Object Frequency Characteristics of Visual Acuity

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PURPOSE. To examine the extent to which visual acuity (VA) for broadband optotypes is scale invariant by determining whether the same object frequencies mediate VA for individuals with different levels of VA.

METHODS. LogMAR (minimum angle of resolution) VA for briefly presented tumbling E's was measured in 10 visually normal individuals and in five patients with VA loss. The E's were either unblurred or blurred with Gaussian low-pass filters that had cutoff frequencies spanning a 1.2-log unit range. The data were fit with a standard equivalent intrinsic blur model to estimate each subject's unblurred VA (MAR₀ in minutes of arc) and equivalent intrinsic blur (σ_{int} in minutes of arc). From these estimates, the high-frequency cutoff of the band of retinal frequencies (cpd_{crit} in cycles per degree) and object frequencies (cpl_{crit} in cycles per letter) mediating VA were derived.

RESULTS. LogMAR₀ was related linearly to log σ_{int} with a slope of 1.47, which is steeper than that predicted by scale invariance. Log cpd_{crit} was related linearly to logMAR₀ by a slope of -0.64, which is shallower than that predicted by scale invariance. This lack of scale invariance is due to a linear relationship between log cpl_{crit} and logMAR₀ that had a slope of 0.36.

CONCLUSIONS. The overall pattern of results is not consistent with the expectation of scale invariance underlying the MAR scale. Optotypes that conform to the expectations of scale invariance are needed to improve vision assessment and to provide equivalency of VA defined in terms of MAR and cpd. (*Invest Ophthalmol Vis Sci.* 2011;52: 9534–9538) DOI:10.1167/iovs.11-8426

The size of letter optotypes on standard visual acuity (VA) charts is typically specified as the logarithm of the minimum angle of resolution (logMAR). A fundamental assumption of the MAR metric is that scale invariance holds for VA. That is, the MAR scale assumes that individuals use the same information to identify letters at the acuity limit, regardless of their VA. In particular, the MAR scale assumes that letter identification is based on the stroke width of the letter at all VA levels.

The stroke width of standard Sloan optotypes corresponds to an object frequency of 2.5 cycles per letter (cpl), given that there are five strokes in each letter and two strokes (one light bar and one dark bar) per cycle. If VA is based on stroke width, then the MAR value of a letter can readily be equated to retinal

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Corresponding author: J. Jason McAnany, Department of Ophthalmology and Visual Sciences, University of Illinois at Chicago, 1855 W. Taylor Street, Chicago, IL 60612; jmcana1@uic.edu. spatial frequency (cycles per degree or cpd) based on the angular subtense of the stroke width. For example, a letter with an MAR of 1 (0 logMAR or 20/20 Snellen equivalent) would correspond to a retinal spatial frequency of 30 cpd, because one cycle would subtend 2 arcmin.

However, evidence indicates that VA for letter optotypes, which are broadband in object frequency content, may not be based on object frequencies near 2.5 cpl. For example, object frequencies less than approximately 1.25 cpl were found to mediate VA in the normal periphery for a task that involved discrimination between letter pairs.¹ A study of the Fourier components of the Landolt C and tumbling E concluded that, whereas VA for the tumbling E could be based on object frequencies near 2.5 cpl, VA for the Landolt C was most likely based on object frequencies in the range of 1.3 cpl.² This proposal for the Landolt C was verified experimentally using band-limited targets in a study of the effect of crowding on VA.3 VA measurements using low- and high-pass-filtered tumbling E's showed that the VA for this target in the normal visual field periphery is based on object frequencies lower than the 2.5 cpl proposed previously.⁴

The finding that VA for broadband targets may be governed by object frequencies lower than 2.5 cpl would not be an issue if scale invariance holds for VA. That is, if individuals with different levels of VA all use the same low object frequencies to identify letters at the acuity limit, then the equivalent retinal frequency would simply be a scalar transform of the nominal retinal frequency that is derived from stroke width. However, there is evidence from studies of contrast sensitivity that the object frequencies used for letter identification vary with letter size.⁵⁻⁷ Specifically, there is a linear relationship between log object frequency and log angular subtense of the letter that has a slope of approximately 1/3, such that contrast sensitivity is mediated by low object frequencies for letters of small angular subtense but higher object frequencies for larger letters. Given that individuals with decreased VA require larger-than-normal letters at the acuity limit, it is possible that they may use higher object frequencies than normally sighted individuals, and scale invariance would not hold for VA.

