BEHAVIORAL MOMENTUM THEORY: EQUATIONS AND APPLICATIONS

JOHN A. NEVIN

UNIVERSITY OF NEW HAMPSHIRE

AND

TIMOTHY A. SHAHAN

UTAH STATE UNIVERSITY

Behavioral momentum theory provides a quantitative account of how reinforcers experienced within a discriminative stimulus context govern the persistence of behavior that occurs in that context. The theory suggests that all reinforcers obtained in the presence of a discriminative stimulus increase resistance to change, regardless of whether those reinforcers are contingent on the target behavior, are noncontingent, or are even contingent on an alternative behavior. In this paper, we describe the equations that constitute the theory and address their application to issues of particular importance in applied settings. The theory provides a framework within which to consider the effects of interventions such as extinction, noncontingent reinforcement, differential reinforcement of alternative behavior, and other phenomena (e.g., resurgence). Finally, the theory predicts some counterintuitive and potentially counterproductive effects of alternative reinforcement, and can serve as an integrative guide for intervention when its terms are identified with the relevant conditions of applied settings.

Key words: behavioral momentum theory, resistance to change, extinction, alternative reinforcement, relapse

———————————

Clinical practitioners often are faced with the task of reducing or eliminating some form of problem behavior (e.g., aggression, disruption, or self-injury). In many cases, the problem behavior has an extensive history and may be extraordinarily resistant to treatment. A standard approach is to conduct a functional analysis (Iwata et al., 1994) to identify reinforcers that maintain problem behavior, such as attention or escape from tasks, and then to withhold reinforcers for problem behavior while functionally equivalent reinforcers are provided for adaptive alternative behavior (e.g., effective communication skills).

If the intervention is truly effective, problem behavior should decrease and be replaced by adaptive behavior, and these changes should persist in different treatment settings and endure when treatment ends. Behavioral momentum theory (BMT) is principally concerned with the resistance to change of operant behavior, and therefore is directly relevant to the reduction or elimination of problem behavior and the establishment of durable adaptive behavior.

Following Thorndike (1911, 1913), BMT asserts that reinforcers increase the strength of a response in the presence of a stimulus, where *strength* is defined not as asymptotic response rate or probability but as resistance to change. BMT also asserts that the resistance to change of a target response in the presence of a stimulus depends directly on the rate or amount of reinforcement signaled by that stimulus (i.e., the stimulus–reinforcer relation), regardless of whether all reinforcers are contingent on the target response. Some data in support of the second assertion will be presented below.

In addition to the everyday language statements above, BMT consists of a set of equations that provide a quantitative structure within which the variables that affect resistance to change can be represented in relation to each other. This representation provides a model for understanding and interpreting data from basic and applied analyses; it is presented as a set of equations for clarity and precision in order to avoid the ambiguities of ordinary language. In basic research and theory, the model may be evaluated by the extent to which large data sets obtained under controlled conditions can be described by equations with relatively few parameters. In applied work, the model's principal value is to serve as an integrative guide for analysis and intervention. Critchfield and Reed (2009) made similar points about the value of quantitative models in translational applications, and noted that applications of BMT may be hampered by the lack of a single standard equation during its development. Here we attempt to remedy that deficiency. Equation 2, presented below, provides the present basis from which theory and application may be developed; like all theoretical expressions, it may be modified in light of new data or adapted for application to new paradigms.

THE METAPHOR OF
BEHAVIORAL MOMENTUM

The metaphor is based on Newton's second law of motion: When an external force is applied to an object in motion, the change in velocity is related directly to the magnitude of the force and is related inversely to the object's inertial mass. Translated into behavioral terms: When a disrupter is applied to ongoing behavior, the decrease in response rate is related directly to the magnitude of the disrupter and is related inversely to the behavioral equivalent of mass. Just as inertial mass is independent of the velocity of a physical body in motion, behavioral mass is independent of response rate, both conceptually and empirically (see examples below). The equation is

$$\Delta B = \frac{-x}{m}, \tag{1}$$

where ΔB is the change in response rate, x represents the value of a current disrupter, and m represents behavioral mass, determined by the subject's history of reinforcement. The sign of x is negative to indicate that disruptions decrease response rate.

In theoretical development of the model and its application to quantitative data, ΔB is expressed as the difference between asymptotic response rate, B_o, and response rate during disruption, B_x, where response rates are transformed to logarithms. Thus, $\Delta B = \log(B_x) - \log(B_o)$ or equivalently, via the rules of logarithms, $\log(B_x/B_o)$, the log proportion of baseline. Behavioral mass m depends on the reinforcer rate in effect during steady-state determination of baseline responding according to a power function. Fits to parametric data sets suggest that $m \sim r^{0.5}$, where r is reinforcer rate (Nevin, 2002). Therefore, Equation 1 becomes

$$\log\left(\frac{B_x}{B_o}\right) = \frac{-x}{r^{0.5}}. \tag{2}$$

The change in behavior, ΔB, is expressed as log proportion of baseline for three reasons. First, logarithms transform equal changes in x (the disrupter) into equal effects on response rate. Second, log proportions have desirable metric properties. Third, transforming response rates during disruption to proportions of baseline facilitates comparison of data from different conditions or individuals by removing differences in the levels of baseline responding. The first and second considerations are not likely to be important in applications; the third, however, is important for illuminating the effects of different schedules and disrupters, and we will use proportions of baseline to illustrate the predictions of Equation 2 and its extensions throughout this paper. As will be shown, the level of baseline responding does not

affect resistance to change measured as proportion of baseline.[1]

To apply Equation 2 to proportions of baseline rather than log proportions, it must be "antilogged" or exponentiated as in Equation 3.[2] Although its form makes it harder to grasp intuitively, Equation 3 is necessarily equivalent to Equation 2, in that proportion of baseline response rate during disruption is an increasing function of r and a decreasing function of x.

$$\frac{B_x}{B_o} = 10^{\left(\frac{-x}{r^{0.5}}\right)} \qquad (3)$$

----

[1] The qualitative assertion that resistance to change depends on the stimulus–reinforcer relation suggests that the reinforcer rate signaled by a stimulus should be expressed relative to the reinforcer rate in the context within which that stimulus is presented. Because the reinforcement value of the contextual environment rarely is specified or controlled in applied analyses, we will assume that it can be represented by a unit constant and thus will not appear in the equations developed in the remainder of this paper. For consistency with quantitative summaries of basic research (e.g., Nevin, 1992), we develop the model's treatment of context here.

If the context is identified with the overall average reinforcer rate in the experimental setting, designated r_a, Equation 2 becomes

$$\log\left(\frac{B_x}{B_o}\right) = \frac{-x}{(r_s/r_a)^{0.5}} \qquad (2a)$$

where x must be dimensionless. The ratio in the denominator, r_s/r_a, expresses the strength of the stimulus–reinforcer relation and is equivalent to the contingency ratio proposed by Gibbon (1981) as a determiner of autoshaped responding. In qualitative terms, if r_s is greater than r_a, the stimulus signals an increase in reinforcement relative to the context, and if it is less than r_a, it signals a decrease. Thus, resistance to change depends directly on the value of r_s relative to r_a (i.e., the strength of the stimulus–reinforcer relation). However, if r_a is constant across conditions with different values of r_s, as in many experiments and most applications, its value is absorbed into x so that Equation 2a is identical to Equation 2.

[2] A numerical example may clarify exponentiation. If $B_x = 50$ and $B_o = 100$, $\log_{10}(B_x/B_o) = -0.30$. To convert to the proportion of baseline B_x/B_o, the base 10 is raised (exponentiated) to the power -0.30, yielding 0.50.

Figure 1 shows simulations of Equations 2 and 3 that illustrate the effects of varying the magnitude of the disrupter x for three values of reinforcer rate r. Simulations are presented as log proportion of baseline (Equation 2) on the left and simple proportions of baseline (Equation 3) on the right. In the left panel, when $x = 0$ (i.e., baseline), log proportion of baseline is 0 and decreases linearly as x increases. The slope depends on the rate of reinforcement r, being steep when reinforcer rate is small (10 per hour) and shallow when it is large (160 per hour). The right panel shows that proportion of baseline decreases from 1.0 toward 0 as x increases, where the rate of decrease is greatest when r is small and least when r is large. These panels illustrate the basic prediction of BMT: The relative decrease in responding produced by a disrupter is a direct function of the magnitude of the disrupter x and an inverse function of the reinforcement rate r prior to disruption. In other words, bigger disrupters decrease behavior more, but the behavior-decreasing effects of a disrupter are lessened by higher rates of reinforcement. The Appendix provides illustrative calculations.

