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Random Stride Intervals with Memory

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Abstract. The stride interval in normal human gait is not strictly constant, but fluctuates from step to step in a random manner. Herein we show that contrary to the traditional assumption of uncorrelated random errors, these fluctuations have long-time memory. However, rather than being a monofractal process as found earlier, there exists a multiplicative time scale that characterizes the process in addition to the fractal dimension. Further, these long-time correlations are interpreted in terms of an allometric control process.

Key words: Stride interval

1. Introduction

It has been known for over a century that there is variation in the stride interval of humans during walking [1]. This variation is approximately 3–4% of the time it takes to complete a step on the average, as measured by the standard deviation, and is so small, that the biomechanical community has historically considered these fluctuations to be an uncorrelated random process. In practice this means that the fluctuations in gait were thought to contain no useful information. In Figure 1a is depicted typical data of the time interval between successive maximum extensions of the leg for a healthy individual walking in a relaxed manner. The fluctuations about the average value in this time series, approximately 0.8 seconds, looks like a random process. However, we can detect a somewhat richer structure in the time series, by comparing these experimental data with a computer-generated data set, for a truly uncorrelated Gaussian process with the same mean and variance, as depicted in Figure 1b. The question is whether there is a quantifiable difference between the two time series and whether we can determine this measure is a simple way. Herein we provide evidence that human gait time series is a modified random fractal process and that a relatively simple data processing technique is all that is necessary to provide quantitative information about this time series to establish this fact.

Previous gait studies by Hausdorff et al. [2, 3] demonstrated for the first time that the stride-interval time series exhibit long-time correlations, suggesting that

