g., 8 items vs. 16 items), order a series of relative quantities (e.g., 2, 3, 4, items), and some (including human infants and preschoolers) have a rudimentary ability to count and engage in simple addition and subtraction[33, 34, 35, 36]. These core abilities provide the foundation for children's early number sense, which is manifested in their ability to (a) apprehend the quantity of sets of 3 to 4 objects or actions without counting[37, 38, 39]; (b) use non-verbal processes or counting to quantify small sets of objects and to add and subtract small quantities to and from these sets[40, 41]; and, (c) estimate the relative magnitude of sets of objects and the results of simple numerical operations[42]. Butterworth and colleagues have proposed that MLD results from deficits in two of these fundamental number sense systems, one that supports the representation and implicit understanding of the *exact quantity* of small collections of objects and of symbols (e.g., Arabic numerals) that represent these quantities (e.g., '3' = ▪▪▪), and the other for representing the *approximate magnitude* of larger quantities[27, 43].

In keeping with this hypothesis, children with MLD and, to a lesser extent, LA children may have deficits or developmental delays in both of these fundamental number representation and processing systems[44, 45, 46, 47]. As an example, Koontz and Berch asked third and fourth graders with MLD and their TA peers to determine if combinations of Arabic numerals (e.g., 3–2), number sets (▪▪-▪▪), or numerals and sets were the same (2-▪▪) or different (3-▪▪)[46]. This simple task provided an assessment of the children's representational system for small, exact quantities. Confirming earlier findings[48], the TA children processed representations of three (e.g., 3, ▪▪▪) as quickly as they processed representations of two. The children with MLD could also quickly process representations of two, but appeared to rely on counting to determine quantities of three. The results suggest that some children with MLD might not have an inherent representation for numerosities of three or the exact representational system does not reliably discriminate two and three.

Follow-up studies in which children were asked to mentally combine sets of objects (e.g., ▪▪▪▪) and Arabic numerals to match a target number (e.g., ▪▪▪▪ + 2 = 5) confirm slow number processing for groups of MLD and LA children[8, 31, 49]. The fluency scores in Figure 2 index the speed and accuracy with which children access and combine these small numerical quantities, and illustrate the five year trends for the same groups represented in Figure 1[31]. The number processing fluency of the children with MLD is at the same level or slightly lower than that of the children in the Low-IQ group. By fifth grade both groups are performing at a level comparable to that of TA third graders. The LA children are about one year behind their TA peers. Equally important, the trends in Figure 3 show no indication that the MLD and LA groups are catching up to their TA peers; if anything the gap widens after third grade.

Children's placements of numerals on a physical number line have been used to make inferences about the nature of their approximate magnitude representational system. Placements that conform to the natural logarithm of the numbers may reflect dependence on the potentially inherent system that represents approximate magnitudes[50, 51]. These placements reflect an expansion of the number line for smaller values, as shown in the middle section of Figure 4, and a contraction for larger ones. The "mental distance" between one and two is much larger than the distance between eight and nine, and thus children more easily distinguish between the difference in the magnitudes between one and two than between eight and nine, even though the actual difference is the same. With instruction, children eventually learn the mathematical number line; the distance between two consecutive numbers is the same regardless of position on the line. Whatever the underlying representational system, accuracy in making linear placements is predictive of later mathematics achievement[52].

In one of our studies, we compared the placements of MLD, LA, and TA first and second graders on the 0 to 100 number line[44]. Group differences emerged using group medians, with trial-by-trail assessments of whether the placement was consistent with a log (suggesting reliance on the approximate magnitude system) or linear (suggesting learning of the mathematical number line) mental representation of position on the number line, and with several measures of absolute error. The overall pattern suggested that children with MLD were more heavily dependent on the approximate representational system – they were not learning the mathematical number line as easily as other children – and consistent with Butterworth's hypothesis their representation of magnitude appeared to be more compressed than that of LA and TA children, as represented by the bottom number line in Figure 3. In other words, the children with MLD are have difficulty discriminating between the magnitudes represented by even small numerals, possibly due to a deficit or delay in the system for representing approximate magnitudes. The follow-up of these children through fifth grade revealed the Low-IQ children (not assessed in the first study) caught up with their TA peers on this task by third grade, the LA children by fourth, but the children with MLD still had not caught up by fifth, although they had closed the gap[31].

