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EQUATIONS AND THE EQUAL SIGN IN ELEMENTARY MATHEMATICS TEXTBOOKS

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Abstract

To promote a relational understanding of the equal sign (=), students may require exposure to a variety of equation types (i.e., 3 = 8 − 5; 2 + 3 = 1 + 4; 9 − 3 = 6). The purpose of this study was to evaluate 8 elementary curricula for degree of exposure to equation types. Across 6 elementary grade levels, curricula were coded for the number of standard and nonstandard equation types appearing within the student textbook. Except in 1 of the 8 curricula, students typically do not receive exposure to nonstandard equation types that promote a relational understanding of the equal sign. An analysis of the accompanying teacher manual for each textbook suggests that students receive minimal instruction on relational definitions of the equal sign, with the majority of instruction occurring in grades K–2 and minimal instruction provided in grades 3–5.

Students in elementary school often misinterpret the equal sign (=) as an operational (i.e., do something or write an answer) symbol even though the equal sign should be viewed as a relational symbol (Sherman & Bisanz, 2009). Students should understand the equal sign as relational, indicating that a relationship exists between the numbers or expressions on each side of the equal sign (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). The number or expression on one side of the equal sign should have the same value as the number or expression on the other side of the equal sign. If the equal sign is interpreted in an operational manner, this typically leads to mistakes in solving equations with missing numbers (e.g., 5 − ___ = 1) and difficulties with algebraic thinking (e.g., $x - 2 = 2y + 4$; Lindvall & Ibarra, 1980; McNeil & Alibali, 2005b). Research has shown, however, that ongoing classroom dialogue (e.g., Blanton & Kaput, 2005; Saenz-Ludlow & Walgamuth, 1998) or explicit instruction (McNeil & Alibali, 2005b; Powell & Fuchs, 2010; Rittle-Johnson & Alibali, 1999) can change students' incorrect interpretations of the equal sign.

One possible reason for misinterpretation of the equal sign is a lack of exposure to a variety of equation types. The purpose of this study was to evaluate eight elementary curricula across grades K–5 to determine the degree to which students receive exposure on nonstandard equation types and to understand how teachers are encouraged to define the equal sign and provide instruction on nonstandard equation types. These nonstandard equations are generally believed to be necessary to promote a relational understanding of the equal sign (McNeil et al., 2006). To date, an evaluation of the types of equations presented in elementary mathematics textbooks has not been conducted.

Before proceeding, I comment briefly on equation terminology. An *expression* is a combination of numbers and operations without an equal sign (e.g., 9 ÷ 3; 1 + 1 + 4; $y \times 6$). An *equation* is a mathematical statement where the equal sign is used to show equivalence

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between a number or expression on one side of the equal sign to the number or expression on the other side of the equal sign. Every equation has two *sides* (i.e., left and right). The dividing point between the sides is the equal sign. *Standard* equations are in the form of operations equal an answer (e.g., 2 + 4 = ___; 2 + 4 = 6; 2 + 2 + 2 = 6). As the equation is read left to right, the equal sign is always in the second-to-last position, and the answer is after the equal sign. Standard equations can be *open* (i.e., incorporating a blank or variable to solve) or *closed* (without any missing information). *Nonstandard* equations occur in any form other than standard (e.g., 6 + 4 = ___ + 8; 6 = 2 + 4) and can also be open or closed. A *mathematical equation* is an equation with zero or one variables (e.g., 9 = 6 + 3; 9 = x + 3), whereas an *algebraic equation* is an equation with two or more variables (e.g., $x - 3 = y$).

Understanding the Meaning of the Equal Sign

Across elementary, middle, and high school, students often misunderstand the equal sign (Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006). Problems with interpreting and working with the equal sign may arise as students implicitly develop ideas about addition and subtraction before entering school (Seo & Ginsburg, 2003) and as students receive early elementary school instruction that exclusively presents equations in the form of a number, operator symbol, number, equal sign, and blank (e.g., 2 + 3 = ___; Capraro, Ding, Matteson, Capraro, & Li, 2007; McNeil, 2008). Often, students' struggles with equivalency stem from a misunderstanding of the equal sign, which has been noted in students as young as kindergarten age (Falkner, Levi, & Carpenter, 1999). Researchers hypothesize that as students see and work with typical teacher- or textbook-presented equations, where an answer always needs to be computed after the equal sign, students come to understand the equal sign as an operational indicator directing them to perform a calculation (McNeil et al., 2006).

Understanding the equal sign in a relational manner is important for two major reasons. First, students need to solve equations. Many times in school, students are introduced to solving equations where only a sum or difference is computed (Behr, Erlwanger, & Nichols, 1980). These equations are easy for students to solve, and students can perceive the equal sign as an operational symbol and still answer the equations correctly. Understanding the equal sign as a relational symbol becomes more important when students begin solving equations with a missing addend, minuend, subtrahend, factor, dividend, or divisor when the equal sign is not in a standard position (i.e., 2 = 7 − ___; 3 × ___ = 5 + 5 + 5). If students believe the equal sign means to perform an operation, equations where the missing part is not the sum, difference, product, or quotient will most likely be answered incorrectly. Second, students should have proper equal sign understanding to work on higher-level math problems such as word problems and algebraic equations (Herscovics & Kieran, 1980; Powell & Fuchs, 2010). Many times students are taught to read word problems and develop a mathematical equation to assist in solving the word problem (Lindvall & Ibarra, 1980; Nathan & Koedinger, 2000). If students struggle to solve simple equations in the early elementary grades and do not learn the correct interpretation of the equal sign, solving mathematical (e.g., $x + 5 = 8$) or algebraic (e.g., $2y = x - 3$) equations and performing other higher-level math calculations will become more difficult as school progresses (Carpenter, Franke, & Levi, 2003). Because more school districts require all students to pass algebra courses, and because mathematics organizations (National Council of Teachers of Mathematics, 2000) emphasize the importance of teaching algebraic thinking across grades K–12, a proper foundation for algebra taught in the early elementary grades becomes more important every year, and understanding the equal sign is foundational to algebraic competence (Gagnon & Maccini, 2001).

Misinterpretations of the Equal Sign

Often, students interpret the equal sign as an operational symbol, not a relational symbol (Baroody & Ginsburg, 1983; Kieran, 1981). In terms of viewing the equal sign as an operational symbol, most elementary students believe the equal sign signals them to "do something" or "find the total," or that the "the answer comes next." Students viewing the equal sign as a signal to do something look at the left side of an equation and decide the equal sign means to do something to the right side of the equal sign (Cobb, 1987). Other students view the equal sign as a clue to find the total, even when finding the total is inappropriate (McNeil & Alibali, 2005a). For example, students may solve a problem such as 5 + 4 = ___ + 3 by adding all three numbers and writing an answer of 12. Some students believe the equal sign means the answer comes next (Sherman & Bisanz, 2009). If students make this mistake, an answer of 9 would be written in the blank space for 5 + 4 = ___ + 3. With all of these mistakes, students have the misconception that the equal sign is an operator symbol.

Role of Standard and Nonstandard Equations

As mentioned earlier, misinterpretation of the equal sign may develop when students are exposed exclusively to standard equations such as 4 + 7 = ___ or 9 − 4 = ___ (Capraro et al., 2007) in which the equal sign does in fact signal a calculation. To examine differences between standard and nonstandard equations, Carpenter and Levi (2000) presented first- and second-grade students with both types of equations. Students had an easy time agreeing that 1 + 1 = 2 was a correct equation, but a much harder time agreeing that 2 = 1 + 1, 2 = 2, or 1 + 1 = 1 + 1 were acceptable. With discussion and instruction from the teacher, however, students changed their ideas about the equal sign as an operator symbol. Cobb (1987) found a similar pattern with first-grade students. When faced with a task of deciding whether 6 + 4 = 4 + 6 was an acceptable equation, students altered the equation to read 6 + 4 = 10. Students could not deem nonstandard equations as acceptable.

