# Dynamics of Calcium Sparks and Calcium Leak in the Heart

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ABSTRACT We present what we believe to be a new mathematical model of Ca $^{2+}$  leak from the sarcoplasmic reticulum (SR) in the heart. To our knowledge, it is the first to incorporate a realistic number of Ca<sup>2+</sup>-release units, each containing a cluster of stochastically gating Ca<sup>2+</sup> channels (RyRs), whose biophysical properties (e.g., Ca<sup>2+</sup> sensitivity and allosteric interactions) are informed by the latest molecular investigations. This realistic model allows for the detailed characterization of RyR Ca<sup>2+</sup>-release properties, and shows how this balances reuptake by the SR Ca<sup>2+</sup> pump. Simulations reveal that SR Ca<sup>2+</sup> leak consists of brief but frequent single RyR openings (~3000 cell $^{-1}$  s $^{-1}$ ) that are likely to be experimentally undetectable, and are, therefore, "invisible". We also observe that these single RyR openings can recruit additional RyRs to open, due to elevated local (Ca $^{2+}$ ), and occasionally lead to the generation of Ca<sup>2+</sup> sparks (~130 cell<sup>-1</sup> s<sup>-1</sup>). Furthermore, this physiological formulation of "invisible" leak allows for the removal of the ad hoc, non-RyR mediated  $Ca<sup>2+</sup>$  leak terms present in prior models. Finally, our model shows how  $Ca<sup>2+</sup>$  sparks can be robustly triggered and terminated under both normal and pathological conditions. Together, these discoveries profoundly influence how we interpret and understand diverse experimental and clinical results from both normal and diseased hearts.

## INTRODUCTION

 $Ca^{2+}$ -induced  $Ca^{2+}$  release (CICR) forms the basis for the translation of electrical signals to physical contraction in cardiac myocytes during the process known as excitationcontraction coupling. In heart, L-type  $Ca^{2+}$  channel current is amplified by triggering  $Ca^{2+}$  release from the intracellular  $Ca^{2+}$  store (i.e., sarcoplasmic reticulum, SR) primarily via the ryanodine receptor, type 2 (RyR)  $Ca^{2+}$  channel. These  $Ca<sup>2+</sup>$ -activated RyRs are located on the SR membrane and largely arranged in paracrystalline arrays (10–300 RyRs) that are separated from the sarcolemmal membrane (SL) by the small (15-nm) dyadic subspace [\(1,2\)](#page-9-0).

During systole, RyRs are activated by  $Ca^{2+}$  influx via adjacent voltage-sensitive L-type  $Ca^{2+}$  channels (LCCs). Together, the LCC and RyR clusters form a functional unit of  $Ca^{2+}$  release known as the  $Ca^{2+}$ -release unit (CRU), which is essential to the local control of  $Ca^{2+}$  release during excitation-contraction coupling [\(3](#page-9-0)). The synchronized opening of clustered RyRs results in elevations of local (i.e., subspace)  $[Ca^{2+}]$  known as " $Ca^{2+}$  sparks". During diastole, in the absence of LCC  $Ca^{2+}$  influx, spontaneous  $Ca^{2+}$  sparks are rare but still easy to observe using a  $Ca^{2+}$ indicator where the  $Ca^{2+}$ -spark rate reflects the finite opening rate of the RyR channel [\(4](#page-9-0)).

Another form of  $Ca^{2+}$  release, "invisible" by standard confocal imaging methods, is the ''nonspark'' event, which

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involves the opening of a single RyR (e.g., " $Ca^{2+}$  quark") or a few RyRs that fail to trigger a full  $Ca^{2+}$  spark ([5\)](#page-9-0).

A third RyR-based  $Ca^{2+}$  release pathway is attributed to a small population of diffusely distributed RyRs termed ''rogue'' RyRs, which are located away from the junctional cleft ([2,6,7\)](#page-9-0). Here, we present a mathematical model that identifies and characterizes these three forms of visible and invisible diastolic RyR  $Ca^{2+}$  release, in a coherent manner. This SR Ca<sup>2+</sup> leak or loss of Ca<sup>2+</sup> from the SR is experimentally observed ([8,9\)](#page-9-0) but flawed in earlier mathematical models. In our fully stochastic model, the simulation now matches the biology and provides what we believe to be new insight into the mechanisms by which SR  $Ca^{2+}$  leak operates in intact cells. This model is fully informed by the latest molecular investigations of heart cells, heart nanoanatomy, and recent characterizations of channels, transporters, and buffers.

SR Ca<sup>2+</sup> leak is attributed to RyRs, Ca<sup>2+</sup>permeant channels whose open probability is controlled by  $[Ca^{2+}]$ ;  $[Ca^{2+}]_{\rm sr}$ , phosphorylation state, and other factors. In this manner, SR  $Ca^{2+}$  content is regulated by  $Ca^{2+}$  leak and the sarco/endoplasmic reticulum  $Ca^{2+}$ -ATPase (SERCA)  $Ca^{2+}$  pump.  $[Ca^{2+}]_{sr}$  is observed to change in response to diverse diseases (e.g., heart failure and arrhythmia) ([10,11](#page-9-0)) and phosphorylation by kinases such as protein kinase A or  $Ca^{2+}$ -calmodulin-dependent kinase II [\(12,13\)](#page-9-0). Additionally, RyR mutations such as those related to catecholaminergic polymorphic ventricular tachycardia can also underlie changes in RyR behavior and thus change SR  $Ca^{2+}$  content ([14\)](#page-9-0). These conditions are frequently found to be arrhythmogenic and contribute to  $Ca^{2+}$  waves,

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<span id="page-1-0"></span> $Ca^{2+}$  alternans, and other forms of cellular instability ([11\)](#page-9-0). The dynamics of SR  $Ca^{2+}$  leak are thus critical to our understanding of heart function during both physiological and pathophysiological conditions. Computational models offer an explicit means to investigate nanoscale events related  $Ca^{2+}$  leak not easily measured under experimental settings.

The physiological mathematical model of  $Ca^{2+}$  leak presented here provides fundamentally new, to our knowledge, and important findings that change our understanding of  $Ca^{2+}$  signaling at the nanoscopic and cell-wide level. The five major new findings of this model include the following:

- 1. We observe the presence of an "invisible"  $Ca^{2+}$  leak that is quantitatively consistent with earlier unexplained experimental findings ([15\)](#page-9-0).
- 2. We find that the fully stochastic activation and termination of RyR-based  $Ca^{2+}$  release within the ventricular myocyte allows us to properly account for SR  $Ca^{2+}$ leak, obviating the inclusion of an ad hoc, non-RyRmediated  $Ca^{2+}$  leak flux.
- 3.  $[Ca^{2+}]_{sr}$  levels are demonstrated to depend critically on RyR open probability  $(P_0)$  and vice versa. This reconciles modeling concepts and findings with new experimental results relating to pump/leak balance.
- 4. Single RyR openings, although brief, are frequent  $(\sim 3000 \text{ cell}^{-1} \text{ s}^{-1})$  and often fail to trigger a full Ca<sup>2+</sup> spark (~130 cell<sup>-1</sup> s<sup>-1</sup>), suggesting that  $Ca^{2+}$  spark fidelity in response to both single LCC and RyR openings is actually quite low.