The present study evaluated the extent to which VA is scale invariant by determining whether the same object frequencies mediate VA for individuals who have different VAs. A novel approach was used, in which the high-frequency cutoff of the object frequency region mediating VA was derived by using an equivalent intrinsic blur paradigm. With this approach, the target is successively blurred with Gaussian low-pass filters until VA is affected, under the assumption that, if the removal of specific high object frequencies impairs VA, then those frequencies must be necessary for the task. Gaussian filters were used in the present study because they are the basis for models of equivalent intrinsic blur⁸⁻¹⁰ and have also been used in studies of the object frequencies mediating contrast sensi-tivity for letter optotypes.^{11,12} The amount of Gaussian blur necessary to reduce VA by a factor of $\sqrt{2}$ was used to estimate the high-frequency cutoff of the object frequencies mediating VA. This degree of Gaussian blur has been termed "equivalent

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In the present study, the VA of both normally sighted individuals and patients with VA loss due to eye disease was measured with a tumbling E. The patients were included to broaden the range of VA. Equivalent intrinsic blur was measured in the spatial domain (in minutes of arc) and converted to the frequency domain (cpd) to estimate the high-frequency cutoff of the band of retinal frequencies mediating VA. The corresponding high-frequency cutoff of the band of object frequencies mediating VA was obtained by converting the cpd estimate to cpl to establish whether the same object frequencies mediate performance at the acuity limit for individuals with different VA levels.

METHODS

Subjects

Ten normally sighted individuals and five patients diagnosed with diabetes mellitus (DM) participated in the study. Subjects with DM were recruited from the Retina Service at the University of Illinois at Chicago and had VA loss due to diabetic macular edema and/or vein occlusions. These patients were included in the study to broaden the range of VA, and comparison among the patients with DM was not undertaken. Table 1 provides the characteristics of the subjects, including sex, age, diagnosis, refraction, and chart acuity (Lighthouse Distance Acuity Chart; Lighthouse International, New York, NY). The chart acuity measurements were made through the best optical correction and through a 3.0-mm artificial pupil that was used to control the retinal illuminance and also to optimize the optical quality of the eye by minimizing the effects of higher order aberrations and diffraction. The study conformed to the tenets of the Declaration of Helsinki, and the experiments were approved by an institutional review board at the University of Illinois at Chicago. Written informed consent was obtained from each subject before testing.

Instrumentation and Stimuli

The instrumentation and stimuli have been described in detail elsewhere.¹⁰ Briefly, stimuli were generated by computer (Macintosh G4; Apple Computer, Cupertino, CA) with commercial software (MatLab; The MathWorks, Natick, MA) and the Psychophysics Toolbox extensions.¹³ The stimuli were displayed on a 22-in. cathode ray tube (CRT) monitor (FE2111SB; NEC, Irving, TX) with a screen resolution of

TABLE	1.	Subject	Characteristics
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 1024×768 and an 85-Hz refresh rate, driven by a video card with 10-bit DAC resolution (ATI Radeon 9000 Pro; AMD, Sunnyvale, CA).