Weaver (1973) also compared how students in first, second, and third grade managed problems where the operator was on the left side (i.e., standard) versus the right side (i.e., nonstandard) of the equal sign. Students experienced a higher success rate for correctly solving standard (i.e., operation-left-side) equations than nonstandard (i.e., operation-right-side) equations. Lindvall and Ibarra (1980) discovered a similar pattern with first- and second-grade students. When the operator was on the right side of an equation, students made many more mistakes than when the operator was on the left.

Role of Symbols

Misinterpretation of the equal sign may also stem from a misunderstanding of symbols. Without formal instruction, students perform relatively well on verbal story problems yet poorly on corresponding symbolic equations. This indicates that symbolic representations and problem structure may hinder students' ability to solve problems (Carpenter, Hiebert, & Moser, 1981). To make this point, Sherman and Bisanz (2009) asked second graders to solve nonstandard operations-both-sides equations (e.g., 1 + 4 = 2 + ___) using manipulatives (or pictures of manipulatives) or numbers and symbols. Students who worked on operations-both-sides equations with manipulatives solved twice as many equations correctly as students who solved equations with numbers and symbols. Similarly, Cobb (1987) presented first graders with a number of felt squares (e.g., 5), told them the total number of felt squares (e.g., 9), and asked them to find the number unseen (e.g., 4). Every student solved the missing-addends task correctly, indicating that students could cognitively solve equations without symbols.

Beatty and Moss (2007) also demonstrated the benefit of using manipulatives over symbols with third-grade students with learning disabilities. To remediate misunderstandings of the mathematical equations, students were presented with manipulatives (i.e., pieces of candy) on a piece of paper folded in half. After working with the manipulatives and making the manipulatives on the two sides of the paper the same, students were shown a corresponding equation written on a white board. Students demonstrated improved performance on equation solving at posttest and a 3-month maintenance test.

Role of Instruction to Improve Equal Sign Understanding

Experimental work conducted in the late elementary grades by McNeil and Alibali (2005b) and Rittle-Johnson and Alibali (1999) demonstrated the effect of explicit equal sign instruction. McNeil and Alibali (2005b) randomly assigned students (ages 7–11) to three conditions: problem structure, equal sign, and control. Students were presented with a closed nonstandard operations-both-sides problem (e.g., 6 + 4 + 7 = 6 + 11). Problem structure students were taught to focus on the location of the equal sign, and equal sign students were provided with explicit instruction on the meaning of the equal sign. At posttest, students in both groups answered nonstandard equations correctly more often than control students. In a similar study at fourth and fifth grade, Rittle-Johnson and Alibali (1999) randomly assigned students to three conditions: conceptual, procedural, and control. Conceptual students were taught a principle for solving equations (i.e., numbers on each side of the equal sign need to be equal). Procedural students received instruction on how to solve for the missing information. At posttest, students who received instruction significantly outperformed controls on conceptual and procedural tests. These studies, along with studies conducted by Alibali (1999) and Matthews and Rittle-Johnson (2009), demonstrate that students respond to instruction on the equal sign symbol within nonstandard equations and can be taught to shift from interpreting the equal sign in an operational to relational manner.

Student Textbooks and Teacher Manuals

Mathematics curricula, which often rely exclusively on standard equation formats (Capraro et al., 2007; McNeil et al., 2006), may contribute to student misunderstanding of the equal sign. This was shown by McNeil et al. (2006) in an evaluation of four middle school textbooks in grades 6–8. McNeil et al. coded 50%of the pages in each textbook for prevalence of equation types (i.e., standard vs. nonstandard). The authors also analyzed whether nonstandard equations had operations on both sides of the equation (e.g., 3 + 4 = 9 − 2), an operation on the right side of the equation (e.g., 5 = 9 − 4), or no operation (e.g., 7 = 7). Across all four textbooks at all three grade levels, standard equations and nonstandard operation-right-side equations appeared most frequently. Nonstandard operations-both-sides equations appeared infrequently, which caused McNeil et al. concern as the other equation types continue to support an operational meaning of the equal sign.

To determine which types of equations work better for teaching students a relational meaning of the equal sign, McNeil et al. (2006) conducted two experiments with middle school students. In the first experiment, students were exposed to standard equations (e.g., 8 − 3 = 5), nonstandard operation-right-side equations (e.g., 3 = 9 − 6), or nonstandard no-operation equations (e.g., 5 = 5). Students were more successful in defining the equal sign in a relational manner when exposed to the nonstandard equation types. In the second experiment, McNeil et al. contrasted nonstandard operation-right-side equations and nonstandard operations-both-sides equations (e.g., 4 + 4 = 2 + 6). After exposure to the operations-both-sides equations, students understood the equal sign relationally almost twice as often as after exposure to operation-right-side equations.

The middle-school textbook analysis of McNeil et al. (2006) demonstrated that students receive minimal exposure to nonstandard equation types. Adding to this literature, Li, Ding, Capraro, and Capraro (2008) compared U.S. texts for preservice teachers to Chinese texts for preservice teachers and found differences in equal sign definitions and activities. Teacher texts in China provide much more instruction on teaching equivalency and a relational meaning of the equal sign than teacher methods books from the United States. Additionally, Li et al. administered a brief test to sixth-grade students in the United States and China. Only 28% of U.S. students solved nonstandard operations-both-sides equations correctly, whereas 98% of the Chinese students answered the same problems correctly.

Purpose of the Present Study

Based on this body of research, it appears that, without formal relational instruction in the equal sign, students view the equal sign as an operator symbol. That is, the equal sign signals students to “do something” or “find the total,” or that “the answer comes next” (Cobb, 1987; McNeil & Alibali, 2005a; Sherman & Bisanz, 2009). Viewing the equal sign as an operational symbol may result from early exposure to basic adding (e.g., 1 + 1 = 2) strategies where the equal sign always means to add or write an answer (Baroody & Ginsburg, 1983; Herscovics & Kieran, 1980) or from misunderstanding the symbol (i.e., equal sign) that represents equivalence (Beatty & Moss, 2007; Sherman & Bisanz, 2009). Textbooks, which often rely exclusively on standard equations, may also contribute to the problem (Capraro et al., 2007; McNeil et al., 2006). Instruction, however, can have a positive effect on student understanding of the equal sign (McNeil & Alibali, 2005b; Rittle-Johnson & Alibali, 1999). The purpose of this study was to extend the work at the middle-school grades of Li et al. (2008) and McNeil et al. (2006) to determine whether similar trends in standard and nonstandard equations exist in elementary school textbooks, to see whether students receive exposure to the nonstandard equation types that assist with a relational understanding of the equal sign, and to gain understanding of the frequency and type of instruction teachers provide to students about the equal sign.

Method

Curricula

Eight elementary mathematics curricula were analyzed and coded by equation types. See Table 1 for a list of textbooks and authors. These curricula were chosen because they represent eight common curricula found in schools throughout the United States and have been used in other mathematics textbook reviews (Agodini & Harris, 2010; Ding & Li, 2010). Six grade levels of each curriculum (i.e., K–5) were included in the analysis, for a total of 48 curricula.

