Published in final edited form as:
J Learn Disabil. 2012 May ; 45(3): 257–269. doi:10.1177/0022219412442167.

The Early Prevention of Mathematics Difficulty: Its Power and Limitations

Lynn S. Fuchs, PhD[1], **Douglas Fuchs, PhD**[1], and **Donald L. Compton, PhD**[1]
[1]Vanderbilt University, Nashville, TN, USA

Abstract

In this article, the authors consider the power and limitations of responsiveness-to-intervention (RTI) for reducing the need for ongoing and intensive services for the segment of the school population traditionally identified as having a learning disability in mathematics. To assess the robustness of RTI, the authors describe four studies with strong demonstrations of efficacy, as they considered the percentage of students who failed to respond, the post-tutoring achievement gap between tutored and not-at-risk students, and the extent of transfer across components of the mathematics curriculum. The authors then discuss implications and additional research questions pertaining to mathematics intervention generally and within the context of RTI. They conclude with a proposal for an expanded conceptualization of RTI.

Keywords

responsiveness-to-intervention; RTI; mathematics; prevention; learning disabilities

The use of multilevel prevention systems constitutes a major education reform of the past decade. Commonly known as responsiveness-to-intervention (RTI), these systems conceptualize school services in terms of a series of increasingly intensive interventions. The first level, primary prevention, is the general education program, where screenings are conducted periodically to identify students with risk for poor learning outcomes. These at-risk students enter a more intensive level of the prevention system that entails one or more tiers of intervention, often involving at least one round of small-group tutoring. This intervention is conceptualized as time limited, cost efficient, and validated through research to be generally effective.

The purpose of such a multilevel prevention RTI system has been the subject of debate (see D. Fuchs, Fuchs, & Stecker, 2010). Some view its exclusive function as the prevention of academic difficulty, with the assumption—or at least hope—that learning disabilities (and the need for special education) will be reduced dramatically (e.g., Torgesen, 2004). Others include a second, complementary purpose as the identification of students with learning disabilities as nonresponders to validated intervention. This second perspective is based on a two-part assumption. The first part is that most at-risk students will respond nicely to a time-limited, standard form of 10–20 weeks of validated intervention and return to general education with a stronger foundation of academic skill that permits them to thrive in the primary prevention program without additional support. The second part of the assumption,

Corresponding Author: Lynn S. Fuchs, Vanderbilt University, 228 Peabody, Nashville, TN 37203, USA, lynn.fuchs@vanderbilt.edu.

Declaration of Conflicting Interests: The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

however, is that some subset of at-risk students, similar in prevalence to the 5% of students traditionally served with a learning disability, will not respond to time-limited intervention in ways that circumvent the need for ongoing, intensive monitoring and intervention. In some RTI systems, such intervention is conceptualized as a more intensive and ongoing level of the prevention system, in which individualized instruction, often in the form of special education, is provided for students with learning disabilities. As with the rest of the RTI system, the goal is to prevent the negative long-term consequences that occur when students exit school without the skills they need to succeed in life.

The purpose of the present article was to consider the power and limitations of RTI for dramatically reducing the need for this intensive, ongoing service. We focused on mathematics for four reasons. First, as with reading, poor mathematics learning in school is associated with serious, lifelong difficulties (e.g., National Mathematics Advisory Panel, 2008; Rivera-Batiz, 1992). Second, the prevalence of mathematics difficulty is high, with estimates of prevalence ranging between 5% and 9% (e.g., Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Shalev, Auerbach, Manor, & Gross-Tsur, 2000). Third, despite similar prevalence and debilitating, lifelong negative consequences, mathematics learning disabilities have received much less emphasis than reading learning disabilities. Fourth, mathematics, more than reading, is potentially complicated by the fact that school curricula are organized in strands within and across the grades, presumed to represent different component skills. In reading, measurement studies (e.g., Mehta, Foorman, Branum-Martin, & Taylor, 2005) provide the basis for five component reading skills: phonological awareness, decoding, fluency, vocabulary, and comprehension. In mathematics, measurement studies are yet to be conducted, but the assumption, as reflected in curricula, is that many more component skills exist. For example, in primary school, curricular strands include concepts, numeration, measurement, basic facts, algorithmic computation, and word problems; in high school, algebra, geometry, trigonometry, and calculus. It is unclear whether strengthening performance on one component skill can be expected to promote strong performance on other components, and a failure to effect strong performance across component skills would create additional challenges to dramatically reducing the need for ongoing, intensive support in mathematics.

To consider the power and limitations of RTI, we described four studies with strong demonstrations of efficacy, as we considered the percentage of students who failed to respond, the post-tutoring achievement gap between tutored and not-at-risk students, and the extent of transfer across components of the mathematics curriculum. We then discussed implications and additional research questions pertaining to mathematics intervention generally and within the context of RTI. We conclude with a proposal for an expanded conceptualization of RTI.

Efficacy Studies, Nonresponders, Achievement Gaps, and Transfer

Recent research provides the basis for optimism about the efficacy of small-group tutoring in mathematics. Studies show that when tutoring incorporates explicit instruction, provides students with a strong conceptual foundation and efficient procedural strategies, and embeds regular, strategic and cumulative practice, student outcomes improve. In this section, we illustrate such efficacy using four large-scale randomized controlled trials. The first study investigated the effects of first-grade tutoring on multiple components of the curriculum (L. S. Fuchs et al., 2005); in the second study, a major focus was the efficacy of tutoring specifically to enhance fluent and accurate performance on math facts among first graders with risk for mathematics learning disability (L. S. Fuchs, Geary, et al., 2012); the third focused again on math facts tutoring, this time in a population of third-grade students (L. S. Fuchs et al., 2009); and the fourth study investigated the separate and combined effects of primary prevention and supplementary tutoring in the domain of word problems (L. S. Fuchs, Fuchs, Craddock, et al., 2008).

First-Grade Tutoring on Multiple Curricular Components

Working in 41 first-grade classrooms in 10 schools, L. S. Fuchs et al. (2005) assessed the efficacy of first-grade tutoring on multiple components of the mathematics curriculum. At the beginning of first grade, 139 children were identified with risk for developing mathematics difficulty based on their low incoming mathematics performance (i.e., in the lowest quintile). These at-risk students were randomly assigned to a control condition (i.e., the general education program without our tutoring) or a condition in which we provided 16 weeks of tutoring three times each week in small groups. Every tutoring session comprised two activities. The first was a 30-min tutor-led lesson with explicit, interactive instruction on a sequence of 17 topics that focused primarily on numeration, number, and operations; each lesson focused on the relevant concepts along with strategies for successful problem solution on that day's topic, while incorporating manipulative activities, practice, and cumulative review. During the final 10 min of each tutoring session, students used a computer program that was designed to help students commit math facts to long-term memory by providing repeated opportunities for holding associations between problem stems and answers in working memory.

Based on measures of math concepts, procedural calculations, word problems, and math facts, which were administered before and after tutoring (or weekly in the case of curriculum-based measurement), findings supported the efficacy of tutoring as an added level of the prevention system to supplement to the primary prevention regular classroom program. To consider the efficacy of this and other tutoring programs we consider in this article, we rely on estimates of effect size (ES). To compute ES, we used Cohen's *d*, which is a *z* score describing how many standard deviations one group scored higher than another group. Cohen (1988) reluctantly defined small, medium, and large ESs as 0.2, 0.5, and 0.8, but warned of the risk “inherent in offering conventional operational definitions of ES in as diverse a field of inquiry as behavioral science” (p. 25). To put ES in the context of education research, the What Works Clearinghouse generally considers an ES of 0.2 to be small but educationally meaningful.

