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Some heat engine cycles in which liquids can work

(heat engine/Stirling/Malone/Brayton/propylene)

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ABSTRACT Liquids can work in heat engine cycles that employ regeneration. Four such cycles are discussed: Stirling, Malone, Stirling–Malone, and Brayton. Both regeneration and the role of the second thermodynamic medium are treated, and the principles are verified by quantitative measurements with propylene in a Stirling–Malone cycle.

A fundamental property of liquids working in heat engines is that the adiabatic temperature change per unit pressure change is very much smaller than it is for gases. This is a result of the large heat capacity per unit volume of liquids and follows from the relation $(\partial T/\partial p)_S = T\beta/\rho c_p$, where S is entropy, T is the absolute temperature, p is pressure, β is the isobaric expansion coefficient, ρ is the mass density, and c_p is the isobaric specific heat. A Carnot cycle spans a temperature difference determined by the adiabatic temperature change ΔT_{ad} and, hence, is unsuitable for use with liquids, as Carnot himself pointed out (1). All heat engines using liquids must incorporate some method for expanding the temperature span ΔT over which they operate to a value greater than ΔT_{ad}. This can be achieved if, in those parts of the engine where there is to be a steady temperature gradient or where heat is to be absorbed or released, a means is provided always to restore the fluid at a given location to the same temperature regardless of the fluid's history. The fundamental principle showing how this can be done was first revealed in 1816 by Robert Stirling (2). The gas in all parts of Stirling's idealized engine experiences only locally isothermal processes. The ideal engine has Carnot efficiency. There is no relationship in principle between ΔT and ΔT_{ad}.

The heat flows necessary for the above restorative actions are achieved either by the transfer of heat from fluid in one part of the engine to fluid in another part or by the transfer of heat from the fluid to a second thermodynamic medium during one part of the cycle and from the second medium to the fluid in another part of the cycle. In practice, the necessary processes can never be made completely reversible, owing to friction and the finite thermal conductivity of materials.

Some examples of cycles in which liquids can work are given below; they are illustrated with schematic drawings in Fig. 1. The term regenerator is used in Fig. 1 to describe the engine component used in transforming the fluid temperature between the ambient temperature T_A and the remote temperature T_R. The ambient and remote heat exchangers provide means for contact with thermal reservoirs at T_A and T_R. The components D are opposite ends of the same displacer; they move together to displace fluid from one end of the engine to the other at constant total volume. Piston P produces volume changes of the fluid and can be controlled separately from D. Components P and D are moved separately. When first one is moved and then the other, we call the cycle "articulated." When the motions of both P and D are sinusoidal but with a phase shift, we call the cycle "harmonic." The symbol C refers to an isentropic fluid compressor; E refers to an isentropic fluid expander.

The Stirling Cycle. In the articulated Stirling cycle, the components P and D are moved alternately at constant local temperature, respectively causing externally and internally (constant volume) produced pressure changes. For $T_R > T_A$, the phasing of P and D can be arranged for the engine to be either a prime mover or a heat pump. For $T_R < T_A$, the phasing is the same as for the prime-mover operation, and the engine is a refrigerator. In the case of prime-mover operation, the fluid first is expanded isothermally from its compressed state, absorbing heat from the heat exchangers and a second thermodynamic medium (the regenerator); then is displaced at constant total volume and constant local temperature through the regenerator towards the ambient end, rejecting heat to the regenerator; then is compressed isothermally, rejecting heat to the exchangers and the regenerator; and finally is displaced again at constant volume and constant local temperature through the regenerator towards the remote end, absorbing heat from the regenerator and completing the cycle. Desirable qualities of a Stirling regenerator are (*i*) a large heat capacity compared with the fluid passing through it during a half-cycle, (*ii*) excellent lateral thermal contact with the fluid, (*iii*) low longitudinal thermal conductance, (*iv*) small fluid volume, and (*v*) a low impedance to fluid flow. By alternately absorbing and releasing heat to the fluid, the Stirling regenerator enables the cycle to operate with $\Delta T = |T_R - T_A| \gg |\Delta T_{ad}|$. If the volumetric displacement V_D is small enough in a Stirling cycle with a liquid medium, then thermal contact will be impaired with the thermal reservoirs at T_R and T_A. This is a consequence of either a reduction in the effective heat transfer area in the heat exchangers or, for V_D small compared with the dead volume between regenerator and exchangers, a change to a type of very ineffective shuttle heat transfer (3) between regenerator and exchangers. Any reduction in the heat transfer to the reservoirs at T_R and T_A diminishes the thermodynamic effectiveness of the cycle. For gases these considerations seldom apply; but for liquids the displaced volume V_D must be kept small so that the heat capacity of the displaced fluid is small compared to that of the second thermodynamic medium, which has a comparable heat capacity per unit volume. In our apparatus, the above deleterious effects are so large with the Stirling arrangement that we have not undertaken detailed experiments. There may be cases, however, where a liquid can be used in a Stirling cycle.