Coding of Textbooks and Search of Teacher Manuals

Every other page (50%) in the student textbook for each curriculum at each grade level was analyzed, and the number of each equation type was coded on a spreadsheet. The student textbook was chosen for coding because the textbook represented the minimum that teachers typically use with students during the school year. Many of the curricula had additional practice, extension, and review pages along with games and activities designed for whole-group, small-group, or individual settings, but many of these pages and activities were described as optional in each curriculum’s teacher manual. Because it was not possible to guarantee that students received exposure to these extra pages and activities, the approach of analyzing the pages in the student textbook presented a representative sample of how each curriculum represented equations. Analyzing the student textbook, and only the student textbook, was also the approach of McNeil et al. (2006).

To gain insight into the types of equations students are exposed to in their textbooks, the equations were split into 12 categories. An equation, as defined in this analysis, was any number sentence with an equal sign. See Table 2 for explanations and examples of the 12 categories of equations. Five equation types were standard; seven were nonstandard. Within the standard operation-left-side categories, equations were presented horizontally with an equal sign as the second-to-last part of the equation. The five categories were split by operation: addition, subtraction, multiplication, division, or mixed. Almost all standard equations were in the format of number, operator symbol, number, equal sign, and number, but a few equations had additional numbers and operator symbols before the equal sign. Within the nonstandard categories, five categories involved operation-right-side equations, with the equal sign as the first symbol in the equation. Almost all of these nonstandard equations were in the format of number, equal sign, number, operator symbol, and number. A few had three or four numbers after the equal sign with additional operator symbols. Again, these five nonstandard operation-right-side equations categories were split by operation. No-operation equations to show identity of numbers (e.g., 23 = 23) and operations-both-sides equations (e.g., 7 − 4 = 1 + 2) rounded out the nonstandard equations. It is the last nonstandard category (i.e., operations-both-sides) that Alibali (1999) and McNeil et al. (2006) believe has the best capacity to demonstrate a relational meaning of the equal sign. These nonstandard operations-both-sides equations were coded under one category because the operator symbol on one side of the equation did not necessarily match the operator symbol on the other side of the equation (e.g., 12 − 4 = 6 + 2). The terminology for equations (i.e., operation-right-side, no-operation, operations-both-sides) is similar to that of McNeil et al. (2006).

For coding, the number of equations falling under each of the 12 categories was counted and entered into a spreadsheet. Both closed and open equations were included in the analysis. For example, 3 + 4 = 7 and 3 + 4 = ___ were both coded as a standard addition type. Both closed and open equations were included because I was interested in the ways students were exposed to the equal sign and not necessarily the ways students worked with the equal sign. Almost all equations for student practice, however, were open equations. Most of the closed equations were worked examples used for demonstration. As mentioned before, 50% of the pages (i.e., every other page) in the 47 student textbooks were coded in the same fashion.

Additionally, at each grade level for each curriculum, the teacher manual was analyzed for equal sign descriptions and explanations. This descriptive analysis was carried out by scanning every page in the teacher manual and consulting the index for the terms *equality*, *equal*, *equals*, *equations*, and *equivalence*. (An *Everyday Mathematics* student textbook at kindergarten was not analyzed because the kindergarten program does not utilize a student textbook. This brought the total number of student textbooks coded by equation types from 48 to 47. The *Everyday Mathematics* kindergarten teacher manual was analyzed for descriptive data.)

Intercoder Agreement

A second coder counted the number of equations falling under the 12 equation categories on every tenth page of the 47 student textbooks, with 19.42% of the total pages checked. Intercoder agreement was 99.56% across the eight curricula and five grade levels.

Data Analysis

See Table 3 for the number of standard and nonstandard equations by curriculum and grade level. As outlined in Table 2, nonstandard equations were categorized seven ways: operation-right-side (addition, subtraction, multiplication, division, mixed), no-operation, and operations-both-sides. The percentage of nonstandard equations relative to standard equations was calculated by curriculum and grade level and is presented in Table 3. For the data analysis, ANOVAs were applied to the percentage of nonstandard occurrences among curricula and grade levels. Post-hoc tests of least significant differences were run to determine which curriculum, if any, demonstrated a significantly greater number of nonstandard equation occurrences.

Results

Equations in Student Textbooks

See Table 4 for a list of significant differences between curricula. For nonstandard operation-right-side addition equations, there was a significant difference among curricula, $F(7, 46) = 2.256$, $p = .050$. Follow-up tests indicated that *Everyday Mathematics* had a significantly higher percentage of nonstandard addition equations than *HSP* ($p = .024$), *Investigations* ($p = .013$), *Math Connects* ($p = .014$), *Saxon Math* ($p = .010$), *SFAW* ($p = .014$), and *Singapore Math* ($p = .011$). There was no significant difference between *Everyday Mathematics* and *Math Expressions* ($p = .510$). Similar to *Everyday Mathematics, Math Expressions* had a significantly higher percentage of nonstandard addition equations than did *Investigations* ($p = .049$), *Saxon Math* ($p = .040$), and *Singapore Math* ($p = .040$). For nonstandard operation-right-side subtraction equations, there was a significant difference among curricula $F(7, 46) = 8.875$, $p < .001$. Follow-up tests indicated that *Everyday Mathematics* had a significantly higher percentage of nonstandard subtraction equations than all seven other curricula, with all $p < .001$. There were no differences among the other seven curricula and no differences among grade levels.

A similar pattern emerged with nonstandard operation-right-side multiplication equations, $F(7, 46) = 2.337$, $p = .043$. *Everyday Mathematics* had a significantly higher percentage of nonstandard equations than every curriculum except for *Singapore Math* ($p = .101$). There were no other differences among curricula. There were significantly more instances of multiplication equations at fourth and fifth grade over kindergarten and first grade, but this result can be attributed to the fact that multiplication is not taught until the late elementary grades. For nonstandard operation-right-side division equations, there was also a significant difference among curricula, $F(7, 46) = 2.591$, $p = .027$, that favored *Everyday Mathematics* over all seven other curricula. There were no significant differences among the other curricula or among grade levels. There were too few instances of operation-right-side mixed equations to conduct an analysis.

In terms of nonstandard no-operation equations (e.g., 12 = 12), there were no significant differences among curricula or grade levels. For operations-both-sides equations (e.g., 12 ÷ 2 = ___ × 2), there were no significant differences among curricula. There were, however, significant differences among grade levels, $F(5, 46) = 4.639$, $p = .002$. Fifth-grade textbooks had a significantly higher percentage of operations-both-sides equations than kindergarten ($p < .001$), first grade ($p < .001$), second grade ($p = .001$), third grade ($p = .008$), and fourth grade ($p = .007$). There were no other differences among grade levels.

Equal Sign Explanations in Teacher Manuals

There were differences among curricula in the types of equations (i.e., standard and nonstandard) presented in student textbooks. For this analysis, I also looked at how the teacher manual of each curriculum explained the equal sign to understand what teachers are asked to teach. It is interesting to note that the equal sign (i.e., =) is referred to as both the *equal sign* and the *equals sign*. Usage of *equal* and *equals* varies by curriculum and grade level and even within curriculum and grade level. (For this description of equal sign explanations, the term used by the teacher manual [i.e., equal or equals] is provided within the description.) Across curricula, the equal sign was mentioned no more than eight times throughout each teacher manual at each grade. In a few teacher manuals, the equal sign was not mentioned at all. (See Table 5 for a description of equal sign definitions and explanations by curriculum and grade level.)