MODELING THE EFFECTS OF DISRUPTERS

Laboratory studies with several species including humans and employing a wide variety of responses, reinforcers, and disrupters have found that reductions in steady-state baseline response rate depend directly on the magnitude of the disrupter and inversely on the rate or amount of reinforcement in accordance with Equation 3 (for review, see Nevin, 1979, 1992; Nevin & Grace, 2000). Most studies have arranged different reinforcer rates or amounts according to variable-interval (VI) schedules, in which obtained reinforcer rates are largely independent of response rates. Also, most studies have employed multiple schedules in which successive components with different rates or amounts of reinforcement are signaled
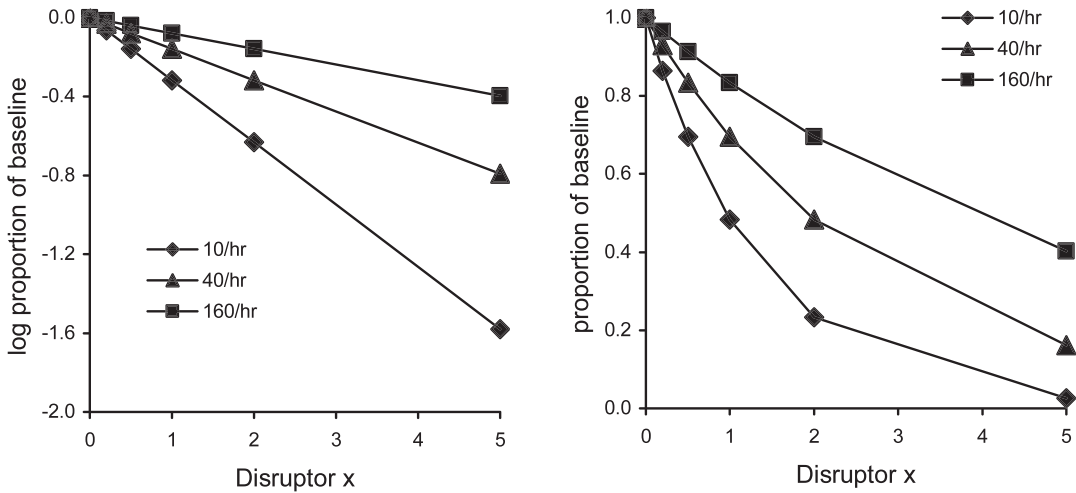
Figure 1.   Simulated predictions of Equations 2 (left) and 3 (right) for the effects of a disrupter (x) at three values of reinforcers per hour (r). Note that as x increases, log proportions of baseline decrease linearly with no lower bound, whereas simple proportions decrease according to nonlinear (exponential) functions with 0 as the lower limit. Note also that for a given value of x, decreases are related inversely to the value of r.

by distinctive stimuli and alternate frequently so that the effects of disrupters can be evaluated within subjects and sessions.

Equation 3 also characterizes the data of translational research that has arranged multiple VI VI schedules with people with developmental disabilities. Working with two adults in a group home, Mace et al. (1990, Study 1) arranged VI 60-s (60 per hour) and VI 240-s (15 per hour) schedules for sorting red and green dinnerware in a paradigm analogous to multiple schedules, where dinnerware color signaled the component reinforcer rate. (For convenience, we will designate the component with more frequent reinforcers as *rich* and the component with less frequent reinforcers as *lean.*) Figure 2 (left) shows that baseline sorting rates were similar for both components, and that sorting rates were substantially lower when the participants watched a favorite TV program; the decrease was greater in the lean component for both participants. Figure 2 (right) shows the disruptive effect of the video expressed as proportions of baseline. In terms of the momentum metaphor, the behavioral mass of sorting was smaller in the lean component than

in the rich component. Therefore, although the video presentation was the same for both components, its disruptive effect was greater in the lean component.

Parry-Cruwys et al. (2011) arranged VI 7-s (514 per hour) and VI 30-s (120 per hour) schedules for various activities for children in a special education classroom, in which the nature of the activity indicated the component reinforcer rate. Figure 3 (left) shows that the rich schedule maintained higher baseline activity rates than the lean schedule for some participants, and lower rates for others. Figure 2 (right) shows that, regardless of the magnitude or direction of differences in baseline rates, the disruptive effects of a video presentation were greater (i.e., proportion of baseline response rates were lower) for the activity maintained by the leaner schedule for all but one participant. These data confirmed the findings of Mace et al. (1990), in that proportions of baseline depended on the component reinforcer rates and not on the baseline response rates.

Figure 4 shows that the overall trend in the data of Mace et al. (1990) and Parry-Cruwys et al. (2011) can be described reasonably well by
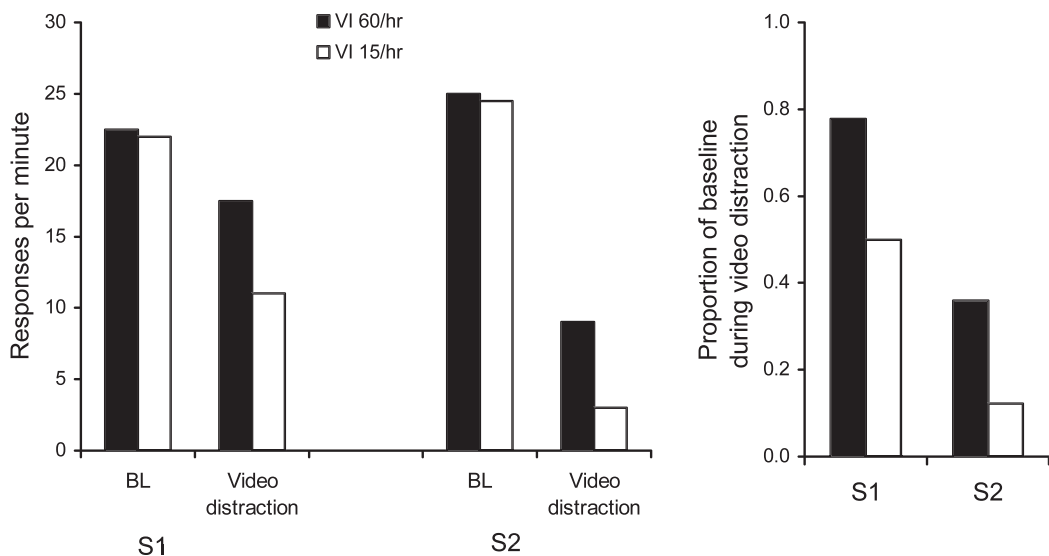
Figure 2. Data of Mace et al. (1990, Study 1) for two adult residents of a group home who received reinforcers for sorting dinnerware according to VI schedules signaled by dinnerware color. The left panel displays sorting rates during baseline and during disruption by a video presentation for both subjects. The right panel displays sorting rates during disruption as proportions of baseline.

Equation 3 with $x = 2.2$, a value chosen by eye to approximate the mean proportions of baseline (indicated as + in the figure). As the rate of reinforcement r increases, so does responding during disruption as a proportion of baseline (i.e., the disrupter has a progressively smaller effect). Intersubject variation within each study precludes evaluation of differences between studies (e.g., the tasks employed or the nature of the video disrupter).
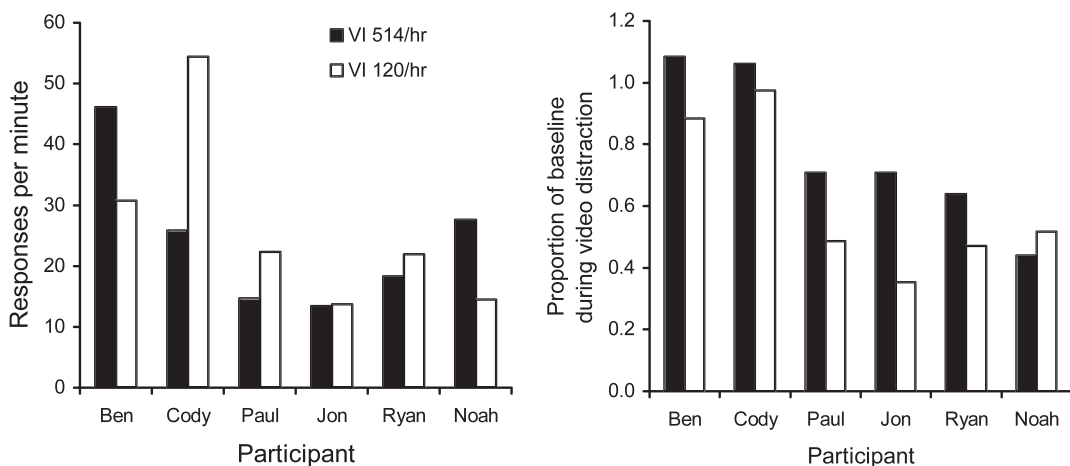

Figure 3. Data of Parry-Cruwys et al. (2011) for six children with developmental disabilities who received reinforcers for engaging in various tasks according to VI schedules signaled by the nature of the task. The left panel presents baseline rates of engagement in the tasks, and the right panel presents proportions of baseline response rates during disruption by a video presentation.
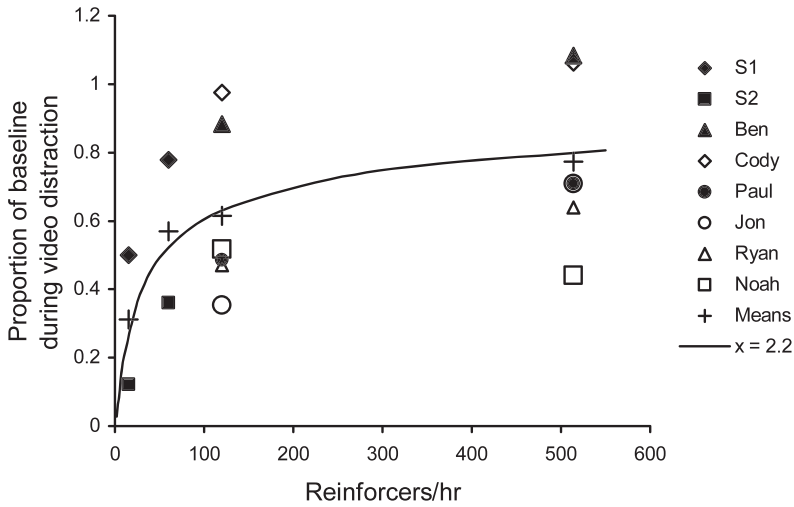
Figure 4. The disruption data of all subjects from Figures 2 and 3, expressed as proportions of baseline, as a function of the reinforcement rate during baseline; averages are shown as + symbols. The solid line shows the predictions of Equation 3 with $x = 2.2$.

