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RESEARCH ARTICLE

Synchronization of chaotic nonlinear continuous neural networks with time-varying delay

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Abstract In this paper, the synchronization problem for delayed continuous time nonlinear complex neural networks is considered. The delay dependent state feed back synchronization gain matrix is obtained by considering more general case of time-varying delay. Using Lyapunov stability theory, the sufficient synchronization criteria are derived in terms of Linear Matrix Inequalities (LMIs). By decomposing the delay interval into multiple equidistant subintervals, Lyapunov-Krasovskii functionals (LKFs) are constructed on these intervals. Employing these LKFs, new delay dependent synchronization criteria are proposed in terms of LMIs for two cases with and without derivative of time-varying delay. Numerical examples are illustrated to show the effectiveness of the proposed method.

Keywords Synchronization · Neural networks · Time-varying delay · Delay decomposition · Maximum admissible upper bound (MAUB)

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Introduction

During the past decade, control and synchronization of chaotic systems have become an important topic, since the pioneering work of Pecora and Carroll in 1990 (Carroll and Pecora 1990, 1991). Chaos synchronization has been widely investigated due to its applications in creating secure communication systems (Yu and Liu 2003; Feki 2003). Both Hopfield Neural Networks (HNNs) and Cellular Neural Networks (CNNs) have attracted considerable attention in recent decades and have been widely applied in number of engineering and scientific fields including image processing, computing technology, solving linear and nonlinear algebraic equations and so on (Lou and Cui 2006; Arik 2003).

Moreover, Haken (2007) has presented a neural net model describing biological activity in visual cortex and coined a problem that synchronization between groups of neurons may be the key to solution of "binding problem". In addition to that noise-induced complete synchronization and frequency synchronization in coupled spiking and bursting neurons studied in Shi et al. (2008). Also in Jirsa (2008) it has been proved that time delay plays a vital role in synchronized states of spiking-burst neuronal networks.

On the other hand, artificial neural networks models can also exhibit chaotic behavior (Lou and Cui 2007; Gilli 1993; Lu 2002) due to the fact that small perturbation in initial conditions may lead to large deviation in system dynamics and so synchronization of chaotic neural networks has also become an important area of study. Some authors have paid attention to the synchronization of neural networks (Chen et al. 2004; Chao Jung et al. 2006; Cui and Lou 2009; Gao et al. 2009; Wang et al. 2010). In Cui and Lou (2009), some sufficient conditions for exponential synchronization of neural networks with time-varying delays have been given in terms of feasible solution in the form of Linear Matrix Inequalities (LMIs). In Gao et al. (2009), based on the Lyapunov method a delay independent sufficient synchronization conditions in term of LMIs for chaotic recurrent neural networks with time-varying delays using nonlinear feedback control have been obtained. Delay-dependent conditions, which contains information concerning time delay, are usually less conservative than delay-independent ones. In addition, synchronization between neurons both in biological neuronal network and artificial neural network is essential for information processing. The study of synchronization problem of delayed neural networks may proceed to study complex synchronization between spike-burst neurons. In this paper, we propose a novel synchronization criterion based on delay decomposition approach to derive a maximum admissible upper bound (MAUB) of the time delay such that two identical chaotic nonlinear continuous neural networks with time-varying delay is synchronized asymptotically. The larger MAUB of time delay implies less conservatism of delay-dependent synchronization criterion. Moreover the gain matrix of the controller for slave system can be determined based on LMIs, which can be easily solved by various convex optimization algorithms (Boyd et al. 1994). In this paper, in order to obtain some less conservative sufficient conditions, we adapted the method proposed by Zhang and Han (2009). Interior point algorithm implemented in MATLAB LMI toolbox is employed to solve the derived LMIs.

To the best of authors knowledge, the delay decomposition approach to delay-dependent synchronization analysis for continuous time nonlinear complex neural networks with time-varying delay has never been tackled in any of the previous literature. Based on LKF approach, some new synchronization criteria are proposed in the form of LMIs, which are dependent on the size of the time delay. Numerical examples are given to illustrate the feasibility and effectiveness of proposed method.

Notations Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote n-dimensional Euclidean space and the set of all $n \times n$ real matrices respectively. I denotes the identity matrix and P^{-1} denotes the inverse matrix of P. The notation $*$ always denotes the symmetric block in one symmetric matrix. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). Matrices, if not explicitly stated, are assumed to have compatible dimensions.

Synchronization problem and preliminaries

Based on the master-slave concept, the unidirectional coupled nonlinear neural networks are described by the following delay differential equation. The master system is

$$\begin{aligned}\dot{x}_i(t) &= -\gamma_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + I_i.\end{aligned} \tag{1}$$

and the slave system is

$$\begin{aligned}\dot{y}_i(t) &= -\gamma_i(y_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij} f_j(y_j(t-\tau(t))) + I_i + u_i(t). \\ &\quad i = 1,2,\ldots,n.\end{aligned} \tag{2}$$

where $n \geq 2$ denotes the number of neurons in the networks, $x_i(t)$ and $y_i(t)$ are the state variables associated with ith neuron of master and slave systems respectively at time t. a_{ij} and b_{ij} indicate the interconnection strength among the neurons without and with time-varying delay respectively. The neuron activation function f_i describes the manner in which the neurons respond to each other. I_i denotes the constant external input and $u_i(t)$ be an unidirectional-coupled term, which is considered as control input and will be appropriately designed to obtain certain control objective. Furthermore, $\tau(t)$ is the time-varying delay such that $0 \leq \tau(t) \leq \bar{\tau}$. System (1) and (2) possess initial conditions $x_i(t) = \phi_i(t) \in \mathbf{C}([-\bar{\tau},0],\mathbf{R})$ and $y_i(t) = \varphi_i(t) \in \mathbf{C}([-\bar{\tau},0],\mathbf{R})$ known as delay history functions for master (1) and slave (2) systems respectively, where $\mathbf{C}([-\bar{\tau},0],\mathbf{R})$ denotes the set of all continuous functions from $[-\bar{\tau},0]$ to $\mathbf{R}$.

We further assume that $\gamma_i(\cdot)$ and $f_j(\cdot)$ satisfy the following conditions:

(A1): Each function $\gamma_i : \mathbf{R} \to \mathbf{R}$ is locally Lipschitz and nondecreasing function, that is, there exists a positive real d_i such that $\gamma_i'(x) = d_i$ for any $x \in \mathbf{R}$ at which γ_i is differentiable function.