5. Simulations predict that increased RyR activity can, somewhat paradoxically, lead to increased SR  $Ca^{2+}$ leak even in the presence of decreased  $[Ca^{2+}]_{sr}$ .

Taken together, this model accounts for how the molecular-level characteristics of individual RyRs influence the emergent property of cellular SR  $Ca^{2+}$  leak, and lays the foundation for fully spatially resolved modeling of  $Ca^{2+}$ signaling in single cells.

## **METHODS**

#### Model formulation

Here we present a whole-cell model for CICR in cardiac myocytes, which builds and expands upon the local SR  $Ca^{2+}$  release model previously intro-duced by Sobie et al. [\(16](#page-9-0)). Both the RyR cytosolic  $[Ca^{2+}]_i$  sensitivity and luminal SR  $[Ca^{2+}]([Ca^{2+}])_{sr}$  dependency have been updated to reproduce recently reported single channel gating behavior [\(17](#page-9-0)). The previous ad hoc formulation of cooperative gating has been replaced with an energetic coupling formulation [\(18\)](#page-9-0) derived from allosteric behavior observed in biological systems ([19,20](#page-9-0)). Our model (seeFig. 1) consists of 20,000 independent  $Ca<sup>2+</sup>$  release units (CRUs), each containing a cluster of stochastically gating RyRs that are instantaneously coupled via local subspace  $[Ca^{2+}]([Ca^{2+}]_{ds})$ . These CRUs are coupled via a bulk myoplasmic  $[Ca^{2+}]$ <sub>i</sub>, which also includes a small fraction (5%) of diffusely distributed, nonjunctional or ''rogue'' RyRs. SERCA pumps  $Ca^{2+}$  from the myoplasm back into the SR [\(21\)](#page-9-0). The SR itself is composed of junctional SR (JSR) and network SR (NSR) components, each with appropriate volumes and  $Ca^{2+}$  buffers. The sarcolemma (SL) has a background  $Ca^{2+}$  leak and two transsarcolemmal calcium transporters (the  $Na<sup>+</sup>-Ca<sup>2+</sup>$  exchanger, and the plasmalemmal  $Ca<sup>2+</sup>-ATPase$ ) to extrude  $Ca^{2+}$  from the cell.



FIGURE 1 Diagram of SR  $Ca^{2+}$  leak model and release site schematic. (A) Model compartments and  $Ca^{2+}$  fluxes (solid arrows). (B) Transitionstate diagram for the two-state Markov chain describing a single RyR. (C) Transition-state diagram for the Markov chain representing the RyR cluster where each state indicates the number of open RyRs  $(N_0)$  in the CRU (e.g., 0, 1, 2, ...48, 49).

Although we also envision a cluster of L-type  $Ca^{2+}$  channels (LCCs) apposing the RyR cluster that trigger the CRUs during electrical activity, the LCCs have been disabled for this simulation that focuses on diastolic  $Ca<sup>2+</sup>$  signaling when LCCs are quiescent. We are assuming minimal or no mitochondrial Ca<sup>2+</sup> buffering ([22\)](#page-9-0), therefore potential contributions of this organelle to  $Ca^{2+}$  handling are omitted. The mathematical description of key elements is presented below. The seeming novelty of this model lies in the details of its formulation and the inclusion of the known properties and functions of each model component that had been overlooked, ignored, or strategically excluded in prior models. We have consequently identified several major findings—believed by us to be entirely new—as described above and presented below.

#### RyR model

In this study, the RyR signaling that underlies cardiac EC coupling is studied by observations of  $Ca^{2+}$  sparks in resting cells in which LCCs are quiescent. The dyad, or dyadic subspace, is a restricted space located between the transverse tubule and JSR membranes that contains a cluster of RyRs that gate stochastically, display coupled gating ([23\)](#page-9-0), and have a  $P_0$  that depends both cytosolic and luminal  $[Ca^{2+}](17)$  $[Ca^{2+}](17)$  $[Ca^{2+}](17)$ . Here, each  $Ca<sup>2+</sup>$ -activated RyR is represented by a two-state Markov chain containing one closed (C) and one open (O) state. Transitions from C to O are dependent on both  $[Ca^{2+}]_{ds}$  and junctional SR  $[Ca^{2+}]([Ca^{2+}])_{jsr}$  (see [Fig. 1](#page-1-0) B), with  $k^+$  as the association rate constant for  $\left[Ca^{2+}\right]_{ds}$  binding with units of  $\mu$ M<sup>-7</sup> s<sup>-1</sup>, and k<sup>-</sup> as the corresponding dissociation rate constant in s<sup>-1</sup>. A luminal regulation function ( $\phi = \phi_m [Ca^{2+}]_{sr} + \phi_b$ ) serves to modify the channel opening rate where  $[Ca^{2+}]_{sr}$  is the luminal  $[Ca^{2+}]$  associated with each RyR (which is  $[Ca^{2+}]_{\text{isr}} [Ca^{2+}]_{\text{sr}}$  and  $[Ca^{2+}]_{\text{nsr}}$  for junctional and nonjunctional RyRs, respectively).

Combining 49 of these identical two-state RyRs into a cluster and assuming they are instantaneously coupled via  $\left[Ca^{2+}\right]_{ds}$  yields a  $M = 50$ state release site where each state indicates the number of open RyRs  $(N<sub>o</sub>)$  for the CRU ( $0 \le N<sub>o</sub> \le 49$ ), as shown in [Fig. 1](#page-1-0) C where terms on the arrows are transition rates. In these rate terms,  $\chi_{oc}$  and  $\chi_{co}$  represent mean-field allosteric coupling factors ([18\)](#page-9-0) given by

$$
\chi_{\rm oc} = \exp\{-a_*0.5[N_{\rm c}\varepsilon_{\rm cc} - (N_{\rm o}-1)\varepsilon_{\rm oo}]\},\qquad(1)
$$

$$
\chi_{\rm co} = \exp\{-a_*0.5[N_0\varepsilon_{\rm oo} - (N_{\rm c}-1)\varepsilon_{\rm cc}]\},\qquad(2)
$$

where  $a_*$  represents the average allosteric connectivity (based on a  $7 \times 7$ grid with nearest-neighbor coupling), and  $\varepsilon_{cc}$  is a dimensionless free energy of interaction (units of  $k_BT$ ) that specifies the change in free energy experienced by a channel in state C when allosterically coupled to another channel in state C, and similarly for  $\varepsilon_{oo}$ . The coefficients  $N_c$  (number of closed RyRs) and  $N_0$  serve to partition allosteric coupling between the forward and reverse transitions. In addition to dyadic RyRs, a small fraction (5%) of RyRs are assumed to be found away from the junction as diffusely distributed, single channels.