In the L. S. Fuchs et al. (2005) study, the weekly rate of improvement of the tutored students on *Curriculum-Based Measurement* First-Grade Computation (L. S. Fuchs, Hamlett, & Fuchs, 1990) exceeded that of the control group by an ES of 0.40, and the tutored students' growth was comparable to that of their not-at-risk classmates (ES = 0.11). On *Woodcock–Johnson III* Calculation (Woodcock, McGrew, & Mather, 2001), results were even more impressive: Improvement for the tutored students exceeded not only that of control group peers (ES = 0.57) but also that of not-at-risk classmates (ES = 0.61). Results were similarly strong on the *First-Grade Concepts and Applications Test* (L. S. Fuchs et al., 1990), which measures skill in numeration, concepts, geometry, measurement, applied computation, charts and graphs, and word problems, on which tutored students improved reliably more than controls (ES = 0.67) and not-at-risk classmates (ES = 0.45). On the *Story Problems Test* (Jordan & Hanich, 2000), the improvement of tutored students exceeded that of controls (ES = 0.70), although it was reliably lower than that of not-at-risk classmates (ES = –0.38).

Because the tutored students manifested impressive growth on computation, story problems, and concepts and applications relative to the control group and, on two of four measures, relative to their not-at-risk classmates, we conclude that preventive tutoring was efficacious when used as an added level of prevention. Yet some patterns in the findings prompt caution about assuming that such tutoring inoculates the population of at-risk learners from developing future difficulty with mathematics. The first cause for concern is the lack of universal response—as is typically the case even when results are statistically significant. To estimate the rate of unresponsiveness, we used 16 approaches. One was low achievement at the end of tutoring (i.e., < 10th percentile) on mathematics concepts and applications, for which 11 of the 64 tutored students were deemed unresponsive. This translates into 5.14% of the larger at-risk and not- at-risk students in the study. With other definitions of unresponsiveness (i.e., focusing on different components of the mathematics curriculum; relying on different tests; applying different decision rules), the rate of unresponsiveness varied. For example, with low final achievement on *Woodcock–Johnson III* Calculation, for which the range of items at first grade is narrow, the rate of unresponsiveness was less than 1% of the larger pool of at-risk and not-at-risk students in the study. By contrast, for low final achievement on math facts and for low slope and low final intercept on *Curriculum-Based Measurement* Computation (both with broader sampling in the first-grade range of performance), unresponsiveness increased to 6.38% and 7.94%, respectively. This suggests that denser sampling of items in the range of students' performance increases discriminations among students, especially at the lower end of the distribution, and thereby increases sensitivity to the lack of response. In any case, it appears that approximately 5% of students may be unresponsiveness to this generally effective form of tutoring.

And a second cause for caution is that, at the end of first grade, the performance of tutored students on every measure remained below that of not-at-risk classmates. At pretest, the ESs contrasting the not-at-risk students against at-risk tutored students ranged from 0.60 to 1.48 standard deviations. At posttest, the gap had decreased but remained sizeable, ESs were between 0.41 and 1.35. Of course, although the performance gap narrowed for the at-risk tutored students, it had increased for the at-risk control children, whose end-year performance was 0.70 to 2.04 standard deviations below not-at-risk classmates. So preventative tutoring enhanced the rate of math development beyond what would have otherwise occurred (as revealed by the at-risk control group), but the not-at-risk students' continued development precluded the tutored children from catching up. The remaining end-year performance gap suggests that, despite the efficacy of preventative tutoring, some of the at-risk learners, perhaps even some of those who demonstrated responsiveness, will require additional mathematics support in subsequent years.

Performance on math fact fluency was the final cause for caution. Although the performance of the tutored group exceeded that of controls by a respectable 0.40 standard deviations, this difference was not statistically significant. We attempted to promote fact fluency by providing explicit tutor-directed conceptual instruction specifically on math facts in 9–15 tutoring sessions (depending on rate of mastery) and by allocating 10 min of each 40-min session to computer-assisted practice on math facts. One of several reasons may explain the lack of statistically significant effects. It is possible that math fact fluency represents an especially difficult type of competence to promote. A signature feature of students with mathematics learning disabilities is the failure to develop memory-based retrieval of facts (Fleishner, Garnett, & Shepherd, l982; Geary, Widaman, Little, & Cormier, 1987; Goldman, Mertz, & Pellegrino, 1988), and prior work suggests that such difficulty is persistent (e.g., Jordan, Hanich, & Kaplan, 2003). In reading, remediation success has occurred with accuracy, even as fluency has proven more difficult to promote (Rashotte, Torgesen, & Wagner, 1997, cited in Torgesen et al., 1999). Our data suggest a similar phenomenon for mathematics learning disabilities. At the same time, it is possible that the design of the computer program in its initial form, which did not provide pictorial representations for promoting understanding (INSERM Cognitive Neuroimagery Unit, 2003) or follow-up paper-and-pencil practice for promoting transfer, could be improved. Still another explanation for the lack of effects is that our fact fluency measures, which represent fact families to 12, may have failed to sample performance where the tutored students improved.

On the computer, students mastered an average of 12.71 families; among the first 20 problems on our math fact fluency measures, only 4 items provided opportunities for students to demonstrate this acquired knowledge.

First-Grade Tutoring to Build Fluent and Accurate Math Facts Performance

We attempted to address these limitations in a later first-grade study (L. S. Fuchs, Geary, et al., 2012), in which a major focus was the efficacy of tutoring specifically to build fluent and accurate math facts performance among students with risk for mathematics learning disability. The overall purpose of this study was to examine the role of domain-general abilities versus basic numerical competencies (sometimes referred to as “number sense”; e.g., Berch, 2005; Gersten, Jordan, & Flojo, 2005) in the development of competence with math facts (see L. S. Fuchs et al., 2010; L. S. Fuchs et al., 2010). In this study, we randomly assigned at-risk students at the beginning of first grade to three conditions. One third were assigned to the control condition (no tutoring from us); two thirds were assigned to one of two conditions in which children received 16 weeks of tutoring three times per week, 30 min per session. Both tutoring conditions included the same 25 min of work on number knowledge. The assumption was that the major determinant of competence with math facts is number sense (L. S. Fuchs et al., 2012), which we operationalized as basic numerical competencies such as counting, numeration, and the quick apperception and manipulation of small quantities (Butterworth, 1999; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Geary, 2007). The second, 5-min component of the session differed across the two tutoring conditions. Our goal was to experimentally test the contribution of domain-general abilities to the development of competence with math facts. At the end of the session, students in the “untimed practice” tutoring condition played games that emphasized the math concepts just covered. In the timed practice tutoring condition, students instead completed 5 min of “timed pratice” designed to compensate for their potential weaknesses in the domain-general abilities associated with difficulty with math facts: inattentive behavior, processing speed, phonological processing, working memory, and reasoning ability (see Fuchs, Fuchs, Stuebing et al., 2008; L. S. Fuchs et al., 2010).