External distractors are valuable as disrupters in probe tests for resistance to change because they leave baseline schedule contingencies intact and are highly relevant to educational settings in which resistance to disruption is important for effective instruction. However, when dealing with clinically significant problem behavior, distractors may not be effective in the long term because they depend on novelty. To produce large and ideally durable decreases in problem behavior, interventions often arrange extinction after a functional analysis has identified an effective reinforcer that can be withheld during treatment. Although most interventions also include other treatment components in addition to extinction, we begin by examining the simpler case by extending Equation 3 to disruption produced by extinction alone. In subsequent sections, we will extend the model to more complex interventions that involve alternative reinforcement.

MODELING RESISTANCE TO EXTINCTION

To characterize extinction as a disrupter in the basic momentum equation, we note that three things happen when reinforcement is discontin-

ued. First, the contingency between responses and reinforcers is suspended, and responding may decrease as a result. Second, the environment no longer includes reinforcers (as stimuli), and responding may decrease due to generalization decrement. Third, time passes, and the effects of contingency suspension and generalization decrement are assumed to increase with the passage of time. To incorporate these aspects of extinction, the numerator of the exponent of Equation 3 is restated as $-t(c+dr)$, where c scales the effects of suspending the contingency, d scales the effects of removing r reinforcers per unit time from the environment, and t represents time or sessions of extinction. Nevin, McLean, and Grace (2001) validated this model of extinction in a series of multiple-schedule experiments with pigeons as subjects. They evaluated c by arranging noncontingent reinforcement (NCR), and then showed that c and d were independent and combined additively during extinction.

Thus, the equation for resistance to extinction, expressed as proportions of baseline, is

$$\frac{B_t}{B_o} = 10^{\left(\frac{-t(c+dr)}{r^{0.5}}\right)} \qquad (4)$$
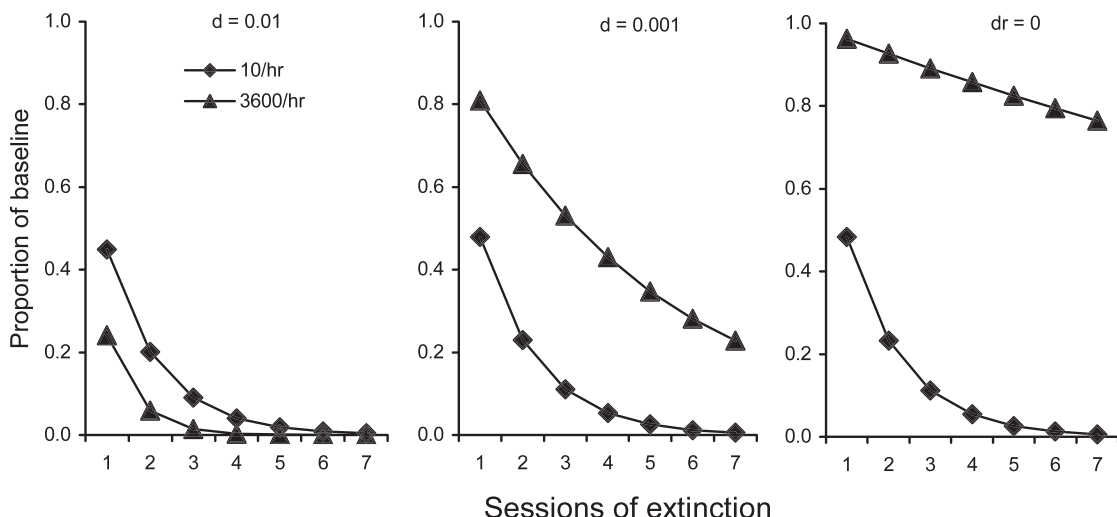
Figure 5. Extinction curves predicted by Equation 4 after training with two different reinforcer rates. In the left panel, the omission of reinforcers is assumed to be relatively discriminable ($d = 0.01$), and extinction proceeds more rapidly after training with the richer schedule. In the center panel, the discriminability of reinforcer omission is assumed to be relatively poor ($d = 0.001$), and extinction proceeds more rapidly after training with the leaner schedule. The right panel illustrates the expected effects of presenting reinforcers noncontingently at the same rate as in training so that $dr = 0$.

where B_t is response rate at time t in extinction and B_o is baseline response rate. By inspection, the dr term has little effect when d is small and reinforcer rate is low, but becomes important when d is large and reinforcer rate is high while c remains constant with respect to reinforcer rate. These properties of Equation 4 are illustrated in Figure 5 for two reinforcer rates: a very rich schedule (3,600 renforcers per hour), approximating continuous reinforcement [CRF]), and a very lean schedule (10 reinforcers per hour, equivalent to VI 6 min). The simulation in Figure 5 uses arbitrary values of d to illustrate extreme cases. When reinforcer omission is highly distinctive (represented as $d = 0.01$ in the left panel), extinction proceeds more rapidly when $r = 3,600$ per hour. These extinction curves exemplify the pervasive partial reinforcement extinction effect (PREE), in which extinction is slower after intermittent reinforcement (PRF) than after CRF. When reinforcer omission is less salient ($d = 0.001$, center panel), the extinction curves reverse, in that slope is shallower when $r = 3,600$ per hour than when $r = 10$ per hour. When the target

response has no consequences but reinforcers are delivered independently of the response at the same rate as in training, the dr term goes to 0, and the effect of contingency suspension c is smaller when $r = 3,600$ per hour than when $r = 10$ per hour (right). The Appendix provides illustrative calculations.

The following example suggests some factors that might determine the value of d in applied settings. Koegel and Rincover (1977) evaluated the effects of extinction in a distinctively different setting after treatment with reinforcement (food plus praise) for simple tasks (e.g., clap hands) with children with autism. Training with CRF, fixed-ratio (FR) 2, or FR 5 was conducted in a therapy room, and extinction testing was conducted outdoors. Responding in the extinction setting decreased rapidly after CRF or FR 2, but persisted for many trials after FR 5, exemplifying the PREE. These ordinal results are predicted by Equation 4 with relatively large d for the simultaneous omission of distinctive reinforcers (food plus praise) and the change from the therapy room to outdoors. With different participants, Koegel and Rincover also examined

the effects of presenting noncontingent reinforcers (NCR) every 20th trial during extinction after FR 2, and found that responding persisted for many more trials than after comparable training without NCR during extinction. They suggested that presenting reinforcers during extinction would make the posttreatment environment more similar to that during treatment. In terms of Equation 4, the presentation of NCR would reduce the value of *dr* because the extinction environment includes some reinforcers, leading to more prolonged extinction responding.

The value of *d* also might be relatively small in applied settings in which social reinforcers such as attention are employed during intervention and extinction follows without interruption after training. With no change in the environment or in the therapist's behavior other than withholding attention, a reverse PREE of the sort illustrated in Figure 5 (middle) might be obtained; this is the typical result in basic research with nonhuman subjects for extinction after training on multiple VI VI schedules with different reinforcer rates in the components. If attention was given independently of the participant's behavior, the value of *dr* would be zero and the attendant reduction in responding would be ascribed to the change from contingent reinforcement and captured by the parameter *c* in Equation 4. Nevin et al. (2001) found that when multiple VI VI schedules were changed to variable-time (VT) schedules that presented noncontingent reinforcers at the same rates as in training, response rate decreased less in the richer component, consistent with the simulation in Figure 5 (right). We now consider the effects of NCR alone or in combination with extinction.

EFFECTS OF ALTERNATIVE REINFORCEMENT

When problem behavior cannot be reduced by extinction, either because it might be exacerbated (e.g., for aggression) or because the reinforcer cannot be withheld (e.g., for self-stimulation), interventions often arrange alternative reinforcement. We concentrate on differential reinforcement of specified alternative behavior (DRA) and NCR.[3] These alternative reinforcement contingencies unquestionably are effective: Data summaries by Iwata et al. (1994) and Asmus et al. (2004) reported 80% to 90% decreases in a wide variety of problem behaviors in a large majority of participants. However, basic research with simple operant behavior has found that when alternative reinforcement is employed to reduce baseline response rates, responding becomes more resistant to subsequent interventions that include extinction, regardless of whether alternative reinforcers are presented independently of responding (NCR) or contingent on a defined alternative response (DRA).