The stimuli consisted of unblurred and blurred tumbling E optotypes. The E's were constructed according to the principles of the Sloan font,¹⁴ such that the stroke width was one fifth of the overall optotype size and the three bars were of equal length. The stroke width ranged from 0.6 to 20 arcmin in 16 steps spaced approximately 0.1 log unit apart. The E at each size was blurred with a set of three 2-D Gaussian filters with standard deviations (σ_{stim}) of 2, 8, and 32 pixels, corresponding to 0.4, 1.6, and 6.4 arcmin at the test distance of 4.5 m. Stimuli were presented for approximately 60 ms (five video frames) in the center of an adapting field that subtended 3.4° horizontally and 2.6° vertically. The luminance of the adapting field was 106 cd/m² and the luminance of the unblurred test stimulus was 1.4 cd/m^2 , yielding a Weber contrast of -99%. The blurred stimuli were presented without rescaling the contrast. The stimulus luminances were measured with a photometer (LS 110; Minolta Osaka, Japan), and the temporal characteristics of the display were confirmed with an oscilloscope and photocell.

Procedure

Before all measurements, the pupil of the tested eye was dilated with 2.5% phenylephrine hydrochloride drops. The subject's pupil was centered on the 3.0-mm artificial pupil using a two-dimensional, two-color alignicator.¹⁵ The subject's task was to judge the orientation of the tumbling E, which was randomly facing either to the right or up on each trial. A brief warning tone signaled the start of each stimulus presentation, and the subject verbally reported the orientation, which was recorded by the examiner. The subjects were given practice trials to become familiar with the task.

Threshold logMAR (logMAR_t) for each value of σ_{stim} (0, 0.4, 1.6, 6.4 arcmin) was determined using a two-alternative, forced-choice staircase procedure. An initial estimate of logMAR_t was obtained by presenting the optotype at a suprathreshold size and then decreasing the size by 0.1 log unit until an incorrect response was recorded. After this initial search, logMAR_t was determined using a two-down, one-up decision rule, which provides an estimate of the 71% correct point on a psychometric function.¹⁶ Each staircase continued until 10 reversals had occurred, and the mean of the last 6 reversals was taken as logMAR_t. The staircase length was typically 40 to 50 trials, which produced stable measurements (the SEM of the last eight reversals was typically less than 0.05 log unit). One staircase measurement of logMAR_t was obtained from each subject for each value of σ_{stim} . The conditions were presented in order of increasing σ_{stim} , but for a given staircase, E's of different sizes were convolved with a constant filter

Subject No.	Sex	Age (y)	Diagnosis	Refraction Sphere (D)	Refraction Cylinder (D × Angle)	Chart VA (logMAR)
1	М	46	Normal	-1.50	0.00	-0.08
2	F	48	Normal	-1.00	$+0.50 imes 90^{\circ}$	-0.08
3	М	53	Normal	-7.75	$+0.50 imes0^{\circ}$	-0.01
4	F	53	Normal	-7.00	$+0.75 imes 90^{\circ}$	0.03
5	F	54	Normal	-5.75	$+0.25 imes100^{\circ}$	0.06
6	F	56	Normal	-0.50	0.00	-0.10
7	М	56	Normal	0.00	0.00	-0.07
8	F	57	Normal	-3.25	$+1.00 imes100^\circ$	-0.01
9	М	58	Normal	0.00	0.00	-0.07
10	М	63	Normal	-4.00	0.00	-0.16
11	М	44	CRVO; CSDME	0.00	0.00	0.66
12	F	58	PDR; CSDME	0.00	0.00	0.22
13	F	64	NPDR; CSDME	+2.00	0.00	0.28
14	F	68	NPDR; CSDME	-0.75	$+0.25 imes100^{\circ}$	0.58
15	Μ	71	BRVO	-0.50	0.00	0.34

CRVO, central retinal vein occlusion; CSDME, clinically significant diabetic macular edema; PDR, proliferative diabetic retinopathy; NPDR, non-proliferative diabetic retinopathy; BRVO, branch retinal vein occlusion.

width ($\sigma_{\rm stim}$) so that the degree of blur varied across trials within the staircase.