In terms of intervention, therefore, the major distinctions between this more recent first-grade study and L. S. Fuchs et al. (2005) include the following: (a) fewer topics in the mathematics curriculum were covered, with a greater emphasis on the concepts underpinning math facts (which may also underpin other components of mathematics skill), and (b) practice in the timed practice condition was more strategic, conducted directly by tutors rather than computers, and designed to compensate for the cognitive weaknesses associated specifically with inadequate development of fluent performance with math facts. In addition, the more recent study had some methodological advantages over Fuchs et al., including additional measures of math fact performance.

Findings indicate that math fact outcomes for children in both tutoring conditions reliably exceeded those of the control group. The performance of the timed practice condition, however, was reliably stronger than that of the untimed practice condition, and the ES contrasting the timed practice group against the control group (i.e., nearly one standard deviation) was substantially larger than in Fuchs et al. In corresponding fashion, a greater percentage of students in the untimed than in the timed practice condition failed to respond, 16.2% versus 8.3%, respectively, denoting unresponsiveness as below the 25th percentile of not-at-risk students in terms of the amount of math facts improvement. On number knowledge, by contrast, both tutoring groups learned comparably and reliably more than the control group. This would be expected given that both groups received the same focus on number knowledge; however, the ESs on number knowledge were smaller than for math facts, suggesting number knowledge may be more difficult to promote. As with L. S. Fuchs et al. (2005), the post-tutoring gap between tutored and not-at- risk students remained substantial on number knowledge. Yet, the timed practice students made ground in catching up to not-at-risk classmates, whereas untimed practice students did not.

In terms of transfer to components of the mathematics curriculum that received relatively little emphasis during tutoring, findings indicate superior performance on procedural calculations, on which both groups performed reliably stronger than the control group, but outcomes were stronger for timed practice than for untimed practice. This suggests that the number knowledge focus of tutoring promoted stronger procedural calculation skill, and that enhanced math facts performance transferred to procedural calculation skill. In terms of transfer to word problems, the pattern of effects was the same as for number knowledge. Thus, findings suggest that transfer occurred from number knowledge to word problems, but not from math facts to word problems.

Acquisition and Transfer Effects of Third-Grade Math Facts Tutoring

We addressed the issue of transfer from math facts to word problems more analytically in L. S. Fuchs et al. (2009) within a randomized field trial in Nashville and Houston. It incorporated two tutoring conditions, one of which focused exclusively on math facts. In the second condition, tutoring focused primarily on word problems, although it did allocate a small amount of time to foundational skills, including counting strategies to help students efficiently answer math facts (needed to solve word problems). The study had three purposes. The first was to assess the efficacy of the two tutoring protocols, which were evaluated against each other and against a no-tutoring control group. By including a control group, we controlled for maturation, historical effects, and business-as-usual schooling. By incorporating two tutoring conditions, we controlled for tutoring time when considering the effects of one protocol against the other. Our second purpose was to explore whether tutoring was differentially efficacious depending on students' difficulty status: mathematics difficulty alone versus mathematics difficulty with concomitant reading difficulty. Third, we assessed the transportability of the tutoring protocols, hence the two sites, one of which was distal to the developers.

Our final purpose, most pertinent to the present article, was to assess transfer from math facts tutoring to word-problem outcomes (we could not do the reverse because word-problem tutoring taught counting strategies for deriving answers to math facts). Third-grade students ($n = 162$) were stratified on site and the nature of their difficulty (math difficulty alone or with reading difficulty) and were then randomly assigned to a control group or one of the two tutoring conditions. In both active conditions, tutoring time was held constant, with 20–30 min per session, and 48 sessions were delivered three times per week for 16 weeks; instruction was interactive and explicit; and the programs relied on the same structured system for ensuring on-task behavior and hard work.

Math facts tutoring comprised five daily activities. In the first, *flash card warm up*, students answered flash cards, addressing the pool of 200 math facts for 2 min. For errors, students “counted up” to derive correct answers using the “min” strategy was addition and the “missing addend” counting strategy for subtraction. The second activity, which lasted 10–15 min, was *conceptual and strategic instruction*, in which tutors (a) introduced or reviewed concepts and strategies underpinning that session's math facts set using the number line and manipulatives and (b) taught and reviewed strategies in which students were encouraged to “Know It or Count Up.” In the third activity, tutors conducted *lesson-specific flash card practice* for 1 min (e.g., if a lesson focused on the 5 set, lesson-specific flash cards were math facts with sums or minuends of 5). Again, students counted up to fix errors, and tried to beat the first session's score on that topic. In the fourth activity, students completed *computerized practice* to build fluency with math facts and to assess mastery with the session's math facts set. This program was a revision of L. S. Fuchs et al. (2005), which added number line representations of math facts. Each session addressed 10 lesson-specific and five review facts. A fact flashed on the screen for 1.3 s. Students rehearsed the fact while it briefly appeared; when it disappeared, students retyped the entire math fact (e.g., addends and answer). If the fact was correct, the student heard applause and earned a point. If incorrect, the student had another chance to enter it correctly. Computerized practice ended when the student answered each of the 10 lesson-specific math facts correctly two times or after 7.5 min. Then students completed a *paper-and-pencil review*, in which they had 1 min to complete 15 lesson-specific facts and another minute to complete 15 review facts.

As already explained, we contrasted this math facts tutoring to a business-as-usual control group and to a word-problem tutoring condition, which was based on schema theory. The word-problem tutoring program included a small focus on math facts, which was limited to three activities: (a) a single lesson in which the counting strategies were taught and practiced, (b) a daily 2-min warm-up flash card activity (identical to the first activity in math facts tutoring), and (c) a correction procedure that required students to count up whenever a math fact error occurred within word problems.

Results demonstrated the efficacy of both tutoring protocols for third-grade students. On math facts, both tutoring conditions effected superior improvement compared to the control group, with no significant difference between tutoring conditions. Compared to the control group, the ES for math facts tutoring was 0.55, and the ES for word-problem tutoring was similar: 0.62. The comparability of outcomes for the two tutoring conditions is notable because math facts tutoring allocated dramatically more time to math facts over the 16-week intervention. On this basis, we concluded that teaching students efficient counting strategies, while providing frequent but small amounts of timed practice to gain efficiency in using those strategies and while contextualizing the use of the strategies within word problems, results in comparable outcomes to an expanded tutoring protocol devoted entirely to math facts.

In terms of procedural calculations, both tutoring conditions again produced superior outcomes compared to the control group. In this case, the ES compared to the control condition was 0.27 for math facts tutoring but almost double that for word-problem tutoring (0.53). This difference was not statistically significant; yet based on these ESs, word-problem tutoring might achieve differential efficacy compared to math facts tutoring with larger samples. Such a finding would not be surprising because only word-problem tutoring allocated direct, albeit limited, time to procedural calculations (i.e., one direct lesson to review procedural calculations; 2 min of paper-and-pencil practice at the end of each session; and completion of procedural calculations while solving word problems, with corrective feedback). It is however interesting that even without direct work on procedural calculations, math facts tutoring resulted in better outcomes on procedural calculations compared to the control group, indicating that transfer occurred.