The Malone Cycle. In 1931, J. F. J. Malone (4) published an account of experimental engines designed specifically for use with liquids. The ideal articulated Malone cycle is similar to the Stirling cycle in that components P and D are moved alternately and a constant local temperature in the regenerator and heat exchangers. Unlike the Stirling cycle with its reciprocating flow, the fluid has a pulsating, unidirectional counterflow inside the regenerator (or "thermodynamic pile" as Malone called it) and

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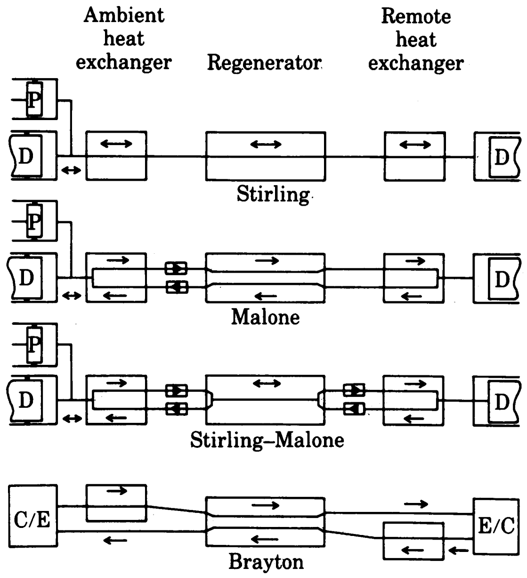

FIG. 1. Schematic drawing of several heat engines in which liquids can be used. P, piston; D, displacer; E/C, expander/compressor; ⊟, check valve. The two components D in a given schematic are opposite ends of one and the same displacer, which has been cut to simplify the appearance of the fluid circuit.

a pulsating, circulating flow inside the heat exchangers—all accomplished with the use of two check valves and two sets of flow channels. The volume of fluid displaced during a half-cycle is small compared with the fluid volume inside the regenerator. Fluid flowing from the remote end towards the ambient end during a half-cycle exchanges heat with the fluid flowing in the opposite direction during the other half-cycle. No nonfluid heat capacity is needed in principle to regenerate the fluid at constant pressure. The circulating flow promotes thermal contact between the fluid and the heat exchangers and between the heat exchangers and the regenerator, overcoming the corresponding difficulties in the Stirling cycle. Owing to the fact that $\Delta T_{ad}/T$ is always small for a liquid, it is immaterial whether the pressure changes adiabatically or isothermally in the open volumes adjacent to the ends of the displacer piston. It is only essential that all processes in the regenerator and exchangers be locally isothermal.

The Stirling–Malone Cycle. The Stirling–Malone cycle combines the regenerative qualities of the Stirling cycle with the improved thermal contact between the fluid and the heat exchangers of the Malone cycle. This is achieved by using a pair of check-valves at each end of the regenerator. As in the Malone cycle, the reciprocating action of the displacer, together with the check valves, causes the fluid leaving the regenerator at a given end to circulate completely through the heat exchangers before entering the same end of the regenerator again. As in the Stirling cycle, a second thermodynamic medium is necessary to regenerate the fluid at constant pressure, and the fluid reciprocates inside the regenerator.