In *Everyday Mathematics*, the kindergarten teacher manual does not provide teachers with explicit instruction on the equal sign. At first grade, the equals sign is discussed as teachers are prompted to define the sign as *is equal to* or *is the same as*. Teachers teach students to use these terms interchangeably. This terminology for the equals sign continues into the second-grade teacher manual. Additionally, teachers are encouraged to use nonstandard equations in their mathematics instruction so students understand the relationship between two sides of an equation. At third, fourth, and fifth grade, however, the teacher manuals provide no explicit instruction on teaching students the meaning of the equal sign.

With *HSP Math*, kindergarten teachers are provided with two definitions of the equal sign: *is equal to* and *the number on one side is equal to the other side*. In one instance, however, the teacher manual asks teachers to explain that *the number in all* is written after the equal sign. Similar definitions of the equal sign (i.e., *is equal to*, *is the same as*) are provided within the first-grade teacher manual. At second and third grade, teachers are given the definition of *is equal to* for the equal sign. Not until fourth grade are teachers encouraged to work with an equabeam balance to concretely demonstrate how to solve an equation such as $4 + m = 9$. Similar to many of the other math curricula, the fifth-grade teacher manual does not include any definitions of the equal sign.

The *Investigations* curriculum includes many examples and definitions of the equal sign. In the kindergarten teacher manual, teachers are encouraged to define the equal sign as *this side is the same as*, but the teacher manual also prompts teachers to ask, “What sign do we use to show 10 altogether?” In the first-grade manual, teachers are given a definition of *both sides are the same*. Teachers are also encouraged, similar to *Everyday Mathematics*, to teach with nonstandard equations so students have opportunities to see if sides have the same amount. It is interesting that teachers are encouraged to use nonstandard equations, yet students are presented with few instances of nonstandard equations in the student textbook. Similar equal sign explanations are in the teacher manuals at second and third grade. Teachers are given an equal sign explanation of *two things are the same*, and teachers instruct students that the equal sign denotes equivalency. In the fourth- and fifth-grade manuals, teachers are provided with equal sign definitions of *equal to* or *one side is equivalent to the other side*. In the fifth-grade materials, teachers are discouraged from allowing students to use multiple equal signs in the same equation to show a linear process of mathematics (i.e., $8 + 2 = 10 + 2 = 12$), but no definition of the equal sign is provided to explain why the equal sign should not be used in instances of $8 + 2 = 10 + 2 = 12$.

Very few equal sign explanations are provided within the *Math Connects* teacher manuals. The glossary of the kindergarten manual explains that the equal sign has two sides and the sides need to be balanced. At first grade, the equals sign is defined for teachers as *having the same value.* Pictures of a pan balance for solving nonstandard equations are provided in the teacher manual at second grade without explicit directions for teaching about nonstandard equations. The equal sign is not mentioned in the teacher manuals at grades 3, 4, and 5.

On the other hand, *Math Expressions* teacher manuals have quite a few equal sign definitions and explanations. At kindergarten, students are introduced to the equals sign and provided with a definition of *the same as*. Teachers are also encouraged to teach about the unequals sign (≠) and provide students with opportunities to use both signs. Teachers also introduce both standard (i.e., $4 + 5 = 9$) and nonstandard (i.e., $9 = 4 + 5$) equations so students can see the equal sign in different places. The definition of *the same as* is continued in first grade along with a definition of *is equal to*, and teachers continue teaching with the equals and unequals signs. At grade 2, the equals sign is mentioned once within the teacher manual as meaning *is equal to*. The teacher manual at third grade asks teachers to discuss the equal sign as meaning *two sides have the same value*. In a similar way, the fourth-grade teacher manual describes the equals sign within an equation where both sides have the same value. Additionally, teachers are asked to point out that the answer in an equation can be written with the answer on the left side and the operation on the right (i.e., nonstandard). There are no equal sign definitions in the fifth-grade teacher manual.

With *Saxon Math*, no equal sign explanations are provided in the teacher manual at kindergarten even though the equal sign is used for addition equations. A definition of *two quantities have the same value* is included within the first- and second-grade teacher manuals in reference to the equals sign. In the third-grade teacher manual, teachers tell students to write an equal sign if *two amounts are the same* and *have the same value*. Similar to *HSP Math*, a balance scale is introduced in fourth grade to show that two amounts are the same. At fifth grade, the teacher manual mentions using comparison symbols, such as the equal sign, but does not provide explicit instruction on the meaning of the equal sign.

SFAW also provides teachers with quite a few equal sign activities. In the kindergarten teacher manual, teachers are instructed to teach students that the equals sign means *one side is the same as*, and students are encouraged to read equations as 3 plus 2 *is the same as* 5. Interestingly, the teacher manual also explains to teachers that the equals sign goes between the numbers added and the sum. This explanation is continued at first grade, where teachers are asked to question students as to what sign comes before the sum. In the same manual, however, teachers are supposed to use a balance to show that two sides of an equation should be the same and teach the equal sign as meaning *the same as.* In the second-grade teacher manual, teachers are provided with equals sign definitions of *equal to* and *one side is the same as the other side*. At third grade, teachers are given a definition of *two expressions are equal* for the equals sign. The balance scale is used again at fourth grade to show that sides of an equation have the same value, but no equal sign instruction is provided within the fifth-grade teacher manual.

In the kindergarten *Singapore Math* teacher manual, teachers are encouraged to use a definition of *to make* for the equal sign. A more detailed explanation is provided at first grade, where teachers are provided with equal sign definitions of *is equal to*, *same amount*, or *same number*. The second-grade teacher manual encourages teachers to use the equal sign multiple times in an equation (e.g., $6 \times 2 = 10 + 2 = 12$) to show equivalence between multiple expressions and numbers, but no definitions of the equal sign are provided for the teachers or students. The third- and fourth-grade teacher manuals have no equal sign explanations, but the fifth-grade manual describes the equal sign as meaning having the *same value*.

Discussion

In terms of exposure to equations, the majority of equations across kindergarten to fifth grade fall into the standard category. Only one curriculum, *Everyday Mathematics*, resisted this trend with addition, subtraction, multiplication, and division equations and, with a greater frequency, used nonstandard equations in the student textbooks across grades. The nonstandard equations that were most often used in *Everyday Mathematics* were operation-right-side equations. This mirrors the middle-school textbook analysis of McNeil et al. (2006) where standard and nonstandard operation-right-side equations appeared in middle-school textbooks the majority of the time. Another curriculum, *Math Expressions*, included a significant number of nonstandard addition equations over three other curricula, but this pattern for *Math Expressions* did not hold true for subtraction, multiplication, and division equations. No curriculum featured no-operation equations more prominently than another curriculum, and no-operation equations appeared much less frequently than any other nonstandard equation type. For operations-both-sides equations, which have been shown to be more effective in fostering a relational understanding of the equal sign (McNeil et al., 2006), there were no significant differences among curricula. There was a trend of more operations-both-sides equations at fifth grade than any other grade, as textbooks begin to focus on some of the properties of mathematics (e.g., commutative, associative, distributive) that are important for algebraic reasoning.