This is theoretically important because math facts are viewed as a signature, bottleneck deficit for students with mathematics disability (Fleishner et al., 1982; Geary et al., 1987; Goldman et al., 1988). The hypothesis is that with a fixed amount of attention, students with math facts deficits allocate available resources to deriving answers to these simple problems instead of focusing on the demands of the more complex mathematics into which math facts are embedded (see Ackerman, Anhalt, & Dykman, 1986; Goldman & Pellegrino, 1987). If math facts represent a signature deficit, performance on more complex mathematics tasks should improve simply as a function of math facts tutoring, just as decoding intervention has been shown to sometimes improve reading comprehension (Blachman et al., 2004; Torgesen et al., 2001). We found support for this hypothesis in the transfer we observed from math facts remediation to procedural calculation outcomes, suggesting that math facts may in fact serve as a “bottleneck” deficit, at least with respect to procedural calculations. This provides support for the notion that transfer may have occurred from math facts to procedural calculations in the first-grade L. S. Fuchs et al. (2012) study.

By contrast, we found no evidence to support this hypothesis on word-problem outcomes. Word-problem tutoring resulted in strong word-problem outcomes, compared to both contrasting conditions, with large ESs. With math facts improvement but in the absence of word-problem tutoring, however, students evidenced no word-problem improvement. This suggests that the source of their difficulty is not diverting attention from the complex mathematics to the math facts embedded in those word problems, but rather failing to comprehend the relations among the numbers embedded in the narratives or to process the language in those stories adequately. This also suggests that math facts are not the bottleneck for word-problem performance. Instead, it indicates a more complicated pattern of difficulty, implicating language—as has been suggested elsewhere (e.g., Fuchs et al., 2008; L. S. Fuchs et al., 2005; L. S. Fuchs et al., 2006; L. S. Fuchs et al., 2010; L. S. Fuchs et al., in press). Given these contradictory findings about transfer from math facts tutoring, in which effects transferred to procedural calculations but not word problems, future work should continue to explore this issue focusing on word problems as well as other components of the mathematics curriculum.

In terms of the present discussion, however, results again suggest that, as with the two previously discussed studies, intervention on one component of the mathematics curriculum may not carry over to other components, at least with respect to math facts and word problems. This is problematic for prevention efforts because the mathematics curriculum comprises many components, with new components routinely introduced as student progress through school. For example, fractions, which is not a major component of the curriculum in the primary grades, represent a strong focus in at the intermediate grades. Moreover, whole-number logic may actually interfere with students' understanding of fractions (e.g., one eighth is not greater than one fourth, even though eight is greater than four; see Hecht, Vagi, & Torgesen, 2007). In addition, although language is not implicated in whole-number procedural calculations, it is a unique predictor of students' development of competence with rational number computation across third through fifth grades (Seethaler, Fuchs, Star, & Hamlett, 2010). As this illustrates and in line with the absence of transfer from math facts tutoring to word-problem outcomes in L. S. Fuchs et al. (2009) and in L. S. Fuchs et al. (2012), prevention activities in mathematics may present challenges to enduring effects; this issue may be less salient in reading. And as with any generally efficacious instructional intervention and as already demonstrated with L. S. Fuchs et al. (2005), not all students who received tutoring responded. To designate unresponsiveness, we set a cut point below the 16th percentile on end-year performance of a representative sample of third graders. This resulted in 6.8% of students in math facts tutoring designated as nonresponders in terms of their math facts outcomes, 4.8% of students in word-problem tutoring in terms of their math facts outcomes, and 3.7% of students in word-problem tutoring in terms of their word-problem outcomes.

Separate and Combined Effects of Primary Prevention and Supplementary Tutoring on Word Problems

Another randomized controlled trial, Fuchs, Fuchs, Craddock, Hollenbeck et al., (2008), also illustrates how generally efficacious mathematics intervention does not denote universal response, this time while considering how tutoring interacts with classroom instruction. We were interested in the issue of whether tutoring for at-risk learners is more effective when conducted with high-quality classroom instruction to help guide schools in designing their RTI systems: If tutoring is differentially efficacious when combined with validated classroom instruction, then both levels of the prevention system are critical, and classroom instruction needs to be designed deliberately with at-risk students in mind, even when they receive tutoring. By contrast, if tutoring promotes comparable outcomes regardless of the classroom instructional context, then tutoring might occur as a replacement for, rather than as a supplement to, classroom instruction. Tutoring as replacement for classroom instruction would make RTI prevention systems more feasible and efficient and would permit resources to be infused at the tutoring level.

Stratifying to represent classroom conditions in a balanced way in each school, we randomly assigned 40 classrooms to control and 80 classrooms to validated schema-broadening word-problem instruction (i.e., Hot Math; L. S. Fuchs et al., 2003). Control classrooms received 3 weeks of researcher-designed general math problem-solving instruction plus 13 weeks of teacher-designed math problem-solving instruction. Validated instruction involved the same 3 weeks of researcher-designed general math problem solving; however, the 13 weeks of word-problem instruction was researcher designed.

Although all students participated in their classroom condition, we selected a representative sample of 1,200 students to enter the study as research participants, from whom we designated 288 with risk for poor word-problem outcomes. These at-risk students were assigned to tutoring conditions (validated schema-broadening tutoring vs. no tutoring from us), while stratifying by classroom condition. In this way, some at-risk students received no validated instruction (neither in their classrooms nor via tutoring), some received validated instruction in their classrooms but not via tutoring, some received validated instruction via tutoring but not in their classrooms, and some received validated instruction both in their classrooms and via tutoring.

Results showed an interaction between the two levels of the prevention system: Tutoring was significantly and substantially more effective when it occurred in combination with validated classroom instruction than when tutoring occurred with conventional classroom instruction, with an ES of 1.34. This suggests that two levels are better than one level of prevention and indicates the importance of providing at-risk students validated instruction in the classroom and then supplementing that instruction with validated tutoring. We note, however, that in this study, the two levels of instruction were closely aligned, both addressing the same types of word problems at the same time and both relying on the same theoretical and operational approach to instruction. It is possible that when the two levels of instruction are less well aligned, as is often the case, results would differ. (It is also possible that aligning primary prevention and tutoring instructional content may differ as a function of academic domain. For this reasons, future studies should assess the value added of validated classroom instruction that is more and less aligned with tutoring in different academic content.)

In terms of the present discussion, in which we consider the durability of prevention activities for long-term success with mathematics, it is also interesting to consider the performance of at-risk students against not-at-risk peers. When at-risk students received validated tutoring, but with conventional classroom instruction, at-risk tutored students improved more than their not-at-risk classmates, narrowing the achievement gap: At preintervention, the ES was 1.09; at postintervention, only 0.13. On the other hand, with two levels of the prevention system (i.e., validated classroom and tutoring), at-risk tutored students and not-at-risk peers achieved comparably (ES = 0.14), with the achievement gap remaining sizable (at preintervention, 1.29; at postintervention, 0.72). On the other hand, when at-risk and not-at-risk students received the same, single level of validated classroom instruction, without tutoring for the at-risk learners, the achievement gap remained the same or grew: The preintervention ES was 1.30; at postintervention, it was 1.55.

Together, findings indicate that validated tutoring is essential for at-risk learners. Without it, the gap between at-risk and not-at-risk students continues to widen, even when not-at-risk students suffer the disappointing effects of conventional problem-solving classroom instruction. Accordingly, results highlight the importance of validated problem-solving instruction in the primary prevention regular classroom and suggest that tutoring occur as a supplement to, not a replacement for, classroom instruction. When at-risk students receive tutoring combined with validated classroom instruction, their learning exceeds that of students who receive tutoring without validated classroom instruction—by a practically important ES of 1.34 standard deviations.