#### SERCA formulation

SERCA consumes ATP to pump  $Ca^{2+}$  into the SR from the myoplasm. Here, we have implemented a SERCA formulation recently developed by Tran et al. [\(21](#page-9-0)). The SERCA flux takes the form,  $J_{\text{serca}} = 2v_{\text{cycle}}A_p$ , where  $v_{\text{cycle}}$ is the cycling rate per pump molecule (see the [Supporting Material\)](#page-9-0) and  $A_n$  is the concentration of SERCA molecules per liter cytosol. The Tran-Crampin SERCA formulation was selected because it provides a realistic representation of SERCA-dependent  $Ca^{2+}$  flux that displays physiological behavior at low  $[Ca^{2+}]$ <sub>i</sub> (see [Fig. S6](#page-9-0) A in the [Supporting Material\)](#page-9-0) and is sensitive to changes in  $[Ca^{2+}]_{sr}$  (see [Fig. S6](#page-9-0) B), which is essential when allowing RyR-based leak to balance SERCA. Whole-cell  $Ca^{2+}$  dynamics obtained using this SERCA formulation is provided in the [Supporting Material](#page-9-0).

#### Concentration balance equations

The Markov chain Monte Carlo model used here consists of  $2N + 2$  ( $N =$ 20,000) ordinary differential equations representing the time-evolution of  $(Ca^{2+})$  in the bulk myoplasm  $([Ca^{2+}]_i)$ , the NSR  $([Ca^{2+}]_{nsr})$ , the JSR  $([Ca<sup>2+</sup>]_{jsr})$ , and the dyadic subspace  $([Ca<sup>2+</sup>]_{ds})$  compartments, and N Markov chains representing the stochastic RyR clusters (see [Fig. 1](#page-1-0) A). The concentration balance equations and formulation for all fluxes can be found in the [Supporting Material.](#page-9-0) The flux through the RyR cluster associated with the  $i^{\text{th}}$  CRU is given by

$$
J_{\text{ryr}}^i = N_0^i \nu_{\text{ryr}} \bigg( \big[ \text{Ca}^{2+} \big]_{\text{jsr}}^i - \big[ \text{Ca}^{2+} \big]_{\text{ds}}^i \bigg), \tag{3}
$$

where  $1 \le i \le N$ ,  $v_{\text{ryr}}$  is the RyR Ca<sup>2+</sup> release rate in s<sup>-1</sup>, and  $N_{o}^{i}$  is the number of open RyR channels at the  $i<sup>th</sup>$  release site. Similar to previous work [\(24,25](#page-9-0)), model parameters lead to rapid equilibrium of  $\lbrack Ca^{2+} \rbrack_{ds}$ with the  $[Ca^{2+}]$ <sub>i</sub> and  $[Ca^{2+}]_{is}$  allowing  $[Ca^{2+}]_{ds}$  to be approximated using an algebraic expression of  $[Ca^{2+}]_{i}$ ,  $[Ca^{2+}]_{jsr}$ , and  $N_0$  (see [Eq. S23](#page-9-0) in the [Supporting Material](#page-9-0)).

#### RESULTS

 $Ca<sup>2+</sup>$  signaling in heart cells involves interactions among many proteins in a spatially and temporally complex environment as suggested in [Fig. 1](#page-1-0) A. Teasing apart the causal links in this cellular environment is aided enormously by mathematical models informed by experimental results. Here, we present results from a whole-cell model of SR  $Ca^{2+}$  signaling that seeks to investigate local  $Ca^{2+}$  dynamics using fresh modeling elements constrained and informed by the recent experimental findings described below.

# RyR  $Ca^{2+}$  sensitivity and allosteric coupling

SR  $Ca^{2+}$  release in cardiac myocytes is a multiscale process ranging from small invisible  $Ca^{2+}$  leak events due to brief openings of a single RyR, to microscopic  $Ca^{2+}$  sparks involving numerous RyRs at single a CRU, to cell-wide  $[Ca^{2+}]$ <sub>i</sub> transients involving release from numerous CRUs within the cell. This process depends critically on the sensitivity of the RyRs to local cytosolic and luminal  $[Ca^{2+}]$  as well as interactions between neighboring channels (i.e., allosteric coupling). It has been shown that  $Ca^{2+}$  sparks result from synchronized openings of clustered RyRs ([4\)](#page-9-0), mediated by  $[Ca^{2+}]_{ds}$  with a strong dependence on  $[Ca^{2+}]_{sr}$ ([26\)](#page-9-0). Clustered RyRs undergo probabilistic interactions here referred to as ''allosteric coupling'' but first described more rigidly as ''coupled gating'' ([23\)](#page-9-0). In our sticky-cluster model ([16\)](#page-9-0), we attempted to capture this known property of RyR gating by including an ad hoc formulation for cooperativity among the RyRs and demonstrated that this feature enabled robust  $Ca^{2+}$  spark termination. Here, we have implemented an energetic coupling formulation [\(18](#page-9-0)) based on models of protein-protein interactions ([19\)](#page-9-0).

[Fig. 2](#page-3-0) shows how the sensitivity of single RyR channels to be triggered by  $Ca^{2+}$  depends on the local  $[Ca^{2+}]$ <sub>i</sub> ([Fig. 2](#page-3-0) A), local  $\left[\text{Ca}^{2+}\right]_{\text{sr}}$  ([Fig. 2](#page-3-0) B), and also on allosteric coupling

<span id="page-3-0"></span>

FIGURE 2 Single channel RyR Ca<sup>2+</sup> sensitivity and influence of allosteric coupling on RyR steady-state open probability  $(P_0)$ . (A) Single channel RyR  $P_0$  as a function of cytosolic  $[Ca^{2+}]$  ( $[Ca^{2+}]_{\text{isr}} = 1 \text{ mM}$ ). (Solid line) Model result in which data points (solid circles) indicate best fit; normalized  $P_0$  ( $K_d = 12.2 \mu M$ ,  $\eta = 2.6$ ) from [Fig. 1](#page-1-0) B in Qin et al. [\(27](#page-9-0)). (B) Single RyR  $P_0$  as a function of luminal  $[Ca^{2+}]$  ( $[Ca^{2+}]_{ds}$  = 8  $\mu$ M). (Solid line) Model result in which data points (solid squares) indi-cate experimental results from Fig. 3 B in Qin et al. ([27\)](#page-9-0). (C)  $P_0$  for a cluster of 49 RyRs as a function of  $[Ca^{2+}]$ <sub>i</sub> with varying levels of average RyR allosteric connectivity ( $a$ <sup>\*</sup>). As  $a$ <sup>\*</sup> increases,  $P_0$  exhibits steeper dependence  $[Ca^{2+}$ ]<sub>i</sub>. (D) Zoom of Fig. 2 C showing lower  $[Ca^{2+}$ ]<sub>i</sub> levels. For reference, markers on x axis indicate the diastolic  $[Ca^{2+}]_i$  and peak  $[Ca^{2+}]_{ds}$  resulting from a single open RyR (blue triangle and red triangle, respectively).

between the individual RyRs within the CRUs (Fig. 2, C and D). When plotted as a function of  $[Ca^{2+}]_i$  (Fig. 2 A), the steady-state single RyR  $P_0$  is an increasing function of  $[Ca^{2+}]$ <sub>i</sub> with a half-maximal point  $(K<sub>m</sub>)$  of ~12  $\mu$ M when  $[Ca^{2+}]_{sr}$  is held constant at 1 mM (solid line). This result corresponds well with results from lipid bilayer measurements (solid circles) that display a  $K<sub>m</sub>$  of 12.2  $\mu$ M and a Hill coefficient of 2.6 ([27\)](#page-9-0). Fig. 2 B shows how the RyR  $P_{\rm o}$  increases as a function of increasing  $\left[{\rm Ca}^{2+}\right]_{\rm sr}$  when  $[Ca^{2+}]$ <sub>i</sub> is held constant at 8  $\mu$ M, which also agrees well with a corresponding experimental observation (solid squares) from Qin et al. [\(17](#page-9-0)).