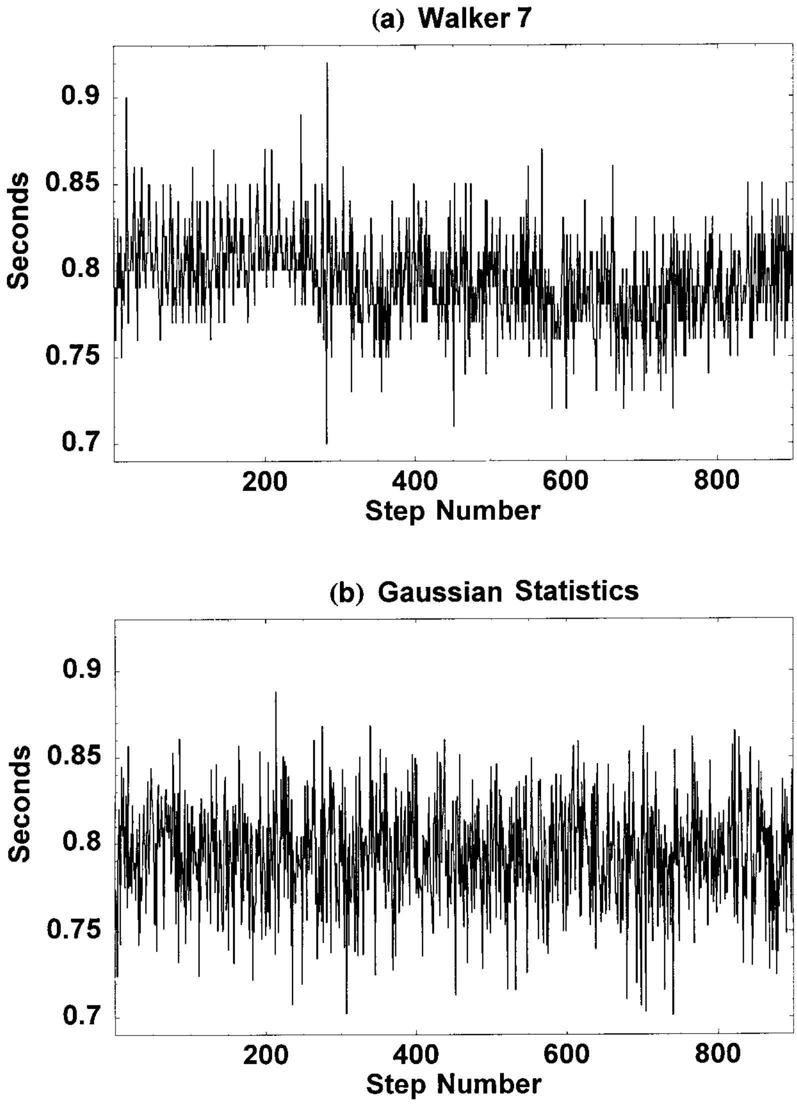

Figure 1. The time intervals between strides for the first 900 steps made by the seventh walker in the experiment are depicted in (a). Lines are used to connect the data points to aid the eye in seeing any patterns. In (b) a computer generated time series having Gaussian statistics with the same mean and variance as in (a) and no correlations is depicted.

the phenomenon of walking may well be a self-similar, fractal activity. Subsequent studies by West and Griffin [4, 5] support these conclusions using a completely different experimental protocol for generating the stride-interval time series data and a very different method of analysis. The argument for the fractal nature of walking is based on the complexity of this apparently simple activity, since the locomotor system synthesizes inputs from the motor cortex, the basal ganglia and the cerebellum, as well as feedback from the vestibular, visual and proprioceptive sources. In these earlier studies the gait time series were argued to be monofractal. Herein we find that things are not quite that simple, and instead of the process having no characteristic time scale, as would be the case for a monofractal, there is a preference for a multiplicative time scale in the physiological control system. An argument based on a renormalization group relation is used to explain this preference.

The procedure we use to analyze the stride interval time series is based on the relative dispersion, the ratio of the standard deviation to the mean, as we aggregate the data. The variation in the relative dispersion with the degree of aggregation gives an unambiguous indication of the scaling behavior of the time series. In Section 2 we review the method of aggregated relative dispersion and show that it yields an inverse power law for a random fractal time series. In Section 3 we apply this method to the ten time series obtained in the present study [6] and determine that the interpretation of human gait as a monofractal process [4, 5], while not wrong, is overly restrictive. A better fit to the data is obtained in Section 5 using an argument involving multiple time scales, leading to a renormalization group relation between the relative dispersion at different levels of aggregation developed in Section 4. In Section 6 we discuss the properties of the random mode-switching model, whose second moment has the same monofractal properties as does the stride interval time series, and may therefore suggest how to interpret the experimental finding. The implications of these results and some conclusions are drawn in Section 7.

2. Aggregating data and relative dispersion

The correlation of discrete time series data is here determined by grouping the data into aggregates of two or more of the original data points and calculating the relative dispersion, the ratio of the standard deviation to the mean. The procedure can be repeated using groupings of two, three, four and more data points. In this way the fractal dimension that is independent of the degree of coarse-graining can be determined. Let us examine how the relative dispersion changes as a function of the number of adjacent data elements we aggregate. To see this we aggregate n-adjacent data points, so that the jth data element in such an aggregation is given by

$$Y_j^{(n)} = Y_{nj} + Y_{nj-1} + Y_{nj-2} + \ldots + Y_{nj-(n-1)}. \tag{1}$$

For example, when $n = 3$ each value of the new variable, defined by (1), consists of the sum of three non-overlapping original data points, and the number of new data points is given by $[N/3]$, where the bracket denotes the closest integer value and N is the original number of data points. In terms of these new data the average is defined as the sum over the total number of data points $[N/n]$ divided by this number

$$\bar{Y}^{(n)} = \frac{1}{[N/n]} \sum_{j=1}^{[N/n]} Y_{j=1}^{(n)} Y_j^{(n)} = n\bar{Y}^{(1)}. \tag{2}$$