Even with these impressive results, as with the other efficacy studies already discussed in this article, preventative tutoring did not ensure individual student response. We operationalized responsiveness with a widely used tool for identifying mathematics disability in the schools, *Woodcock–Johnson III* Applied Problems (Woodcock et al., 2001). This measure operationalizes math problem solving broadly to include counting, telling time or temperature, and word problems. Using the 15th percentile as a cut point, 12.8% of at-risk tutored students (or 3.9% of the at-risk and not-at-risk participants) were deemed unresponsive. (Using the same method, 26.6% of at-risk students who did not receive validated tutoring, or 6.8% of the at-risk and not-at-risk participants, met this criterion of unresponsiveness. It is also important to note that validated classroom instruction, in the absence of tutoring, did not reduce the rate of unresponsiveness.)

Implications and Additional Research

The set of studies described in this article provides the basis for conclusions about the power as well as the limitations of RTI for dramatically reducing the need for ongoing and intensive intervention in the area of mathematics. We focus first on the power. As all four studies reveal, it is possible to design tutoring programs to enhance the outcomes of students who are at risk of poor mathematics development. Across interventions, these programs incorporate explicit instruction, provide students with a strong conceptual foundation and efficient procedural strategies, and embed regular, strategic, and cumulative practice. It is interesting that we found that teaching students efficient counting strategies for deriving answers to math facts, while providing frequent but small amounts of timed practice to gain efficiency in using those strategies and while contextualizing the use of the strategies within word problems, produces comparable outcomes to an expanded tutoring protocol devoted entirely to math facts. Given the ever-changing nature of the mathematics curriculum, this argues for efficient methods for addressing multiple components of the mathematics curriculum. Such an integrated approach may, at least partly, address the challenges to transfer we observed in the four efficacy trials described in this article. We note that additional research is needed to identify methods for such integration, even as studies investigate whether subgroups of students, perhaps those with co-occurring reading and mathematics difficulties, require more expanded intervention on individual mathematics skills (L. S. Fuchs, 2010).

In any case, across studies, results clearly demonstrate that students at risk for poor mathematics development suffer reliably and substantially less positive mathematics outcomes if left in the general education program without such tutoring (as represented in the control conditions in these studies). Moreover, when at-risk students do not receive these preventative tutoring services, the gap between their level of mathematics performance and that of not-at-risk classmates grows, making it increasingly difficult for these children to profit from classroom instruction. By contrast, at-risk students as a group, who receive high-quality tutoring, make progress toward catching up to their classmates, and, for some of these children, the scaffolding provided through such short-term validated tutoring creates a strong foundation for them to experience long-term success with their mathematics schooling. Clearly, reliable screening of risk to identify students for 15–20 weeks of accurately implemented, validated tutoring, as represented in the secondary prevention level (typically at Tier 2 or Tier 3) of the RTI system, is a valuable and important service.

At the same time, it is important for policy makers and schools to recognize the limitations of RTI for dramatically reducing the need for ongoing and intensive services for some segment of the school population. We highlight four such limitations: the lack of universal response, the variability in the groups of students designated unresponsive as a function of the method used to determine response, the sizeable post-tutoring mathematics achievement gap between tutored and not-at-risk students, and questions about transfer across the components of the mathematics curriculum.

In terms of lack of universal response, each of the four studies described in this article illustrates that we cannot expect, even with generally efficacious tutoring programs, all students will respond. Across the four studies, the modal rate of unresponsiveness on the components of the curriculum targeted for intervention approximated 4% of the general population. This is similar to the prevalence of learning disabilities in the United States when IQ–achievement discrepancy is used as the method of identification (although research indicates that the groups of students identified via RTI methods of identification versus the IQ–achievement discrepancy differ; D. Fuchs, Fuchs, Compton, Fuchs, & Davis, 2008; L. S. Fuchs et al., 2005). This rate of unresponsiveness suggests the limitations of RTI preventative services for dramatically reducing the need for ongoing, intensive services for students traditionally identified as having a learning disability. This is the case when 15–20 weeks of small-group tutoring is provided. If it were possible to provide a longer duration of tutoring or deliver that tutoring individually, it would perhaps be possible to reduce the rate of unresponsiveness further. Of course, with longer runs of one-to-one tutoring, services begin to resemble the level of intensity expected within special education, which prompts concerns about due process and how schools might fund such a level of intensity without special education resources.

It is also important to consider, however, that the rate of unresponsiveness documented in these four studies probably underestimates the actual percentage when RTI is practiced in the schools. This is the case not only because, in these randomized controlled trials, we ensured that validated tutoring procedures were implemented with strong fidelity but also because the estimates of unresponsiveness were based on performance immediately following tutoring. In actual practice, it is likely that fidelity of implementation will be lower, with reduced effects. In addition, as students continue in school, the effects of tutoring can be expected to diminish, and without additional support, some responders will reemerge with difficulty.

It is also interesting to consider implications of the variability in the rates of unresponsiveness as well as in the groups of students so designated, within and across studies. This occurs at least in part as a function of the method by which response is designated. The most prominent methods for determining response within RTI systems, at least as represented in research, are low final achievement at the end of tutoring or low rate of improvement (i.e., slope or posttest vs. pretest performance) during tutoring or some combination of low final status and low improvement. In L. S. Fuchs et al. (2005), we explicitly contrasted these options. For methods based on low final achievement, the groups of students labeled unresponsive manifested uniformly severe difficulty compared to that of responders, with ESs between these groups between 0.75 and 1.03. The prevalence of unresponsiveness was, however, unrealistically low when designating response based on the *Woodcock–Johnson III* subtests (less than 1% of at-risk and not-at-risk students). On the other hand, estimates for *Curriculum-Based Measurement* (CBM) Computation and for Addition Facts Fluency were disturbingly high (9.40% and 6.38%, respectively). The one math measure where a realistic prevalence rate (5.14%) was combined with severe difficulty (ES = 0.92 contrasting students with and without response) was First-Grade Concepts/ Applications.

Of course, relying on low final achievement to denote unresponsiveness identifies some students as responsive even though they make strong improvement during tutoring. This may occur when initial achievement is very low. It seems unfortunate to categorize these children as unresponsive— hence, an index of improvement as the primary basis for denoting response. L. S. Fuchs et al. (2005) contrasted three identification methods based on improvement (all using CBM Computation, which was collected weekly and thereby permitted the calculation of slope). The first approach, as proposed by Vellutino et al. (1996), sets a cut point for unresponsiveness as the median slope for tutored students. This requires 50% of tutored students to be identified as unresponsive, which was not in line with any other method (and resulted 38.67% of not-at-risk children designated as unresponsive).

Given the unrealistically high prevalence rate for an RTI method using a median cut on tutored students' slopes, we considered CBM slope discrepancy, another method for designating response based on improvement. In this case, using a cut point of below the 10th percentile, the percentage of nonresponders was still high (8.16%). As argued elsewhere (L. S. Fuchs & Fuchs, 1998), however, improvement alone may be an inadequate basis for designating response because it permits students with low slopes, who begin and complete first grade with respectable achievement standing, to be identified as unresponsiveness. For this reason, we also considered a dual discrepancy, which permits classification as unresponsive only when students experience a low slope over the course of tutoring and complete tutoring with low final achievement. This approach reduced the rate of unresponsiveness to 4.43%, while retaining a moderate degree of severity in mathematics difficulty with an average ES of 0.62.