(A2): Each function $f_j : \mathbf{R} \to \mathbf{R}$ is monotonic nondecreasing and globally Lipschitz, that is, there exists a positive real l_j such that

$$0 \le \frac{(f_j(x) - f_j(y))}{(x-y)} \le l_j \quad \text{for any } x, y \in \mathbf{R}, \text{with } x \ne y \text{ and } j = 1, 2, \ldots, n.$$

Define the synchronization error $e_i(t) = x_i(t) - y_i(t)$. Thus the error dynamic system can be represented as

$$\dot{e}_i(t) = -\beta_i(e_i(t)) + \sum_{j=1}^{n} a_{ij} g_j(e_j(t), y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(e_j(t-\tau(t), y_j(t-\tau(t))) - u_i(t). \tag{3}$$

where $\beta_i(e_i(t)) = \gamma_i(x_i(t)) - \gamma_i(y_i(t))$ and $g_j(e_j(\cdot), y_j(\cdot)) = f_j(e_j(\cdot) + y_j(\cdot)) - f_j(y_j(\cdot))$.

For notational purpose, we denote $g_j(e_j(\cdot), y_j(\cdot))$ as $g_j(e_j(\cdot))$. From (A2), one can obtain that $g_j(e_j(\cdot))$ satisfying

$$0 \le e_j(\cdot) g_j(e_j(\cdot)) \le l_j e_j^2(\cdot) \tag{4}$$

and from (A1) and according to Lebourg theorem (see *Theorem* 2.3.7 in Clarke 1983), there exists $c_i \ge d_i$ such that $\beta_i(e_i(t)) = c_i e_i(t)$.

In order to ensure synchronization of coupled neural networks, the control input $u_i(t)$ is designed as follows

$$u_i(t) = \sum_{j=1}^{n} w_{ij}(x_j(t) - y_j(t)).$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & \ldots & w_{1n} \\ w_{21} & w_{22} & \ldots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \ldots & w_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) - y_1(t) \\ x_2(t) - y_2(t) \\ \vdots \\ x_n(t) - y_n(t) \end{bmatrix} = Ke(t). \tag{5}$$

where $e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T$, $\mathrm{K} = (w_{ij})_{n \times n} \in \mathbf{R}^{n \times n}$ is the state feedback control gain matrix to be determined for synchronizing both master and slave systems. Thus from Eqs. 3 and 5, we rewrite the error dynamic system as

$$\dot{e}_i(t) = -c_i e_i(t) + \sum_{j=1}^{n} a_{ij} g_j(e_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(e_j(t - \tau(t))) - \sum_{j=1}^{n} w_{ij} e_j(t). \tag{6}$$

Transforming Eq. 6 into compact form as

$$\begin{aligned} \dot{e}(t) &= -De(t) + Ag(e(t)) + Bg(e(t-\tau(t))) - Ke(t). \\ &= -Ce(t) + Ag(e(t)) + Bg(e(t-\tau(t))) \end{aligned} \tag{7}$$

where $C = D + K$, $D = diag\{c_i\}$, $A = (a_{ij})_{n \times n}$, $B = (b_{ij})_{n \times n}$, and $g(e(\cdot)) = [g_1(e_1(\cdot)), g_2(e_2(\cdot)), \ldots, g_n(e_n(\cdot))]^T$.

Now we are stating the following Lemmas, which will be more useful in the sequel.

Lemma 2.1 [Han (Zhang and Han 2009)] *For any constant matrix $R \in \mathbb{R}^{n \times n}, R = R^T > 0$, scalar h with $0 \le \tau(t) \le h < \infty$ and a vector-valued function $\dot{x} : [t-h, t] \to \mathbb{R}^n$, the following integration is well defined, then*

$$-\tau(t) \int_{t-\tau(t)}^{t} \dot{x}^T(s) R \dot{x}(s) ds \le \begin{bmatrix} x(t) \\ x(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} -R & R \\ & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \end{bmatrix}.$$

Lemma 2.2 (Schur complement) *Let P, Q, R be given matrices of appropriate dimensions such that $R > 0$.* Then

$$\begin{bmatrix} P & Q \\ Q^T & -R \end{bmatrix} < 0 \Leftrightarrow P + Q^T R^{-1} Q < 0$$

Based on the available information on the time-varying delay, we will consider the following two cases.

Case I $\tau(t)$ is a continuous function satisfying

$$0 \le \tau(t) \le \bar{\tau} < \infty, \quad \forall \quad t \ge 0, \tag{8}$$

Case II $\tau(t)$ is a differentiable function satisfying

$$0 \le \tau(t) \le \bar{\tau} < \infty, \quad \dot{\tau}(t) \le \mu < \infty, \quad \forall \quad t \ge 0, \tag{9}$$

where $\bar{\tau}$ and μ are scalars.

Synchronization criteria

In this section we introduce LKFs to derive some new delay-dependent synchronization criterion for nonlinear continuous neural networks with time-varying delay system described by Eqs. 1 and 2.

Theorem 3.1 *Under case I and hypotheses (A1)–(A2), for a given scalar $\bar{\tau} > 0$, the master-slave neural networks (1) and (2) are completely synchronized with control gain $K = \tilde{Y}\tilde{P}^{-1}$ if there exist positive definite symmetric matrices $\tilde{P} = \tilde{P}^T > 0, \tilde{Q}_i = \tilde{Q}_i^T > 0, \tilde{R}_i = \tilde{R}_i^T > 0, (i = 1, 2, \ldots, N)$, any matrix $\tilde{Y}$ and diagonal matrices $\tilde{S}_1 > 0$, $\tilde{S}_2 > 0$ such that the LMI (10) holds for all $k \in \{1, 2, \ldots, N\}$,*