The variance, for a monofractal time series, is similarly given by [8]

$$Var\bar{Y}^{(n)} = n^{2H} Var\bar{Y}^{(1)} \tag{3}$$

where the superscript (1) on the average variable indicates that it was determined using all the original data without aggregation and the superscript (n) on the average variable indicates that it was determined using the aggregation of n data points. Thus, the relative dispersion for an aggregated data set is

$$RD^{(n)} = \frac{\sqrt{Var\bar{Y}^{(n)}}}{\bar{Y}^{(n)}} = \frac{\sqrt{n^{2H} Var\bar{Y}^{(1)}}}{n\bar{Y}^{(1)}} = n^{H-1} RD^{(1)} \tag{4}$$

which is exactly an inverse power law in the aggregation number for the Hurst exponent in the interval, $0 \leq H \leq 1$. It is well established [7, 8], that the exponent in such scaling equations is related to the fractal dimension, D, of the underlying time series by $D = 2 - H$.

A simple monofractal time series, therefore, satisfies the inverse power-law relation for the relative dispersion given by (4). This technique has been applied to a number of biomedical data sets [8] and most recently to the stride interval time series using the linear regression relation [4, 5, 6]

$$\log RD^{(n)} = \log a + b \log n \tag{5}$$

where $b = 1 - D$. The significance of the fractal dimension can be determined using the auto-correlation function, which for nearest-neighbor data points can be written [10]

$$r_1 = 2^{3-2D} - 1. \tag{6}$$

If the fractal dimension is given by $D = 1.5$, the exponent in (6) is zero and consequently $r_1 = 0$, indicating that the data points are linearly independent of one another. This would be the case for a slope of $b = -0.5$ in (5) and in the log-log plots of the relative dispersion versus aggregation number, n. This fractal dimension corresponds to an uncorrelated random process with normal statistics, often referred to as Brownian motion. If, one the other hand, the nearest neighbors are perfectly correlated, $r_1 = 1$, the irregularities in the time series are uniform at all times and the fractal dimension is determined by the exponent in (6) to be $D = 1.0$. A fractal dimension of unity implies that the time series is regular, such as it would be for simple periodic motion.

Most time series have fractal dimensions that fall somewhere between the two extremes of Brownian motion with $D = 1.5$, and complete regularity with $D = 1.0$. Let us now examine the gait time series data to determine if human gait is more like the former or more like the latter.

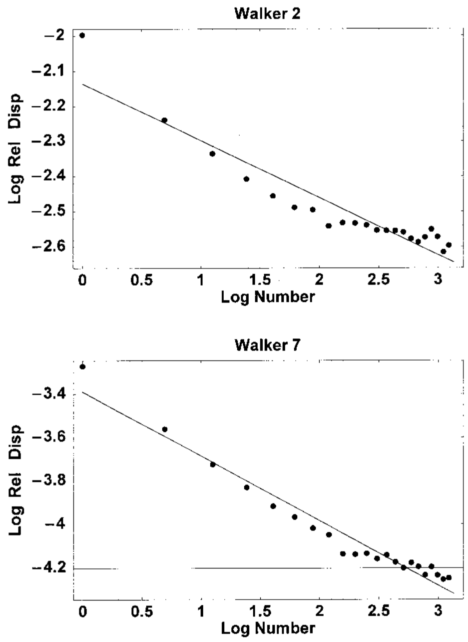

Figure 2. The stride interval data, for two of the walkers in the experiment, are used to construct the coarse-grained relative dispersion as indicated by the dots. The log of the relative dispersion is plotted versus the log of the number of points used in the aggregating process for a maximum of $n = 46$. The solid lines are the best mean-square linear regression to the aggregated data points using (5), fit over the entire range data.