Fig. 2 C displays the steady-state  $P_0$  for a cluster of RyRs versus  $[Ca^{2+}]$ <sub>i</sub> with varying levels of average RyR allosteric connectivity  $(a_*)$  and Fig. 2 D shows the same behavior for low  $\lceil Ca^{2+} \rceil$  levels. Note that decreasing  $a_*$  is equivalent to removing allosteric interactions between some RyRs. In the absence of coupling (i.e.,  $a_* = 0$ ), RyRs are coupled only by  $[Ca^{2+}]_{ds}$  and exhibit higher  $P_o$  at low local  $[Ca^{2+}]$ <sub>i</sub>, similar to isolated RyR channels. However, with the allosteric coupling levels used in this study (i.e.,  $a_* =$ 0.07; blue line), clustered RyRs display decreased  $P_0$  at low cytosolic  $[Ca^{2+}]$ <sub>i</sub> due to stabilization of closed channels by the allosteric interactions. Because closed channels have a stabilizing influence on their neighbors, the RyR cluster is more likely to remain closed at low  $[Ca^{2+}]$ <sub>i</sub> and transition rapidly from all-closed to all-open at elevated  $[Ca^{2+}]_i$ . Note, we also observed that  $N_{\text{RyR}}$  has a modest impact on CRU  $P<sub>o</sub>$  (see [Fig. S2\)](#page-9-0).

# Local  $Ca^{2+}$  dynamics

In our previous model [\(16](#page-9-0)), we showed that SR  $Ca^{2+}$  release events via a cluster of cooperatively gating RyRs with both cytosolic and luminal Ca<sup>2+</sup> sensitivity provided robust Ca<sup>2+</sup> sparks with characteristics comparable to those observed experimentally including frequency, duration, termination, and restitution ([4\)](#page-9-0). However, this single CRU model did not attempt to characterize the leak of  $Ca^{2+}$  out of the SR nor was the dynamic  $Ca^{2+}$  sensitivity examined. The formulation of RyR gating presented here as new and the inclusion of a thermodynamically constrained SERCA formulation allows significant insights into the dynamics of SR  $Ca^{2+}$ leak shown below.

Fig. 3 and [Fig. S3](#page-9-0) show diastolic  $Ca^{2+}$  sparks and nonspark  $Ca^{2+}$ -release events as measured by  $[Ca^{2+}]_{ds}$ (Fig. 3 A and [Fig. S3](#page-9-0) A), stochastic RyR gating as measured by  $N_0$  (Fig. 3 B and [Fig. S3](#page-9-0) B), and Ca<sup>2+</sup> blinks as measured by  $[Ca^{2+}]_{\text{isr}}$  (Fig. 3 C and [Fig. S3](#page-9-0) C). In Fig. 3, all 20,000 CRUs were simulated and the traces shown depict release activity recorded in a subpopulation of randomly selected CRUs (i.e., 10% of total) over a period of 200 ms. In this subpopulation, out of the 58 CRUs that exhibited release activity, only five produced  $Ca^{2+}$  sparks during that period of time. This is representative of the whole-cell behavior,



FIGURE 3 Local Ca<sup>2+</sup> dynamics. (A) Subspace  $[Ca^{2+}]([Ca^{2+}]_{ds})$  associated with each RyR cluster during spontaneous, diastolic  $Ca^{2+}$  release events. (B)  $N_0$  at each CRU. (C) Junctional SR  $[Ca^{2+}]$  ( $[Ca^{2+}]_{\text{isr}}$ ) present on the luminal side of RyR cluster. (Each colored line represents a different CRU where a  $Ca^{2+}$  spark is observed and nonspark RyR activity is shown in shading.) Note: For visual clarity, only 10% of total CRU activity is displayed here.

<span id="page-4-0"></span>which resulted in 132  $Ca^{2+}$  sparks cell<sup>-1</sup> s<sup>-1</sup>. The five CRUs displaying  $Ca^{2+}$  sparks did not interact directly and were coupled only via  $[Ca^{2+}]_i$ . [Fig. 3](#page-3-0) shows not only the  $[Ca^{2+}]_{ds}$  changes due to  $Ca^{2+}$  sparks (*colored lines*) but also the changes in  $[Ca^{2+}]_{ds}$  due to the opening of individual RyRs within CRUs that did not lead to  $Ca^{2+}$  sparks (gray *lines*). In the latter case, CRUs that do not display  $Ca^{2+}$ sparks are still capable of SR  $Ca^{2+}$  leak through uncoordinated openings of single or small groups of RyRs that would likely be invisible experimentally. We also tested a wide range of CRUs sizes (16  $\leq N_{RvR} \leq 100$ ) and found robust  $Ca^{2+}$ -spark triggering and termination (see [Fig. S3](#page-9-0) and [Fig. S4\)](#page-9-0).

## Dynamics of  $Ca^{2+}$ -spark triggering and termination

The model demonstrates that diastolic  $Ca^{2+}$  sparks arise from elevations in  $[Ca^{2+}]_{ds}$  caused by brief but frequent openings of single RyRs. The stochastic nature of RyR gating also allows for prolonged openings of single RyRs that increase the probability of a  $Ca^{2+}$  spark due to sustained periods of elevated  $\left[\text{Ca}^{2+}\right]_{\text{ds}}$ . Fig. 4 A shows the details of the RyR currents ( $I_{RvR}$ ) that underlie the Ca<sup>2+</sup> spark dynamics in [Fig. 3,](#page-3-0) and Fig. 4 B provides a closer view of these  $I_{RvR}$  events. The current associated with a  $Ca^{2+}$  spark  $(I<sub>spark</sub>)$  peaks at ~10 pA (see Fig. 4 A) whereas the current associated with an individual RyR  $(I<sub>quark</sub>)$  peaks at 0.2 pA (see Fig.  $4B$ ). These values are consistent with experimental results ([28\)](#page-9-0). Examination of the upstroke of  $I_{\text{spark}}$  reveals a stepwise increase in  $I_{RvR}$  that precedes the regenerative CICR rise to the peak of the  $I_{\text{spark}}$ .  $Ca^{2+}$  sparks arise from frequent I<sub>quark</sub> events within each CRU. However, there are many  $I_{\text{quark}}$  records that do not initiate an  $I_{\text{snark}}$  and some that even trigger one or more RyR to open without triggering a  $Ca^{2+}$  spark. This recruitment pattern is clearly consistent with the stochastic nature of local CICR and with the inference that RyR clusters are relatively difficult to trigger [\(29,30\)](#page-9-0).