Analysis

LogMAR_t for each subject was plotted as a function of log σ_{stim} , and the data were fit with the log form of the following equation⁸:

$$MAR_{t} = MAR_{0}(1 + (\sigma_{stim}/\sigma_{int})^{2})^{1/2}$$
(1)

where MAR₀ represents VA in the absence of blur (i.e., $\sigma_{\text{stim}} = 0$), and σ_{int} (corresponding to equivalent intrinsic blur) is the value of σ_{stim} at which MAR = MAR₀ · $\sqrt{2}$. This value of MAR was defined as MAR_{crit}. MAR₀ and σ_{int} were adjusted to minimize the mean squared error between the fitted function and the data.

Object frequency was derived from MAR_{crit} and σ_{int} in two steps. First, the Gaussian function representing equivalent intrinsic blur was converted from the spatial domain to the frequency domain. In the frequency domain, the SD of the Gaussian function is a measure of the high-frequency cutoff of the band of retinal frequencies mediating VA (cpd_{crit}). The value of σ_{int} was converted to cpd_{crit} as follows:

$$\text{cpd}_{\text{crit}} = 1/(2\pi \cdot \sigma_{\text{int}}/60) = 9.55/\sigma_{\text{int}}.$$
 (2)

Next, the high-frequency cutoff of the band of object frequencies mediating VA (cpl_{crit}) was derived from MAR_{crit} and cpd_{cri} using the following relationship:

$$cpl_{crit} = MAR_{crit} \cdot cpd_{crit}/12.$$
 (3)

RESULTS

Figure 1 presents logMAR_t as a function of log σ_{stim} for one representative normally sighted subject (subject 10). For reference, the right *y*-axis shows the corresponding Snellen equivalents of the logMAR_t values. The curve represents the least-squares best fit of equation 1 to the data (R^2 values for the individual subjects ranged from 0.8 to 1.0). According to equation 1, logMAR_t is approximately constant for small values of log σ_{stim} , whereas logMAR_t increases linearly with a slope of 1 for substantially larger values of log σ_{stim} . In this figure, logMAR₀ and log σ_{int} are indicated by the arrows. For refer-



FIGURE 1. Threshold logMAR as a function of log σ_{stim} for one representative normally sighted subject (subject 10). The right *y*-axis shows the Snellen equivalents of the logMAR_t values. *Solid line* represents least-squares best fit of equation 1 to the data. *Arrows*: log σ_{int} , logMAR_{crit}, and logMAR₀.



FIGURE 2. LogMAR₀ as a function of log σ_{int} for the normally sighted subjects and patients with DM. The right *y*-axis shows the Snellen equivalents of the logMAR₀ values and the top *x*-axis shows the linear values of σ_{int} . *Solid line* represents least-squares best-fit linear regression line.

ence, the value of logMAR_{crit} is also indicated. For this subject, MAR₀ and σ_{int} were approximately 0.7 and 1.1 arcmin, respectively.

Figure 2 shows the relationship between logMAR₀ and log $\sigma_{\rm int}$ for the 10 normally sighted subjects (circles) and five DM patients (squares). The right *y*-axis shows the corresponding Snellen equivalents of the logMAR₀ values. The individual values of logMAR₀ and log $\sigma_{\rm int}$ are listed in Table 2. The values of logMAR₀ in this table correlated significantly with the chart acuity values given in Table 1 (r = 0.92, P < 0.01), demonstrating the validity of using logMAR₀ as an index of VA. As expected from previous studies,^{8,10,17} logMAR₀ was re-

As expected from previous studies,^{8,10,17} logMAR₀ was related linearly to log σ_{int} , such that subjects with lower values of logMAR₀ (better VA) had lower equivalent intrinsic blur. The data of the normally sighted subjects alone were best fit with a linear regression line with a slope of 1.0, which is in agreement with previous studies.^{8,10,17} However, when the patients with