$$
\Xi=\begin{bmatrix}
\tilde{\alpha}_1 & \tilde{R}_1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & A\tilde{P}+\tilde{S}_1^TL & 0 & 0 & \dots & 0 & 0 & B\tilde{P} & -\tilde{P}D^T\delta-\tilde{Y}^T\delta\\
 & \tilde{\alpha}_2 & \tilde{R}_2 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
 & * & \tilde{\alpha}_3 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
 & * & * & \dots & \tilde{\alpha}_k & 0 & \dots & 0 & 0 & \tilde{R}_k & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
 & * & * & \dots & * & \tilde{\alpha}_{k+1} & \dots & 0 & 0 & \tilde{R}_k & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
 & * & * & \dots & * & * & \dots & \tilde{\alpha}_N & \tilde{R}_N & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
 & * & * & \dots & * & * & \dots & * & \tilde{\alpha}_{N+1} & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0\\
 & * & * & \dots & * & * & \dots & * & * & -2\tilde{R}_k & 0 & 0 & 0 & \dots & 0 & 0 & \tilde{S}_2^TL & 0\\
 & * & * & \dots & * & * & \dots & * & * & * & \tilde{\beta}_1 & 0 & 0 & \dots & 0 & 0 & 0 & \tilde{P}A^T\delta\\
 & * & * & \dots & * & * & \dots & * & * & * & * & \tilde{\beta}_2 & 0 & \dots & 0 & 0 & 0 & 0\\
 & * & * & \dots & * & * & \dots & * & * & * & * & * & \tilde{\beta}_3 & \dots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
 & * & * & \dots & * & * & \dots & * & * & * & * & * & * & \dots & \tilde{\beta}_N & 0 & 0 & 0\\
 & * & * & \dots & * & * & \dots & * & * & * & * & * & * & \dots & * & \tilde{\beta}_{N+1} & 0 & 0\\
 & * & * & \dots & * & * & \dots & * & * & * & * & * & * & \dots & * & * & -2\tilde{S}_2 & \tilde{P}B^T\delta\\
 & * & * & \dots & * & * & \dots & * & * & * & * & * & * & \dots & * & * & * & \tilde{\mathcal{R}}-2\tilde{P}
\end{bmatrix}<0 \quad (10)
$$

where

$$
\tilde{\alpha}_i=\begin{cases}-2D\tilde{P}-2\tilde{Y}+\tilde{Q}_1-\tilde{R}_1 & i=1,\\ \tilde{Q}_i-\tilde{Q}_{i-1}-\tilde{R}_i-\tilde{R}_{i-1}, & i=2\dots N,\\ -\tilde{Q}_N-\tilde{R}_N, & i=N+1.\end{cases} \quad (11)
$$

$$
\tilde{\beta}_i=\begin{cases}\tilde{Q}_1-2\tilde{S}_1 & i=1,\\ \tilde{Q}_i-\tilde{Q}_{i-1}, & i=2\dots N,\\ -\tilde{Q}_N, & i=N+1.\end{cases} \quad (12)
$$

$$
\tilde{\mathcal{R}}=\tilde{R}_1+\tilde{R}_2+\dots+\tilde{R}_N, \quad (13)
$$

$$
\mathrm{L}=diag(l_1,l_2,\dots,l_n). \quad (14)
$$

Proof Let $N>0$ be an integer. We decompose the delay interval $[-\bar{\tau},0]$ into N equidistant subintervals, that is,

$$
[-\bar{\tau},0]=\bigcup_{j=1}^{N}[-j\delta,-(j-1)\delta],
$$

where $\delta=\bar{\tau}/N$. Then choosing different matrix pairs (Q_j, R_j) on $[-j\delta,-(j-1)\delta], (j=1,2,\dots,N)$, we construct the following new LKF:

$$
V(e(t))=V_1(e(t))+V_2(e(t)+V_3(e(t)) \quad (15)
$$

where

$$
V_1(e(t))=e^T(t)Pe(t),
$$

$$
V_2(e(t))=\sum_{j=1}^{N}\int_{-j\delta}^{-(j-1)\delta}\begin{bmatrix}e(t+s)\\ g(e(t+s))\end{bmatrix}^T Q_j\begin{bmatrix}e(t+s)\\ g(e(t+s))\end{bmatrix}ds,
$$

$$
V_3(e(t))=\sum_{j=1}^{N}\delta\int_{-j\delta}^{-(j-1)\delta}\int_{t+\theta}^{t}\dot{e}^T(s)R_j\dot{e}(s)dsd\theta
$$

with $P=P^T>0, Q_j=Q_j^T>0$ and $R_j=R_j^T>0$, $(j=1,2,\dots,N)$.

Taking the derivative of $V(e(t))$ in Eq. 15 with respect to t along the trajectory of Eq. 7 yields

$$
\dot{V}(e(t))=\dot{V}_1(e(t))+\dot{V}_2(e(t))+\dot{V}_3(e(t)). \quad (16)
$$

where

$$
\begin{aligned}\dot{V}_1(e(t))&=2e^T(t)P\dot{e}(t)\\ &=2e^TP[-Ce(t)+Ag(e(t))+Bg(e(t-\tau(t)))]\end{aligned} \quad (17)
$$

$$\dot{V}_2(e(t)) = \sum_{j=1}^{N}[e^T(t-(j-1)\delta)Q_j e(t-(j-1)\delta) - e^T(t-j\delta)Q_j e(t-j\delta) + g^T(e(t-(j-1)\delta))Q_j g(e(t-(j-1)\delta)) - g^T(e(t-j\delta))Q_j g(e(t-j\delta))] \tag{18}$$

$$\dot{V}_3(e(t)) = \sum_{j=1}^{N}\delta^2\dot{e}^T(t)R_j\dot{e}(t) - \sum_{j=1}^{N}\delta\int_{t-j\delta}^{t-(j-1)\delta}\dot{e}^T(s)R_j\dot{e}(s)ds. \tag{19}$$

$$\sum_{j=1}^{N}\delta^2\dot{e}^T(t)R_j\dot{e}(t) = \xi^T(t)[\delta^2\Gamma^T\mathcal{R}\Gamma]\xi(t). \tag{20}$$

where $\mathcal{R} = \sum_{j=1}^{N} R_j$,

$$\xi(t) = [e^T(t) \quad e^T(t-\delta)\ldots e^T(t-N\delta) \quad e^T(t-\tau(t)) \quad g^T(e(t)) \quad g^T(e(t-\delta)) \quad g^T(e(t-2\delta)) \ldots g^T(e(t-N\delta)) \quad g^T(e((t-\tau(t))))]^T \tag{21}$$

$$\text{and } \Gamma = [-C \quad 0 \quad \ldots \quad 0 \quad A \quad 0 \quad \ldots \quad 0 \quad B] \tag{22}$$