This effect of alternative reinforcement on resistance to change has been found in studies with goldfish, pigeons, rats, and humans and with a variety of reinforcers (e.g., Cohen, 1996; Grimes & Shull, 2001; Harper, 1999; Igaki & Sakagami, 2004; Pyszczynski & Shahan, 2011; Shahan & Burke, 2004). It is exemplified by Experiment 1 of Nevin, Tota, Torquato, and Shull (1990). In Condition 1, pigeons were trained on a two-component multiple schedule with VI 1-min (60 reinforcers per hour) schedules in both components. In Condition 2, response-independent reinforcers (NCR) were presented at variable times averaging 30 s (VT 30 s; 120 reinforcers per hour) in one of the components. Figure 6 (left) shows that in Condition 1, baseline response rates were about equal, and that in Condition 2, response rate was moderately lower in the VI+VT component (all subjects exhibited this difference). Figure 6 (right) shows that response rate was substantially more resistant to extinction, relative to baseline, in the VI+VT component. Thus, although NCR reduced the

---

[3]Although differential reinforcement of other behavior is an effective contingency of alternative reinforcement, we do not consider it here because the obtained reinforcer rate depends inversely on the participant's response rate and thus cannot serve as an independent term in the momentum equations.
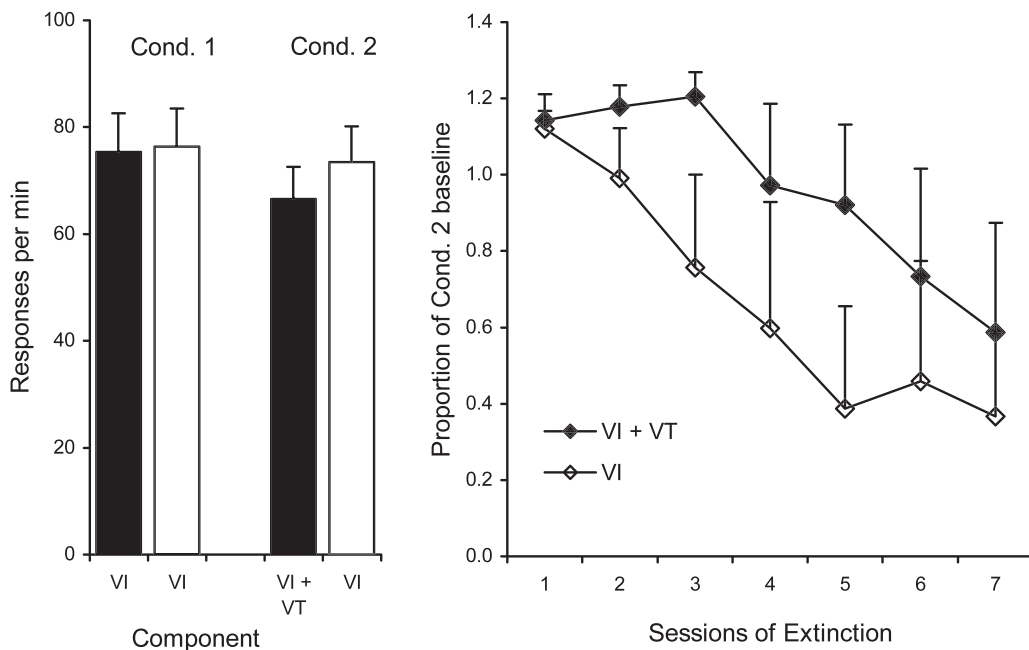
Figure 6. Data of Nevin et al. (1990, Experiment 1), averaged for three pigeons; error bars represent standard errors. The left panel shows baseline response rates in multiple-schedule components with equal VI schedules (Condition 1) and the effects of adding NCR in one component (Condition 2); the right panel shows extinction responding in both components expressed as proportions of baseline.

rate of a target response, the added reinforcers in the discriminative stimulus context also increased subsequent resistance to extinction.

Mace et al. (1990, Study 2) arranged similar multiple-schedule contingencies with two adults with developmental disabilities who lived in a group home. The task was sorting red and green dinnerware, where dinnerware color signaled the component reinforcer rates and contingencies as in Study 1 described above (see Figure 2). One color signaled a standard VI 60-s schedule (60 per hour), and the other signaled an identical VI 60-s schedule with added reinforcers presented noncontingently according to a VT 30-s schedule (120 per hour). Figure 7 (left) shows that, although baseline response rates were lower in the VI+VT component, response rates were higher in that component during disruption with a video presentation. Figure 7 (right) shows resistance to disruption by the video as proportions of baseline and shows that, as in Experiment 1 of Nevin et al. (1990; Fig. 6), added

response-independent reinforcers increase resistance to change.

Experiment 2 by Nevin et al. (1990) arranged concurrent reinforcement for a defined alternative response (DRA) rather than NCR. Pigeons were trained in a three-component multiple schedule with concurrent VI 240 s (15 per hour) VI 80 s (45 per hour) on the right and left keys in Component A; VI 240 s (15 per hour) and extinction on the right and left keys in Component B; and VI 60 s (60 per hour) and extinction on the right and left keys in Component C. Thus, Components A and B arranged the same rate of reinforcement for a target behavior (i.e., pecking the right key), but Component A also included additional reinforcement for an explicit alternative behavior (i.e., pecking the left key). Like Component B, Component C included reinforcement for only the target behavior, but at a rate that equaled the sum of reinforcer rates for the two behaviors in Component A. Figure 8 (left) shows that
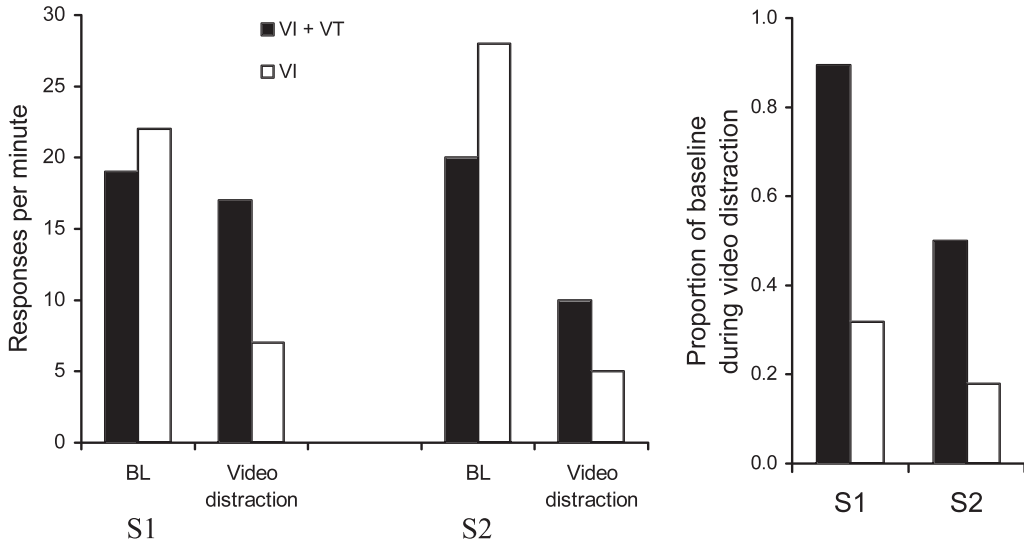
Figure 7.   Data of Mace et al. (1990, Study 2) for two adult residents of a group home who received reinforcers for sorting dinnerware according to equal VI schedules, in which one dinnerware color also signaled NCR. The left panel displays sorting rates during baseline and during disruption by a video presentation for both subjects. The right panel displays sorting rates during disruption as proportions of baseline. The schedules are the same as those arranged by Nevin et al. (1990, Experiment 1; see Figure 6).

during baseline, target (i.e., right key) response rates were substantially lower in Component A, in which DRA reinforcement was available, than in Component B, in which DRA reinforcement was absent. Thus, the addition of DRA reinforcement in Component A was effective in decreasing the rates of the target behavior. Response rates were highest in Component C, in which the highest rates of reinforcement were provided for the target behavior in the absence of DRA. Figure 8 (right) shows that, when the data are expressed as proportions of baseline, resistance to extinction of the target behavior was greater in Component A than in Component B, replicating the findings of Experiment 1, in which NCR was provided in one component. Resistance to extinction in Component C was similar to that in Component A, which provided the same overall rate of reinforcement. Thus, DRA increased resistance to extinction of the target behavior because it increased the overall rate of reinforcement obtained in the discriminative stimulus context.