The Brayton Cycle. In the Brayton cycle engine shown in Fig. 1, fluid circulates continuously through the engine. The pressure is constant at any particular point in the engine but is changed isentropically at two places in the fluid circuit by a compressor and an expander. A given mass of fluid completes two adiabatic processes and two isobaric processes in a cycle. The regenerator of a Brayton engine is actually a counterflow heat exchanger with a pressure difference between the two sides. No nonfluid heat capacity is needed in this cycle. Its chief problems in use with liquids are technical: (*i*) it is necessary to provide continuously operating and thermodynamically reversible high-pressure compressors and expanders for a fluid of low compressibility; and (*ii*) a problem of lesser conceptual difficulty, the counterflow heat exchanger must be capable of withstanding a large pressure differential.

Experiments on the regeneration at constant pressure of liquid propylene (C_3H_6)

Measurements of the ineffectiveness of regeneration at constant pressure, or regenerative loss, were made on two regenerators in the Malone and Stirling–Malone configurations and also in a quasi-continuous counterflow arrangement by using a method which we will describe elsewhere. The experiments on Stirling–Malone regeneration were done with the arrangement shown in Fig. 2, but with P stationary. The arrangement used to study Malone regeneration was similar to that in Fig. 2, except that the two check valves at the remote end were removed and that the internal division of the flow passages into two sets was restored. The displacer was designed so that its contribution to the thermal losses is small. One regenerator was comprised of 30 parallel-plate fluid channels, each 0.025 cm thick, with flow in adjacent channels in opposite directions in the Malone arrangement. The plates separating the channels were no. 304 stainless steel, each 0.152 cm thick. This regenerator had an active length of 57.2 cm, an active fluid volume of 261 cm^3, and a total fluid volume of 338 cm^3. The other regenerator was made from a stack of 3710 copper-screen annuli of 4.13-cm outside diameter and 2.54-cm inside diameter. The axial fluid flow was separated radially into two concentric annular regions by means of compressed 0.5-mm-diameter solder wire rings of 3.43-cm mean diameter. (The screen was "100 mesh," meaning that it

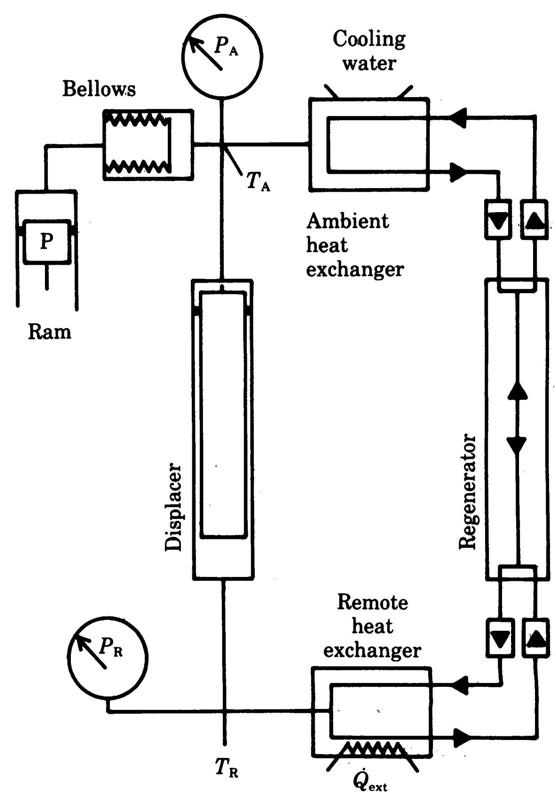

FIG. 2. Schematic drawing of an experimental Stirling–Malone heat engine. Bellows separates hydraulic fluid in ram from propylene in engine.

was woven from 0.11-mm-diameter copper wire on 0.25-mm centers.) This regenerator had an active length of 75 cm and an active fluid volume of 307 cm^3.