We now disclose the interrelationship between $e(t-\tau(t))$ and $e(t), e(t-\delta), \ldots, e(t-N\delta)$ by utilizing the integral terms in Eq. 19. Since $\tau(t)$ is a continuous function satisfying Eq. 8 $\forall \quad t \geq 0$, there should exist a positive integer $k \in \{1, 2, \ldots, N\}$ such that $\tau(t) \in [(k-1)\delta, k\delta]$. In this situation,

$$\begin{aligned} &-\delta\int_{t-k\delta}^{t-(k-1)\delta}\dot{e}^T(s)R_k\dot{e}(s)ds \\ &= -\delta\int_{t-k\delta}^{t-\tau(t)}\dot{e}^T(s)R_k\dot{e}(s)ds - \delta\int_{t-\tau(t)}^{t-(k-1)\delta}\dot{e}^T(s)R_k\dot{e}(s)ds \\ &\leq -[k\delta-\tau(t)]\int_{t-k\delta}^{t-\tau(t)}\dot{e}^T(s)R_k\dot{e}(s)ds \\ &\quad -[\tau(t)-(k-1)\delta]\int_{t-\tau(t)}^{t-(k-1)\delta}\dot{e}^T(s)R_k\dot{e}(s)ds. \end{aligned} \tag{23}$$

Applying Lemma 2.1 to the last two integral terms in Eq. 23 and after simple manipulations, we have

$$-\delta\int_{t-k\delta}^{t-(k-1)\delta}\dot{e}^T(s)R_k\dot{e}(s)ds \leq \eta^T(t)\begin{bmatrix} -2R_k & R_k & R_k \\ & -R_k & 0 \\ & * & -R_k \end{bmatrix}\eta(t), \tag{24}$$

where

$$\eta(t) = [e^T(t-\tau(t)) \quad e^T(t-(k-1)\delta) \quad e^T(t-k\delta)]^T.$$

For $j \neq k$, we also have the following inequality by Lemma 2.1:

$$\begin{aligned} &-\delta\int_{t-j\delta}^{t-(j-1)\delta}\dot{e}^T(s)R_j\dot{e}(s)ds \\ &\leq \begin{bmatrix} e(t-(j-1)\delta) \\ e(t-j\delta) \end{bmatrix}^T \begin{bmatrix} -R_j & R_j \\ & -R_j \end{bmatrix} \begin{bmatrix} e(t-(j-1)\delta) \\ e(t-j\delta) \end{bmatrix}. \end{aligned} \tag{25}$$

Combining Eqs. 24 and 25, we have

$$-\sum_{j=1}^{N}\delta\int_{t-j\delta}^{t-(j-1)\delta}\dot{e}^T(s)R_j\dot{e}(s)ds \leq \xi^T(t)(\Psi)\xi(t). \tag{26}$$

$$\left[\begin{array}{ccccccccccccc}
-R_1 & R_1 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
 & -R_2-R_1 & R_2 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
 & * & -R_3-R_2 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & * & * & \ldots & -R_k-R_{k-1} & 0 & \ldots & 0 & 0 & R_k & 0 & \ldots & 0 \\
 & * & * & \ldots & * & -R_{k+1}-R_k & \ldots & 0 & 0 & R_k & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & * & * & \ldots & * & * & \ldots & -R_N-R_{N-1} & R_N & 0 & 0 & \ldots & 0 \\
 & * & * & \ldots & * & * & \ldots & * & -R_N & 0 & 0 & \ldots & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & -2R_k & 0 & \ldots & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & * & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & \ldots & 0
\end{array}\right]_{(2N+4)\times(2N+4)} \tag{27}$$

where Ψ is given in Eq. 27 (see previous page). From the sector condition, the following inequalities hold

$$-2g^T(e(t))S_1g(e(t)) + 2e^T(t)LS_1g(e(t)) \geq 0. \tag{28}$$

$$\begin{aligned} &-2g^T(e(t-\tau(t)))S_2g(e(t-\tau(t))) \\ &\quad + 2e^T(t-\tau(t))LS_2g(e(t-\tau(t))) \geq 0. \end{aligned} \tag{29}$$

Therefore, using Eqs. 17–26 in Eq. 16 and adding Eqs. 28, 29 to Eq. 16 we have

$$\dot{V}(e(t)) \leq \xi^T(t)[\Phi + \delta^2\Gamma^T\mathcal{R}\Gamma]\xi(t). \tag{30}$$

where Φ is given in Eq. 31, and

matrices $P = P^T > 0, Q_i = Q_i^T > 0, R_i = R_i^T > 0\,(i = 1, 2..., N)$ of appropriate dimensions, such that

$$\dot{V}(e(t)) \leq \xi^T(t)[\Phi + \delta^2\Gamma^T\mathcal{R}\Gamma]\xi(t) \tag{34}$$

$$\leq -(\lambda)e^T(t)e(t) < 0 \quad \forall t \neq 0 \quad \text{with } \lambda > 0. \tag{35}$$

In order to guarantee Eq. 34, we require the following condition

$$[\Phi + \delta^2\Gamma^T\mathcal{R}\Gamma] < 0, \tag{36}$$

which can be written by Lemma 2.2 as

$$\begin{bmatrix} \Phi\delta\Gamma^T & \mathcal{R} \\ & -\mathcal{R} \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix}
\alpha_1 & R_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & PA + S_1^TL & 0 & \ldots & 0 & 0 & PB \\
& \alpha_2 & R_2 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
& * & \alpha_3 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
& * & * & \cdots & \alpha_k & 0 & \cdots & 0 & 0 & R_k & 0 & 0 & \ldots & 0 & 0 & 0 \\
& * & * & \cdots & * & \alpha_{k+1} & \cdots & 0 & 0 & R_k & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
& * & * & \cdots & * & * & \cdots & \alpha_N & R_N & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
& * & * & \cdots & * & * & \cdots & * & \alpha_{N+1} & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
& * & * & \cdots & * & * & \cdots & * & * & -2R_k & 0 & 0 & \ldots & 0 & 0 & S_2^TL \\
& * & * & \cdots & * & * & \cdots & * & * & * & \beta_1 & 0 & \cdots & 0 & 0 & 0 \\
& * & * & \cdots & * & * & \cdots & * & * & * & * & \beta_2 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
& * & * & \cdots & * & * & \cdots & * & * & * & * & * & \cdots & \beta_N & 0 & 0 \\
& * & * & \cdots & * & * & \cdots & * & * & * & * & * & \cdots & * & -\beta_{N+1} & 0 \\
& * & * & \cdots & * & * & \cdots & * & * & * & * & * & \cdots & * & * & -2S_2
\end{bmatrix} \tag{31}$$

$$\alpha_i = \begin{cases} -2PC + Q_1 - R_1 & i = 1 \\ Q_i - Q_{i-1} - R_i - R_{i-1}, & i = 2..N \\ -Q_N - R_N, & i = N+1 \end{cases} \tag{32}$$

$$\beta_i = \begin{cases} Q_1 - 2S_1 & i = 1 \\ Q_i - Q_{i-1}, & i = 2..N \\ -Q_N, & i = N+1 \end{cases} \tag{33}$$

A sufficient condition for synchronization of the master-slave systems described by Eqs. 1 and 2 is that there exist real diagonal matrices S_1, S_2 and positive semi definite

where Φ is defined in Eq. 31.

