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Perspective

Multijurisdictional economies, the Tiebout Hypothesis, and sorting

*Myrna H. Wooders**

Department of Economics, University of Toronto, Toronto, Canada M5S 3G7; and Department of Economics, University of Warwick, Coventry, United Kingdom CV4 7AL

The Tiebout Hypothesis asserts that, when it is efficient to have multiple jurisdictions providing local public goods, then competition between jurisdictions for residents will lead to a near-optimal outcome. Research from cooperative game theory both provides a foundation for the hypothesis and extends the hypothesis to diverse situations where small groups of participants are effective.

The Tiebout Hypothesis (1) asserts that, in economic situations where it is optimal to have many jurisdictions offering competing packages of public goods, the movement of consumers to jurisdictions where their wants are best satisfied and competition between jurisdictions for residents will lead to near-optimal, "market-like" outcomes. A jurisdiction (or club) is a group of individuals who collectively provide public goods for themselves exclusively (the public goods are local). Tiebout also suggested that individuals would sort into taste-homogeneous jurisdictions.

This article primarily reports on research interpreting and extending the Tiebout Hypothesis through cooperative game theory: in large economies with relatively small effective coalitions, there are outcomes in the core, that is, there are feasible states of the economy that cannot be improved upon by any coalition. (Note that a coalition may consist of many jurisdictions.) Moreover, the core has the equal treatment property—outcomes in the core do not discriminate between identical individuals, a "market-like" feature. When the effect of an individual on others is determined by his crowding type (his observable characteristics, including profession, appearance, age, gender, and lifestyle), the core dictates not only that identical individuals are treated identically but also that, in their interactions with others, individuals with the same crowding types are treated equally (2–4). These features all are in stark contrast to the situation with pure public goods (such as radio or national defense), for which relatively small jurisdictions are inefficient.

The results apply more broadly than perhaps anticipated by Tiebout. As in the literature on clubs, initiated by Buchanan (5), individuals may be affected by both the numbers and characteristics of other individuals in the same jurisdiction. The only coalition able to achieve all gains to cooperation may be the total population. There may be spillovers between jurisdictions—pollution, for example, produced in one jurisdiction may spill over into others. Individuals may belong to multiple jurisdictions providing different public goods packages. The results discussed depend primarily on one crucial property, small group effectiveness (SGE) (6), dictating that all or almost all gains to cooperation can be realized by coalitions that are small relative to the total population.

In general, taste homogeneity holds only for individuals of the same crowding type in the same jurisdiction (7). Interestingly, this taste homogeneity implies that optimal prices (or taxes) for local public goods are equal for all individuals in the same jurisdiction with the same crowding type.

The equivalence of the core and price-taking equilibrium outcomes provides another approach to interpreting the Tiebout Hypothesis. A brief list of results is presented.

Cooperative Games and ε-Cores

Some features of economies satisfying SGE are independent of details of the economy. To demonstrate this, I begin with the structure of a cooperative game. Let N be a finite set of players and let V be a function mapping nonempty subsets of N, called coalitions, into subsets of R^N. The pair (N, V) is a game. For any coalition $S \subset N$, the set $V(S)$ describes the set of payoffs or utilities that the members of S can achieve by using their own resources. A point in $V(N)$ is an outcome of the game. One possibility open to a coalition is to divide into smaller coalitions. Thus a coalition S always can achieve any outcomes achievable by coalitions in a partition of S. Cooperative games can represent diverse economic models, including economies where players form multiple, possibly overlapping jurisdictions and those where players are affected by the characteristics of other members of the coalition, their social skills, their education levels, and their productive abilities.

Unless certain mathematical conditions are satisfied (8), the core may be empty—there may be no outcomes that are stable against cooperative coalition formation. An explanation is that it may not be possible to partition agents into "optimal" coalitions. For example, suppose any two players can earn a dollar, all other coalitions are ineffective, and the total number of players is odd. Every outcome will have one player left out of any two-person coalition; this left-out player creates instability—the core is empty.