In typical measurements of regenerative loss, we measured the externally applied electrical power $\dot{Q}_{ext}$ (Fig. 2) needed to keep the average value of $\Delta T = T_R - T_A$ constant, with T_A held constant by adjusting the cooling water. The dependence of displaced volume on time was periodic, and either a sinusoidal or a symmetrical triangular wave with known peak-to-peak volumetric displacement V_D and period τ. The nonfluid heat loss $\dot{Q}_{loss} = a\,\Delta T$ due to conduction and radiation, with "a" roughly temperature independent, could be evaluated empirically. The fluid is heated by friction as it passes through the regenerator and consequently heats each of the exchangers at a rate $\dot{Q}_{fric}$ given approximately by the formula

$$\dot{Q}_{fric} = \frac{\pi}{8}\frac{V_D}{\tau}\,\delta p, \qquad [1]$$

where δp is the maximum measured pressure drop $|p_R - p_A|_{max}$ (Fig. 2) and where half of the total frictional heating is assumed to be applied to each exchanger. The regenerative loss can then be calculated from

$$\dot{Q}_{reg} = \dot{Q}_{ext} - \dot{Q}_{loss} + \dot{Q}_{fric}\,. \qquad [2]$$

Measurements of $\dot{Q}_{reg}$ made with the triangular displacer drive are multiplied by a factor $\pi^2/8$ to allow direct comparison with measurements made with the sinusoidal drive. This factor is the ratio of the time-averaged squares of the spatially averaged fluid velocities for sinusoidal and triangular drives.

Frictional heating can be neglected in the parallel-plate regenerator, but in the copper-screen regenerator with the Stirling–Malone configuration, $\dot{Q}_{fric}$ and $\dot{Q}_{reg}$ are comparable. $\dot{Q}_{fric}$ varied somewhat more rapidly than $(V_D/\tau)^2$, presumably reflecting a somewhat stronger dependence than linear of δp on V_D/τ as a result of nonlaminar flow in the check valves and interconnecting lines. It was about twice as large in the Malone as in the Stirling–Malone configuration for a given V_D/τ, owing to the doubling of the regenerator flow impedance in the former case. However, fluid friction was fractionally less important in the Malone case due to larger regenerative losses.

In experiments with both regenerators, in the Malone, Stirling–Malone, and quasi-continuous flow configurations, the regenerative loss per cycle, $Q_{reg} = \tau \dot{Q}_{reg}$, obeys the empirical relation

$$Q_{reg} = g(\tau)\, V_D{}^2 \Delta T\ , \qquad [3]$$

where $g(\tau)$ is a function of τ and independent of V_D that depends on the configuration of the apparatus, the geometry of the regenerator, and the thermophysical properties of the fluid. For both the quasi-continuous counterflow and parallel-plate Malone regeneration in the long τ limit, theory predicts $g(\tau) \propto \tau^{-1}$. Measurements of $Q_{reg}/\Delta T V_D{}^2$ for the equivalent sinusoidal displacement are plotted as a function of τ in Fig. 3 for the parallel-plate regenerator in the quasi-continuous counterflow and Malone configurations and for the copper-screen regenerator in the counterflow, Malone, and Stirlong–Malone configurations.

All data are for propylene in the same general range of temperature but with some differences in detail. Several points should be noted. (*i*) The copper-screen regenerator is superior; the regenerative loss is nearly half that of the parallel-plate regenerator in the counterflow configuration. (*ii*) In the counterflow configuration, both regenerators have the expected $g(\tau) \propto \tau^{-1}$ behavior. In the parallel-plate regenerator, heat transfer is limited by the fluid; in the screen regenerator, heat transfer is limited by the copper. (*iii*) In the Malone configuration, the parallel-plate regenerator approaches the expected $g(\tau) \propto \tau^{-1}$ behavior for large τ. (*iv*) The approximate $\tau^{-1/2}$ dependence of $g(\tau)$ for the copper-screen regenerator in the Malone configuration suggests that heat is transferred between the two sides by diffusive temperature waves of wave vector $(2\pi/\kappa_e\tau)^{1/2}$, where κ_e is an effective thermal diffusivity of the copper wires and fluid. Estimates of $\dot{Q}_{reg}$ based on this idea and on known properties of the regenerator support this explanation. (*v*) Finally, the regenerative loss is reduced for the copper-screen regeneration in the Stirling–Malone configuration by nearly an order of magnitude at small τ compared to the Malone case. Perhaps more importantly, the dependence of Q_{reg} on τ has been changed qualitatively, so that increasing the speed of the engine actually reduces the heat loss per cycle in the regenerator. This suggests that the usual losses are now very small for small periods and that a new loss mechanism must be considered. (The larger scatter of the experimental data in this case is related to the much smaller values of Q_{reg}, which are then difficult to measure precisely.)