Although not conclusive, these studies suggest that many children with MLD and to a lesser extent their LA peers do not have a strong intuitive sense of numerical magnitude that this deficit is not related to intelligence or reading ability. Follow-up studies are needed but the development trends shown in Figure 2 suggest that these difficulties will extend well beyond the elementary school years. Whether these result from early neurodevelopmental deficits in the fundamental systems for representing small, exact quantities and approximate larger quantities, as hypothesized by Butterworth, remains to be determined. Either way, poor performance on these simple number tasks is predictive of below average mathematics achievement, above and beyond the influence of intelligence, working memory, or reading ability[49].

ARITHMETIC

Typical development—By the time they begin formal schooling most children have coordinated their number knowledge and counting skills with an implicit understanding of addition and subtraction, and as a result can begin to use number words and Arabic numerals to solve formal addition and subtraction problems (e.g., "How much is 3 + 2?")[53, 54, 55]. Although children of this age will use a mix of problem solving strategies, the most common approaches involve counting, sometimes with and sometimes without use of their fingers[56]. The min and sum procedures are two common ways children count[57]. The min procedure involves stating the larger-valued addend and then counting a number of times equal to the value of the smaller addend; for example, stating "five" and then counting "six, seven, eight" to solve "5+3=?". The sum procedure involves counting both addends starting from 1. The use of counting results in the development of long-term memory representations of basic facts[56]. Once formed, these representations support the use of memory-based processes; specifically, direct retrieval of arithmetic facts and decomposition. The latter involves reconstructing the answer based on the retrieval of a partial sum; for example, 6+7 might be solved by retrieving the answer to 6+6 and then adding 1 to this partial sum.

Development, however, is not simply a switch from use of less sophisticated counting to more sophisticated retrieval strategies[58]. Rather, at any time children can use any of the many strategies they know to solve different problems; they may retrieve the answer to 3+1 but count to solve 5+8. What changes is the mix of strategies, with sophisticated ones used more often and less sophisticated ones less often[59].

Children with MLD and LA—The same methods developed to study TA children's arithmetical competencies have been applied to the study of children with MLD and LA children, and have revealed similarities and a few notable differences[8, 13, 60, 61, 62, 63]. Children with MLD and LA children use the same types of problem solving approaches as their TA peers, but are delayed in the development of procedural skills and have more persistent difficulties remembering basic arithmetic facts.

Procedural Competence: Children with MLD and their LA peers commit more procedural errors than same-grade TA children when they solve simple (e.g., 4+3) and complex (e.g., 745–198) arithmetic problems (4+3), as well as word problems[8, 60, 64, 65]. A common error for first graders with MLD is to under count when using the min procedure; for the problem "5 + 3 = ?", they will state "five, six, seven". They correctly count three number words, the min addend, but do not use "five" to represent the cardinal value of the larger addend. Even when these children do not commit errors, they tend to use developmentally immature procedures[60, 62, 66]. By first grade, most TA children can count silently ("in their head") and using the min procedure to solve simple addition problems, but first graders with MLD use their fingers to help them keep track of counting and use the sum procedure more frequently than their TA peers. LA children also use their fingers more often than TA children, but use the min procedure more frequently than do children with MLD[8, 62, 67, 68]. For simple arithmetic, this translates into roughly a two- to three-year developmental delay for children with MLD and about a one year delay for LA children.

The deficits and delays of children with MLD and LA children when solving simple problems become more evident when they attempt to solve more complex ones[69, 70]. During the solving of multistep arithmetic problems, such as 45×12 or 126+537, fourth graders with MLD committed more errors than their IQ-matched TA peers[64]. The errors include the misalignment of numerals while writing down partial answers or while carrying or borrowing from one column to the next. Common subtraction errors included subtracting the larger number from the smaller one (e.g., 83–44 = 41), failing to decrement after borrowing from one column to the next (e.g., 92 – 14 = 88; the 90 was not decremented to 80), and borrowing across 0s (e.g., 900–111 = 899) [66]. These patterns were found for children with MLD and LA children, regardless of their reading achievement. Again, the errors are largely developmental delays and not persistent problems; they are committed by younger TA children[71], and children with MLD and LA children eventually learn the correct procedures, albeit several years later than their TA peers.

Memory for Basic Facts: The most consistent research finding is that most children with MLD and a subset of LA children have persistent difficulties committing basic arithmetic facts to long-term memory or retrieving them once they are committed[14, 60, 62]. It is not that these children cannot memorize or retrieve any basic facts, but rather they show persistent differences in the frequency with which they correctly retrieve them and in the pattern of retrieval errors. Three different mechanisms have been proposed as the potential source of these retrieval difficulties.