Recent research has shown similar effects of DRA on resistance to extinction in applied settings. Mace et al. (2010) employed DRA contingencies to reduce problem behaviors (i.e., hair pulling, food stealing, and aggression) in three children with developmental disabilities. After baseline determinations of the frequency of problem behavior, all three children were exposed to treatment with DRA followed by extinction and also to extinction directly after baseline without DRA treatment; the order of conditions varied across children. For example, during baseline, Andy received verbal reprimands for hair pulling. During DRA, reprimands for hair pulling continued, and in addition, Andy was prompted to play with toys and received praise for doing so. During extinction, hair pulling was blocked and all reinforcers were withheld. After baseline conditions were restored, extinction was repeated with no intervening DRA treatment. As shown in Figure 9 (top left), the frequency of hair pulling was lower during treatment with DRA
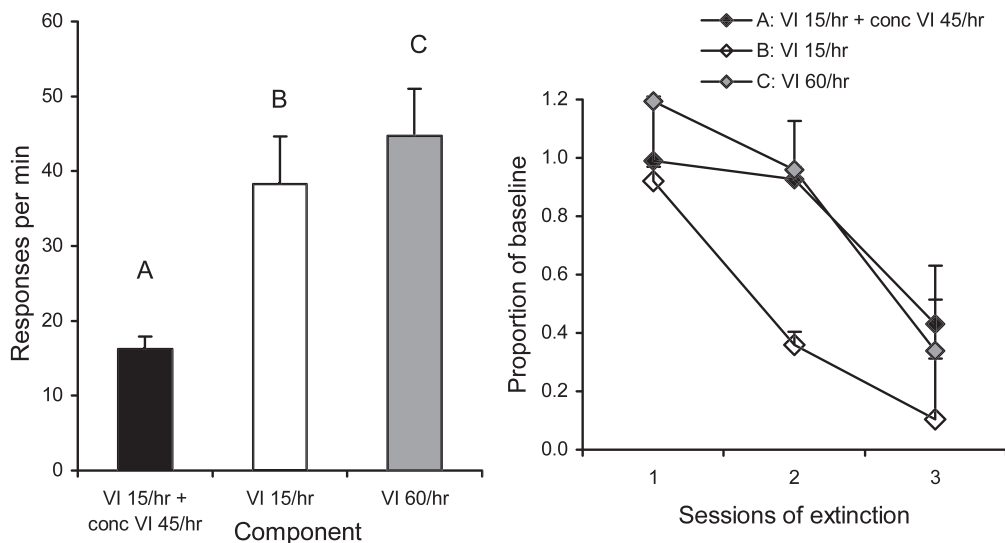
Figure 8. Data from Nevin et al. (1990, Experiment 2), averaged for three pigeons; error bars represent standard errors. The left panel shows response rates on a target key in components with DRA (left column, A), no DRA (center column, B), or the same total reinforcer rate as with DRA (right column, C). The right panel shows the persistence of target responding during extinction in these three components. See text for further explanation.

than in baseline only. Figure 9 (top right) shows that when extinction directly followed baseline, hair pulling decreased more rapidly and to a lower level than when it followed treatment with DRA, where extinction responding is expressed as a proportion of the immediately preceding baseline. Figure 9 (middle and bottom) shows that comparable results were obtained for the other two participants. Note that in many sessions, extinction responding after DRA was substantially greater than during the intervention with DRA (i.e., proportions of baseline were often greater than 1.0). Thus, although differential reinforcement of desirable alternative behavior generally decreased rates of problem behavior, it increased resistance to extinction, as in Experiment 2 by Nevin et al. (1990; Figure 8).

MODELING THE EFFECTS OF ALTERNATIVE REINFORCEMENT

During training with reinforcement for a target response, the effects of alternative reinforcement may be analogous to the effects of disruption in the studies by Mace et al. (1990) and Parry-Cruwys et al. (2011). However, the disruptive effects of visual distracters are likely to be short lived, whereas the disruptive effects of alternative reinforcement on a target response are likely to persist as long as those reinforcers are presented. To account for the effects of disruption by alternative reinforcers, we replace the generic disrupter parameter x in Equation 3 with pR_a:

$$\frac{B_x}{B_o} = 10^{\left(\frac{-pR_a}{(r+R_a)^{0.5}}\right)}, \qquad (5)$$

where B_x represents response rate during alternative reinforcement and B_o represents the response rate before introduction of alternative reinforcers. The parameter p scales the disruptive impact of alternative reinforcement. Because alternative reinforcers are presented in the same stimulus situation as the reinforcers contingent on the target response B, they also must be included in the denominator of Equation 5. When alternative reinforcers are introduced,
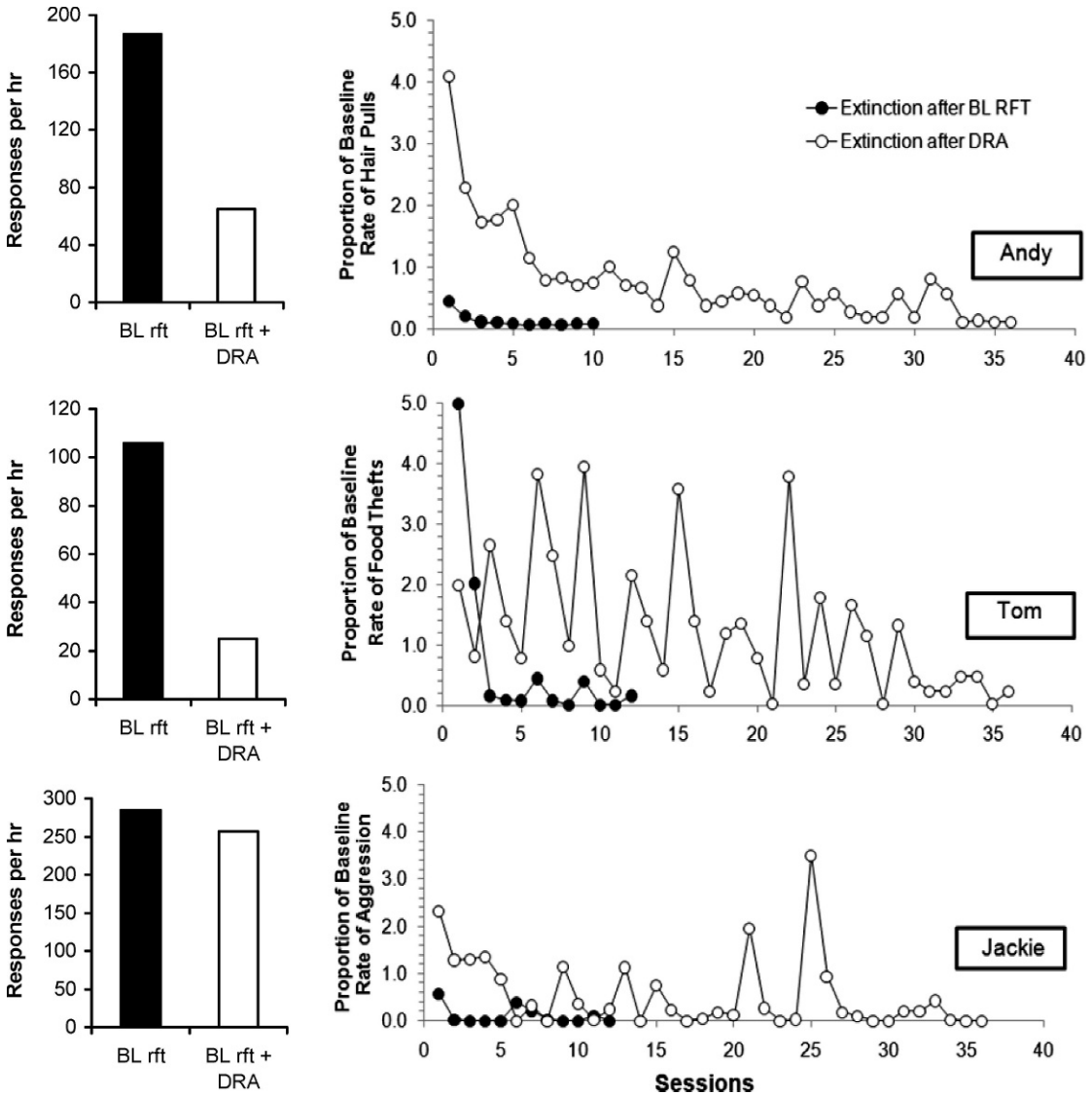
Figure 9.   Data from Mace et al. (2010) for problem behaviors of three children with developmental disabilities. For each child, the left panels show the baseline rates of problem behavior with and without DRA for desirable behavior, and the right panels compare rates of problem behavior relative to baseline when extinction followed DRA and when it directly followed baseline.

increases in the value of pR_a lead to reductions in response rate exactly as increases in the value of x reduced responding in Figure 1. This disruptive effect of alternative reinforcement is apparent in the left panels of Figures 6 and 9.