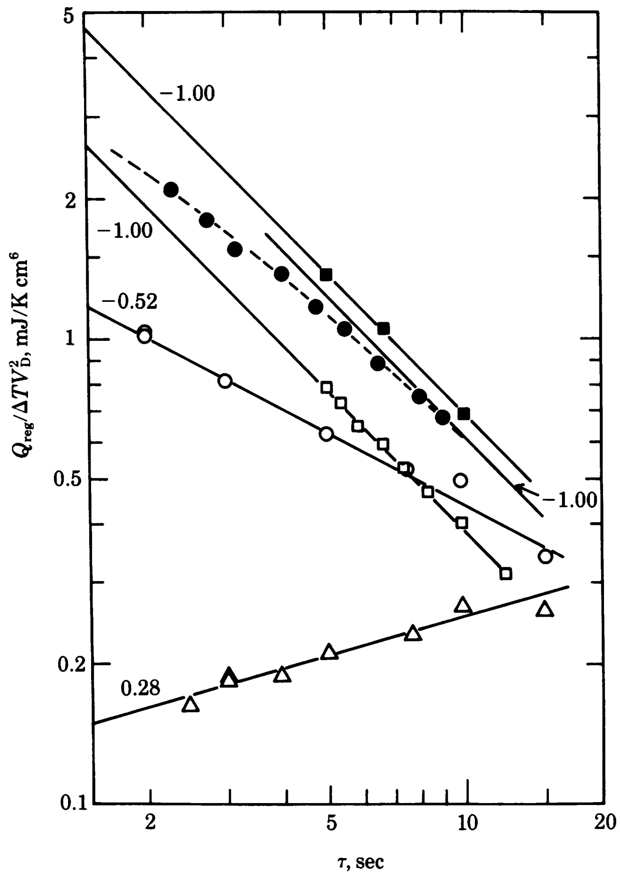

FIG. 3. Logarithm of $g(\tau) = Q_{reg}/V_D{}^2\,\Delta T$ (Eq. 3) as a function of the logarithm of the period τ for the parallel-plate regenerator (solid symbols) and the copper-screen regenerator (open symbols) in several configurations using propylene. Numbers are the exponent in the relation $g(\tau) \propto \tau^n$ and are the slopes of the lines fit to the data. ■, Quasi-continuous counterflow, triangular-drive data adjusted to sinusoidal drive (T_A = 298.6K, T_R = 329.4 K, p = 34.5 bar); ●, Malone, sinusoidal drive (T_A = 297.9 K, T_R = 328.6 K, p = 96.5 bar); □, quasi-continuous counterflow, triangular-drive data adjusted to sinusoidal drive (T_A = 297.1 K, T_R = 326.7 K, p = 34.5 bar); ○, Malone, sinusoidal drive (T_A = 298.5 K, T_R = 338.2 K, p = 86.2 bar); △, Stirling–Malone, sinusoidal drive (T_A = 298.3 K, T_R = 338.2 K, p = 124.1 bar).

That the usual losses are small is explained by the elimination of the thermal resistance of the copper and by the factor-of-two reduction in fluid velocity for the Stirling–Malone configuration as compared with the Malone arrangement. The idealized shuttle heat-loss mechanism (3) can give a heat loss per cycle proportional to $\tau^{1/2}$ or τ, depending on the nature of the problem. The $\tau^{0.28}$ dependence shown on Fig. 3 no doubt reflects a complicated situation. One might speculate that a shuttle heat loss is rooted in the thermal lag between the oscillating longitudinal temperature gradient in the copper screens and the surrounding stainless steel pressure vessel.