Looking across these methods for designating response, L. S. Fuchs et al. (2005) concluded that low final achievement on First-Grade Concepts/Applications and CBM dual discrepancy appear promising. Yet additional research with other samples and other forms of preventative tutoring is clearly warranted (for some studies in reading, see Barth et al., 2008, and D. Fuchs et al., 2008). The larger point for the present discussion, however, is the variability itself. Findings suggest that, as in reading, different methods for designating response to mathematics tutoring yield different sets of students designated as unresponsive, with varying prevalence rates and fluctuations in the severity of students' mathematics difficulty. Research on RTI may benefit from agreement on a small number of uniformly applied strategies for designating response, which would facilitate comparisons in the literature about the degree of unresponsiveness.

To further complicate this issue, it is possible that many schools determine responsiveness without any formal measurement of the construct, relying instead on informal judgments about response. Research is needed to determine what methods schools use and how well the resulting judgments of responsiveness correspond to students' future trajectories and long-term outcomes. One important reason for the emergence of RTI as an education reform was the 2004 reauthorization of the Individuals with Disabilities Education Act, which permits states to use unresponsiveness to evidence-based practices for the purpose of identifying learning disabilities. Yet as documented in the present article and as just discussed, varying methods for operationalizing response will yield different sets of students identified with learning disabilities and will result in varying rates of prevalence, with fluctuations in the severity academic difficulty. Such heterogeneity in the learning disabled population may plague RTI methods of identification, as occurred with the IQ–achievement discrepancy approach to identification. Therefore, work is needed to provide the field guidance about how to determine which students are and are not responding to preventative tutoring.

At the same time, to date, all methods for determining responsiveness rely on norm-referenced criteria (i.e., a percentile cutoff on final achievement, improvement, or both). As the normative reference group changes, however, so will the groups of students identified as nonresponsive. The field of learning disabilities has struggled in the past about whether local or national reference groups are more defensible. A local normative framework provides the basis for providing intensive services to students who are discrepant with respect to their classroom peers and to the level of instruction provided in those classrooms. On the other hand, local norms increase heterogeneity in the groups of students designated for intensive intervention across districts and states. This raises questions about the tenability of the learning disability construct itself, even as it creates practical problems for schools: As students move to new locations, those who required service in one context may be capable of profiting from a new level of classroom instruction (or vice versa). An alternative to reliance on norms entirely may, however, exist. Some minimum level of academic competence is required for a successful school and life experience, and students who perform below such a criterion after secondary prevention in the RTI system clearly require more long-term and intensive intervention. Such a criterion-referenced approach for designating response, which would preserve meaning across districts and states, may provide a more rationale basis for identifying students for intensive intervention. The research base for deriving such a criterion in mathematics (and reading) does not presently exist, and important questions arise concerning whether such criteria change with development and whether they must be framed separately for different components of the curriculum. A research program on the development of such criteria seems timely.

The results of these randomized controlled trials also illustrate that even with efficacious tutoring programs, where students improve reliably and substantially better than control group at-risk students (or those in competing tutoring groups), the post-tutoring mathematics achievement gap between tutored and not-at-risk students remains substantial. Depending on the mathematics outcome and the tutoring group, the post-tutoring gap ranged from 0.41 to 1.35 standard deviations in L. S. Fuchs et al. (2005), from 0.54 to 1.35 in L. S. Fuchs et al. (2012), and from 0.13 to 0.72 in Fuchs, Fuchs, Craddock, Hollenbeck et al., (2008). (We did not have a strong basis for estimating this in L. S. Fuchs et al., 2009.) These achievement gaps prompt caution about whether at-risk students derive sufficient benefit from tutoring to transition back to the general education with long-term mathematics success. Research is needed to identify the long-term outcomes of students who receive validated tutoring programs.

The final issue that represents a potential challenge to preventing long-term mathematics difficulty with small-group tutoring concerns questions about transfer across components of the mathematics curriculum. As the studies described in this article illustrate, although transfer may occur from math facts tutoring to procedural calculations, there is no basis to presume that math facts tutoring enhances word-problem performance, even when those word problems require students to answer math facts to derive solutions. Mathematics, more than reading, is potentially complicated by the fact that the school curriculum comprises multiple components within and across the grades. In the primary grades, the focus is on whole numbers, with students expected to develop competence with (among other things) concepts, numeration, measurement, math facts, procedural calculations, and word problems. By the intermediate grades, the focus shifts dramatically to rational numbers, which, as already discussed, requires students to move beyond whole-number logic. In the United States, the majority of students, not just those who labor with the primary-grade curriculum, suffer from poor understanding of and procedures with fractions (Mullis et al., 1997). For example, half of eighth-grade students cannot order the magnitude of three fractions (National Council of Teachers of Mathematics, 2006), and most fifth graders cannot determine whether .274 or .83 is greater (Rittle-Johnson, Siegler, & Alibali, 2001). Moving to high school, however, creates new and even greater challenges. Although some speculate that competence with fractions provides the scaffolding needed for success with algebra (e.g., National Mathematics Advisory Panel, 2008), research to document this possibility is lacking. Moreover, over the course of high school, the components of the mathematics curriculum, which include geometry, trigonometry, calculus, as well as algebra, diverge more dramatically than in earlier grades.

Research on the connection among these many components of the mathematics curriculum, which is only beginning to emerge, engenders caution that the cognitive abilities underpinning competence in different components of the mathematics curriculum may differ. Three previous large-scale investigations, for example, considered the role of a large battery of cognitive abilities for more than one mathematics outcome. Assessing 353 first through third graders, Swanson and Beebe-Frankenberger (2004) found that working memory contributed to competence with both math facts and word-problem skill, but unique abilities as a function of math component were also identified: for math facts, phonological processing; for word problems, math facts and fluid intelligence and short-term memory. Swanson (2006) then followed these students' development of math fact and word-problem skill over the next school year. He identified controlled attention, vocabulary knowledge, and visuospatial working memory as predictors of math facts, but the executive control component of working memory for word problems. With 312 third graders, L. S. Fuchs et al. (2006) examined the concurrent cognitive correlates of math facts versus word problems, this time controlling for the role of number combination skill in word problems. Teacher ratings of inattentive behavior correlated with both components of the mathematics curriculum, but the remaining abilities differed: for math facts, phonological decoding and processing speed; for word problems, nonverbal problem solving, concept formation, and language. Across studies, some findings recur; others are idiosyncratic. But together, results suggest that different combinations of cognitive abilities underlie the development of different forms of mathematical competence. This raises questions about whether we can expect students who profit from tutoring in one component of the curriculum to necessarily thrive in the general education program, as the curriculum takes its twists and turns.

In the present review, we considered transfer only from math facts to procedural calculations and to word problems. Findings were encouraging for transfer to procedural calculations but not to word problems, and little is known about transfer across other components of the mathematics curriculum. Such research is needed. In the meantime, we can hope that preventative tutoring strengthens students' foundation for benefiting from the general education classroom mathematics program and, in this way, inoculates students from further difficulty. For some students, this is undoubtedly the case. Yet for other students, it is unlikely that preventative tutoring on a limited set of components in the mathematics curriculum can be expected to promote strong performance on other components.