Given $\varepsilon \geq 0$, an outcome is in the ε-core if no coalition of players can increase the utility of each of its members by at least ε (9, 10). Suppose the following conditions are satisfied:

SGE. Given $\beta > 0$, there is a number $\eta_0(\beta)$ such that, if coalitions are restricted to contain fewer than $\eta_0(\beta)$ members, the loss of utility is at most β for each player.

Per Capita Boundedness. Average feasible payoff is uniformly bounded. This condition is eminently plausible and holds even in models of large economies with pure public goods (see, for example, ref. 11). Per capita boundedness allows ever-increasing but bounded per capita gains to coalition formation.

Substitution. For most players in the economy, there are many similar players. This substitution allows the possibility that there are players who are truly unique—who have unique and possibly large impacts on outcomes—but there can be only a few such players.

Strong Comprehensiveness (or, Alternatively, Convexity of Payoff Sets). Strong comprehensiveness is a technical condition ensuring that there is some means of transferring payoff (not necessarily at a one-to-one rate) from some individuals to others.

The following results all use the above conditions.

Nonemptiness. For any $\varepsilon > 0$ there is an integer η_1 such that all economies with more than η_1 players have nonempty ε-cores (10, 12–13). (Under fewer conditions, analogous results hold for weaker notions of approximate cores; see especially ref. 12.)

In the face of global phenomena such as widespread pollution and the worldwide web, cooperation of the total economy may be

*To whom reprint requests should be addressed. E-mail: mwooders@chass.utoronto.ca.

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required to achieve all gains to cooperation, so it is important that SGE allows ever-increasing returns to coalition size. Nevertheless, when SGE holds, large games can be approximated by games with bounded effective coalition sizes and large populations can be partitioned into jurisdictions so that most players are in optimal or nearly optimal jurisdictions. I introduce the following condition.

Strict SGE (SSGE). There is a bound B such that all payoffs feasible for any coalition S can be realized by cooperation restricted to elements of some partition of S into smaller coalitions, each containing no more than B members (12, 14). (For expository purposes, this is a particularly strong form of SSGE. A discussion of alternative conditions of SSGE is provided in ref. 15.)

Partitioning. Under SSGE, when there is a fixed number of exact player types (that is, all players of the same type are identical to each other) and the total population is large, then players can be partitioned into coalitions so that most players (all but a small fraction of the population) are in optimal coalitions (10). This result is true independently of the function V; it depends only on the number of types and the bound B (12, 14).

To motivate nonemptiness, by the definition of SGE, games satisfying SGE can be approximated by games satisfying SSGE. Strong comprehensiveness or convexity makes it possible to compensate leftover players, if any, when optimal or near-optimal coalitions are formed.

Equal Treatment. If a game satisfies SSGE with bound B and there are more than B players of each (exact) type, then all outcomes in the core have the equal treatment property (10).

If a core outcome treated two identical players unequally, then there would be two identical players in different coalitions who experience different utility levels. The new coalition formed by replacing the best-off player by the worst off could improve upon the given outcome for its members, which would be a contradiction.

Equal Treatment with Limited Side Payments. Under the first four conditions above, if large transfers between coalitions are ruled out, then an ε-core payoff must treat all similar players similarly (16).

For games with side payments (that is, for each coalition S there is a number $v(S)$ such that $x \in V(S)$ if and only if $\Sigma_{i\in S}x_i \leq v(S)$) an additional result holds.

Equal Treatment with Side Payments. For games with side payments, ε-core payoffs treat most players of the same type nearly equally (15).