3. First results

The data obtained, by an individual walking at a relaxed pace, consists of the time interval for each stride and the number of strides in a sequence of steps. The maximal extension of the right leg, the ‘stride interval’ versus the stride number, plotted on a graph, has all the characteristics of a time series, cf. Figure 1a. There were ten participants in the study (four males and six females), all in good health, with no acute injuries, ranging in age from 20 to 46 years old with a median age of 29 years. Relaxed walking was monitored for the ten participants, and an electrogoniometer was used to collect kinematic data on the angular extension of the right leg. The signal from the electrogoniometer was recorded at 100 Hz by a computer contained in a ‘fanny pack’ attached to the walker. These data were downloaded to a PC after twelve to fifteen minutes and the interval between successive maximal extensions of the right leg in the analog signal was digitized and used as the time series data, which we then processed. The time series for the interstride interval, for a typical walker, is depicted in Figure 1a. In this figure we can see that the fluctuations closely resembles that of a simple random walk process, shown in Figure 1b. These experimental data are from the first 900 steps of a particular individual in the study. Preliminary studies by West and Griffin [4, 5] have shown that the control of walking is apparently a fractal random process which can be characterized by a power-law index, the fractal dimension. Similar results were found somewhat earlier by Hausdorff et al. [2, 3]. In Figure 2 we plot the aggregated relative dispersion of the data for two walkers versus the number of aggregated data points on log-log graph paper. In this figure the data aggregation sizes $n = 1, 2, \ldots, 46$ are used. The fit to the aggregated relative dispersion is of the form (5).

In Figure 2 we depict the best straight line fit to the stride interval data using (5) for two of the walker data sets. In the graph using the second walker data we see that the relative dispersion has a slope of $b = -0.16$ and an intercept of $\ln a = -3.39$, or an amplitude of $a = 0.03$, so that taking the antilogarithm of the fitting equation we obtain for the relative aggregated dispersion

$$RD^{(n)} = \frac{0.03}{n^{0.30}}. \tag{7}$$

The fractal dimension is therefore determined to be $D = 1.30$. A similar result was found for the seventh walker as well. However, it is clear from Figure 2 that the data systematically deviates from this inverse power-law fit.

We found that some individuals are better represented by an inverse power law form for the aggregated relative dispersion than are others. Since at least half of the ten individuals in the study display aggregated relative dispersions that deviate systematically from the inverse power law form, as we shall show, we concluded that the analysis based on the assumption of a monofractal time series is not adequate to describe the phenomenon. Thus, in the next section we consider an argument first made in an economic context by Montroll and Shlesinger [11] that gives rise to the kind of behavior observed.

4. Multiplicative preferential scale

The deviation of the aggregated relative dispersion data from the best inverse power-law fit, shown in Figure 2, clearly suggests that an underlying mechanism exits that is producing this effect. Consider a random function $f(n, \lambda)$ that is related to the theoretical relative dispersion at aggregation level n, but which is characterized by a time scale λ. We know that the relative dispersion does not have a characteristic time scale, in fact, at each level n there are a number of such scales present. The fluctuations in $f(n, \lambda)$ can then be characterized by a distribution of λ's, that is, $p(\lambda)d\lambda$ is the probability that a particular scale is present in the relative dispersion. The theoretical relative dispersion at the nth aggregation level is then

$$\overline{RD}^{(n)} = \int f(n, \lambda)p(\lambda)d\lambda \tag{8}$$

where we average out the dependence on time scales. However, if the distribution of scales contains finite central moments, then the theoretical relative dispersion given by (8) will contain this scale. So how do we get around this difficulty?

Consider any distribution $p(\lambda)$ that has finite central moments, say a mean value $\bar{\lambda}$. Now following Montroll and Shlesinger [11], we apply an amplification mechanism such that $p(\lambda)$ has a new mean value $\bar{\lambda}/b$:

$$p(\lambda/\bar{\lambda}) \rightarrow p(b\lambda/\bar{\lambda})$$

and we assume this amplification occurs with relative frequency $1/a$. We apply this amplification a second time so that the scaled mean is again scaled and the new mean is $\bar{\lambda}/b^2$ and occurs with relative frequency $1/a^2$. This amplification process is repeated over and over again and eventually generates the new (unnormalized) distribution:

$$P(\xi) = p(\xi) + \frac{1}{a}p(b\xi) + \frac{1}{a^2}p(b^2\xi) + \dots \tag{9}$$