The second thermodynamic medium

In regenerating the fluid at constant pressure, an important difference between the Stirling or Stirling–Malone cycles and the Malone or Brayton cycles is that no nonfluid heat capacity in the form of a second thermodynamic medium is essential in the latter two. But for there to be a net thermodynamic effect, the Stirling, Malone, and Stirling–Malone reciprocating cycles must have an appropriately phased time-varying pressure. Consequently, all the reciprocating cycles, including the Malone cycle, require the heat capacity of a second thermodynamic medium in order to absorb or reject within the regenerator and exchangers on a locally isothermal basis the heats of compression or expansion caused by the pressure changes induced by motions of the piston and displacer. Consider, for example, the Malone engine shown schematically in Fig. 1 operating as a prime mover in an articulated cycle. Assume for simplicity that the heat capacity of the exchangers is infinite, that their thermal contact with the fluid is perfect, that the displaced volume is small compared with the volume V_r of the regenerator, that the fluid is essentially incompressible, and that the temperature and pressure variations of the fluid parameters can be neglected over the temperature difference ΔT_{ad} and the pressure difference caused by the compression, Δp. Then fluid moving from the regenerator's remote end to the displacer after a compression has an excess temperature

$$\delta T = \frac{(T\beta)_R V_r \Delta p}{C_F + C_2}, \quad [4]$$

where $(T\beta)_R$ is the value of this quantity at T_R, $C_F = V_r (\rho c_p)_R$ is the total heat capacity of the fluid in the regenerator taken at T_R, and C_2 is the heat capacity of the regenerator in contact with the fluid. The heat rejected to the remote heat exchanger when the volume V_D is displaced into or through it is

$$Q_1 = V_D (\rho c_p)_R \delta T = \frac{C_F}{C_F + C_2} (T\beta)_R V_D \Delta p. \quad [5]$$

Expansion through pressure difference Δp cools the fluid in the remote end of the displacer. When this fluid is displaced into or through the remote exchanger it absorbs heat

$$Q_2 = (T\beta)_R V_D \Delta p. \quad [6]$$

The total heat absorbed from the remote heat exchanger each cycle is $Q = Q_2 - Q_1$:

$$Q = \Gamma (T\beta)_R V_D \Delta p, \quad [7]$$

where $\Gamma = C_2/(C_F + C_2)$. The heat absorbed is reduced by the factor Γ from the ideal case of a regenerator with infinite heat capacity. The work done per cycle is also reduced by the factor Γ.

The above arguments apply equally well to the Stirling and Stirling–Malone cycles. The Brayton cycle does not require the use of a second thermodydnamic medium because the pressure at any point of the heat exchangers is constant in time; the pressure-induced temperature differences are confined to the compressor and expander. Although in the Malone cycle regeneration at constant pressure remains efficient as $C_2/C_F \rightarrow 0$, this advantage cannot be realized, as the second thermodynamic medium is still needed to restore the temperature of the fluid during pressure changes.

Experiments with liquid propylene (C_3H_6) working in a Stirling–Malone cycle

We tested the above concepts quantitatively by using the apparatus shown schematically in Fig. 2 with sinusoidal displacer and ram motions phased at 90° for prime-mover action. The ram displaces 67.8 cm^3; the total engine fluid volume is about 900 cm^3. All results presented here are for an engine period $\tau \simeq 3.0$ sec. The external power $\dot{Q}_{ext}$ and cooling water were adjusted to keep the time-averaged temperatures $\bar{T}_R$ and $\bar{T}_A$ constant. Because of the adiabatic heating and cooling of the fluid in the displacer by the pressure change Δp, the average temperature drop across the regenerator is $\Delta T = \bar{T}_R - \bar{T}_A + \Delta\bar{T}_i$, where $\Delta\bar{T}_i$ depends on the details of the cycle and is proportional to the adiabatic temperature change, $(T\beta/\rho c_p)\,\Delta p$. For equilibrium, the power $\dot{Q}_{ext}$ is given by

$$\dot{Q}_{ext} = \dot{Q}_R + \dot{Q}_{reg} + \dot{Q}_{loss} - \dot{Q}_{fric}, \quad [8]$$