Concluding Thoughts on Reenvisioning RTI

Preventative small-group tutoring, when conceptualized, designed, and delivered well, is a valuable resource that permits many students to gain sufficient ground to enjoy a “new start” in the general education classroom and experience long-term success there. At the same time, we identified four sources of concern about whether RTI, as presently conceptualized, can eliminate the need for ongoing and intensive services for some portion of the population. Therefore, in this final section, we consider a reenvisioned RTI that incorporates rather than excludes special education (as is often argued; see D. Fuchs et al., 2010), that reenvisions special education as part of the continuum of prevention services. The idea is to use the RTI umbrella to build a more comprehensive prevention system that not only addresses the needs of the at-risk “responder” population, as is presently the case, but also is designed to prevent the long-term failure of the “nonresponders.”

This more inclusive RTI system incorporates three levels of intensity. *Primary prevention* is restricted to the instructional practices general education teachers should be expected to conduct independently and with competence: the core program, with classroom routines that provide opportunities for instructional differentiation, with accommodations that permit access for all students, and with motivational strategies to engage students who have relevant skills but opt against using them. Most core programs are designed using instructional principles derived from research, but few are validated because of the challenges associated with conducting controlled studies of complex, multicomponent programs. *Secondary prevention*, by contrast, involves some type of intervention, which may include one or more round of the kinds of tutoring described in this article: time-limited runs of small-group instruction that relies on a validated tutoring program to specify instructional procedures along with the duration (typically 10 to 15 weeks of 20- to 40-min sessions) and frequency (3 or 4 times per week) of tutoring. The intensity of secondary prevention is distinguishable from primary prevention because secondary prevention is empirically validated (whereas primary prevention is research principled) and because an adult delivers secondary prevention in a standard manner to small groups of students (whereas primary prevention relies primarily on whole-class instruction with or without differentiated activities, which that may involve peer tutoring or independent learning centers). When a validated tutoring program is used at secondary prevention with fidelity, most students are expected to benefit—as revealed in the responsiveness rates we described. In this way, *validation* provides a basis for two critical, interrelated assumptions. First, a student's unresponsiveness to a validated protocol is not the result of poor instruction but rather child characteristics (that may involve a disability). Second, students who do not benefit from secondary prevention demonstrate a need for *non*standard instruction. As written in federal law, students who have a disability and display a need for nonstandard instruction are entitled to special education.

Hence, a comprehensive evaluation follows to confirm the presence of a disability, making *tertiary prevention* synonymous with special education. Tertiary prevention differs from secondary prevention in that (a) in tertiary prevention, teachers establish clear, individual, and ambitious year-end goals in instructional material that match the student's needs and (b) tertiary instruction is individualized (because the student has demonstrated insufficient response to standard forms of instruction at primary and secondary prevention). To individualize, the teacher begins with a more intensive version of a standard protocol (e.g., longer sessions, smaller group size), but instead of assuming it will meet the student's needs, the teacher uses frequent progress monitoring to quantify the effects of the protocol. When the rate of improvement forecasts the student will not achieve the year-end goal, the teacher revises the protocol while monitoring the effects of those revisions. In this way, the teacher inductively and recursively designs an effective, individualized instructional program.

To make special education a meaningful component of the RTI prevention system, however, two innovations are required. First, special educators must be afforded the time and resources to implement the kinds of individualized and intensive intervention just described. Second, tertiary prevention needs to be conceptualized as a level of service that students with learning disabilities enter and exit on a flexible basis to address their present needs. When a student shows strong response at tertiary prevention, he or she returns to secondary or primary prevention, even as the special educator continues to monitor that student's

NIH-PA Author Manuscript

NIH-PA Author Manuscript

progress. With ongoing monitoring, the teacher can detect renewed difficulty as soon as it occurs, pinpoint the area in which difficulty is occurring, and move the student to the appropriate level of the prevention system for efficient remediation. Schools might designate a required period of long-term success in the general education program that dictates the termination of the IEP, at which time the monitoring is limited to the RTI system's regular screening mechanism. When special education is reconceptualized as an individualized, intensive service that moves students flexibly between the levels of the prevention as needed, it can serve as an integral component of the RTI system, with the goal of preventing the long-term failure of students who prove unresponsive to a more limited RTI system.

Acknowledgments

Funding: The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by Grant HD053714 and Core Grant HD15052 from the Eunice Kennedy Shriver National Institute of Child Health & Human Development to Vanderbilt University. The content is solely the responsibility of the authors and does not necessarily represent the official views of the Eunice Kennedy Shriver National Institute of Child Health & Human Development or the National Institutes of Health.

References

Ackerman PT, Anhalt JM, Dykman RA. Arithmetic automatization failure in children with attention and reading disorders: Associations and sequaelae. Journal of Learning Disabilities. 1986; 19:222–232. [PubMed: 3701218]

Barth AE, Stuebing KK, Anthony JL, Denton CA, Mathes PG, Fletcher JM, Francis DJ. Agreement among response to intervention criteria for identifying responder status. Learning and Individual Differences. 2008; 18:296–307. [PubMed: 19081758]

Berch DB. Making sense of number sense. Journal of Learning Disabilities. 2005; 38:333–339. [PubMed: 16122065]

Blachman BA, Schatschneider C, Fletcher JM, Francis DJ, Clonan SM, Shaywitz BA, Shaywitz SE. Effects of intensive reading remediation for second and third graders and a 1-year follow up. Journal of Educational Psychology. 2004; 96:444–461.

Butterworth, B. The mathematical brain. London, UK: Macmillan; 1999.

Cohen, J. Statistical power analysis for the behavioral sciences. 2nd. Hillsdale, NJ: Lawrence Erlbaum; 1988.

Dehaene, S. The number sense: How the mind creates mathematics. New York, NY: Oxford University Press; 1997.

Dirks E, Spyer G, van Lieshout CDM, de Sonneville L. Prevalence of combined reading and arithmetic disabilities. Journal of Learning Disabilities. 2008; 41:460–473. [PubMed: 18768777]

Feigenson L, Dehaene S, Spelke E. Core systems of number. Trends in Cognitive Science. 2004; 8:307–314.

Fleishner JE, Garnett K, Shepherd MJ. Proficiency in arithmetic basic fact computation of learning disabled and nondisabled children. Focus on Learning Problems in Mathematics. 1982; 4:47–56.

Fuchs D, Compton DL, Fuchs LS, Davis GC. Making “secondary intervention” work in a three-tier responsiveness-to-intervention model: Findings from the first-grade longitudinal study at the National Research Center on Learning Disabilities. Reading and Writing: A Contemporary Journal. 2008; 21:413–436.

Fuchs D, Fuchs LS, Stecker PM. The “blurring” of special education in a new continuum of general education placements and services. Exceptional Children. 2010; 76:301–323.

Fuchs, LS. The intersection of reading and mathematics disabilities: Intervention. Presentation at the Eunice Kenney Shriver National Institute on Child Health and Human Development workshop on the intersection of reading and mathematics disabilities; Rockville, MD: 2010 Aug.

Fuchs LS, Compton DL, Fuchs D, Paulsen K, Bryant JD, Hamlett CL. The prevention, identification, and cognitive determinants of math difficulty. Journal of Educational Psychology. 2005; 97:493–513.

Fuchs, LS.; Compton, DL.; Fuchs, D.; Seethaler, PM.; Hamlett, CL.; Bryant, JD.; et al. Wilson, J. An experimental evaluation of the numerical vs. general cognitive abilities underlying the development of fluency with math facts: Preliminary findings. Paper presented at the annual meeting of the Pacific Coast Research Conference; Coronado, CA. 2012.