Although there is no guarantee that teachers read and follow teacher manuals that are provided with a curriculum, the description of how the equal sign is discussed within each curriculum is important. First and foremost, the definitions provided for the equal sign (i.e., *equal to*, *is the same as*, *two sides are the same*) are fairly consistent across curricula. The definitions, for the most part, were relational in meaning. No curriculum, however, provides the same definition at all grade levels, and some curricula provide different definitions across grade levels or within the same grade level. This could prove confusing to students who learned one definition in first grade and are provided with another in second grade without understanding that the definitions may have similar meanings. Second, the equal sign, when mentioned, is mentioned infrequently or only once throughout an entire year-long curriculum. In fact, the terms *equality* or *equal sign* were mentioned at most eight times in the teacher manuals, and more often than not, the equal sign was only mentioned once. In three of the eight curricula (i.e., *Everyday Mathematics*, *Math Connects*, and *Saxon Math*), the equal sign was not introduced when students received initial exposure to addition number sentences, and if it was introduced with addition number sentences, a review of the equal sign was not provided when students were introduced to subtraction. Third, explanations of the meaning of the equal sign occurred most often in kindergarten and first grade, with some work into second grade. The equal sign was discussed infrequently at grades 3, 4, and 5. Although it makes sense that the equal sign is discussed more frequently when students are learning the basic symbols of mathematics, students should receive continuous instruction on the meaning of the equal sign into the late elementary grades because, as demonstrated by Alibali (1999) and Matthews and Rittle-Johnson (2009), students in the late elementary grades often view the equal sign in an operational manner. Once instruction is provided, however, this misunderstanding can be corrected (Rittle-Johnson & Alibali, 1999). Therefore, explicit instruction at all grade levels is beneficial until researchers demonstrate that students do not continue to misinterpret the equal sign. Fourth, a few curricula provide incorrect definitions of the equal sign (i.e., *the number in all* and *the sign between the addends and the sum*). As demonstrated by Cobb (1987) and McNeil and Alibali (2005a), given that students already have existing misconceptions of the equal sign, which they view as an operational symbol, it is unfortunate that curricula reinforce this misunderstanding. Fifth, three of the eight curricula encourage teachers to use nonstandard equations in instruction, but only one curriculum (i.e., *Everyday Mathematics*) reflects this suggestion with a fair number of nonstandard equations across operations presented within the student textbooks. Sixth, a balance scale is presented in four of the eight curricula as a way of teaching that two sides of an equation should be the same. The scale, however, is introduced at various grade levels and not consistently used across curricula or grade level or even used multiple times throughout the year.

Overall, although some curricula provide more opportunities for students to work with nonstandard equations, no curriculum appears to provide the complete package of equal sign understanding. That is, no curriculum provided relational definitions of the equal sign (across the school year and across grade levels) and ample opportunities for exposure to nonstandard equations. *Everyday Mathematics* provided more examples of nonstandard equations, but the equal sign was not discussed often in the teacher manuals across grade levels. Additionally, *Everyday Mathematics* presented nonstandard equations with an operation on the right side much more frequently than nonstandard no-operation equations and operations-both-sides equations. This is important because McNeil et al. (2006) and Rittle-Johnson and Alibali (1999) demonstrated that nonstandard operations-both-sides equations were important for fostering a relational meaning of the equal sign. Some curricula like *HSP Math*, *Investigations*, *Saxon Math*, and *SFAW* provide more equal sign definitions across grades, yet these curricula did not present students with many opportunities in their textbooks to practice equal sign understanding with nonstandard equations. *Math Expressions* did present many nonstandard addition equations to students, but most of these examples were from the kindergarten student textbook, and these equation types did not appear in later-grade materials. *Singapore Math* provided quite a few operation-right-side multiplication equations at fourth grade, but the teacher manuals offered minimal definitions of the equal sign. *Math Connects* provided very few definitions of the equal sign in the teacher manuals or nonstandard equations in the student textbooks.

Implications for Curriculum Developers and Teachers

Currently, most curricula incorporate equal sign instruction into teacher manuals and student textbooks with relative infrequency. The definitions, when provided, are generally relational in meaning, but there are inconsistencies in definitions within and across grade levels. Additionally, most curricula do not present students with many opportunities to see or solve nonstandard equations in their student textbooks. McNeil et al. (2006) demonstrated that nonstandard equations, especially operations-both-sides equations, foster a relational understanding of the equal sign. Nonstandard equations do not appear often in the teacher manuals to help students understand how the equal sign denotes a relationship between two sides of an equation.

It would be relatively easy to provide equal sign definitions and instruction within elementary mathematics curricula via teacher manuals. Providing more instances of nonstandard equations might prove a more time-consuming task (and teacher manuals would have to contain instruction on solving nonstandard equations), but these nonstandard equation types would help students with equal sign understanding. It is important, however, that curricula present consistent definitions of the equal sign within and across grade levels and provide exposure to nonstandard equations in similar proportions to standard equations within and across grade levels. If curriculum developers decide not to expand each curriculum’s lessons on the equal sign, then classroom teachers may have to deviate from a curriculum and provide their own instruction. Classroom teachers would want to provide relational definitions (i.e., *the same as* and *two sides are the same*) and repeatedly discuss and work with the equal sign with various equation types across the school year and across grade levels.

Suggestions for Further Research

Results from the present study indicate that, for the most part, students receive little to no exposure to equations that are atypical (i.e., nonstandard equations) in student textbooks. Further research needs to be conducted to investigate the correlation between exposure and opportunities to solve nonstandard equation types in elementary mathematics textbooks and equal sign understanding. If a connection between an ability to work with the equal sign and exposure to a variety of equations is established, then curriculum developers may be more inclined to incorporate a combination of standard and nonstandard equations within a curriculum. Further research should also investigate which methods are best for teaching the relational meaning of the equal sign. Perhaps explicit instruction and a change in vocabulary is necessary, or perhaps students need exposure to a variety of equation types.

The equal sign is a small piece of the mathematics puzzle, but because it is used in all computation equation types (i.e., addition, subtraction, multiplication, and division), elementary students use the equal sign more than any other symbol. For this reason, it is important that students understand what the symbol means. Not only do students need to understand the equal sign for basic addition and subtraction problems, but as students learn to solve mathematical and algebraic equations, it is of utmost importance that students learn that the equal sign means to balance the two sides of the equation (Jacobs et al., 2007). While instruction on a relational meaning of the equal sign could be taught when algebra instruction begins, instruction provided in the elementary grades may correct misconceptions or even inhibit misunderstandings from developing.

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References

Agodini R, Harris B. An experimental evaluation of four elementary math curricula. Journal of Research on Educational Effectiveness. 2010; 3:199–253.

Alibali MW. How children change their minds: Strategy change can be gradual or abrupt. Developmental Psychology. 1999; 35:127–145. [PubMed: 9923470]

Baroody AJ, Ginsburg HP. The effects of instruction on children’s understanding of the “equals” sign. Elementary School Journal. 1983; 84:199–212.

Beatty, R.; Moss, J. Teaching the meaning of the equal sign to children with learning disabilities: Moving from concrete to abstractions. In: Martin, WG.; Strutchens, ME., editors. The learning of mathematics: Sixty-ninth yearbook, 27–41. Reston, VA: National Council of Teachers of Mathematics; 2007. p. 161-187.

Behr M, Erlwanger S, Nichols E. How children view the equals sign. Mathematics Teaching. 1980; 92:13–15.

Bell, J.; Bell, M.; Beer, DW.; Freedman, D.; Goodsell, NG.; Hanvey, N.; Morrison, K. Everyday mathematics kindergarten. Chicago: Wright Group/McGraw-Hill; 2007.

Bell, M.; Bell, J.; Bretzlauf, J.; Dairyko, ME.; Dillard, A.; Hartfield, R.; Saecker, P. Everyday mathematics grade 3. Chicago: Wright Group/McGraw-Hill; 2007.