where

$$\dot{Q}_R = \Gamma \frac{\pi}{4} \overline{(T\beta)}_R \left(\frac{V_D}{\tau}\right) \Delta p \quad [9]$$

is the "thermodynamic" heating rate from Eq. 7, $\overline{(T\beta)}_R$ being

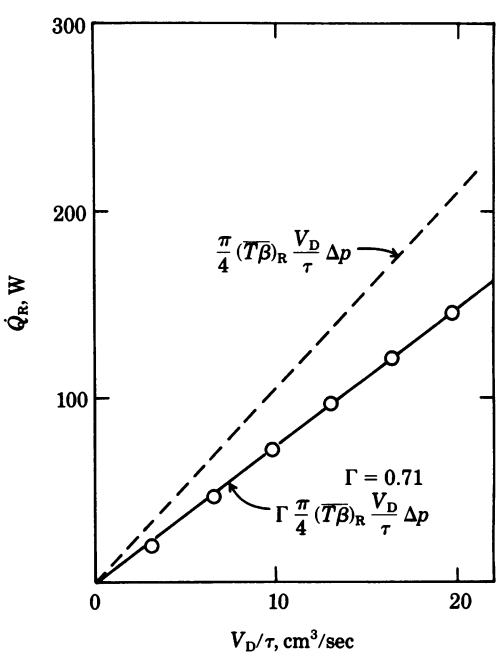

FIG. 4. Measured (O) thermodynamic heating rate for $\bar{T}_R - \bar{T}_A = 0.0$ K as a function of V_D/τ. The lines are calculated by using experimental data from Eq. 9 for $\Gamma = 1.00$ (---) and $\Gamma = 0.71$ (—). $\tau = 3.0$ sec, $\bar{T}_A = 298.3$K, $\bar{p} = 110.3$ bar, $\overline{\Delta p} = 167.6$ bar, and $\Delta\bar{T}_i = 6.6$ K.

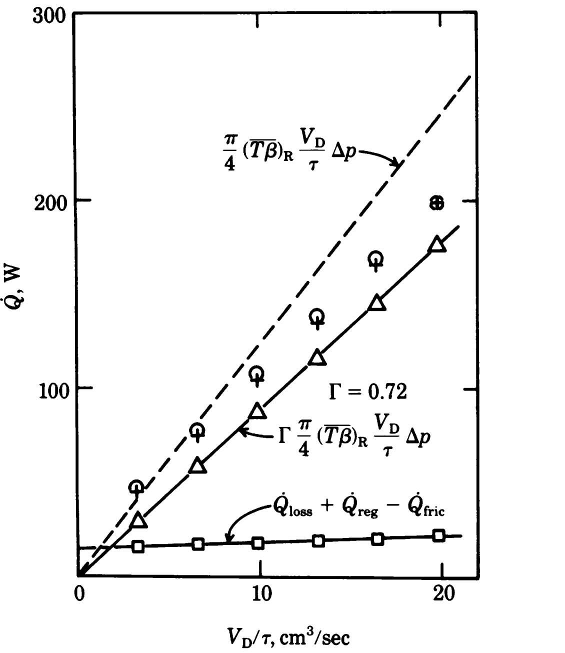

FIG. 5;. Measured (○) and calculated (+) power supplied to the remote heat exchanger as a function of V_D/τ for $\bar{T}_R - \bar{T}_A = 39.8$ K, $\bar{T}_A = 298$ K, $\bar{p} = 124.3$ bar, $\overline{\Delta p} = 131.7$ bar and $\overline{\Delta T_i} = 6.3$ K. The sum $\dot{Q}_{loss} + \dot{Q}_{reg} - \dot{Q}_{fric}$ is calculated from experimental data; the quantity $\Gamma \frac{\pi}{4}(\overline{T\beta})_R \frac{V_D}{\tau} \Delta p$ is calculated by using experimental measurements of the factors and $\Gamma = 0.72$. $\tau = 3.0$ sec.

the pressure-averaged value of $(T\beta)$ for the temperature $\bar{T}_R$. The term $\dot{Q}_{fric}$ is calculated from measurements by means of Eq. 1; $\dot{Q}_{loss}$ is calculated from $a\Delta T$, where "a" is measured; and $\dot{Q}_{reg}$ is given by a modification of Eq. 3,

$$\dot{Q}_{reg} = f\, g\,(\tau)\, V_D^2\, \Delta T/\tau, \qquad [10]$$

in which $f \leq 1$ is a factor we introduced (5) to account for the possible effect on regeneration of the pressure dependence of c_p. Aside from the factor f, for the purpose of evaluating the right side of Eq. 8 we take $\dot{Q}_{reg}$ to be that measured in constant-pressure experiments on regeneration.