in terms of the dimensions variable $\xi = \lambda/\bar{\lambda}$. We use the distribution (9) to evaluate the observed relative dispersion in terms of the theoretical relative dispersion

$$RD^{(n)} = \overline{RD}^{(n)} + \frac{1}{a}\overline{RD}^{(n/b)} + \frac{1}{a^2}\overline{RD}^{(n/b^2)} + \dots. \tag{10}$$

or in a more compact form

$$RD^{(n)} = \frac{1}{a}RD^{(n/b)} + \overline{RD}^{(n)}. \tag{11}$$

Equation (11) has the form of a renormalization group relation.

We can solve (11) approximately by neglecting the second term on the right-hand side of the equation. Alternatively we can separate the exact solution into an analytic and a singular part, the latter part dominating. The singular part of the solution, denoted by the s subscript, satisfies the functional equation

$$RD_s^{(n)} = \frac{1}{a}RD_s^{(n/b)}. \tag{12}$$

The solution to (12) is known to be, see, for example [9]

$$RD_s^{(n)} = \frac{A(n)}{n^{\alpha}} \tag{13}$$

where by direct substitution we obtain for the power-law index

$$\alpha = \frac{\ln a}{\ln b} \tag{14}$$

and the modulation function

$$A(n) = A(n/b) = \sum_{k=-\infty}^{\infty} A_k e^{-ik2\pi\frac{\ln n}{\ln b}}. \tag{15}$$

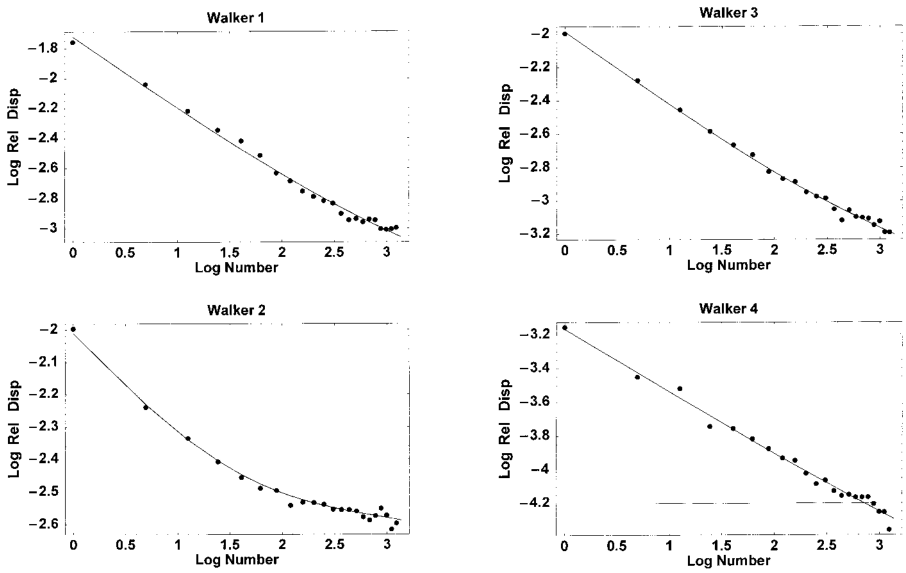

Figure 3. The stride interval data for the ten walkers are used to construct the coarse-grained relative dispersion, as indicated by the dots, for the log of the relative dispersion plotted versus the log of the number of points used in the aggregating process for a maximum of $n = 46$. The solid lines are the best mean-square linear regression to the aggregated data points using the solution to the renormalization group relation (16), fit over the entire range of data.

Thus, the relative dispersion has a dominant inverse power law with an index given by (14) and is modulated by a function that varies logarithmically with aggregation number with a period $\ln b$. Note that the solution (13) does not depend on the specific form of the distribution of scales, only on the condition that the distribution has a finite mean. This mean is then leveraged by means of the amplification mechanism to yield the modulated inverse power law.