TABLE 2. Parameters Derived from Equation and Estimates of Retinal

 and Object Frequency for Each Subject

Subject No.	LogMAR ₀	σ _{int} (arcmin)	Retinal Frequency (cpd _{crit})	Object Frequency (cpl _{crit})
1	-0.13	0.87	10.96	0.95
2	-0.10	1.02	9.33	0.89
3	-0.01	0.89	10.72	1.23
4	0.04	1.35	7.08	0.91
5	0.14	1.38	6.92	1.10
6	0.10	1.45	6.61	0.95
7	-0.10	1.02	9.33	0.87
8	0.04	1.23	7.76	1.02
9	-0.11	0.98	9.77	0.89
10	-0.14	1.10	8.71	0.72
11	0.61	3.16	3.02	1.45
12	0.48	2.09	4.57	1.62
13	0.37	1.95	4.90	1.35
14	0.57	2.75	3.47	1.55
15	0.64	3.02	3.16	1.62

VA loss are included, the best-fit regression line (Fig. 2, solid line) has a slope of 1.47 ($R^2 = 0.94$), which is significantly steeper than 1.0 (t = 5.1, P < 0.01).

The relationship between log cpd_{crit} and $logMAR_0$ is shown in Figure 3, with the individual values of cpd_{crit} given in Table 2. The dashed line in Figure 3 has a slope of -1.0 and describes the relationship between retinal spatial frequency and VA, assuming that scale invariance holds and that a logMAR of 0 (20/20 Snellen VA) is equivalent to a retinal spatial frequency of 30 cpd. It is apparent from Figure 3 that there was a linear relationship between log cpd_{crit} and logMAR₀, but the slope of the best-fit regression line (solid line, $R^2 = 0.94$) was -0.64. This slope is significantly shallower than the slope of -1.0 (t =-8.7, P < 0.01) predicted by scale invariance (dashed line).

The basis for the relatively shallow slope of the function shown in Figure 3 can be appreciated by inspection of Figure 4, which plots log cpl_{crit} as a function of logMAR₀ for the normally sighted subjects and DM patients. The individual values of cpl_{crit} are given in Table 2. The solid line in Figure 4 represents the best-fit regression line ($R^2 = 0.92$), which has a slope of 0.36. This slope is significantly steeper than a slope of 0 (t = 8.7, P < 0.01). This non-0 slope indicates a dependence of object frequency on VA, such that observers with worse VA (higher logMAR₀) used higher object frequencies than subjects with better VA. This lack of scale invariance accounts for the absence of a one-to-one relationship between log cpd_{crit} and logMAR₀ (Fig. 3).

DISCUSSION

In this study, we evaluated the extent to which the object frequency information mediating VA for broadband optotypes is scale invariant across individuals who have different VA levels. If the object frequency information mediating VA depends on the individual's VA level, then this would complicate



FIGURE 3. Log retinal frequency (cpd_{crit}) as a function of $logMAR_0$ for the normally sighted subjects and patients with DM. The right *y*-axis shows the linear values of cpd_{crit} and the top *x*-axis shows the Snellen equivalents of the $logMAR_0$ values. *Solid line* represents least-squares best-fit linear regression line; *dashed line* represents scale invariance, with the assumption that 30 cpd is equivalent to 0 logMAR.



FIGURE 4. Log object frequency (cpl_{crit}) as a function of $logMAR_0$ for the normally sighted subjects and patients with DM. The right *y*-axis shows the linear values of object frequency and the top *x*-axis shows the Snellen equivalents of the $logMAR_0$ values. *Solid line* represents least-squares best-fit linear regression line.

the relationship between VA and retinal frequency. The potential difficulty of relating MAR and retinal frequency was noted previously in a study of the effects of optical blur on letter and grating acuity.¹⁸