During I<sub>spark</sub>,  $[Ca^{2+}]_{\text{isr}}$  declines because  $Ca^{2+}$  moves from the JSR to the dyadic subspace via the open RyRs, causing  $I_{RvR}$  to decrease. This can be readily appreciated by examining the fluctuations in  $I_{RyR}$  produced during the initial phase of a  $Ca^{2+}$  spark as compared to fluctuations visible near the end of each  $Ca^{2+}$  spark (see Fig. 4 B). Consequently, fluctuations in the release current due to the gating of individual RyRs becomes increasingly small during the final phase of I<sub>spark</sub>. Eventually  $\left[Ca^{2+}\right]_{\text{isr}}$  (see [Fig. 3](#page-3-0) C) is depleted to a point where the  $Ca^{2+}$  spark terminates stochastically facilitated by allosteric coupling and reduced RyR reopenings due to lower  $\left[Ca^{2+}\right]_{ds}$  and  $\left[Ca^{2+}\right]_{isr}$ .

Although this result is consistent with the termination mechanism proposed in Sobie et al. [\(16](#page-9-0)), many of the key elements appear novel. For example, the clear character of  $I<sub>quark</sub>$  and how it initiates  $I<sub>spark</sub>$  reveals that triggering of



FIGURE 4 Dynamics of diastolic  $Ca^{2+}$  spark triggering and termination. (A) Spontaneous RyR Ca<sup>2+</sup> current in pA during a 1-s simulation. (B) Zoom of RyR current showing spark initiation and termination profiles. (Colored lines) Different CRUs, respectively, where a  $Ca^{2+}$  spark is observed. (Shading) Nonspark RyR activity for the remaining CRUs. Note: For visual clarity, only 10% of the whole cell activity is displayed here. (C) Number of  $Ca<sup>2+</sup>$  sparks as function of the strength of allosteric coupling (coupling strength is assumed to be symmetrical, i.e.,  $\varepsilon_* = \varepsilon_{cc} = \varepsilon_{oo}$ ). (D) Average  $Ca^{2+}$  spark duration versus  $\varepsilon_*$ .

a  $Ca^{2+}$  spark is not an all-or-none process as we had previously hypothesized [\(16](#page-9-0)). Significantly, Fig. 4, A and B, shows that the overwhelming majority of single RyR gating activity does not lead to  $Ca^{2+}$  sparks. The duration of these nonspark, single-channel events are consistent with the biophysical properties of the RyR (e.g., its inherent closing rate) that display mean open times in the range of 1–4 ms in lipid bilayer experiments [\(31](#page-9-0)).  $Ca^{2+}$  released via this mechanism is likely to go unaccounted for experimentally and lends itself to the concept of invisible leak. A closer examination of this nonspark leak mechanism is presented below.

The effect of allosteric coupling strength  $(\varepsilon*)$  on frequency and duration of  $Ca^{2+}$  sparks is examined in Fig. 4, C and D. Note that decreasing  $\varepsilon_*$  is analogous to increasing the amount by which RyR interactions stabilize both closed and open channel pairs.  $Ca^{2+}$ -spark rate rises exponentially and  $Ca^{2+}$ -spark duration decreases as coupling strength is lowered (i.e., increased  $\varepsilon$ \*). This demonstrates that RyRs exhibit elevated activity in the presence of weak allosteric coupling—caused by the increased <span id="page-5-0"></span>opening rate of uncoupled channels (see [Fig. 2](#page-3-0) C). Although the Ca<sup>2+</sup>-spark rate is very sensitive to  $\varepsilon$ <sup>\*</sup>, the average spark duration is much less sensitive to  $\varepsilon$ \*, suggesting that RyR luminal  $[Ca^{2+}]$  dependency is playing the dominant role for  $Ca^{2+}$ -spark duration and termination. Note, simulations indicate that complete removal of allosteric coupling yields  $Ca^{2+}$ -spark rates much higher than those observed in FK506-based uncoupling experiments ([32\)](#page-9-0). This supports the idea that RyR-RyR coupling may involve factors in addition to FKBP12.6.

# Visibility of SR  $Ca^{2+}$  leak

Although SR  $Ca^{2+}$ -release events originating from a fully activated cluster of RyRs are easily detectable experimen-tally ([4\)](#page-9-0),  $Ca^{2+}$  release by an individual or even a few RyRs is well below the experimental detection threshold ([16\)](#page-9-0).  $I_{quark}$  is comparable to unitary LCC current at far less than that of a  $Ca^{2+}$  sparklet ([33\)](#page-9-0)—which is detect- $-30$  mV (i.e., 0.2 pA). This local Ca<sup>2+</sup>-signaling mass is able because of its long mean open time  $(\geq 10 \text{ ms due to})$ channel agonists) and high external  $(Ca^{2+})$  (20 mM) or special optical conditions (e.g., the 100-nm-thin optical section of total internal reflection microscopy) or both. In Fig. 5, the visibility of  $Ca^{2+}$  release due to I<sub>spark</sub> and I<sub>quark</sub> is assessed in the presence of realistic noise (see Sobie et al. ([16\)](#page-9-0)). In Fig. 5 A, a simulated linescan image of the SR Ca<sup>2+</sup> release activity shown in Fig. 5 B (black line, right *axis*) is given. Although the signal due to the  $I_{\text{spark}}$  event is clearly visible, the signal emitted by the two  $I_{\text{quark}}$  events (generated by one and four open RyRs) that precede the  $Ca^{2+}$  spark cannot be discerned. The fluorescent profile



FIGURE 5 Experimental detection of SR Ca<sup>2+</sup> release. (A) Simulated linescan of  $I_{RyR}$  activity. (B) Time course of  $I_{RyR}$  exhibiting both spark  $(I<sub>snark</sub>)$  and nonspark  $(I<sub>onark</sub>)$  events (*black line*) and the corresponding fluorescence profile ( $F/F<sub>0</sub>$ ) of Fluo3 (blue line). The  $F/F<sub>0</sub>$  profile was obtained by averaging fluorescence from a 1- $\mu$ m-wide region (blue box). Simulated linescan based on previously published methods (see Sobie et al. [\(16\)](#page-9-0) and Smith et al. ([40\)](#page-9-0)).

(Fig. 5 B (blue line, left axis)) suggests that such SR  $Ca^{2+}$ release events that fail to induce a full  $Ca^{2+}$  spark are likely to go undetected. We also generated a simulated  $Ca^{2+}$  spark/ blink pair (see [Fig. S5\)](#page-9-0) for comparison with recent experimental measurements ([34\)](#page-9-0). Below, we examine the propensity of these nonspark occurrences during diastole.