The first is a deficit in the ability to form phonetic, language-sound based representations in long-term memory[24]. This hypothesis follows from children's early reliance on counting when they are first learning to solve arithmetic problems, as counting depends on the phonetic and semantic systems of the language domain. Any disruption in the ability to represent or retrieve information from these systems should, in theory, result in difficulties in forming problem/answer associations for arithmetic problems during the act of counting, as well as result in comorbid word retrieval problems during the act of reading. Studies of arithmetic deficits following brain injury suggests that the retrieval of addition facts is indeed supported by a system of neural structures that appear to support phonetic and semantic representations and are engaged during incrementing processes, such as counting[72, 73]. These findings need to be interpreted with caution, however, because they are based on studies of adults and the brain and cognitive systems that support early learning differ in important ways from those that support the same competence in adulthood[74, 75].

The second mechanism is a deficit in the ability to inhibit irrelevant associations from entering working memory during the process of fact retrieval[76]. These intrusions are often assessed by asking children to only try and remember the answer and not use counting or any other procedure for problem solving[26]. If intrusions disrupt children's ability to retrieve the correct answer, then the corresponding retrieval errors should be associated with the numerals in the presented problem. Examples include the retrieval of 36 when trying to solve 6×5, or 8 when trying to solve 4+7. The first is called a table-related error because it is a correct answer to a similar (6×6) problem in the multiplication table, and the second is called a counting-string error because the retrieved answer follows one of the addends in the counting string (8 follows 7)[77, 78]. Both types of intrusions occur for TA children, as do cross-operation intrusions; e.g., recalling 40 for 8+5[79]. All of these types of intrusions are more common and more persistent across more grades for children with MLD and some LA children. Adolescents with MLD have frequent table-related intrusions when they solve simple multiplication problems, and in elementary school counting-string intrusions are common for addition retrieval.[14, 76].

The third proposed mechanism is the deficits or delays in the number systems that support the exact representations of small quantities and the approximate representations of larger magnitudes[27]. The reasoning is that children's early learning of arithmetic may be dependent on this intuitive understanding of number: Their ability to estimate the approximate answers when first learning to solve arithmetic problems may be dependent, in part, on the approximate magnitude representational system. In other words, these basic number systems are hypothesized to provide part of the foundation for learning above numbers (e.g., the base-10 system) and arithmetic in school. In this view, the retrieval deficits are secondary to a more basic deficit in the approximate representational system. Empirical evaluation will require longitudinal studies to determine if there is a relation between number processing deficits during the preschool years and retrieval deficits in the elementary school years. Although this type of longitudinal study remains to be conducted, analyses of the relation between the number fluency deficits shown in Figure 2 and the retrieval deficits of children with MLD and LA children with frequent intrusion errors suggest these two deficits are not related. For some LA children, slow number processing, for instance, is found in the absence of retrieval deficits[80].

Overall, it is clear that difficulties learning or retrieving basic arithmetic facts are a common and persistent difficulty for children with MLD and for a subset of LA children. Of the proposed mechanisms, the evidence is strongest for intrusion errors, that is, the retrieval deficits are related in part to the intrusion of related, but task irrelevant information into working memory when these children are attempting to remember arithmetic facts. Not all of their errors are due to intrusions, however, suggesting multiple mechanisms may be involved and that different children may have retrieval deficits for different reasons. Whether these alternative mechanisms involve the language system and number processing deficits remains to be determined.

Before moving to **Domain General Deficits**, I note that retrieval errors in and of themselves cannot be used to determine if a child has MLD. This is because many mathematics curricula used in the United States de-emphasize the learning of basic facts and thus many children do not know all of them. Children with MLD and many of their LA peers commit more of these errors than TA children, and, as noted, more of these are intrusion errors. Evidence that these children's difficulty with fact retrieval is a real deficit and not the result of limited practice comes from studies conducted in countries that emphasize the memorization of basic facts. Children in Hong Kong, where memorization is emphasized, have the same procedural delays and retrieval deficits for groups of MLD and LA children as found in the United States and many other countries[81]. Common mathematics curricula in the United States do not cause these difficulties but they do make them more difficult to detect.

DOMAIN GENERAL DEFICITS

By definition, learning disabilities are determined by the child's performance on academic achievement tests and in school more generally, and for adults by the effects poor reading and mathematical competencies have on their day-to-day functioning, including their employment[1]. For both children and adults it is therefore important to not only assess specific deficits (e.g., in number processing) but also other cognitive factors that predict school achievement and performance at work. These domain general learning abilities include fluid intelligence, working memory, and processing speed. Although measures of these different abilities are typically correlated with one another, they all assess unique competencies that are potentially important for academic learning[82, 83, 84, 85].

