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Composite fermions and bosons: An invitation to electron masquerade in Quantum Hall

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ABSTRACT The two-dimensional electron gas formed at the semiconductor heterointerface is a theater for many intriguing plays of physics. The fractional quantum Hall effect (FQHE), which occurs in strong magnetic fields and low temperatures, is the most fascinating of them. The concept of composite fermions and bosons not only is beautiful by itself but also has proved highly successful in providing pictorial interpretation of the phenomena associated with the FQHE.

Elementary particles are classified into bosons and fermions. The distinction is made on the basis of the symmetry of their wavefunction upon particle exchange. In quantum mechanics, the physical state of a group of particles is represented by a wavefunction $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$, where \mathbf{r}_i represents the position of the i th particle. As the identical particles are indistinguishable, exchange of two particles changes the wavefunction at most by a factor C , i.e., $\Psi(\dots\mathbf{r}_i, \dots\mathbf{r}_j, \dots) = C\Psi(\dots\mathbf{r}_j, \dots\mathbf{r}_i, \dots)$. Doing the two-particle exchange twice brings the system back to the original situation, so that $C^2 = 1$, and hence $C = \pm 1$. The choice of the sign of C is intimately linked to the spin quantum number of the particles, namely, particles having an integer spin quantum number take $C = +1$ and are called bosons, whereas those with a half-odd spin take $C = -1$ and are called fermions. This can be expressed by writing $C = e^{i\theta}$, in terms of the change in the phase θ of the wavefunction by 0 or π (mod 2π) upon particle exchange. Fermions and bosons exhibit quite distinct types of behavior at low temperatures. Bosons, for example, undergo Bose–Einstein condensation and become a superfluid.

Electrons having spin $1/2$ are fermions. In our three-dimensional (3D) world, the distinction between fermions and bosons is strictly enforced. In a two-dimensional (2D) world, however, the rule can be relaxed by a trick called flux attachment. One can transform a fermion to a boson (or *vice versa*) by attaching to it fictitious magnetic flux(es), which takes care of the change in the phase of its wavefunction. The reason why this sort of trick works lies in the following feature of particle exchange operation in the 2D world. When we exchange two particles in two dimensions, clockwise and counterclockwise exchange operations are distinct, as shown in Fig. 1. In three dimensions, by contrast, they are not distinguishable, because one can be transformed into the other by a continuous transformation of the exchange path. This is linked to the fact that *winding number* is a conserved quantity in two dimensions, but not in three dimensions.

If we imagine that there is a fictitious vector potential field $\mathbf{a}(\mathbf{r})$, an electron picks up an extra phase, $\Delta\theta = (e/\hbar)\int_{\text{path}} \mathbf{a}(\mathbf{r})\cdot d\ell$ as it moves along the path. This phase adds to the fermionic (π) or bosonic (0) phase, and it can alter the overall

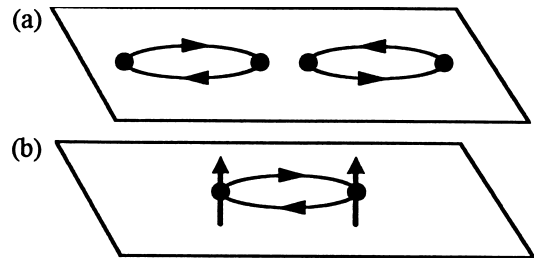


FIG. 1. (a) Clockwise and counterclockwise exchange operations are distinct in two dimensions. (b) The way the phase of the wavefunction is changed upon particle exchange can be altered by attaching a fictitious flux to each particle.

phase change upon particle exchange. If the path encloses a single flux quantum $\phi_0 \equiv h/e$, the above-mentioned extra phase is π , so fermionic electrons can be formally converted to bosons. This trick can be applied to every electron if each carries an odd number of fictitious fluxes of unit flux quantum. Likewise, attaching an even number of unit fluxes converts fermions to fermions and bosons to bosons. Such particles with fictitious fluxes attached are called composite particles (composite fermions and composite bosons).

The idea of composite particles has evolved from the study of the quantum Hall effect. The phenomenon occurs in a 2D electron gas (2DEG) at the semiconductor interface subjected to a strong magnetic field at low temperatures. The quantum Hall effect was discovered in 1979 in Si-MOS (metal-oxide-semiconductor) devices and was subsequently observed in GaAs/AlGaAs HEMT (high electron mobility transistor) devices.

When 2D electrons are subjected to a vertical magnetic field, their energy states are quantized to a series of discrete levels called Landau levels. Each Landau level can accommodate up to $eB/h = 1/2\pi\ell^2$ ($\ell = \sqrt{\hbar/eB}$) electrons per unit area. The electron density divided by this quantity gives the number of Landau levels occupied by electrons, and is called the filling factor ν . The filling factor is the ratio of the number of electrons to the number of flux quanta.

The salient features of the quantum Hall effect (QHE) are as follows: (i) the Hall conductivity σ_{xy} as a function of magnetic field exhibits plateaus at around integer values of ν ; (ii) the values of the Hall conductance at the plateaus are ν (integer) times the universal constant $e^2/h = (25.813\text{k}\Omega)^{-1}$; and (iii) the longitudinal conductivity σ_{xx} vanishes concomitantly. That a quantity such as conductivity which generally depends on many parameters of a particular sample takes a universal value was a big surprise. The physical origin of the

phenomenon is explained in terms of electron localization in a random potential (Anderson localization). The phenomenon is now called integer quantum Hall effect (IQHE), to distinguish it from fractional quantum Hall effect (FQHE), which was discovered a few years later.