Equation 37 contains bilinear matrix inequalities, which may not be solved efficiently if used directly. Thus the novel matrix transformation for LMIs is used. Pre and post multiply Eq. 37 with $diag\{P^{-1}, P^{-1}, P^{-1}, \ldots, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, \ldots, P^{-1}, P^{-1}, P^{-1}, \mathcal{R}^{-1}\}$ and applying change of variables $P^{-1}Q_iP^{-1} = \tilde{Q}_i, P^{-1}R_iP^{-1} = \tilde{R}_i, P^{-1}S_1 \quad P^{-1} = \tilde{S}_1, P^{-1}S_2P^{-1} = \tilde{S}_2, KP^{-1} = \tilde{Y}, P^{-1} = \tilde{P}$ and $C = (D + K)$, we get Φ_1 which has represented as an Eq. 38.

$$\left[\begin{array}{cccccccccccccccccc}
\tilde{\alpha}_1 & \tilde{R}_1 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & A\tilde{P}+\tilde{S}_1^T L & 0 & 0 & \ldots & 0 & 0 & B\tilde{P} & -\tilde{P}D^T\delta-\tilde{Y}^T\delta \\
 & \tilde{\alpha}_2 & \tilde{R}_2 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
 & * & \tilde{\alpha}_3 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & * & * & \ldots & \tilde{\alpha}_k & 0 & \ldots & 0 & 0 & \tilde{R}_k & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \\
 & * & * & \ldots & * & \tilde{\alpha}_{k+1} & \ldots & 0 & 0 & \tilde{R}_k & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & * & * & \ldots & * & * & \ldots & \tilde{\alpha}_N & \tilde{R}_N & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
 & * & * & \ldots & * & * & \ldots & * & \tilde{\alpha}_{N+1} & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & -2\tilde{R}_k & 0 & 0 & 0 & \ldots & 0 & 0 & -\tilde{S}_2{}^T L & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & * & \tilde{\beta}_1 & 0 & 0 & \ldots & 0 & 0 & 0 & \tilde{P}A^T\delta \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & \tilde{\beta}_2 & 0 & \ldots & 0 & 0 & 0 & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & * & \tilde{\beta}_3 & \ldots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & * & * & \ldots & \tilde{\beta}_N & 0 & 0 & 0 \\
 & * & * & \text{etc.} & * & * & \ldots & * & * & * & * & * & * & \ldots & * & \tilde{\beta}_{N+1} & 0 & 0 \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & * & * & \ldots & * & * & -2\tilde{S}_2 & \tilde{P}B^T\delta \\
 & * & * & \ldots & * & * & \ldots & * & * & * & * & * & * & \ldots & * & * & * & \mathcal{R}^{-1}
\end{array}\right]_{(2N+5)\times(2N+5)} < 0 \tag{38}$$

It is noted that Eq. 38 is not to be an LMI condition because of the term $\mathcal{R}^{-1}$, which is equal to $\tilde{P}\tilde{\mathcal{R}}^{-1}\tilde{P}$. It can be written as $-\tilde{P}\tilde{\mathcal{R}}^{-1}\tilde{P} \leq \tilde{\mathcal{R}} - 2\tilde{P}$ resulting from $(\tilde{P}-\tilde{\mathcal{R}})^T\tilde{\mathcal{R}}^{-1}(\tilde{P}-\tilde{\mathcal{R}}) = \tilde{P}\tilde{\mathcal{R}}^{-1}\tilde{P} - 2\tilde{P} + \tilde{\mathcal{R}} \geq 0$.

This leads to LMI (10). Considering all possibilities of k in the set $\{1, 2, \ldots, N\}$, we arrive at the conclusion that Eq. 10 holds for all $k \in \{1, 2, \ldots, N\}$. This completes the proof. □

Remark 3.2 Since $f_j(x(\cdot))$ describe the neuron behavior, the results proposed in this paper can be easily applied to Hopfield neural networks ($f_j(x(\cdot)) = \tanh(x(\cdot))$) and to Cellular neural networks ($f_j(x(\cdot)) = 0.5(|x(\cdot)+1| - |x(\cdot)-1|)$) etc., Moreover unlike in Gao et al. (2009) and Hu (2009), we have presented less conservative relaxed sufficient synchronization criterions by removing the constraints $\mu = \dot{\tau}(t) < 1$.

Case II $\tau(t)$ is a differentiable function satisfying Eq. 9. In this case, the derivative of the time-varying delay is available. We will use this additional information to provide a less conservative result. For this goal, we will modify $V(e(t))$ as

$$\hat{V}(e(t)) = V(e(t)) + \int_{t-\tau(t)}^{t} \begin{bmatrix} e(s) \\ g(e(s)) \end{bmatrix}^T S \begin{bmatrix} e(s) \\ g(e(s)) \end{bmatrix} ds, \tag{39}$$

where $S = S^T > 0$ and $\tilde{S} = P^{-1}SP^{-1}$.

Remark 3.3 Notice that for constant time delay case, Theorem 1, can be utilized without using the Eq. 23 (without time-varying delay interrelationship). On the other hand using the information on derivative of delay $\dot{\tau}(t) = \mu = 0$ which is based on case II, synchronization criteria for constant time delay neural networks can be obtained from the following theorem.