A useful heuristic is Carroll's hierarchical organization of these competences[86]. Fluid intelligence is at the top and represents processes that affect learning across contexts and content, especially the ease of learning new and complex concepts[83, 87, 88]. Working memory and processing speed are at the second level, and are broad abilities that affect learning in many but not all domains. At the third level are more restricted domains of competence, including mathematics. Studies of performance on a wide range of paper-and-pencil tests have revealed at least two core mathematical domains, Numerical Facility, which assesses competence in arithmetic and for young children their number and counting knowledge, and Mathematical Reasoning which assess more abstract mathematical knowledge[89].

Intelligence—Fluid intelligence is the best individual predictor of academic achievement[90, 91]. As one example, a five year prospective study of more than 70,000 students revealed that intelligence at age 11 explained nearly 60% of the variation on national mathematics tests administered at age 16[92]. Intelligence is also heritable and there appear to be shared genes contributing to the correlation between intelligence and mathematics achievement[93]. One possibility then is that the slow mathematical growth of children with MLD and their LA peers and the partial heritability of these disorders might be related to intelligence.

Although this may be a contributing factor for children with MLD, it does not appear to be the primary one. As noted in **CAUSES** and returning to Figure 1, the mathematics achievement of children with MLD is significantly lower than that of children with much lower intelligence scores. If anything, children with MLD should have higher mathematics scores than they do, if intelligence was the primary source of their learning disability. Intelligence cannot be a factor at all for LA children, given that their intelligence is average. This is not to say that children or adults with lower intelligence scores do not have difficulties learning mathematics, they do. Indeed, controlling for intelligence closed the mathematics achievement gap comparing the TA and Low-IQ groups shown in Figure 1, but did not contribute to the gap between the TA and MLD groups; that is, the performance of the Low-IQ children was consistent with their intelligence scores, but that of the children with MLD was lower than expected. The point is there are many children and presumably adults who struggle with some areas of mathematics for reasons unrelated to their intelligence.

Working Memory—Working memory is the ability to use attentional focus to keep information in mind while engaging in other mental activities; to filter out information that is irrelevant to the task at hand; and, to switch back and forth from one task to another. Cognitive scientists have determined that working memory is dependent on three core systems. A central executive that provides top-down control of information that is active (i.e., one is consciously aware of it) in two representational systems[94, 95]. These are a language-based phonological loop and a visuospatial sketch pad[96, 97]. There is a fourth system, the episodic buffer that contributes to the integration of language and visuospatial information and for the recall of memories of personal experiences, but not as much is known about this system as the other three[98].

A relation between working memory capacity and performance on mathematics achievement tests and on mathematical cognition tasks is well established[8, 99]. Whether assessed concurrently or one or more years earlier, the higher the capacity of the central executive the better the performance on measures of mathematics achievement and mathematical cognition[100, 101, 102]. The importance of the phonological loop and visuospatial sketch pad varies with content of the assessed mathematics[8, 100]. The phonological loop appears to support processes that involve the articulation of numbers, as in counting, the solving of mathematical word problems, and may be related to arithmetic fact retrieval[24, 99, 102]. The visuospatial sketch pad supports learning in a broader number of mathematical domains, such as the number line and aspects of translating word problems into mathematical equations[104, 105].

Children with MLD and LA children: Children with MLD have working memory deficits in each of the three core systems which in turn contributes to their slow progress in learning mathematics above and beyond the contributions of intelligence and processing speed[8, 67, 100, 106, 107]. Their compromised central executive is especially important[8, 108, 109], but this relation is complicated by at least three subcomponents of the central executive, each of which may affect mathematical learning in different ways. These include competence at maintaining information in working memory, task switching, and inhibiting the retrieval of irrelevant information[9, 110, 111, 112].

In any case, difficulties inhibiting the activation of irrelevant information in working memory have been independently related to poor mathematics achievement by several research groups[110, 111, 112]. As noted in ***Memory for Basic Facts***, deficits in this component of the central executive may explain children with MLD's high frequency of intrusion errors during the act of arithmetic fact retrieval and may be a contributing factor to the comorbidity of MLD and RD in some children; poor readers are less able to suppress context-irrelevant meanings of ambiguous words (e.g., river bank, bank teller), the meanings of similar sounding words (e.g., patients, patience), and retrieve more contextual information than is appropriate for the read passage[113]. Although the content is different for arithmetic and reading, the underlying causes of some (but not all) of the learning difficulties in these areas may be the same. The intrusion errors that occur for some LA children are also consistent with such a deficit, but their central executive scores are typically in the average range. However, the central executive measures used in these studies primarily assessed the maintenance and task-switching components and not the inhibitory control component[8, 106]. Thus, a direct link between the inhibitory control component of the central executive and the intrusion errors that contribute to the poor fact retrieval of children with MLD and their LA peers remains to be forged.