In the present study, the high-frequency cutoff (cpl_{crit}) of the band of object frequencies mediating VA for subjects with different VA levels was derived using an equivalent intrinsic blur paradigm. An estimate of equivalent intrinsic blur was obtained for each subject, and this estimate was used to derive the high-frequency cutoff of the band of retinal frequencies (cpd_{crit}) mediating VA. The value of cpd_{crit} was then converted to the corresponding object frequency (cpl_{crit}). The results showed that the object frequency information mediating VA is not scale invariant. Subjects with worse VA (higher values of logMAR₀) had higher values of cpl_{crit} than subjects with better VA. Because of this lack of scale invariance, VA defined in terms of MAR and VA defined in terms of equivalent retinal frequency were not proportionally related by a slope of -1.0(Fig. 3). For example, subjects with MAR values that differed by 1.0 log unit differed in equivalent retinal frequency by only 0.66 log units.

The value of cpl_{crit} for our sample of subjects ranged from 0.7 to 1.6 (Fig. 4). These values are lower than those reported in a previous study that investigated the effect of low-pass and high-pass filtering on orientation judgments of the tumbling E in the normal visual field periphery.⁴ That study reported that object frequencies between approximately 1.25 and 2.25 cpl mediated performance for the tumbling E. However, a direct comparison of our results with those of this previous study is complicated by the fact that, whereas the previous investigators based their estimate of object frequency on the filter cutoffs required to affect VA, the values reported here are dependent on the point on the function relating logMAR, to log $\sigma_{\rm stim}$ that is selected for analysis (Fig. 1). We chose to use the point at which MAR_t was elevated by $\sqrt{2}$ above MAR₀, which corresponds to the standard measure of equivalent intrinsic blur. Selecting a lower point on the curve would result in a higher estimate of cpl_{crit} , which would be more similar to the values reported previously.⁴ It is important to note, however, that altering the chosen point on the curve would not affect the non-0 slope of the line relating log cpl_{crit} and $logMAR_0$ (Fig. 4). Consequently, there is a lack of scale invariance for VA, regardless of the point that is selected as the basis for the derivation of critical object frequency.

The slope of the line relating log cpl_{crit} and logMAR_0 was approximately 1/3 (Fig. 4). A similar slope has been reported in previous studies of contrast sensitivity for broadband optotypes, including orientation judgments of the Sloan N (a twoalternative, forced-choice task similar to that of the present study), letter detection, letter discrimination, and letter identification tasks.^{5-7,11,12} The contrast sensitivity data of these previous studies together with the VA data of the present study suggest that an increase in object frequency with increasing target size is a general characteristic of the measurement of visual function with broadband optotypes. A linear relationship between object frequency and letter size with a slope of approximately one third was also observed in a study of contrast sensitivity in amblyopic subjects.¹⁹ This latter finding suggests that the lack of scale invariance for VA found in the present study would probably generalize to other patient populations beyond the DM patients studied here. However, additional work is needed to confirm this hypothesis.

In conclusion, the present results demonstrate that scale invariance cannot necessarily be assumed in VA measurements that use standard broadband optotypes. This lack of scale invariance complicates the interpretation of acuity measurements for individuals with different VA values. Scale invariance could be achieved by using band-limited optotypes, and there have been previous attempts to use band-limited optotypes in VA measurements. However, these targets typically have limitations. For example, VA has been measured with sine-wave grating targets,^{20,21} but these stimuli are typically unfamiliar to patients, and VA measurements made in the periphery with these stimuli can be affected by spurious resolution and aliasing.²² VA has also been measured with "vanishing optotypes" that have pseudo-high-pass spatial characteristics.^{23,24} However, untrained subjects, patients with central field loss, or patients with unsteady fixation may have trouble localizing these targets in space at sizes near the acuity limit, which would increase spatial uncertainty. Thus, further study is needed to identify optotypes that maintain the desirable characteristics of letters but conform to the expectations of scale invariance, which would provide a better assessment of VA.

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