## $Ca<sup>2+</sup>$ -spark latency and fidelity

The cardiac SR Ca<sup>2+</sup>-leak pathway includes both Ca<sup>2+</sup> sparks and nonspark  $SR Ca<sup>2+</sup>$ -release events. The nonspark events occur when RyR channels open but do not trigger enough RyRs to produce a  $Ca^{2+}$  spark. Statistical analysis of all spontaneous  $Ca^{2+}$  release events provides information on SR Ca<sup>2+</sup>-leak dynamics such as  $Ca^{2+}$ -spark latency (the time from spark initiation to spark peak) and fidelity (the probability of individual RyR openings triggering a full  $Ca^{2+}$  spark). During diastolic simulations (see [Figs. 3](#page-3-0)) [and 4](#page-3-0)) the time required to reach the peak of a  $Ca^{2+}$  spark varies from less than a millisecond to several milliseconds and decreases as a function of  $N<sub>o</sub>$  (Fig. 6 A). The fidelity of  $Ca^{2+}$ -spark triggering has been anticipated by the examples of  $I<sub>quark</sub>$  and  $I<sub>spark</sub>$  shown in [Fig. 4](#page-4-0), A and B. Single RyR openings occur at a rate of  $\sim 3000$  cell<sup>-1</sup> s<sup>-1</sup>; however, most fail to trigger a  $Ca^{2+}$  spark.

Fig. 6 B shows a histogram of the number of  $Ca^{2+}$ -release events versus the  $N_0$  involved in the event. Some events serve as the base for triggering additional RyRs to open (solid bars) whereas other events peak at  $N_0$  and then



FIGURE 6 Analysis of  $Ca^{2+}$  spark latency and fidelity. (A) Average mean time to spark peak as a function of the  $N<sub>o</sub>$ . (B) Number of Ca<sup>2+</sup> release events versus the  $N<sub>o</sub>$  where events are segregated between events that lead to additional RyR openings (solid bars) and events that do not (shaded bars). Note that the bar above  $N_0 = 10$  also corresponds to the number of  $Ca^{2+}$ sparks during the simulation.  $(C)$  Probability of triggering a spark as a function of peak  $N<sub>o</sub>$ . (D) Histogram of the probability of a single RyR opening triggering a spark as a function of  $[Ca^{2+}]_{sr}$ .

<span id="page-6-0"></span>terminate (shaded bars). Note that most (85%) of the 20,000 CRUs have no RyRs that open at all during the 1-s simulation. There are between 100 and 150 diastolic  $Ca^{2+}$  sparks cell<sup>-1</sup> s<sup>-1</sup> in rat ventricular myocytes ([4\)](#page-9-0) and this is faithfully reproduced by the model. Of the ~3000 RyR opening events, only ~130 lead to  $Ca^{2+}$  sparks, indicating that the fidelity of an  $I<sub>quark</sub>$  triggering an  $I<sub>spark</sub>$  is low. This is further illustrated in [Fig. 6](#page-5-0) C, which shows the likelihood of a  $Ca^{2+}$ spark increases with  $N_o$ , but is quite low (~5%) when  $N_o = 1$ . This indicates the probability of a single or a few  $N_0 = 1$ . This indicates the probability of a single or a few open channels to trigger a Ca<sup>2+</sup> spark is small under normal conditions but increases rapidly as  $[Ca^{2+}]_{sr}$  is increased (see [Fig. 6](#page-5-0) D). In summary, Fig. 6 shows that  $Ca^{2+}$  sparks are a dynamic balance between the regenerative nature of CICR and the stabilizing influence of allosteric coupling on the RyR channels.

### SR Ca<sup>2+</sup> leak in permeabilized cells

SR  $Ca^{2+}$  content is known to influence the rate of diastolic loss of  $Ca^{2+}$  from the SR [\(9](#page-9-0)). To examine SR  $Ca^{2+}$  leak and its dependence on  $Ca^{2+}$  sparks and I<sub>RyR</sub>, we mimicked the inhibition of SERCA by thapsigargin (TG) and observed the loss of  $Ca^{2+}$  from the SR under permeabilized conditions (similar to experiments by Zima et al. ([8\)](#page-9-0)). Fig. 7 A shows the temporal decline in  $[\text{Ca}^{2+}]_{\text{sr}}$  and  $\text{Ca}^{2+}$ -spark rate after SERCA inhibition. The high cytosolic  $[Ca^{2+}]_i$  (150 nM) used in these permeabilized cell experiments results in an elevated Ca<sup>2+</sup> spark rate (~1000 sparks cell<sup>-1</sup> s<sup>-1</sup>)—six times the  $Ca^{2+}$  spark rate seen in rat myocytes during diastole. In Fig. 7 A  $Ca^{2+}$  sparks cease after  $[Ca^{2+}]_{sr}$  falls below 700  $\mu$ M. However, even in the absence of Ca<sup>2+</sup> sparks, Ca<sup>2+</sup> continues to leak from the SR. The model suggests that this is due to the background  $I_{RvR}$  that is experimentally "invisible" (see [Fig. 5\)](#page-5-0), and the SR loses half of its  $Ca^{2+}$  within 5 min of SERCA inhibition.



FIGURE 7 Effect of SERCA inhibition on SR  $[Ca^{2+}]$ . (A) Simulated effect of SERCA inhibition by thapsigargin (TG,  $A_p = 0$ ) on  $\left[Ca^{2+}\right]_{nsr}$ (black line) and whole-cell  $Ca^{2+}$  spark rate (red circles). (B) Simulated effect of a RyR-specific antagonist, ruthenium red (RuR) on  $\left[Ca^{2+}\right]_{nsr}$ (black line) and whole-cell  $Ca^{2+}$  spark rate (red circles), before and after SERCA inhibition by TG. In each case,  $[Ca^{2+}]$ <sub>i</sub> was fixed at 150 nM to mimic experimental conditions in permeabilized cells (see Zima et al. [\(8](#page-9-0))). RuR is modeled here by disabling all but 2% of the RyRs in the model cell and giving the functional RyRs the kinetics of nonjunctional, rogue RyRs ( $v_{\text{ryr}} = 0$ ,  $v_{\text{ryr,nj}} = 1.01 \text{ s}^{-1}$ ).

To further investigate invisible leak, 98% of the RyRs in the cell were disabled by ruthenium red (RuR)—completely preventing the generation of  $Ca^{2+}$  sparks (Fig. 7 B). During the brief period when most RyRs were blocked but SERCA remained active,  $\left[\text{Ca}^{2+}\right]_{\text{sr}}$  increases to nearly twice its initial value due to the shift in pump/leak balance. However, after SERCA inhibition,  $Ca^{2+}$  still leaks out of the SR via the nonspark pathway even as  $[Ca^{2+}]_{sr}$  declines below 1 mM, albeit at a much slower rate than in Fig. 7 A. These simulations suggest that the unidentified leak pathway highlighted by Zima et al. [\(8](#page-9-0)) may, in fact, be incomplete inhibition of the cell's nearly 1,000,000 RyRs.

# Dependence of SR Ca<sup>2+</sup> leak on SR Ca<sup>2+</sup> content

The opening of RyRs depends on both  $[Ca^{2+}]$ <sub>i</sub> and  $[Ca^{2+}]$ <sub>sr</sub> and makes experimental analysis of SR  $Ca^{2+}$  leak complicated. However, the model allows us to directly examine how SR Ca<sup>2+</sup> leak varies based on changes in  $[Ca^{2+}]_{sr}$ . Fig. 8 shows the characteristics of SR  $Ca^{2+}$  leak in our whole-cell SR Ca<sup>2+</sup> release model and its Ca<sup>2+</sup>-spark and -nonspark  $Ca^{2+}$  leak-pathways via junctional and nonjunctional rogue RyRs.