The FQHE is similar to the IQHE, except that it occurs at fractional values of ν , such as $1/3$, $2/3$, $2/5$. Electron-electron interaction plays an essential role in the FQHE. In a strong magnetic field such that all electrons are accommodated in the lowest Landau levels, the kinetic energy of electrons is quenched so that the electron-electron interaction becomes important. At a series of fractional values of filling factor with odd denominators, the electrons take special configurations that are particularly effective in minimizing the repulsive Coulomb interaction. Fig. 2 shows the Hall resistivity ρ_{xy} and longitudinal resistivity ρ_{xx} of a high-quality GaAs/AlGaAs 2DEG sample. The FQHE features at $\nu = 1/3, 2/5, 3/7, \dots$ and $\nu = 2/3, 3/5, 4/7, \dots$ show up strongly.

The wavefunction describing the $\nu = 1$ state is written as

$$\Psi_{\nu=1}(z_1, z_2, \dots, z_N) = \prod_{i,j} (z_i - z_j) \exp\left(-\sum_i |z_i|^2 / 4\ell^2\right),$$

where $z_i = x_i + iy_i$ represents the position of the i th electron. It is readily seen that this wavefunction vanishes whenever two electrons come to the same position, as required by the Pauli exclusion principle.

The ground state at $\nu = 1/3$ is well described by the following wavefunction proposed by Laughlin:

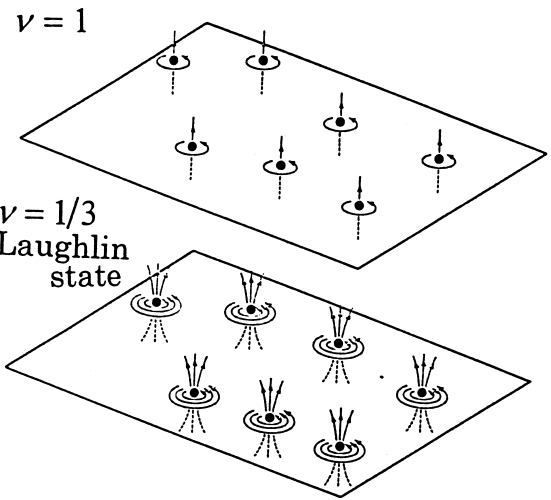


FIG. 3. Schematic drawings of the $\nu = 1$ and $\nu = 1/3$ states.

$$\Psi_{\nu=1/3}(z_1, z_2, \dots, z_N) = \prod_{i,j} (z_i - z_j)^3 \exp\left(-\sum_i |z_i|^2 / 4\ell^2\right),$$

which is depicted in Fig. 3 Lower. As seen by comparing Fig. 3 Upper and Lower, the $\nu = 1/3$ state can be generated from the $\nu = 1$ state by attaching two more fluxes to each electron. It is readily seen that the operation of attaching two more fluxes converts a ν state to a $(\nu^{-1} + 2)^{-1}$ state. Therefore, applying the same flux attachment operation to the IQHE states $\nu = 1, 2, 3, \dots$ generates a series, $\nu = 1/3, 2/5, 3/7, \dots$, which is the major sequence of the FQHE states. In other words, the FQHE states can be viewed as the IQHE states of composite fermions, each of which consists of an electron and two fluxes. In this scheme, the $\nu = 1/2$ state corresponds to $\nu = \infty$ (zero magnetic field) state of the composite fermions.

An alternative view of the QHE states involves composite bosons. As mentioned earlier, an electron accompanied by an odd number of fluxes can be regarded as a composite boson. Therefore, the $\nu = 1$ and $\nu = 1/3$ QHE states depicted in Fig. 3 can be viewed as a composite boson gas in zero magnetic field, and the QHE states can be interpreted as superfluid states of those composite bosons.

I thank Profs. Hideo Aoki, Mansour Shayegan, and Hideo Ohno for their efforts to make the session successful.

1. Willet, R., Eisenstein, J. P., Störmer, H. L., Tsui, D. C., Gossard, A. C. & English, J. H. (1987) *Phys. Rev. Lett.* **59**, 1776–1779.

Suggested Readings

- Prange, R. E. & Girvin, S. M., eds. (1990) *The Quantum Hall Effect* (Springer, New York), 2nd Ed.
- Das Sarma, S. & Pinczuk, A., eds. (1997) *Perspectives in Quantum Hall Effects* (Wiley, New York).

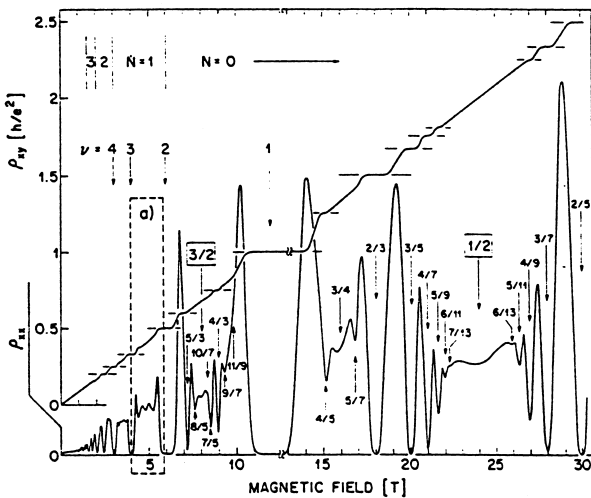


FIG. 2. The Hall and longitudinal resistivity of a GaAs/AlGaAs heterojunction sample at low temperatures, exhibiting rich structures associated with the integer and fractional QHEs. Reproduced with permission from ref. 1, © 1987, American Institute of Physics.