We have found that LA children have average scores on measures of the phonological loop and visuospatial sketch pad[8], but some of these children may have subtle visuospatial deficits[106]. As noted, children with MLD have deficits in both of these working memory systems that in turn may contribute to their slow progress in specific areas of mathematics[8, 44]. As an example, the poor visuospatial working memory of children with MLD may contribute to their slow number processing and poor performance on the number line task, relative to IQ-match LA children. The potential contribution of visuospatial working memory to these specific deficits is intriguing because the exact and approximate representational systems for magnitude are believed to be located in an area of the brain that also contributes to the ability to form visuospatial representations. In contrast, the frequent errors committed by children with MLD when they use counting to solve simple addition problems is related to their poor phonological working memory, that is, their ability to keep language sounds (e.g., number words) in mind while engaged in another task (e.g., keep track of the counting process).

There is clearly much that remains to be learned about the relation between the multiple components of working memory and individual differences in learning across different areas of mathematics in general and the contributions of these working memory systems to the poor achievement of children with MLD and their LA peers. At this point, we can conclude that children with MLD have pervasive deficits across all of the working memory systems that have been assessed, but our understanding of the relations between specific components of working memory and specific mathematical cognition deficits is in its infancy. Many LA children, in contrast, appear to have a normal phonological working memory, especially if reading achievement is average or better, and a normal ability to use the attentional control functions of the central executive to maintain information in working memory. Many of these children also appear to have an intact visuospatial working memory system, but a subset of them may have more subtle deficits. The most promising results suggest that LA children have subtle deficits in the inhibitory control component of the central executive[8, 9], but we await confirmation.

Processing speed—Faster processing speed is associated with higher achievement scores, although the strength of these relations is smaller than that found between intelligence, working memory, and achievement[87, 88]. Cognitive scientists are currently debating whether individual differences in working memory are driven by more fundamental differences in speed of cognitive processing and decision making, or whether the attentional focus associated with the central executive speeds information processing[114, 115]. Whatever the direction of the relation, processing speed has several subcomponents that are independent of working memory[86], and is sometimes found to be a better predictor of mathematics outcomes than working memory or an independent predictor after controlling for working memory and intelligence[116, 117]. Developmentally, processing speed increases rapidly for many simple tasks during the early elementary school years and then asymptotes to near adult levels in adolescence[118]. The mechanisms underlying this pattern are not fully understood but may include substantial improvements in the attentional focus component of the central executive and rapid increases in neuronal white matter (which speeds neural transmission) during this age range[119].

Children with MLD and LA children take more time to solve problems, on average, than do their TA peers[120], but this is not necessarily an indication of a slower fundamental processing speed[24]. Their slow speed of solving problems is due in part to the fact retrieval difficulties of many of these children, which results in reliance on slower procedures for problem solving; e.g., it takes longer to count than to retrieve when trying to solve simple addition problems. Mathematical modeling can be used to break processing speed into component parts, such as speed of encoding numbers into working memory and speed of implicitly counting. Use of these techniques has revealed a more nuanced picture of the processing speed of children with MLD and their LA peers[13, 120, 121]. Studies sometimes suggest children with MLD are slower at implicit counting than their TA peers but sometimes there are no differences. A more consistent finding is that young children with MLD are slower at more basic processes, such as encoding numbers into working memory.

Use of Rapid Automatized Naming (RAN), where children are asked to name a series of well-learned letters or numbers as quickly as they can, is a better approach to the question of whether children with MLD and LA have a slower fundamental speed of encoding and processing information[122]. Because the processed information is very simple, the results are not confounded by different strategic approaches (e.g., counting vs. retrieval). Slower performance on RAN tasks is consistently related to lower reading achievement scores, potentially mediated by ease of encoding and representing language sounds in the phonological loop[123, 124]. In our studies, we have found that children with MLD start school with a much slower speed of number processing than their TA peers, with children in the LA and Low-IQ groups in between, but the gap closes rapidly. For some LA children, their slow processing may be related to the attentional control component of the central executive rather than to a more fundamental difference in processing speed per se. For children with MLD, in contrast, there may be a more fundamental difference in the mechanisms (e.g., white matter development) that support speed of information processing but any such difference appears to be more of a developmental delay than a persistent deficit. Brain imaging studies will be needed to determine if this is the case.