We now consider the effects of alternative reinforcement on subsequent resistance to extinction during which all reinforcers are withheld, in which the target response rate established during treatment with alternative reinforcement serves as the preextinction baseline. The model is exactly like Equation 4, with the addition of alternative reinforcers in the denominator of the exponent, as in Equation 5; pR_a is omitted from the numerator because alternative reinforcers
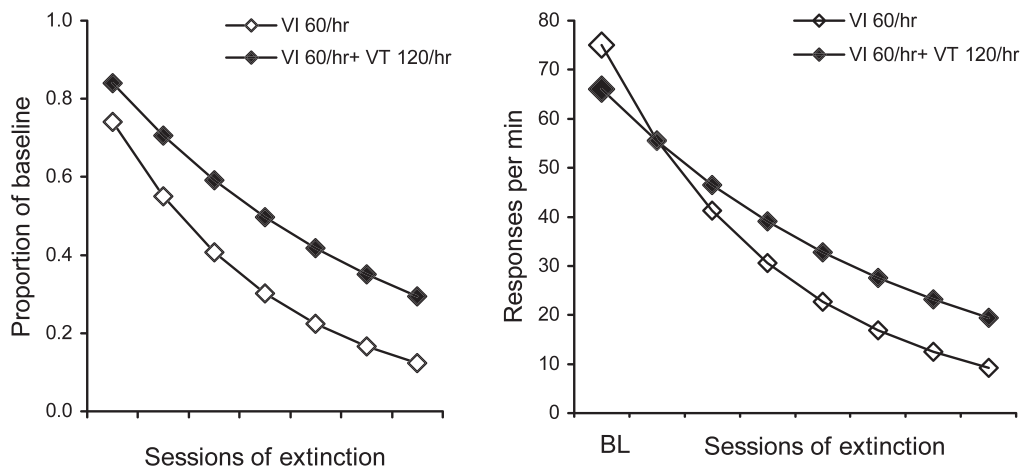
Figure 10. Predictions of Equation 6, showing that DRA during extinction reduces proportions of baseline throughout a series of sessions (left) but leads to a shallower extinction curve when data are expressed as response rates (right); baseline response rates are shown as enlarged diamonds.

are no longer operative as disrupters during extinction.

$$\frac{B_t}{B_o} = 10^{\left(\frac{-t(c+dr)}{(r+R_a)^{0.5}}\right)}. \qquad (6)$$

Equation 6 suggests that including alternative reinforcers in the same stimulus situation as the target behavior should increase resistance to change of the target behavior. Figure 10 shows simulated results for the schedules arranged by Nevin et al. (1990, Experiment 1; see Figure 6). The left panel shows predicted proportions of baseline, and the right panel shows absolute response rates in extinction with baseline rates (B_o) of 74 per minute in the VI 60 reinforcers per hour component and 66 per minute in the VI 60 per hour plus VT 120 per hour component (the empirical values displayed in the right panel of Figure 6, Condition 2). Note that the levels of the predicted extinction curves reverse early in extinction. This sort of reversal often appears in basic research with other disrupters as well as extinction when results are displayed as response rates rather than proportions of baseline (for discussion, see Nevin et al., 1990). Thus, if problem behavior is reduced by alternative

reinforcement and then extinguished, its rate actually may be higher than if alternative reinforcement had not been arranged before extinction. The effect is especially clear in the DRA data of Mace et al. (2010; Figure 9) for resistance to extinction; it also appears in the NCR data of Mace et al. (1990; Figure 7) for resistance to disruption. In this respect, interventions that arrange alternative reinforcement to reduce problem behavior may be countertherapeutic when that behavior is challenged in some way.

ALTERNATIVE REINFORCEMENT, EXTINCTION, AND RESURGENCE

To provide maximally effective treatment of problem behavior, extinction often is arranged concurrently with DRA (e.g., Iwata et al., 1994; Petscher, Rey, & Bailey, 2009). The combination of extinction and DRA can be characterized by Equation 7:

$$\frac{B_t}{B_o} = 10^{\left(\frac{-t(c+dr+pR_a)}{(r+R_a)^{0.5}}\right)}. \qquad (7)$$

Equation 7 differs from Equation 6 in the third term of the numerator, where pR_a represents
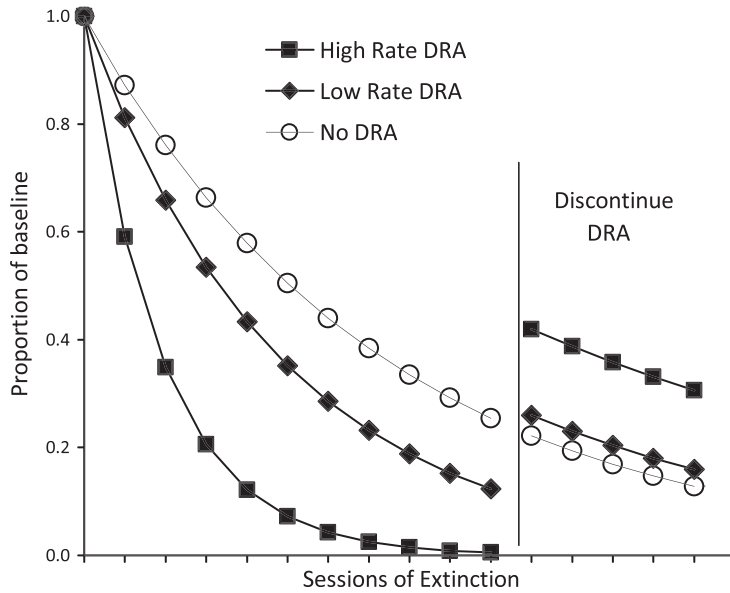
Figure 11. Predictions of Equation 7 for rich (120 per hour) and lean (15 per hour) schedules of DRA during extinction, showing that reductions in proportion of baseline are directly related to the rate of DRA. When DRA is discontinued, response rate is predicted to increase (resurgence), and the magnitude of the increase is directly related to the rate of DRA. Adapted from Shahan and Sweeney (2011).

disruption by the rate of reinforcement for alternative behavior R_a, multiplied by p to scale its disruptive impact on the target behavior (as in Equation 5); it is multiplied by t, like other disrupters during extinction, to indicate that its effectiveness increases as extinction progresses. Figure 11 shows a simulation of the effects of adding DRA during extinction of a target behavior previously maintained by 60 reinforcers per hour (i.e., VI 60 s). The curves to the left of the vertical line show responding during extinction with no DRA, low-rate DRA (15 per hour; VI 240 s), and high-rate DRA (120 per hour; VI 30 s). The addition of DRA reduces responding more rapidly than extinction alone, and high-rate DRA reduces responding more rapidly than low-rate DRA. The Appendix provides illustrative calculations.

The use of DRA during extinction, however, comes with a potential side effect if DRA is subsequently removed. The curves to the right of the vertical line in Figure 11 show the effects of discontinuing DRA while extinction remains in effect. When DRA is discontinued, pRa in the numerator of Equation 7 goes to zero and the target behavior increases as a result of this reduction in disruption. Such an increase in responding during extinction with the termination of alternative reinforcement is commonly known as *resurgence.* Indeed, Shahan and Sweeney (2011) have suggested that Equation 7 can serve as a general model of resurgence. Although the presence of higher rate DRA generates lower rates of responding during extinction, removal of higher rate DRA also produces larger increases in responding than does removal of lower rate DRA. The reason is that, although higher rate DRA produces a greater disruption of the target behavior when it is present (i.e., pR_a), it also introduces more reinforcement into the stimulus situation (i.e., R_a in the denominator of the exponent of Equation 7), and thus, increases the future strength of the target behavior. The strengthening effects of high-rate DRA reinforcers become apparent only when their counteracting

disruptive effects are removed from the numerator. Equation 7 does a good job describing data from basic research with pigeons (Leitenberg, Rawson, & Mulick, 1975) that showed the effects of different rates of DRA on target responding during extinction and on resurgence when DRA is removed (see Shahan & Sweeney for review).

For applied work, resurgence is important because it means that the termination of DRA, a frequently employed intervention, can lead to reappearance of problem behavior after an intervention (e.g., Lieving, Hagopian, Long, & O'Connor, 2004; see Lattal & St. Peter Pipkin, 2009, for review). For example, Volkert, Lerman, Call, and Trosclair-Lasserre (2009) observed resurgence in three children with autism or other developmental disabilities who exhibited self-injury, aggression, or disruption. After baseline evaluation, they received functional communication training (FCT), a form of DRA in which participants can obtain reinforcers with an alternative manding response. After FCT performances were well established, the FCT schedule was changed abruptly from FR 1 to extinction. In every case, problem behavior increased relative to its levels during extinction plus FCT. Volkert et al. also examined an abrupt switch from an FR 1 schedule of FCT reinforcement to an FR 12 schedule. With the switch to the FR 12 schedule, the participants experienced few FCT reinforcers and problem behavior also showed resurgence.

The resurgence of responding obtained by Volkert et al. (2009) with the change from FR 1 to FR 12 alternative reinforcement is consistent with related basic research with pigeons showing that a reduction in the rate of alternative reinforcement can produce resurgence (Lieving & Lattal, 2003). Equation 7 predicts this effect because decreases in pR_a reduce the disruption term in the numerator and thus increase target responding. Furthermore, because it is the reduction in disruption in the numerator that produces resurgence, a change from reinforcement

contingent on an explicitly defined alternative response to NCR at the same rate would not be expected to produce resurgence. This prediction is consistent with additional pigeon data from Lieving and Lattal showing that a change from a VI 30-s schedule to a VT 30-s schedule of alternative reinforcement did not produce resurgence. Thus, in applied settings, resurgence of problem behavior following the use of DRA in combination with extinction should be expected only when DRA reinforcement is reduced or eliminated, reflecting compromises to treatment integrity.