To study the effect of the second medium under simple conditions, we made measurements for the case $\bar{T}_R - \bar{T}_A = 0$. We measured all quantities in Eq. 8 except $\dot{Q}_R$ and then calculated it from Eq. 8, the results being shown in Fig. 4 for various values of V_D/τ. Also shown in Fig. 4 are plots of Eq. 9 calculated from thermodynamic data for $T\beta$ and for the measured Δp and V_D/τ. A value $\Gamma = 0.71$ is a good fit to the data; we calculate a value of 0.69 from estimates of the heat capacities of copper screens, solder rings, and fluid in the regenerator. We conclude that Eq. 9 describes the thermodynamic heating rate with adequate accuracy.

As a further test of our understanding of the energy balance, we made measurements for $\bar{T}_R - \bar{T}_A = 39.8$ K. These are shown in Fig. 5, which includes as a function of V_D/τ both the measured values of $\dot{Q}_{ext}$ and the values computed for this quantity from the right side of Eq. 8. The agreement is excellent. We used $\Gamma = 0.72$, corresponding to the somewhat different fluid properties from those at zero temperature difference. We also used $f = 1$ in Eq. 10, which allows a good fit to the data and may reflect the fact (Fig. 3) that regenerative loss for the Stirling–Malone cycle has some loss mechanism other than the common temperature defect. Additionally, we found no evidence for a heat loss $\dot{Q}_0$ due to ram motion alone (5). The data on Fig. 5 give further weight to Eq. 9, which takes into account the effect of the finite heat capacity of the second medium on the net thermodynamic heating rate.

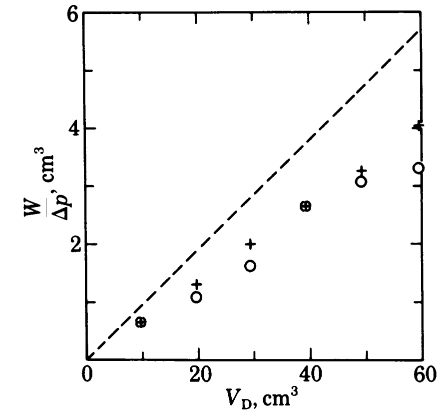

FIG. 6. Measured (○) and calculated (+) values of $W/\Delta p$ as a function of V_D for the same conditions as given in the caption to Fig. 5. ---, values calculated with $\Gamma = 1.0$.

The ideal thermodynamic efficiency of the engine remains unchanged for $\Gamma < 1$, so that the work per cycle is reduced by the same factor Γ as is the thermodynamic heat. Using the approximate expression for work W from ref. 5, we thus expect

$$W \simeq \left[\Gamma \frac{\pi}{4}(\overline{T\beta})_R\, V_D\, \Delta p\right]\left(\frac{\beta_A + \beta_R}{2\beta_R}\right)\left(1 - \frac{T_A}{T_R}\right), \qquad [11]$$

where the first factor is the thermodynamic heat at the remote end, the second factor (involving the pressure-averaged expansion coefficients at ambient and remote ends) reflects the intrinsic inefficiency of liquids working in heat engines, and the third factor is Carnot's efficiency. In Fig. 6 we plot the measured values of $W/\Delta p$ and values calculated using Eq. 11 and a value of $\Gamma = 0.72$ (Fig. 5). The measured values of the work are obtained by subtracting from the difference between the indicated work with the displacer moving and with it stopped, the same difference for $\bar{T}_R - \bar{T}_A = 0$. Agreement is within the experimental error in measuring these differences.

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