Bell, J.; Bell, M.; Bretzlauf, J.; Dillard, A.; Hartfield, R.; Isaacs, A.; Saecker, P. Everyday mathematics grade 1. Chicago: Wright Group/McGraw-Hill; 2007a.

Bell, M.; Bell, J.; Bretzlauf, J.; Dillard, A.; Hartfield, R.; Isaacs, A.; Saecker, P. Everyday mathematics grade 2. Chicago: Wright Group/McGraw-Hill; 2007b.

Bell, M.; Bretzlauf, J.; Dillard, A.; Hartfield, R.; Isaacs, A.; McBride, J.; Sconiers, S. Everyday mathematics grade 4. Chicago: Wright Group/McGraw-Hill; 2007a.

Bell, M.; Bretzlauf, J.; Dillard, A.; Hartfield, R.; Isaacs, A.; McBride, J.; Carroll, W. Everyday mathematics grade 5. Chicago: Wright Group/McGraw-Hill; 2007b.

Blanton ML, Kaput JJ. Characterizing a classroom practice that promotes algebraic reasoning. Journal for Research in Mathematics Education. 2005; 36:412–446.

Capraro MM, Ding M, Matteson S, Capraro RM, Li X. Representational implications for understanding equivalence. School Science and Mathematics. 2007; 107:86–88.

Carpenter, TP.; Franke, ML.; Levi, L. Thinking mathematically: Integrating arithmetic and algebra in elementary school. Portsmouth, NH: Heinemann; 2003.

Carpenter TP, Hiebert J, Moser JM. Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. Journal for Research in Mathematics Education. 1981; 12:27–39.

Carpenter, TP.; Levi, L. Developing conceptions of algebraic reasoning in the primary grades. Madison: University of Wisconsin–Madison, National Center for Improving Student Learning and Achievement in Mathematics and Science; 2000. (Report No. 00–2)

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects grade 1. Columbus, OH: Macmillan/McGraw-Hill; 2012a.

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects grade 2. Columbus, OH: Macmillan/McGraw-Hill; 2012b.

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects grade 3. Columbus, OH: Macmillan/McGraw-Hill; 2012c.

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects grade 4. Columbus, OH: Macmillan/McGraw-Hill; 2012d.

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects grade 5. Columbus, OH: Macmillan/McGraw-Hill; 2012e.

Carter, JA.; Cuevas, GJ.; Day, R.; Malloy, CE. Tennessee math connects kindergarten. Columbus, OH: Macmillan/McGraw-Hill; 2012f.

Charles, RI.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade 1. Glenview, IL: Pearson Education; 2006a.

Charles, RI.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade 2. Glenview, IL: Pearson Education; 2006b.

Charles, RI.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade 3. Glenview, IL: Pearson Education; 2006c.

Charles, IR.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade 4. Glenview, IL: Pearson Education; 2006d.

Charles, RI.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade 5. Glenview, IL: Pearson Education; 2006e.

Charles, RI.; Crown, W.; Fennell, F. Mathematics: Scott Foresman Addison Wesley grade K. Glenview, IL: Pearson Education; 2006f.

Cobb P. An investigation of young children's academic arithmetic contexts. Educational Studies in Mathematics. 1987; 18:109–124.

Ding M, Li X. A comparative analysis of the distributive property in U.S. and Chinese elementary mathematics textbooks. Cognition and Instruction. 2010; 28:146–180.

Falkner KP, Levi L, Carpenter TP. Children's understanding of equality: A foundation for algebra. Teaching Children Mathematics. 1999; 6:232–236.

Fuson, KC. Math expressions grade 1. Orlando, FL: Houghton Mifflin Harcourt; 2009a.

Fuson, KC. Math expressions grade 2. Orlando, FL: Houghton Mifflin Harcourt; 2009b.

Fuson, KC. Math expressions grade 3. Orlando, FL: Houghton Mifflin Harcourt; 2009c.

Fuson, KC. Math expressions grade 4. Orlando, FL: Houghton Mifflin Harcourt; 2009d.

Fuson, KC. Math expressions grade 5. Orlando, FL: Houghton Mifflin Harcourt; 2009e.

Fuson, KC. Math expressions kindergarten. Orlando, FL: Houghton Mifflin Harcourt; 2009f.

Gagnon JC, Maccini P. Preparing students with disabilities for algebra. Teaching Exceptional Children. 2001; 31:8–15.

Hake, S. Saxon math intermediate 3. Orlando, FL: Harcourt Achieve; 2008a.

Hake, S. Saxon math intermediate 4. Orlando, FL: Harcourt Achieve; 2008b.

Hake, S. Saxon math intermediate 5. Orlando, FL: Harcourt Achieve; 2008c.

Herscovics N, Kieran C. Constructing meaning for the concept of equation. Mathematics Teacher. 1980; 73:572–580.

Jacobs VR, Franke ML, Carpenter TP, Levi L, Battey D. Professional development focused on children's algebraic reasoning in elementary school. Journal for Research in Mathematics Education. 2007; 38:258–288.

Kheong, FH.; Ramakrishnan, C.; Choo, M. Math in focus: The Singapore approach grade 2. Singapore: Marshall Cavendish Education; 2009a.

Kheong, FH.; Ramakrishnan, C.; Choo, M. Math in focus: The Singapore approach grade 3. Singapore: Marshall Cavendish Education; 2009b.

Kheong, FH.; Ramakrishnan, C.; Soon, GK. Math in focus: The Singapore approach grade 4. Singapore: Marshall Cavendish Education; 2009.

Kheong, FH.; Ramakrishnan, C.; Wah, BLP. Math in focus: The Singapore approach grade 1. Singapore: Marshall Cavendish Education; 2009.

Kheong, FH.; Soon, GK.; Ramakrishnan, C. Math in focus: The Singapore approach grade 5. Singapore: Marshall Cavendish Education; 2009.

Kieran C. Concepts associated with the equality symbol. Educational Studies in Mathematics. 1981; 12:317–326.

Knuth EJ, Stephens AC, McNeil NM, Alibali MW. Does understanding the equal sign matter? Evidence from solving equations. Journal for Research in Mathematics Education. 2006; 37:297–312.

Larson, N. Saxon math 1. Orlando, FL: Harcourt Achieve; 2008a.

Larson, N. Saxon math 2. Orlando, FL: Harcourt Achieve; 2008b.

Larson, N. Saxon math K. Orlando, FL: Harcourt Achieve; 2008c.

Li X, Ding M, Capraro MM, Capraro RM. Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. Cognition and Instruction. 2008; 26:195–217.

Lindvall CM, Ibarra CG. Incorrect procedures used by primary grade pupils in solving open addition and subtraction sentences. Journal for Research in Mathematics Education. 1980; 11:50–62.

Maletsky, EM.; McLeod, J. HSP math grade 1. Orlando, FL: Harcourt; 2009a.

Maletsky, EM.; McLeod, J. HSP math grade 2. Orlando, FL: Harcourt; 2009b.

Maletsky, EM.; McLeod, J. HSP math grade 3. Orlando, FL: Harcourt; 2009c.

Maletsky, EM.; McLeod, J. HSP math grade 4. Orlando, FL: Harcourt; 2009d.

Maletsky, EM.; McLeod, J. HSP math grade 5. Orlando, FL: Harcourt; 2009e.

Maletsky, EM.; McLeod, J. HSP math kindergarten. Orlando, FL: Harcourt; 2009f.