Further Results. Another way to interpret the Tiebout Hypothesis is through the equivalence of markets—private goods exchange economies where all players have concave utility functions—and games. Markets and games with nonempty cores are equivalent (17). Large games satisfy SGE, including games derived from Tiebout economies, are also equivalent to markets (6). Some results for games with side payments, including equal treatment, are applied to economies with firms in ref. 18 and to economies with possibly memberships in multiple jurisdictions and many public and private goods in ref. 19; a remarkably general application to such economies is provided in ref. 12. With a continuum of agents and finite coalitions, a limit model of a game satisfying SGE, the core is nonempty (ref. 20 and earlier research, referenced therein).

Crowding Types

For the results of this and the next section, the economy is assumed to be described by preferences, endowments, and production possibilities. There is a fixed number of taste types and crowding types of players. The crowding type of a player determines his direct effects on other players in the same jurisdiction while his taste type determines his preferences. Suppose that individuals care only about the crowding types of members of their jurisdictions and not about their (unobservable) tastes. The following nondiscrimination results require only one private good or exogenously given prices for private goods. They also hold for the models with a continuum of players of refs. 2 and 21 discussed below.

Nondiscrimination. Under SSGE, if two players, i and j, are in the same jurisdiction in a core outcome and have the same crowding types, then they must make the same contribution to public goods provision (3). As Robert Aumann (22) wrote, describing joint work with Jacques Dreze: "Discrimination—distinguishing agents by their tastes—is widely regarded as an anticompetitive practice. . . . The point is not to give such agents (those with the same physical endowments) equal opportunity by fiat." Instead, equal opportunity should be a consequence of the situation. The core of the economy satisfies the Aumann-Dreze criterion for nondiscrimination; when SSGE holds, players who have the same crowding types are taxed identically. The argument continues the one above, showing that the core has the equal treatment property.

Near Nondiscrimination. Under SGE the nondiscrimination result also holds in an approximate sense for ε-cores (4).

Nondiscrimination with Endogenous Crowding Types. The nondiscrimination result holds even if crowding types (education level, for example) are the result of investment decisions of individual players (7).

Taste Homogeneity and Sorting

Knowing the structure of optimal jurisdictions sheds light on many important policy questions. It helps us judge, for example, whether it is preferable to have musicians trained by the Julliard School of Music and economists by the London School of Economics or to have both types of students trained at integrated universities with many specialties.

Homogeneous Jurisdictions. For economies with anonymous crowding—that is, utilities and costs depend only on numbers of players and not their characteristics—the core sorts players into jurisdictions consisting of players with the same implicit demands for public goods and crowding (23). If complementarities between types are sufficiently weak, then core jurisdictions will consist of individuals with the same demands (24). But in general, even in the crowding types model, where it has the best chance, players do not necessarily sort into taste homogeneous jurisdictions (7).

Homogeneity Within Crowding Types Holds in General. The nondiscrimination results imply that in a core outcome, the members of each jurisdiction who have the same crowding types have the same demands for public goods—effectively, they have the same tastes in the core outcome (refs. 3 and 7 and earlier results of the authors referenced there).

This result supports the assumption (or outcome) often seen in economic theory that individuals with similar crowding types, say Englishmen or businessmen, have similar preferences over public policies. It also indicates that optimal taxes (or prices for public goods) can be the same for all players with the same crowding types.

Price-Taking Equilibrium and the Core

Another approach to formalizing Tiebout's conjecture is to ask whether the core is equivalent to the set of price-taking equilibrium outcomes. This question was raised and answered affirmatively in ref. 23 for a model with anonymous crowding and one private good under economic assumptions ensuring SSGE. A special number-theoretic assumption (satisfied if, for example, there are two consecutive optimal jurisdiction sizes for each type of player) is required to ensure existence of equilibrium. For large economies, however, approximate cores are nonempty and converge to equilibrium outcomes (25). This result raises the question of how to show convergence to equilibrium when the equilibrium may not exist. Under assumptions ensuring (some form of)

SSGE, for any economy with many players, there will be a large subset of players for which the core is nonempty and equilibrium exists. By taxing each of the players in this subset some small amount and transferring this to the leftover players, an approximate equilibrium can be obtained. For sufficiently large economies and small taxes, the approximate equilibrium outcomes are in approximate cores. See ref. 26 for further results and a survey of models with anonymous crowding.