COGNITIVE INTERVENTIONS

Unfortunately, there are few scientifically validated treatment programs to address the mathematical cognition deficits of children with MLD and their LA peers. On the basis of the few high quality mathematics interventions for learning disabled students, broadly defined, the National Mathematics Advisory Panel determined that direct, teacher-guided explicit instruction on how to solve specific types of mathematics problems was the most effective intervention[20]. Effective interventions always involved multiple sessions extending over several weeks to six months and resulted in large improvements in students' ability to solve mathematical word problems, computational arithmetic problems, and novel word and arithmetic problems. Generally, however, many of these intervention effects to do not generalize, meaning that improvement of computational skills, for example, requires direct intervention on computational skills, not interventions for general problem solving or even other mathematical competencies. Generalization can occur, nonetheless, if the skill that was the target of the intervention is a component of a more complex mathematical problem.

Interventions designed to address the specific cognitive delays or deficits identified in **CAUSES** are currently being designed and evaluated[125, 126]. One intervention focuses on the frequency and accuracy with which children with MLD use the min counting procedure to solve addition problems and a corresponding procedure to solve subtraction problems[126]. The associated study included one-on-one tutoring across 48, 20 to 30 minute sessions. The tutoring included explicit instruction on how to use min counting, illustrated with a number line. For some of the children, the instruction was followed by deliberate practice; specifically, if the child could not answer a simple addition or subtraction problem correctly within one minute, they were instructed to use min counting to solve the problem. Other children were provided the same instruction, but read numerals afterwards instead of engaging in deliberate practice. The combination of explicit instruction and deliberate practice of min counting resulted in improved competence in solving simple addition and subtraction problems and more complex problems in which simple ones were embedded.

There are also interventions being developed to improve working memory, which would be particularly helpful for children with MLD[127, 128, 129, 130, 131]. A typical intervention involves asking children to engage in tasks that tax their working memory capacity, that is, tasks that require simultaneous processing and manipulation of information that is close to the maximum they can effectively handle. In one recent study, it was demonstrated that children who engaged in an intervention that matched task difficulty to their current working memory capacity, but not an easier intervention, showed large gains in the phonological and visuospatial components of working memory following about 20 training sessions (35 minutes each). Importantly, they retained these gains at a six month follow-up, and showed a modest gain on a mathematical reasoning test at follow-up. The source or sources of these gains are not fully understood, but may involve improved top-down attentional control through the central executive; this would improve the ability to keep verbal and visuospatial information in mind during problem solving.

Several of these interventions have also focused on the inhibitory control component of working memory. Unfortunately, the results for these interventions studies are mixed; recall the attentional control subcomponent of the central executive is involved in keeping several things in mind during problem solving and can be separated from the inhibitory control subcomponent of the central executive. At this point, these interventions and especially those that seem to improve attentional control hold considerable promise for addressing the deficits and delays of children with MLD and their LA peers. The next step is to combine working memory interventions with interventions that target critical mathematical competencies.

In final approach is the multi-tiered response to intervention (RtI)[132]. The first tier involves screening of all children for risk of MLD or LA. Students identified as at risk will then participate in general mathematics education (tier one) and small-group interventions (tier two) that target areas of risk. Students who do not improve with the tier two intervention then move to a more intensive, often one-on-one, tier three intervention (see reference 132 for details). It is not currently known how many MLD and LA children are “treatment resistant” with this approach, and or whether treatment resistance or the need for a tier three intervention is a good approach to diagnosing MLD.

SUMMARY AND IMPLICATIONS FOR CLINICAL PRACTICE AND FUTURE RESEARCH

There have been considerable advances in our understanding of the cognitive delays and deficits underlying the slow mathematics learning of the approximately 7% of children with MLD and the 10% of children with persistent low achievement in mathematics, despite average intelligence and reading ability[1, 10]. Although many of the underlying deficits may be the same, albeit to differing degrees, researchers in the field are moving toward distinguishing between MLD and LA, with a diagnostic cutoff for MLD at or below the 10th percentile on a mathematics achievement test for more than one grade and a range between the 11th and 25th percentiles, inclusive, for LA and again for more than one grade[8, 9]. It is important to include achievement across grades before making any such diagnosis, as many children with scores these ranges in one grade will be average in the next. These children, in turn, do not have the cognitive delays and deficits that have been identified in studies of MLD and LA[13].