Matthews P, Rittle-Johnson B. In pursuit of knowledge: Comparing self-explanations, concepts, and procedures as pedagogical tools. Journal of Experimental Child Psychology. 2009; 104:1–21. [PubMed: 18937952]

McNeil NM. Limitations to teaching children 2+2=4: Typical arithmetic problems can hinder learning of mathematical equivalence. Child Development. 2008; 79:1524–1537. [PubMed: 18826540]

McNeil NM, Alibali MW. Knowledge change as a function of mathematics experience: All contexts are not created equal. Journal of Cognition and Development. 2005a; 6:285–306.

McNeil NM, Alibali MW. Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. Child Development. 2005b; 76:883–899. [PubMed: 16026503]

McNeil NM, Grandau L, Knuth EJ, Alibali MW, Stephens AC, Hattikudur S, Krill DE. Middle-school students' understanding of the equal sign: The books they read can't help. Cognition and Instruction. 2006; 24:367–385.

Nathan MJ, Koedinger KR. Teachers' and researchers' beliefs about the development of algebraic reasoning. Journal for Research in Mathematics Education. 2000; 31:168–190.

National Council of Teachers of Mathematics. Principles and standards for school mathematics. Reston, VA: Author; 2000.

Powell SR, Fuchs LS. Contribution of equal-sign instruction beyond word-problem tutoring for third-grade students with mathematics difficulty. Journal of Educational Psychology. 2010; 102:381–394. [PubMed: 20640240]

Rittle-Johnson B, Alibali MW. Conceptual and procedural knowledge of mathematics: Does one lead to the other? Journal of Educational Psychology. 1999; 91:175–189.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade 1. Glenview, IL: Pearson Education; 2008a.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade 2. Glenview, IL: Pearson Education; 2008b.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade 3. Glenview, IL: Pearson Education; 2008c.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade 4. Glenview, IL: Pearson Education; 2008d.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade 5. Glenview, IL: Pearson Education.; 2008e.

Russell, SJ.; Economopoulos, K. Investigations in number, data, and space grade K. Glenview, IL: Pearson Education; 2008f.

Saenz-Ludlow A, Walgamuth C. Third graders' interpretations of equality and the equal symbol. Educational Studies in Mathematics. 1998; 35:153–187.

Seo, K.; Ginsburg, HP. "You've got to carefully read the math sentence …": Classroom context and children's interpretations of the equals sign. In: Baroody, AJ.; Dowker, A., editors. The development of arithmetic concepts and skills: Constructing adaptive expertise. Mahwah, NJ: Erlbaum; 2003. p. 161-187.

Sharpe, P. Math in focus: The Singapore approach kindergarten. Singapore: Marshall Cavendish Education; 2009.

Sherman J, Bisanz J. Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. Journal of Educational Psychology. 2009; 101:88–100.

Weaver JF. The symmetric property of the equality relation and young children's ability to solve open addition and subtraction sentences. Journal for Research in Mathematics Education. 1973; 4:45–56.

Table 1

List of Textbooks

Textbook	Company	Authors and Year
Everyday Mathematics	Wright Group/McGraw-Hill	K: Bell, Bell, Beer, Freedman, et al., 2007
		1: Bell, Bell, Bretzlauf, Dillard, et al., 2007a
		2: Bell, Bell, Bretzlauf, Dillard, et al., 2007b
		3: Bell, Bell, Bretzlauf, Dairyko, et al., 2007
		4: Bell, Bretzlauf, Dillard, Hartfield, et al., 2007a
		5: Bell, Bretzlauf, Dillard, Hartfield, et al., 2007b
HSP Math	Harcourt	K–5: Maletsky & McLeod, 2009a, 2009b, 2009c, 2009d, 2009e, 2009f
Investigations in Number, Data, and Space	Pearson Education	K–5: Russell & Economopoulos, 2008a, 2008b, 2008c, 2008d, 2008e, 2008f
Math Connects	Macmillan/McGraw-Hill	K–5: Carter, Cuevas, Day, & Malloy, 2012a, 2012b, 2012c, 2012d, 2012e, 2012f
Math Expressions	Houghton Mifflin Harcourt	K–5: Fuson, 2009a, 2009b, 2009c, 2009d, 2009e, 2009f
Saxon Math	Harcourt Achieve	K–2: Larson, 2008a, 2008b, 2008c
		3–5: Hake, 2008a, 2008b, 2008c
Scott Foreman Addison Wesley	Pearson Education	K–5: Charles, Crown, & Fennell, 2006a, 2006b, 2006c, 2006d, 2006e, 2006f
Singapore Math	Marshall Cavendish Education	K: Sharpe, 2009
		1: Kheong, Ramakrishnan, & Wah, 2009
		2: Kheong, Ramakrishnan, & Choo, 2009a
		3: Kheong, Ramakrishnan, & Choo, 2009b
		4: Kheong, Ramakrishnan, & Soon, 2009
		5: Kheong, Soon, & Ramakrishnan, 2009

Table 2

Types of Equations for Analysis

Type of Equation	Description	Operation	Form	Examples
Standard	operation-left-side	addition	$a + b = c$	4 + 6 = 10
	operation-left-side	subtraction	$a - b = c$	11 − 6 = 5
	operation-left-side	multiplication	$a \times b = c$	7 × 6 = 42
	operation-left-side	division	$a \div b = c$	25 ÷ 5 = 5
	operation-left-side	mixed	$a + b - c = d$	(4 + 3) × 2 = 14
Nonstandard	operation-right-side	addition	$c = a + b$	10 = 4 + 6
	operation-right-side	subtraction	$c = a - b$	5 = 11 − 6
	operation-right-side	multiplication	$c = a \times b$	42 = 7 × 6
	operation-right-side	division	$c = a \div b$	5 = 25 ÷ 5
	operation-right-side	mixed	$d = (c \div b) \times a$	25 = (10 ÷ 2) × 5
	no-operation	none	$a = a$	4 = 4
	operations-both-sides	any combination of +, −, ×, ÷	$a + b = c - d$ $a - b = c - d$ $a \times b = c \div d$	4 + 3 = 2 + 5 12 ÷ 3 = 2 × 2 9 − 3 = 24 ÷ 4

Table 3

Number of Equations by Curriculum and Grade Level

			Standard Equations					Nonstandard Equations							Percentages	
			Operation-Left-Side					Operation-Right-Side								
Curricula	Grade	Pages Coded	Add.	Sub.	Mult.	Div.	Mixed	Add.	Sub.	Mult.	Div.	Mixed	No-Operation	Operations-Both-Sides	Standard Equations	Nonstandard Equations
Everyday Mathematics	K	0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
	1	105	72	51	0	0	0	13	5	0	0	0	0	1	86.6	13.4
	2	155	72	24	55	35	0	19	14	2	0	0	0	3	83.0	17.0
	3	141	44	12	63	4	5	9	12	13	3	0	0	3	76.2	23.8
	4	172	54	37	61	41	22	21	13	22	9	13	0	5	72.1	27.9
	5	216	70	47	74	20	37	17	5	19	2	9	0	24	76.5	23.5
HSP Math	K	172	31	22	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	273	309	245	0	0	0	11	2	0	0	0	2	1	97.2	2.8
	2	280	111	63	27	10	0	13	0	0	0	0	5	0	92.1	7.9
	3	316	128	59	335	210	0	0	0	55	41	0	2	39	84.2	15.8
	4	347	84	54	181	117	5	6	2	0	0	0	3	27	92.1	7.9
	5	341	47	37	78	51	5	4	2	8	4	0	5	22	82.9	17.1
Investigations	K	39	5	0	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	188	15	22	0	0	0	2	2	0	0	0	0	0	90.2	9.8
	2	265	109	33	0	0	0	0	0	0	0	0	0	0	100.0	.0
	3	266	243	101	127	5	1	0	0	0	0	0	0	0	100.0	.0
	4	287	42	16	371	24	3	2	0	0	0	0	0	0	99.6	.4
	5	300	144	64	116	66	0	3	0	0	0	0	0	24	93.5	6.5
Math Connects	K	280	8	5	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	293	266	223	0	0	0	0	0	0	0	0	0	0	100.0	.0
	2	310	225	178	43	15	0	13	0	0	0	0	2	8	95.2	4.8
	3	322	83	43	155	91	0	0	0	0	0	0	1	27	93.0	7.0
	4	341	56	52	143	135	15	0	0	0	0	0	0	19	95.5	4.5
	5	354	35	27	70	69	4	6	1	10	0	0	1	12	87.2	12.8
Math Expressions	K	193	41	19	0	0	0	110	0	0	0	0	8	0	33.7	66.3
	1	189	275	88	4	0	4	9	0	0	0	0	0	0	97.6	2.4