FIGURE 8 Effect of SR Ca<sup>2+</sup> load ( $\left[Ca^{2+}\right]_{\text{sr}}$ ) on SR Ca<sup>2+</sup> leak. (A) Total integrated RyR flux during a 1-s simulation.  $(B)$  Integrated nonspark RyR flux via junctional RyRs (solid line, solid circles) and nonjunctional RyRs (solid line, open circles). (C) Fraction of RyR flux not associated with a  $Ca^{2+}$  spark (e.g., nonspark RyR flux divided by total RyR flux). (D) Average Ca<sup>2+</sup> spark duration. (E) Ca<sup>2+</sup> spark rate. (F) Number of nonspark events versus  $[Ca^{2+}]_{sr}$ . The junctional nonspark flux is defined as RyR activity that does not precede a  $Ca^{2+}$  spark. (Data points) Average over 10 simulations. (*Error bars*) Standard deviation from the mean. Note that  $[Ca^{2+}]$ <sub>i</sub> was held constant at 90 nM.

A pivotal role of  $[Ca^{2+}]_{sr}$  is governing the overall RyR-mediated SR Ca<sup>2+</sup> efflux as shown in [Fig. 8](#page-6-0)A. The low overall  $Ca^{2+}$  leak seen at low  $[Ca^{2+}]_{sr}$  (  $\leq 800 \,\mu\text{M}$ ) is due not only to decreased RyR opening rates but also decreased  $Ca^{2+}$ -spark triggering fidelity caused by reduced  $I_{RvR}$ . The contribution of nonspark  $Ca^{2+}$  leak to this process is shown in [Fig. 8](#page-6-0) B. For  $[Ca<sup>2+</sup>]$ <sub>sr</sub> levels used here (i.e., rodent),  $Ca<sup>2+</sup>$  sparks dominate SR Ca<sup>2+</sup> leak whereas at lower  $[Ca^{2+}]_{sr}$  levels (i.e., larger mammals) SR  $Ca^{2+}$  leak is primarily via the invisible, nonspark pathway. Not surprisingly, the frequency of both  $Ca^{2+}$  sparks and nonspark  $Ca^{2+}$ -release events are increasing functions of  $\left[Ca^{2+}\right]_{sr}$  ([Fig. 8,](#page-6-0) E and F), with  $Ca^{2+}$  sparks exhibiting an approximately exponential increase. Note that this rapid rise of  $Ca^{2+}$ -spark rate at elevated  $[Ca^{2+}]_{sr}$  levels would likely eventually give way to propagating  $Ca^{2+}$  waves in a fully spatial model. This rapid increase in  $Ca^{2+}$ -spark rate at elevated  $[Ca^{2+}]_{sr}$  levels is due to increased  $Ca^{2+}$ -spark fidelity (ability of I<sub>quark</sub> to trigger I<sub>spark</sub>) and RyR  $[Ca^{2+}]$ <sub>i</sub> sensitivity. In summary,  $Ca^{2+}$  sparks dominate SR  $Ca^{2+}$ leak under normal SR Ca<sup>2+</sup> load; however, at lower  $\lbrack Ca^{2+}\rbrack _{\rm sr}$ , the nonspark-based leak contributes a greater fraction of the overall leak. Note that rogue RyRs can be excluded from this model with modest effect on results.

### SR Ca<sup>2+</sup> leak under pathological conditions

Pathological conditions such as heart failure are often associated with SR  $Ca^{2+}$  mishandling due to downregulation of SERCA and/or excessive phosphorylation of RyR. With this in mind, simulations were conducted with the model to study the effects of these two changes, the goal being to acquire novel insights into the development of the pathology. Fig. 9 demonstrates the effect of phosphorylation on SR Ca<sup>2+</sup> leak, simulated via changes in the RyR opening rate  $(k^+)$  and changes in SERCA's  $[Ca^{2+}]$ <sub>i</sub> sensitivity  $(K_{d,i})$ . In all simulations,  $[Ca^{2+}]_i$  was fixed at 90 nM and  $[Ca^{2+}]_{sr}$  was allowed to equilibrate to its steady-state value. The whole-cell  $Ca^{2+}$ -leak rate is shown to increase with RyR phosphorylation levels (Fig. 9 A), and similarly for nonspark-based leak (Fig. 9 B). This response, along with the decrease in steady-state  $[Ca^{2+}]_{sr}$  shown in Fig. 9 C, is expected and is attributed to the phosphorylation-induced increase of the RyR open probability. The model suggests that  $Ca^{2+}$  sparks produced at these diminished SR  $Ca^{2+}$ levels are of longer durations (Fig. 9 D) and exhibit greater variability—which is, to our knowledge, a novel prediction that has yet to be tested experimentally. Also, simulations show that increased RyR activity can, somewhat paradoxically, lead to increased SR  $Ca^{2+}$  leak even in the presence of decreased  $[Ca^{2+}]_{sr}$  (see Fig. 9, A and C).

#### **DISCUSSION**

We have presented what we believe to be a new mathematical model that successfully and robustly characterizes  $Ca^{2+}$ 



FIGURE 9 Effect of pathology on SR  $Ca^{2+}$  leak modeled by adjusting RyR opening rate  $(k^+)$  and SERCA's  $[Ca^{2+}]_i$  sensitivity  $(K_{d,i})$ . (A) Total integrated RyR flux during a 1-s simulation.  $(B)$  Integrated nonspark RyR release flux via both junctional and nonjunctional RyRs. (C) Steady-state  $[Ca^{2+}]_{sr}$  (D) Average  $Ca^{2+}$  spark duration versus  $k^+$ . In all simulations,  $[Ca^{2+}]$ <sub>i</sub> was fixed at 90 nM and  $[Ca^{2+}]_{\text{nsr}}$  was allowed to equilibrate to its steady-state value. (Data points) Average of 10 simulations. (Error bars) Standard deviation from the mean. (Vertical dashed lines) Parameter values under normal conditions.

sparks and  $Ca^{2+}$  leak in the heart during diastole. The model includes a realistic number of CRUs each containing a cluster of stochastically gating RyRs with properties constrained by experimental observations. The CRUs function independently of each other but are sensitive to the  $[Ca^{2+}]$ in the cytosol, through which they combine to produce whole-cell phenomena as observed experimentally. The model presented here improves upon our previous  $Ca^{2+}$ -spark model ([16\)](#page-9-0) that modeled  $Ca^{2+}$  sparks from a single CRU incorporating RyR regulation by subspace and luminal calcium. It incorporates improved RyR  $Ca^{2+}$  sensitivity and allosteric interactions as well as complete descriptions of diastolic  $Ca^{2+}$  cycling. It embodies several wellcharacterized properties of  $Ca^{2+}$  sparks (duration, termination, activation, and sensitivity to  $Ca^{2+}$  and  $[Ca^{2+}]_{sr}$ ). In doing this, it describes and characterizes SR  $Ca^{2+}$  balance including both  $Ca^{2+}$  spark- and nonspark-based SR  $Ca^{2+}$ leak. The work provides a conceptual framework and tools for an improved understanding of a number of perplexing issues previously presented by us and others as noted below.