A distinction between MLD and LA is important because the former group has extensive working memory deficits that are not typically found in the latter group – the one potential exception is poor inhibitory control – which has implications for remediation. Specifically, children with MLD will benefit from working memory interventions, as well as interventions that target the specific mathematical areas in which they are showing delays or deficits. LA children will also benefit from the latter, but most of them will not need the working memory intervention. The subset of LA children with intrusion errors during the process of fact retrieval may benefit from working memory interventions that target inhibitory control, once these interventions are fined tuned and consistently shown to be effective.

Because interventions are most effective when they target specific, well-defined areas of deficit, the cognitive science research on these children is critical. These studies have revealed several of their core deficits and delays. Children with MLD and, to a lesser extent, LA children show a deficit or delay in their processing of numbers, learning of arithmetic procedures, and in memorizing basic arithmetic facts. These learning difficulties are related in part to low-average intelligence (i.e., 90–95) and below average working memory capacity for children with MLD but this is not the whole story. These children also have number representation and processing deficits that do not appear to be related to intelligence or working memory, and although we know that working memory contributes to their procedural delays and may contribute to their retrieval deficit, these may not be the sole causal mechanism.

Neither intelligence nor broad working memory deficits are viable explanations of the poor mathematics achievement of LA children. These children largely appear to have a below average facility in dealing with numbers (e.g., adding ●●● + 2 = ?), use immature arithmetic procedures, and a subset of them have particular difficulty retrieving basic facts from long-term memory[8, 14, 26]. Whatever the underlying causes, the number processing and procedural difficulties appear to be more of a developmental delay (improves across grades) than a deficit (shows little grade-to-grade improvement), with LA children lagging one year behind their TA peers and children with MLD two to three years behind[117]. The difficulties remembering arithmetic facts are more persistent for children with MLD and for a subset of children with LA[62]. These deficits might be related to a poor ability to inhibit irrelevant information from intruding into working memory during the act of retrieval, although this is not likely to be the only source of fact-retrieval deficits.

One area in which there has been little or if any progress is with regard to the social and emotional functioning of children with MLD and their LA peers. Studies of children with RD suggest heighted risk for comorbid social and emotional problems, but otherwise we know little about these issues. A final task for coming decades is to more fully explore the sources of the comorbidity of MLD, RD and other disorders that affect learning. We know that comorbid disorders are common in these children, but we do not understand why this is the case.

The bottom line for the practicing pediatrician is to (a) routinely obtain mathematics (and reading) achievement scores for their patients and (b) refer children who score at or below the 25th national percentile in more than one grade for an educational assessment; (c) the assessment should include intelligence and working memory tests, as well as tests that assess specific mathematical skills (these are available in many standardized tests). Local schools should provide interventions that include the characteristics (e.g., explicit instruction) described in **COGNITIVE INTERVENTIONS.**

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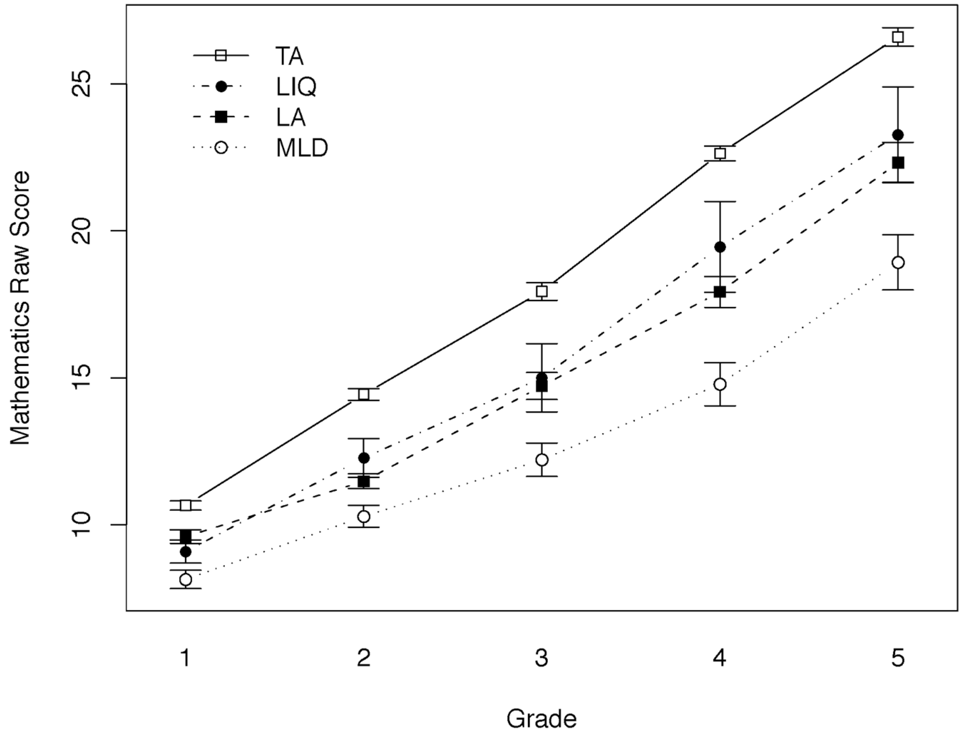