Equation 7 makes some additional predictions about the effects of DRA reinforcement on extinction and resurgence that should be considered in applied settings. For example, because it is the rate of alternative reinforcement that appears in Equation 7 (i.e., R_a), the effects of alternative reinforcement during extinction and resurgence should not depend on the details of how the alternative reinforcers are scheduled. Alternative reinforcers arranged by a variety of different schedules, including NCR, should produce similar decreases in target behavior when present during extinction and similar amounts of resurgence when they subsequently are removed. Basic research with rats and pigeons (e.g., Leitenberg et al., 1975; Winterbauer & Bouton, 2010) is consistent with this prediction (see Shahan & Sweeney, 2011, for review).

Other predictions of Equation 7 that are relevant to applied settings stem from the fact that time plays a critical role in the effects of alternative reinforcement on resurgence. In Equation 7, the numerator continues to grow with the passage of time (i.e., t). As a result, longer exposure to extinction plus alternative reinforcement is expected to produce lower response rates and less resurgence when the alternative source of reinforcement is reduced or eliminated (see Shahan & Sweeney, 2011, for details). Leitenberg et al. (1975) demonstrated this effect of duration of exposure to extinction plus alternative reinforcement with rats.
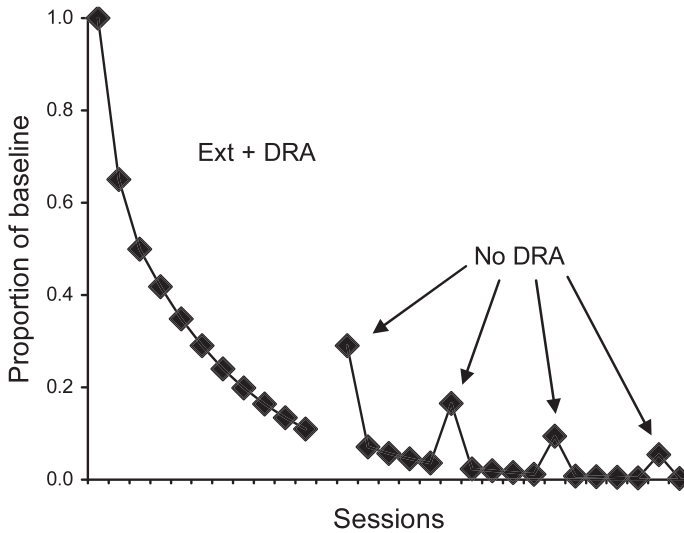
Figure 12. Predictions of Equation 7 for proportions of baseline when DRA is discontinued in probe sessions separated by sessions with extinction plus DRA, showing that the magnitude of resurgence is predicted to decrease over successive probes.

Because resurgence results from the removal of alternative reinforcers, occasional lapses in treatment integrity that lead to omission of the alternative reinforcer during extinction of problem behavior should produce resurgence. It is important to note that, because resurgence decreases with increasing time in extinction, the magnitude of resurgence with repeated removals of the alternative reinforcer should also decrease with time. Figure 12 shows a simulation of such reductions in the magnitude of resurgence, expressed as proportions of the preextinction baseline, with discontinuation of DRA every fifth session.

Wacker et al. (2011) obtained results similar to those simulated in Figure 12 with developmentally disabled children who engaged in destructive behavior that was maintained at least in part by escape from task demands. After determination of baseline during which all responses, including destructive behavior, were placed on extinction, FCT was used to teach the children to request breaks from task demands by pressing a switch. Over a series of sessions with FCT, the children's aggression decreased, their requests for breaks became more frequent,

and their compliance with demands increased markedly. However, when requests for breaks went unreinforced for brief periods, aggression showed resurgence and compliance decreased. Wacker et al. continued this sort of training for up to 16 months with occasional extinction probes and found that resurgence of aggression decreased to zero over the course of four successive tests (as suggested by the simulation in Figure 12). Indeed, they showed that Equation 7 accounted well for their data. Thus, resurgence of problem behavior when alternative reinforcement is discontinued, as might occur during an inadvertent lapse in treatment integrity, can be eliminated with longer durations of exposure to extinction plus alternative reinforcement.

Finally, because Equation 7 is a version of behavioral momentum theory based on Equation 3, it also suggests that the effects of DRA on both extinction and resurgence should depend on the preextinction baseline reinforcement rate r. Specifically, higher baseline reinforcement rates should produce greater resistance to the disruptive effects of DRA during extinction and less resurgence when DRA is eliminated. However,
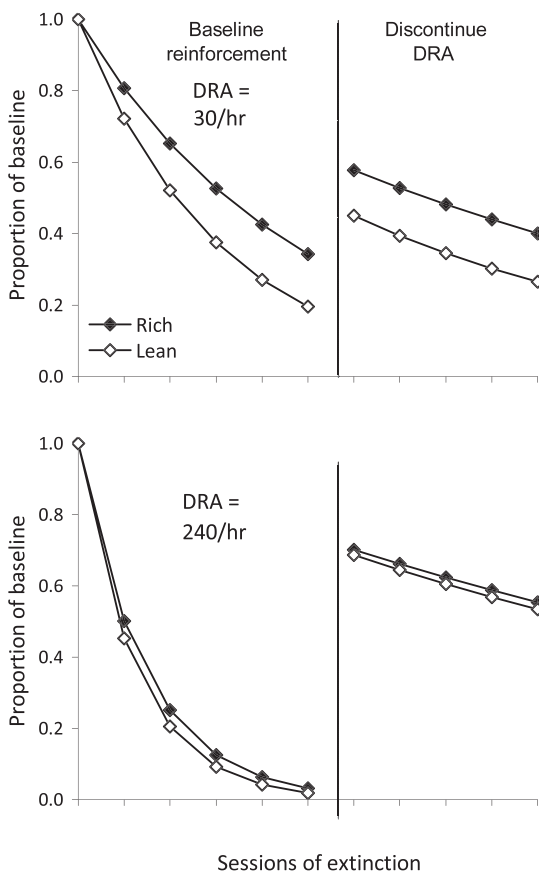
Figure 13. Predictions of Equation 7 showing the effects of low-rate DRA (30 per hour, top) and high-rate DRA (240 per hour, bottom) during extinction following training with a rich VI (120 per hour) and a lean VI (30 per hour) schedule for a target response during baseline. As in Figure 11, resurgence is predicted to be greater when a rich DRA schedule is discontinued, regardless of pre-extinction reinforcer rates. Adapted from Shahan and Sweeney (2011).

Equation 7 also suggests that effects of baseline reinforcement rate on resistance to disruption by DRA and on resurgence interact with the rate of DRA. As a result, the effects of baseline reinforcement rate can be rather small if DRA is provided at a high rate. Figure 13 shows a simulation of how baseline reinforcement rate is predicted to interact with the rate of DRA. The simulation assumes that during baseline, target behavior obtained 120 reinforcers per hour in a rich condition and 30 reinforcers per hour in a

lean condition. The top panel shows the impact of less frequent DRA (30 per hour) on extinction and on subsequent resurgence when the DRA is eliminated. The bottom panel shows the impact of more frequent DRA (240 per hour). As noted in connection with Figure 11 above, more frequent DRA produces greater suppression of responding during extinction and greater resurgence when it is removed. In addition, more frequent DRA reduces the expected difference in resistance to disruption generated by differences in baseline reinforcement rate. In fact, with high rates of DRA reinforcement like those in Figure 13 (bottom), baseline reinforcement rates would be expected to have little effect on resistance to extinction plus DRA or on resurgence (see Shahan & Sweeney, 2011, for review of experimental data that confirm this prediction).

These matters are relevant for application because problem behavior may have a long history with unknown rates of reinforcement. If high rates of alternative reinforcement are arranged during treatment, the unknown pre-treatment reinforcement rate might have little effect on the rate of response elimination. However, as shown in Figure 13, the magnitude of resurgence is predicted to be greater with higher rates of alternative reinforcement, and the possibility of this counterproductive side effect should be considered when implementing a rich schedule of DRA.

SUMMARY

Behavioral momentum theory is concerned with the persistence of reinforced behavior, whether it be adaptive or problematic. The theory states that reinforcers increase the persistence of behavior in the face of challenges or disrupters such as distraction or discontinuation of reinforcement. BMT also states that the persistence of a target response in a given situation is increased by alternative reinforcers presented in that situation, regardless of whether they are contingent on an alternative response. When BMT is expressed as a

quantitative model (a system of equations with terms representing reinforcers and disrupters), it accounts for a number of findings in applied as well as experimental settings, and predicts some counterintuitive and potentially countertherapeutic effects of alternative reinforcement. Because it makes explicit the ways in which reinforcers and disrupters combine to determine persistence, the model can serve as an integrative guide for intervention when its terms are identified with the conditions and contingencies of applied settings.

REFERENCES

Asmus, J. M., Ringdahl, J. E., Sellers, J. A., Call, N. A., Andelman, M. S., & Wacker, D. P. (2004). Use of a short-term inpatient model to evaluate aberrant behavior: Outcome data summaries from 1996 to 2001. *Journal of Applied Behavior Analysis*, 37, 283–304.