Importantly also, the stochastic model of  $Ca^{2+}$  sparks and  $Ca^{2+}$  leak during diastole, which (to our knowledge) we present here as new, lays the foundation for more complex dynamic models with fully integrated, spatially resolved, time-dependent  $Ca^{2+}$  signaling.

### Inactivation of RyR not needed

There is no need for  $Ca^{2+}$ -dependent RyR inactivation to terminate  $Ca^{2+}$  sparks as demonstrated here and in our previous model [\(16](#page-9-0)).  $Ca^{2+}$  sparks terminate stochastically

facilitated by  $[Ca^{2+}]_{\text{isr}}$  depletion and RyR allosteric coupling. Although the physical self-association of RyRs in biochemical preparations of RyRs supports the physical interactions among the homotetramers, superresolution imaging and additional experiments are needed to explore this aspect. Finally, to date, no  $Ca^{2+}$ -dependent inactivation process involving RyRs has been demonstrated experimentally with the right timing to account for the termination of  $Ca^{2+}$  efflux from a CRU. Moreover, recent experimental studies on  $Ca^{2+}$ -spark termination argue against the involvement of  $Ca^{2+}$ -dependent inactivation in spark termination ([35,36](#page-9-0)).

#### Leaky RyR

That RyRs open during diastolic conditions as individual channels to produce  $I_{quark}$  and as ensembles to produce  $I_{spark}$ is consistent with this new  $Ca^{2+}$  spark model. Indeed, it is necessary to include such properties of RyRs in the model to reproduce the recent experimental observation that when SERCA is blocked,  $[Ca^{2+}]_{sr}$  decreases due to both spark-mediated and nonspark-mediated mechanisms ([8\)](#page-9-0). The need for pump-leak balance in the SR has been nicely demonstrated both experimentally ([8,9](#page-9-0)) and in the model here. The model also suggests how increased phosphorylation of the RyR during heart failure might increase the leakiness of the RyRs. Additionally, despite nonjunctional RyRs having increased  $P_0$  due to the lack of allosteric coupling, they contribute only a small fraction of the total SR  $Ca<sup>2</sup>$ leak in this model.

### **SERCA**

The Tran-Crampin SERCA formulation reproduces the experimental observation that  $Ca^{2+}$  uptake into the SR depends on both  $[Ca^{2+}]$ <sub>i</sub> and  $[Ca^{2+}]$ <sub>sr</sub> [\(37,38\)](#page-9-0). The model predicts virtually no backflux of  $Ca^{2+}$  under physiological conditions, allowing entirely RyR-based leak to balance SERCA—providing a simple yet physiological system of pump/leak balance to the model. However, we acknowledge that the system may indeed be more complex (i.e., other leak pathways such as IP<sub>3</sub>Rs may also contribute to SR Ca<sup>2+</sup> leak).

## $Ca^{2+}$ -induced  $Ca^{2+}$  release

The manner by which  $Ca^{2+}$  sparks are produced suggests that the process is dominated by  $Ca^{2+}$ -induced  $Ca^{2+}$  release (CICR). The initial probabilistic opening of a single RyR depends on its sensitivity to both  $[Ca^{2+}]$ <sub>i</sub> and  $[Ca^{2+}]$ <sub>sr</sub>. The opening and closing rates also depend on modulatable properties of RyR and those of interacting proteins (see above). The increase in  $[Ca^{2+}]_{ds}$  produced by the initial RyR opening is the primary trigger for diastolic  $Ca^{2+}$ sparks. However, during EC coupling the triggering is due largely to the increase in  $[Ca^{2+}]_{ds}$  from the opening LCCs in the sarcolemmal or transtubule membranes opposite the RyR cluster in the CRU. Our simulations suggest that when either a single RyR or LCC opens, the local RyR cluster is unlikely to be triggered. This feature, which is required here to reproduce the phenomenon of invisible RyR-mediated leak [\(7,8,15](#page-9-0)), has been suggested previously based on analysis of voltage-clamp experiments ([29,30](#page-9-0)). As the intrinsic sensitivity of RyR increases due to phosphorylation, mutation, or other change, the remarkable stability of the system can change.  $Ca^{2+}$  instability of CICR may arise when  $[Ca^{2+}]_{sr}$  becomes elevated or when RyR  $Ca^{2+}$ sensitivity increases. Exploration of the features of instability will be addressed in a spatially resolved  $Ca^{2+}$ -spark model.

### $Ca<sup>2+</sup>$ -spark characteristics

An important part of this model is to fit all major features of the experimental findings, in respect to  $Ca^{2+}$  sparks and SR  $Ca^{2+}$  leak, with the simplest model possible. These goals were achieved even as we constrained the behavior of RyRs and SERCA by their measured biophysical properties. The  $Ca^{2+}$ -spark rate is similar to that observed in rats during diastole.  $Ca^{2+}$ -spark characteristics for other species are largely the same, except for the  $Ca^{2+}$ -spark rate, which is lower. Only modest changes to the model are needed to lower the  $Ca^{2+}$ -spark rate. For example, slight reductions in diastolic  $\left[Ca^{2+}\right]_{sr}$  dramatically decrease  $Ca^{2+}$ -spark rate.

### Trigger characteristics

The likelihood of a  $Ca^{2+}$  spark occurring due to the opening of a single RyR is  $\langle 10\%$ . During the upstroke of a cardiac action potential, RyRs are triggered by openings of the voltage-gated LCCs. Given that a single LCC opening is known to contribute roughly the same  $Ca^{2+}$  current as an individual RyR and LCC openings are brief  $(-0.5 \text{ ms})$ ([33\)](#page-9-0), therefore many LCCs must open during systole in order to trigger a spark. However, during systole, LCC openings are well synchronized by depolarization, ensuring adequate  $Ca^{2+}$  influx to trigger  $Ca^{2+}$  sparks. In fact, the presence of seven LCCs per CRU (consistent with the LCC/RyR ratio of 1:7 ([39\)](#page-9-0)) allows for robust triggering of the CRUs during a simulated depolarization (see [Fig. S1](#page-9-0)).

In summary, the relatively simple mathematical model of  $Ca^{2+}$  sparks put forward by Sobie et al. [\(16](#page-9-0)) has been advanced significantly by the improvements in RyR  $Ca^{2+}$ sensitivity, inclusion of a physiological SERCA formulation, and the incorporation of these  $Ca^{2+}$ -handling mechanisms into a detailed whole-cell model of CICR. This self-consistent, and to our knowledge, new model is highly constrained by experimental observations and forms the basis for a new generation of  $Ca^{2+}$  signaling simulations.

#### <span id="page-9-0"></span>SUPPORTING MATERIAL

Three tables, six figures, and equations are available at [http://www.](http://www.biophysj.org/biophysj/supplemental/S0006-3495(11)00879-4) [biophysj.org/biophysj/supplemental/S0006-3495\(11\)00879-4.](http://www.biophysj.org/biophysj/supplemental/S0006-3495(11)00879-4)

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