Crowding may be differentiated or, equivalently, nonanonymous (that is, preferences and production possibilities may depend on the composition as well as the size of the set of players with whom the public goods are shared) and there may be costs of forming coalitions (27, 28). Under a weak form of SSGE, called minimum efficient scale, or, equivalently, exhaustion of gains to scale, existence of equilibrium holds for a subsequence of replica economies (27–34), where each economy in the sequence has the same distribution of player types, and in the limit, the core is equivalent to the set of equilibrium states. (I note that the working paper versions of refs. 27 and 28 were published in 1984 and 1985, before ref. 29. Also, with the exception of refs. 29, 30, and 32, all of these papers allow multiple private and public goods.) Under the milder assumption of per capita boundedness, analogues of the results are obtained for approximate equilibrium and approximate cores (27, 28, 31). The admission price equilibrium introduced in ref. 29, with a price for every pair consisting of a description of the club membership and a public goods package, has better existence properties than the two-part equilibrium used in a prior paper (35). From nondiscrimination, admission prices can be defined to depend only on the crowding types of players (3, 7). With a continuum of players and bounded club sizes, equilibrium exists and the core coincides with the set of equilibrium outcomes (ref. 2 and, for the multiple membership case, ref. 21). Current research with John Conley relaxes the boundedness of jurisdiction sizes.

Bewley's Critique

Some possible imitations of the Tiebout Hypothesis are collected in ref. 36: there must be as many jurisdictions as types of players, taxes cannot depend on tastes, and equilibrium typically will not exist except under special assumptions. The arguments are not compelling. In general, it may be optimal for there to be several types of players within a jurisdiction. Finite numbers of types of players and many players of each type are intended to model situations with many players of each of a relatively few approximate types; thus, how many jurisdictions are required depends on how close we require approximate types to be. Tiebout economies cannot be expected to have better existence properties than economies with only private goods; existence of exact equilibrium in private goods economies requires special assumptions, for example, convexity of preferences, divisibility of commodities, or a continuum of agents. The crowding types model resolves the problem that taxes (or cost-sharing schemes) cannot depend on private information of players (their tastes). The possible emptiness of the core is resolved through the effects of large numbers of participants. Profit maximizing governments (refs. 23 and 36, among others) also help in realizing efficient outcomes. The difference between the conclusions of ref. 36 and the research reported above may derive from the fact that Bewley, like some other writers, does not consider economies with many jurisdictions. For further discussion of ref. 36, see ref. 37.

A Perspective

The nature of competition between jurisdictions is crucial. If jurisdictions compete for tax base rather than residents, the fiscal externality problem arises (ref. 38, for example). Each jurisdiction, in attempting to increase its revenue, may cut taxes on industry, leading to undesirably low revenues and low levels of public goods. It is suggested that competition between jurisdictions may thus lead to a "race to the bottom," to outcomes far from those attainable by cooperation, rather than a "race to the top," as suggested by the Tiebout Hypothesis; see ref. 39 for an interesting nontechnical discussion. Recent literature suggests that the fiscal externality problem may be alleviated by tax coordination (40) and the choice of constitutional rules (41).

To me, the root of the fiscal externality problem appears to be that, in Tiebout economies, it is optimal to tax players, including firms, for their external effects. If jurisdictions try to tax players in excess of the external effects they impose, there is a potential for profitable secession or opting out. Because of the assumed mobility of players in Tiebout economies, the redistribution implicit in such taxation cannot be sustained. (Some relevant papers include refs. 28, 42, and 43.) Integrating the objective of equitable redistribution with the constraints imposed by multijurisdictional competition presents an interesting challenge to economic theory.

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