Theorem 3.4 *Under case II and hypotheses (A1)–(A2), for given scalars $\tau > 0$ and $\mu > 0$, the master-slave neural networks (1) and (2) are completely synchronized with control gain $K = \tilde{Y}\tilde{P}^{-1}$ if there exist positive definite symmetric matrices $\tilde{P} = \tilde{P}^T = P^{-1} > 0, \tilde{S} = \tilde{S}^T > 0, \tilde{Q}_i = \tilde{Q}_i^T > 0, \tilde{R}_i = \tilde{R}_i^T > 0, (i = 1, 2, \ldots, N)$ and diagonal matrices $\tilde{S}_1 > 0, \tilde{S}_2 > 0$* such that the following LMI holds for all $k \in \{1, 2, \ldots, N\}$,

$$\hat{\Xi} < 0. \tag{40}$$

where $\hat{\Xi} = \Xi + diag\{\tilde{S}, 0, \ldots, 0, -(1-\mu)\tilde{S}, \tilde{S}, 0, \ldots, 0, -(1-\mu)\tilde{S}, \}$ with Ξ is defined in Eq. 10.

Synchronization algorithm

The main aim of the present study is to design a linear error state feedback controller of the form (5) such that the master

(1) and the slave (2) systems synchronized asymptotically. Theorem 3.1 and 3.4 provide new criteria for synchronization which are dependent on the delay. In adequate to the above results, finding the MAUB of $\bar{\tau}$ can be formulated as a optimization problem for the symmetric, positive definite decision variables $\tilde{P}$, $\tilde{Q}_i, \tilde{R}_i, \tilde{S}_1$, and $\tilde{S}_2$ for $i = 1, 2, \ldots, N$, and for all $k \in \{1, 2, \ldots, N\}$. For example, consider the problem of finding MAUB for case I from Theorem 3.1 as

$$\begin{aligned} \max \quad & \bar{\tau} \\ s.t. \quad & \text{LMI (10)}. \end{aligned} \tag{41}$$

If the problem described in Eq. 41 has a feasible solution set for all i, and k, then there is a delay limit $\bar{\tau}$ and the corresponding control gain K exists such that the master (1) and the slave (2) systems synchronized asymptotically. The suboptimal problem can be easily solved by interior point algorithm given in Matlab LMI toolbox or cone-complementary algorithm implemented in YALMIP using SeDuMi solver or any other LMI solvers. In order to obtain the control gain K while maximizing the delay $\bar{\tau}$, an iterative algorithm is presented as follows.

Step 1: Fix the number of decomposition N'. Set $j = 0$, $N = N'$, and $\delta = 0$.
Step 2: Solve the LMI feasibility problem given in Eq. 41 for the positive definite matrices $\tilde{P}, \tilde{Q}_i, \tilde{R}_i, \tilde{S}_1, \tilde{S}_2$, and any matrix $\tilde{Y}$ for $i = 1, 2, \ldots, N$, and for all $k \in \{1, 2, \ldots, N\}$
Step 3: If a feasible solution exists and positive value for δ exists, then $\bar{\tau} = N{*}\delta$ and the control gain is $K = \tilde{Y}\tilde{P}^{-1}$.
Step 4: Set $j = j + 1$. If K and $\bar{\tau}$ are desirable end the process. Else Go to Step 2 by taking $N = N' + 1$.

Numerical examples

In this section, two examples are provided to show the effectiveness of the proposed methods.

Example 4.1 Consider two dimensional neural network with time varying delay given in Gao et al. (2009).

$$\begin{aligned} \dot{x}_i(t) = & -c_i x_i(t) + \sum_{j=1}^{2} a_{ij} g_j(x_j(t)) \\ & + \sum_{j=1}^{2} b_{ij} g_j(x_j(t - \tau(t))). \quad i = 1, 2. \end{aligned} \tag{42}$$

where $c_i = 1, A = (a_{ij})_{2\times 2} = \begin{bmatrix} 2 & -0.1 \\ -5 & 2 \end{bmatrix}, B = (b_{ij})_{2\times 2} = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1.5 \end{bmatrix}$, $g_j(x_j(\cdot)) = \tanh(x_j(\cdot))$. The response system is designed as follows:

$$\begin{aligned} \dot{y}_i(t) = & -c_i y_i(t) + \sum_{j=1}^{2} a_{ij} g_j(y_j(t)) \\ & + \sum_{j=1}^{2} b_{ij} g_j(y_j(t - \tau(t))) + u_i(t). \quad i = 1, 2. \end{aligned} \tag{43}$$

Based on the proposed criteria, the MAUBs of $\bar{\tau}$ of time-varying delay $\tau(t)$ and the corresponding control gain K such that the master (42) and slave (43) systems are synchronized to be determined, which is not dealt in the existing literature. Solving the suboptimal problem given in Eq. 41, the calculated MAUBs and corresponding controller gain matrices for various N are listed in the Table 1 for case 1. When information on derivative of time varying delay is available, using Theorem 3.4 under case 2, calculated MAUBs are given in Table 2 for various values of μ which include $\mu \geq 1$. The state trajectories and phase-space plot of the master system (42) are given in Figs. 1 and 2, respectively. For simulations, initial condition is taken as $\phi(t) = [-1;\ -1.5]^T$ and the time-varying delay is considered as $\tau(t) = 1.3 + 0.48|\sin(t)|$.

In Gao et al. (2009), the authors illustrated the exponential synchronization feasibility of the system (42) and (43) and for a given exponential convergent degree (ECD) $\alpha = 0.6$, the controller gain matrix has been chosen as $K = \begin{bmatrix} 11.6 & 0 \\ 0 & 11.6 \end{bmatrix}$.

The error dynamics between master (42) and slave (43) is plotted for 10 s in Fig. 3. Applying the computed control gain matrix for $N = 6, K = \begin{bmatrix} 10.9142 & -5.3239 \\ -0.3899 & 10.6373 \end{bmatrix}$ to error system, the master and slave systems are synchronized effectively which is given in Fig. 4. Clearly it shows that the control gains are small and the proposed criteria provides efficient design method for linear error state feedback controller to attain synchronization. Computation of desired MAUB of $\bar{\tau}$ depends on the number of decompositions and CPU time (also known as elapsed time in

Table 1 MAUB of $\tau(t)$ and controller gains for Example 1

N	MAUB	CPU time	K
2	0.9750	11.42	$\begin{bmatrix} 6.7596 & -2.8921 \\ -0.1659 & 6.3865 \end{bmatrix}$
3	1.2936	18.89	$\begin{bmatrix} 7.8202 & -3.5507 \\ -0.2939 & 7.7009 \end{bmatrix}$
4	1.5544	67.01	$\begin{bmatrix} 9.2343 & -4.3795 \\ -0.3376 & 9.1301 \end{bmatrix}$
5	1.7750	128.36	$\begin{bmatrix} 10.9142 & -5.3239 \\ -0.3899 & 10.6373 \end{bmatrix}$
6	1.9686	177.37	$\begin{bmatrix} 12.7158 & -6.3187 \\ -0.4436 & 12.2130 \end{bmatrix}$