Curricula	Grade	Pages Coded	Standard Equations: Operation-Left-Side: Add.	Sub.	Mult.	Div.	Mixed	Nonstandard Equations: Operation-Right-Side: Add.	Sub.	Mult.	Div.	Mixed	No-Operation	Operations-Both-Sides	Percentages: Standard Equations	Nonstandard Equations
	2	243	586	396	18	0	0	28	2	0	0	0	0	0	97.1	2.9
	3	275	100	19	1108	856	4	4	4	0	0	0	0	10	99.1	.9
	4	244	89	25	336	181	1	6	2	0	0	0	1	13	96.6	3.4
	5	256	73	9	238	18	9	3	1	4	4	0	0	24	90.6	9.4
Saxon Math	K	143	32	0	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	355	324	110	0	0	0	0	0	0	0	0	2	3	98.9	1.1
	2	402	483	283	164	7	0	0	1	0	0	0	0	15	98.3	1.7
	3	294	81	30	58	18	0	7	0	0	0	0	0	1	95.9	4.1
	4	378	77	40	39	6	0	2	0	1	0	0	1	10	92.0	8.0
	5	396	43	30	47	16	0	2	0	2	0	0	0	2	95.8	4.2
SFAW	K	151	17	13	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	246	335	201	0	0	0	6	0	0	0	0	2	0	98.5	1.5
	2	249	307	147	48	8	0	39	12	0	0	0	12	39	83.3	16.7
	3	365	141	55	148	89	0	3	0	0	0	0	4	34	91.4	8.6
	4	367	111	61	136	86	2	0	0	0	0	0	2	15	95.9	4.1
	5	374	86	37	71	75	5	4	5	15	5	0	2	24	83.3	16.7
Singapore Math	K	107	6	9	0	0	0	0	0	0	0	0	0	0	100.0	.0
	1	274	177	137	0	0	0	9	0	0	0	0	0	9	94.6	5.4
	2	297	151	152	195	45	0	4	8	0	0	0	0	6	96.8	3.2
	3	328	92	74	253	62	0	1	0	0	0	0	0	19	96.0	4.0
	4	259	33	25	68	36	0	0	0	27	0	0	0	1	85.3	14.7
	5	311	28	23	31	48	0	2	0	4	0	0	0	64	65.0	35.0

Table 4

Results from Coding of Equations

Equation Types	Results	Mean Difference	Standard Error	Significance
Operation-right-side addition	EM > HSP	.154	.066	.024
	EM > INV	.171	.066	.013
	EM > MC	.168	.066	.014
	EM > Saxon	.178	.066	.010
	EM > SFAW	.169	.066	.014
	EM > Sing	.176	.066	.011
	ME > INV	.127	.062	.049
	ME > Saxon	.133	.062	.040
	ME > Sing	.133	.062	.040
Operation-right-side subtraction	EM > HSP	.247	.041	> .001
	EM > INV	.249	.041	> .001
	EM > MC	.257	.041	> .001
	EM > ME	.204	.041	> .001
	EM > Saxon	.262	.041	> .001
	ME > SFAW	.230	.041	> .001
	ME > Sing	.254	.041	> .001
Operation-right-side multiplication	EM > HSP	.096	.041	.024
	EM > INV	.135	.041	.002
	EM > MC	.114	.041	.008
	EM > ME	.133	.041	.002
	EM > Saxon	.124	.041	.004
	EM > SFAW	.106	.041	.013
Operation-right-side division	EM > HSP	.101	.041	.019
	EM > INV	.140	.041	.002
	EM > MC	.140	.041	.002
	EM > ME	.110	.041	.011
	EM > Saxon	.140	.041	.002
	EM > SFAW	.129	.041	.003
	ME > Sing	.140	.041	.002
No-operation		No significant differences		
Operations-both sides		No significant differences		

Note.—EM (*Everyday Mathematics*); HSP (*HSP Math*); INV (*Investigations in Number, Data, and Space*); MC (*Math Connects*); ME (*Math Expressions*); SFAW (*Scott Foresman Addison Wesley*); Sing (*Singapore Math*).

Table 5

Summary of Descriptions in Teacher Manuals

Curriculum	Grade	Terminology	Description
Everyday Mathematics	K	equal	equal sign only used on calculators
	1	equals	*is equal to*
			is the same as
	2	equals	*is the same as* or *is equal to*
			encourages use of nonstandard equations
	3		
	4	equals	
	5		
HSP Math	K	equal	*is equal to*
			numbers on one side equal to other side
			the number in all is after the equal sign
	1		*is equal to*
			is the same as
	2	equal	*is equal to*
	3		*equal to*
	4		equabeam balance used to show balancing equations 4 + m = 9: *equation is true if both values are equal*
	5		
Investigations in Number, Data, and Space	K	equal	“What sign do we use to show 10 altogether?” *this side is the same as*
	1	equal	*both sides are the same*
			either side has the same amount
			the same as
			encourages use of nonstandard equations
	2	equal	*two things are the same*
			the equal sign shows equivalency
	3		*equal to, or has the same sum*
	4	equal	*one side equivalent to the other side*
	5	equal	
Math Connects	K	equal	“equivalent equations are a balance between two sides”
	1	equals	*having the same value*
	2		picture of a balance to show nonstandard equations
	3		
	4		
	5		
Math Expressions	K	equals	*same as*
			encourages teaching unequals sign “9 = 4 + 5 and 4 + 5 = 9 are partners, either way is correct”
	1	equals	*is equal to*
			is the same as
	2	equals	*is equal to*

Curriculum	Grade	Terminology	Description
	3	equal	*two sides have the same value*
	4	equals	"Can write equations with answer on left."
			both sides have the same value
	5		
Saxon Math	K		
	1	equals	*two quantities have the same value*
	2	equals	*two quantities have the same value*
	3	equal	*if two amounts are the same, write the equal sign*
			have the same value
	4	equal	*equation shows two quantities are the same*
			shows balance scale
	5		equal sign introduced as comparison symbol
Scott Foreman Addison Wesley	K	equals	*is the same as*
			goes between numbers added and sum
	1	equals	*equals*
			"What sign comes before the sum?"
			is the same as
			balance used for equations
	2	equals	*one side is the same as other side*
			equal to
	3	equals	*two expressions are equal*
	4	equal	*equation is like a pan balance, sides have same value*
	5		
Singapore Math	K		*makes*
	1	equal	*is equal to*
			same amount or number
	2		
	3		
	4		
	5		*same value*