Figure 1.
Raw Mathematics Achievement scores from first to fifth grade, inclusive. Brackets are standard errors. LIQ = Low IQ, MLD = mathematical learning disability, LA= low achieving, TA = typically achieving.

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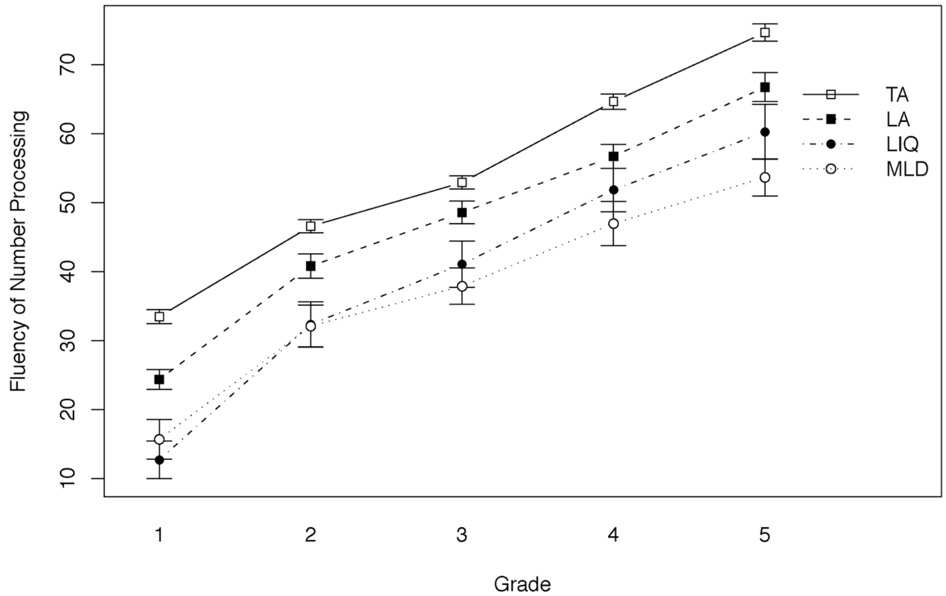

Figure 2.

Fluency scores for identifying and combining quantities associated with sets of objects (e.g., ▪▪▪) and Arabic numerals (e.g., ▪▪▪ + 2 = 5) from first to fifth grade, inclusive. Brackets are standard errors. LIQ = Low IQ, MLD = mathematical learning disability, LA= low achieving, TA = typically achieving.

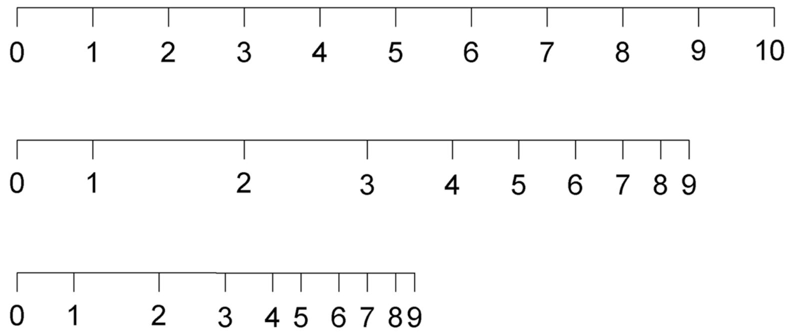

Figure 3.
Standard mathematical number line (top), compressed number line (center), and very compressed number line for children with MLD. The two latter lines depict the mental distance between quantities represented in the approximate magnitude representational system and the more the comprehension the more difficult it is to discriminate between larger quantities.