Table 2 MAUB of $\tau(t)$ for Example 1 with various values of μ

N	$\mu = 0$	$\mu = 0.5$	$\mu = 0.7$	$\mu \geq 1$
2	1.5176	1.4420	1.3760	0.9750
3	2.2764	2.1630	2.0640	1.2936
5	3.7940	3.6050	3.4400	1.7750

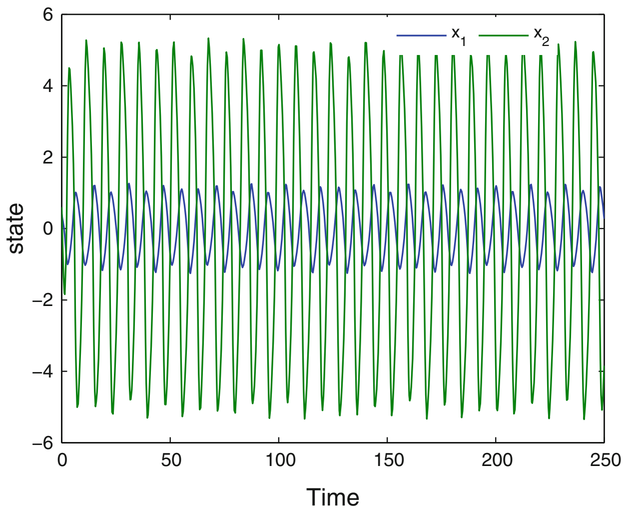

Fig. 1 State Trajectories of Master system (42)

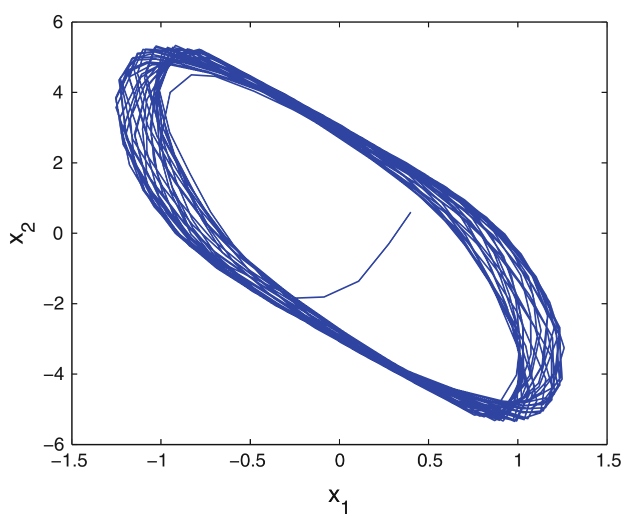

Fig. 2 Phase space portrait of Master system (42)

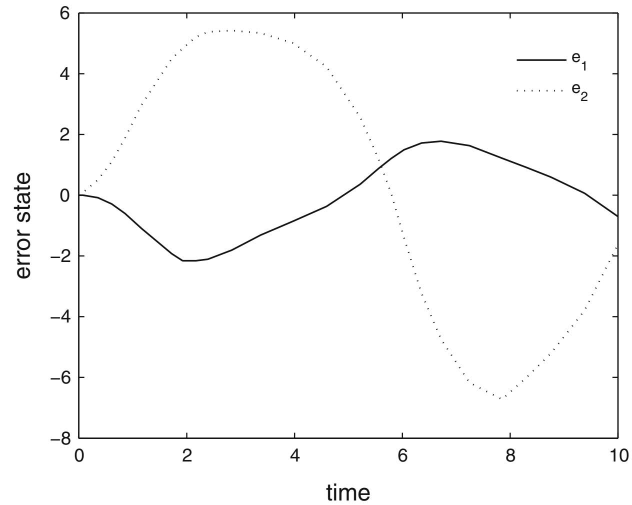

Fig. 3 Error trajectory between Eqs. 42 and 43 with out any control

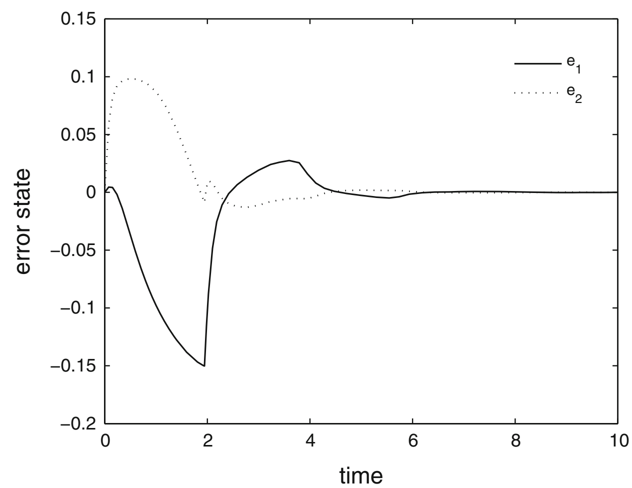

Fig. 4 Error trajectory after applying the controller. Plot shows Eqs. 42 and 43 are synchronized

Matlab) to solve the LMIs. When number of delay decomposition increases (N), MAUB grows larger and attains its analytical delay limit (Zhang and Han 2009). For this example, up to $N = 15$, the calculated MAUBs are plotted against the decompositions which is given in Fig. 5.

Example 4.2 Consider the example given in Hu (2009). A two dimensional nonlinear neural networks with time varying delays is given by

$$\dot{x}_i(t) = -\gamma_i(x_i(t)) + \sum_{j=1}^{2} a_{ij} g_j(x_j(t)) + \sum_{j=1}^{2} b_{ij} g_j(x_j(t - \tau(t))) + J_i. \quad i = 1, 2. \tag{44}$$

where $\gamma_1(v) = 2v - \sin(v)$, $\gamma_2(v) = 2v + \sin(v), J_1 = J_2 = 0$, and $A = (a_{ij})_{2\times 2} = \begin{bmatrix} 2.1 & -0.12 \\ -5.1 & 3.2 \end{bmatrix}$, $B = \begin{bmatrix} -1.6 & -0.1 \\ -0.2 & -2.4 \end{bmatrix}$, $g_j(x_j(\cdot)) = \tanh(x_j(\cdot))$ respectively. Clearly it is seen that $\gamma_1(v)$ and $\gamma_2(v)$ satisfy **A1** (Hu 2009). Similarly, applying the procedure followed in this paper, Table 3 presents