Fuchs LS, Fuchs D. Treatment validity: A unifying concept for reconceptualizing the identification of learning disabilities. Learning Disabilities Research & Practice. 1998; 13:204–219.

Fuchs LS, Fuchs D, Compton DL, Powell SR, Seethaler PM, Capizzi AM, et al. Fletcher JM. The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. Journal of Educational Psychology. 2006; 98:29–43.

Fuchs LS, Fuchs D, Craddock C, Hollenbeck KN, Hamlett CL, Schatschneider C. Effects of small-group tutoring with and without validated classroom instruction on at-risk students' math problem solving: Are two tiers of prevention better than one? Journal of Educational Psychology. 2008; 100:491–509. [PubMed: 19122881]

Fuchs LS, Fuchs D, Prentice K, Burch M, Hamlett CL, Owen R, et al. Janeck D. Explicitly teaching for transfer: Effects on third-grade students' mathematical problem solving. Journal of Educational Psychology. 2003; 95:293–304.

Fuchs LS, Fuchs D, Stuebing K, Fletcher JM, Hamlett CL, Lambert WE. Problem solving and calculation skill: Shared or distinct aspects of mathematical cognition? Journal of Educational Psychology. 2008; 100:30–47. [PubMed: 20057912]

Fuchs, LS.; Geary, DC.; Compton, DL.; Fuchs, D.; Schatschneider, C.; Hamlett, CL.; DeSelms, J.; Seethaler, PM.; Wilson, J.; Craddock, CF.; Bryant, JV.; Luther, K.; Changas, P. First-grader number knowledge tutoring with timed versus untimed practice. 2012. Manuscript submitted for publication

Fuchs LS, Geary DC, Compton DL, Fuchs D, Hamlett CL, Bryant JV. The contributions of numerosity and domain-general abilities to school readiness. Child Development. 2010; 81:1520–1533. [PubMed: 20840238]

Fuchs LS, Geary DC, Compton DL, Fuchs D, Hamlett CL, Seethaler PM, et al. Schatschneider C. Do different types of school mathematics development depend on different constellations of numerical and general cognitive abilities. Developmental Psychology. 2010; 46:1731–1746. [PubMed: 20822213]

Fuchs, LS.; Hamlett, CL.; Fuchs, D. Curriculum-Based Measurement in math. Vanderbilt University; Nashville, TN 37203: 1990. Available from L. S. Fuchs, 228 Peabody

Fuchs LS, Powell SR, Seethaler PM, Cirino PT, Fletcher JM, Fuchs D, et al. Zumeta RO. Remediating number combination and word problem deficits among students with mathematics difficulties: A randomized control trial. Journal of Educational Psychology. 2009; 101:561–576. [PubMed: 19865600]

Geary DC. An evolutionary perspective on learning disability in mathematics. Developmental Neuropsychology. 2007; 32:471–519. [PubMed: 17650991]

Geary DC, Widaman KF, Little TD, Cormier P. Cognitive addition: Comparison of learning disabled and academically normal elementary school children. Cognitive Development. 1987; 2:249–269.

Gersten R, Jordan NC, Flojo JR. Early identification and interventions for students with mathematics difficulties. Journal of Learning Disabilities. 2005; 38:293–304. [PubMed: 16122059]

Goldman SR, Mertz DL, Pellegrino JW. Extended practice of addition facts: Strategy changes in learning-disabled students. Cognition and Instruction. 1988; 5:223–265.

Goldman SR, Pellegrino JW. Information processing and educational microcomputer technology: Where do we go from here? Journal of Learning Disabilities. 1987; 20:249–269.

Hecht, S.; Vagi, KJ.; Torgesen, JK. Fraction skills and proportional reasoning. In: Berch, DB.; Mazzocco, MMM., editors. Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities. Baltimore, MD: Brookes; 2007. p. 121-132.

INSERM Cognitive Neuroimagery Unit. Brain research and learning sciences: Mini-symposium on the design of rehabilitation software for dyscalculia. Orsay, France: Center for Educational Research and Innovation; 2003 Sep.

Jordan NC, Hanich L. Mathematical thinking in second-grade children with different forms of LD. Journal of Learning Disabilities. 2000; 33:567–578. [PubMed: 15495398]

Jordan NC, Hanich LB, Kaplan D. Arithmetic fact mastery in young children: A longitudinal investigation. Journal of Experimental Child Psychology. 2003; 85:103–119. [PubMed: 12799164]

Mehta PD, Foorman BR, Branum-Martin L, Taylor WP. Literacy as a unidimensional multilevel construct: Validation, sources of influence, and implications in a longitudinal study in grades 1–4. Scientific Studies of Reading. 2005; 9:85–116.

Mullis, IVS.; Martin, MO.; Beaton, AE.; Gonzalez, EJ.; Kel-ley, DL.; Smith, TA. Mathematics achievement in the primary school year: IEA's Third International Mathematics and Science Study (TIMMS). Boston, MA: Boston College; 1997.

National Council of Teachers of Mathematics. Curriculum focal points for prekindergarten through grade 8 mathematics. Washington, DC: Author; 2006. Retrieved from http://www.nctm.org/standards/content.aspx?id=270

National Mathematics Advisory Panel. Foundations for success: Final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education; 2008. Retrieved from http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Rittle-Johnson B, Siegler RS, Alibali MW. Developing conceptual understanding and procedural skill in mathematics: An iterative process. Journal of Educational Psychology. 2001; 93:346–362.

Rivera-Batiz FL. Quantitative literacy and the likelihood of employment among young adults in the United States. Journal of Human Resources. 1992; 27:313–328.

Seethaler PM, Fuchs LS, Star JR, Bryant JR. The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. Learning and Individual Differences. 2011; 21:536–542. [PubMed: 21966180]

Shalev RS, Auerbach J, Manor O, Gross-Tsur V. Developmental dyscalculia: Prevalence and prognosis. European Child and Adolescent Psychiatry. 2000; 9(2):58–64.

Swanson HL. Cross-sectional and incremental changes in working memory and mathematical problem solving. Journal of Educational Psychology. 2006; 98:265–281.

Swanson HL, Beebe-Frankenberger M. The relationship between working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. Journal of Educational Psychology. 2004; 96:471–491.

Torgesen JK. Individual differences in response to early interventions in reading: The lingering problem of treatment resisters. Learning Disabilities Research & Practice. 2004; 15:55–64.

Torgesen JK, Alexander AW, Wagner RK, Rashotte CA, Voeller KS, Conway T. Intensive remedial instruction for children with severe reading disabilities: Immediate and long-term outcomes from two instructional approaches. Journal of Learning Disabilities. 2001; 34:33–58. [PubMed: 15497271]

Torgesen JK, Wagner RK, Rashotte CA, Rose E, Lindamood P, Conway T, Garvan C. Preventing reading failure in young children with phonological processing disabilities: Group and individual responses to instruction. Journal of Educational Psychology. 1999; 91:579–593.

Vellutino F, Scanlon DM, Sipay ER, Small SG, Pratt A, Chen R, Denckla MB. Cognitive profiles of difficulty-to-remediate and readily remediated poor readers: Early intervention as a vehicle for distinguishing between cognitive and experiential deficits as basic cause of specific reading disability. Journal of Educational Psychology. 1996; 88:601–638.

Woodcock, RW.; McGrew, KS.; Mather, N. Woodcock– Johnson III. Itasca, IL: Riverside; 2001.