Cohen, S. L. (1996). Behavioral momentum of typing behavior in college students. *Journal of Behavior Analysis and Therapy*, 1, 36–51.

Critchfield, T. S., & Reed, D. D. (2009). What are we doing when we translate from quantitative models? *The Behavior Analyst*, 32, 339–362.

Daniels, F. (1928). *Mathematical preparation for physical chemistry*. New York: McGraw-Hill.

Gibbon, J. (1981). The contingency problem in autoshaping. In C. M. Locurto, H. S. Terrace, & J. Gibbon (Eds.), *Autoshaping and conditioning theory* (pp. 285–308). New York: Academic Press.

Grimes, J. A., & Shull, R. L. (2001). Response-independent milk delivery enhances persistence of pellet reinforced lever pressing by rats. *Journal of the Experimental Analysis of Behavior*, 76, 179–194.

Harper, D. N. (1999). Drug-induced changes in responding are dependent on baseline stimulus-reinforcer contingencies. *Psychobiology*, 27, 95–104.

Igaki, T., & Sakagami, T. (2004). Resistance to change in goldfish. *Behavioral Processes*, 66, 139–152.

Iwata, B. A., Pace, G. M., Dorsey, M. F., Zarcone, J. R., Vollmer, T. R., Smith, R. G., et al. (1994). The functions of self-injurious behavior: An experimental-epidemiological analysis. *Journal of Applied Behavior Analysis*, 27, 215–240.

Koegel, R. L., & Rincover, A. (1977). Research on the difference between generalization and maintenance in extra-therapy responding. *Journal of Applied Behavior Analysis*, 10, 1–12.

Lattal, K. A., & St. Peter Pipkin, C. (2009). Resurgence of previously reinforced responding: Research and application. *The Behavior Analyst Today*, 10, 254–265.

Leitenberg, H., Rawson, R. A., & Mulick, J. A. (1975). Extinction and reinforcement of alternative behavior. *Journal of Comparative and Physiological Psychology*, 88, 640–652.

Lieving, G. A., Hagopian, L. P., Long, E. S., & O'Connor, J. (2004). Response-class hierarchies and resurgence of severe problem behavior. *The Psychological Record*, 54, 621–634.

Lieving, G. A., & Lattal, K. A. (2003). Recency, repeatability, and reinforcer retrenchment: An experimental analysis of resurgence. *Journal of the Experimental Analysis of Behavior*, 80, 217–233.

Mace, F. C., Lalli, J. S., Shea, M. C., Lalli, E. P., West, B. J., Roberts, M., et al. (1990). The momentum of human behavior in a natural setting. *Journal of the Experimental Analysis of Behavior*, 54, 163–172.

Mace, F. C., McComas, J. J., Mauro, B. C., Progar, P. R., Ervin, R., & Zangrillo, A. N. (2010). Differential reinforcement of alternative behavior increases resistance to extinction: Clinical demonstration, animal modeling, and clinical test of one solution. *Journal of the Experimental Analysis of Behavior*, 93, 349–367.

Nevin, J. A. (1979). Reinforcement schedules and response strength. In M. D. Zeiler & P. Harzem (Eds.), *Reinforcement and the organization of behaviour* (pp. 117–158). Chichester, UK: Wiley.

Nevin, J. A. (1992). An integrative model for the study of behavioral momentum. *Journal of the Experimental Analysis of Behavior*, 57, 301–316.

Nevin, J. A. (2002). Measuring behavioral momentum. *Behavioural Processes*, 57, 187–198.

Nevin, J. A., & Grace, R. C. (2000). Behavioral momentum and the law of effect. *Behavioral and Brain Sciences*, 23, 73–130.

Nevin, J. A., McLean, A. P., & Grace, R. C. (2001). Resistance to extinction: Contingency termination and generalization decrement. *Animal Learning and Behavior*, 29, 176–191.

Nevin, J. A., Tota, M. E., Torquato, R. D., & Shull, R. L. (1990). Alternative reinforcement increases resistance to change: Pavlovian or operant contingencies? *Journal of the Experimental Analysis of Behavior*, 53, 359–379.

Parry-Cruwys, D. E., Neal, C. M., Ahearn, W. H., Wheeler, E. E., Premchander, R., Loeb, M. B., et al. (2011). Resistance to disruption in a classroom setting. *Journal of Applied Behavior Analysis*, 44, 363–367.

Petscher, E. S., Rey, C., & Bailey, J. A. (2009, May–June). A review of empirical support for differential reinforcement of alternative behavior. *Research in Developmental Disabilities*, 30, 409–425.

Pyszczynski, A. D., & Shahan, T. A. (2011). Behavioral momentum and relapse of ethanol seeking: Non-drug reinforcement in a context increases relative reinstatement. *Behavioural Pharmacology*, 22, 81–86.

Shahan, T. A., & Burke, K. A. (2004). Ethanol-maintained responding of rats is more resistant to change in a context with added non-drug reinforcement. *Behavioural Pharmacology*, 15, 279–285.

Shahan, T. A., & Sweeney, M. M. (2011). A model of resurgence based on behavioral momentum theory. *Journal of the Experimental Analysis of Behavior*, 95, 91–108.

Thorndike, E. L. (1911). Animal intelligence: An experimental study of the associative processes in animals. *Psychological Review Monographs Supplement*, 2, (No. 8).

Thorndike, E. L. (1913). *Educational psychology: Vol. 2. The psychology of learning*. New York: Columbia University, Teachers College Press.

Volkert, V. M., Lerman, D. C., Call, N. A., & Trosclair-Lasserre, N. (2009). An evaluation of resurgence during treatment with functional communication training. *Journal of Applied Behavior Analysis*, 42, 145–160.

Wacker, D. P., Harding, J. W., Berg, W. K., Lee, J. F., Schieltz, K. M., Padilla, Y. C., et al. (2011). An evaluation of maintenance during long-term treatment with functional communication training. *Journal of the Experimental Analysis of Behavior*, 96, 261–282.

Winterbauer, N. E., & Bouton, M. E. (2010). Mechanisms of resurgence of an extinguished instrumental behavior. *Journal of Experimental Psychology: Animal Behavior Processes*, 36, 343–353.

APPENDIX

Illustrative Calculations

The calculations of numerical predictions by some of the foregoing equations are illustrated here. Parameter values and reinforcer rates have been chosen for ease of calculation.

Equation 3 is

$$\frac{B_x}{B_o} = 10^{\left(\frac{-x}{r^{0.5}}\right)},$$

where B_x is response rate during disruption, B_o is baseline response rate, x represents the impact of a disrupter, and r is the rate of reinforcement. If $r = 100$ reinforcers per hour and $x = 5$, the exponent of the equation is $-5/100^{0.5} = -5/10 = -0.5$. Thus, predicted proportion of baseline, B_x/B_o, is $10^{-0.5}$ or equivalently $1/(10^{0.5})$ (i.e., 1 divided by the square root of 10), which is 0.316 (this value is obtainable from a hand calculator, slide rule, computer spreadsheet, or numerical tables; see, e.g., Daniels, 1928, for calculation methods).

Equation 4 is

$$\frac{B_t}{B_o} = 10^{\left(\frac{-t(c+dr)}{r^{0.5}}\right)},$$

where B_t is response rate at time t in extinction, B_o is baseline response rate, c represents the disruptive effects of suspending the response–reinforcer contingency, and dr represents the discriminability of omitting r reinforcers from the context. If $r = 100$ reinforcers per hour, extinction has been in effect for 2 hr, $c = 1$, and $d = 0.001$, the exponent of the equation is $-2(1+0.001*100)/100^{0.5} = -2(1.1)/10 = -0.22$. Thus, predicted proportion of baseline after 2 hr of extinction is $10^{-0.22}$ or equivalently $1/(10^{0.22})$, which is 0.603.

Equation 7 is

$$\frac{B_t}{B_o} = 10^{\left(\frac{-t(c+dr+pR_a)}{(r+R_a)^{0.5}}\right)},$$

where pR_a represents the disruptive effect of concurrent presentation of alternative reinforcers R_a, and all other terms are as for Equation 4. If $r = 100$ reinforcers per hour, $R_a = 300$ reinforcers per hour, extinction has been in effect for 2 hr, $c = 1$, and $d = 0.001$ as for Equation 4 above, and if $p = 0.01$, the exponent of the equation is $-2(1+0.001*100+0.01*300)/(100+300)^{0.5} = -2(4.1)/20 = -0.41$. Thus, proportion of baseline after 2 hr of extinction is predicted to be $10^{-0.41}$ or equivalently $1/(10^{0.41})$, which is 0.389. Note that this is lower than the proportion of baseline predicted by Equation 4, which did not include alternative reinforcers during extinction of target behavior. Note also that when the disruptive effects of concurrent alternative reinforcers are removed from the numerator, the exponent is $-2(1+0.001*100)/(100+300)^{0.5} = -2(1.1)/20 = -0.11$, and the predicted proportion of baseline is 0.776, exemplifying resurgence.