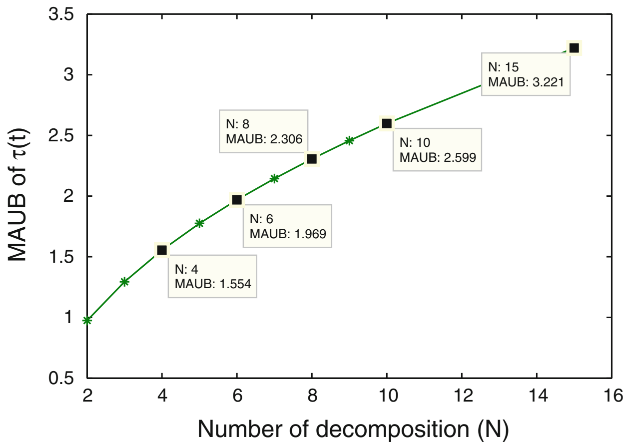

Fig. 5 *N* versus *MAUB* plot for Example 1

Table 3 MAUB of $\tau(t)$ and controller gains for Example 2

N	MAUB	CPU time	K
2	0.7056	10.08	$\begin{bmatrix} 14.8987 & -3.2481 \\ -0.3909 & 11.4185 \end{bmatrix}$
3	0.9051	22.12	$\begin{bmatrix} 19.8651 & -3.9132 \\ -0.4952 & 14.3872 \end{bmatrix}$
4	1.0724	52.83	$\begin{bmatrix} 24.8487 & -4.5732 \\ -0.6048 & 17.3627 \end{bmatrix}$
5	1.2185	120.55	$\begin{bmatrix} 29.7319 & -5.2229 \\ -0.7130 & 20.3627 \end{bmatrix}$
6	1.3494	248.75	$\begin{bmatrix} 34.5689 & -5.8055 \\ -0.8618 & 23.3669 \end{bmatrix}$

MAUB of time varying delay and controller gain matrices found by solving LMIs. For numerical simulations, consider $\tau(t) = 1$ and $\phi(t) = [-1; \ -1.5]^T$. The simulations are presented in Figs. 6, 7 and 8. Figure 8, clearly depicts that the master and slave systems are synchronized with the control applied.

Conclusion

In this paper, the problem of synchronization condition for chaotic nonlinear continuous neural networks has been studied. Applying the delay decomposition approach, a new synchronization criterion have been given in terms of LMIs, which is dependent on the size of the time delay. The delay decomposition, delay-dependent synchronization analysis for chaotic nonlinear continuous neural networks are new and novel. In this paper, the determined the MAUBs for $\bar{\tau}$ of delay $\tau(t)$ and corresponding controller gain matrices, which is not dealt in the existing literature. So we unable to compare the numerical results with other research papers. Finally, two numerical examples have

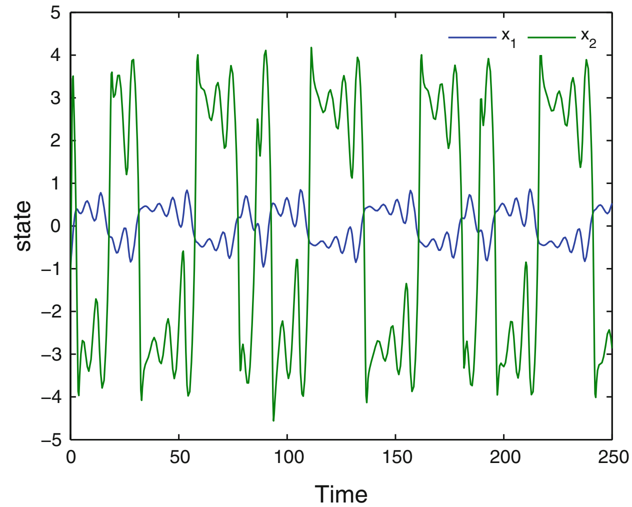

Fig. 6 State trajectory of system (44)

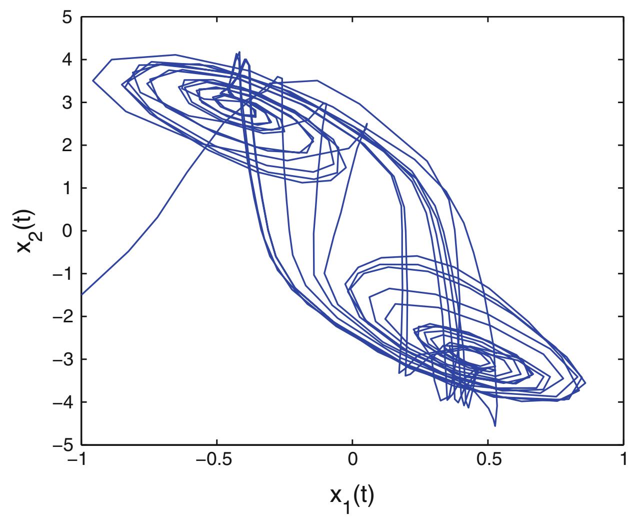

Fig. 7 Phase-space portrait of Eq. 44

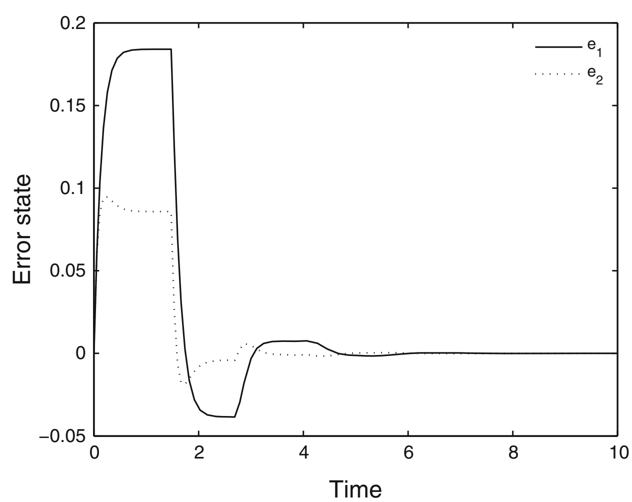

Fig. 8 Synchronized error trajectory after applying state feedback control for Example 2

been presented which illustrate the effectiveness and usefulness of the proposed method.

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