If the function $A(n)$ is constant then (13) reduces to (4) and the underlying gait phenomenon becomes monofractal. In the literature it often appears that one has only two choices, either a process is a monofractal or it is a multifractal. The latter choice, applied to a time series, would imply that the fractal dimension changes over time, ultimately leading to a distribution of fractal dimensions [7]. That is not the situation here, however. The aggregated relative dispersion given by (13) indicates that the process has a preferential scale length, b, in addition to the monofractal behavior determined by the inverse power-law index, α. Thus, there is the interleaving of two mechanisms, one that is scale free and produces the monofractal, the other has equal weighting on a logarithmic scale and is sufficiently slow as to not disrupt the much faster fractal behavior.

We shall see in the next section that the scales in the stride-interval time series are tied together in two distinct ways. The inverse power law, or monofractal, as

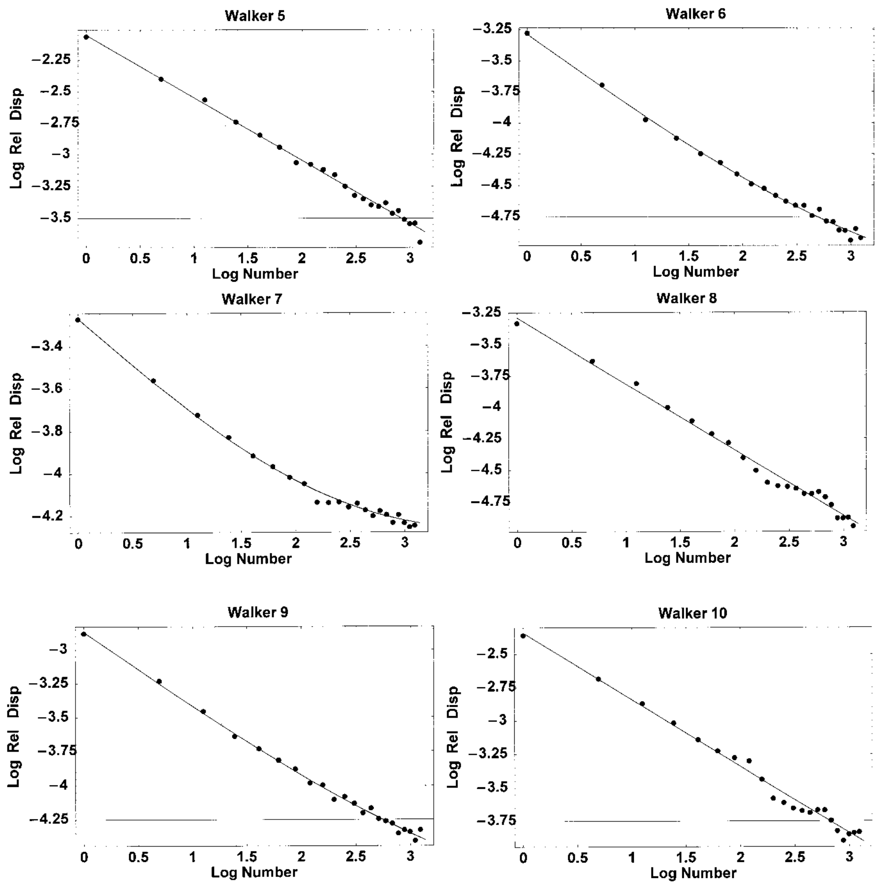

Figure 3. Continued.

obtained earlier [2, 4, 5], and the multiplicative time scale giving rise to the log-periodicity.

5. Data analysis

In the preceding section we used a renormalization argument to obtain an expression for the aggregated relative dispersion that is the product of an inverse power law and a modulation function that varies as the logarithm of the aggregation number, with a period determined by the scaling parameter b. Here we use (13) to motivate a fitting function of the form

$$\log RD^{(n)} = \log a_1 + a_2 \log n + a_3 \sin[a_4 \log n] \tag{16}$$

where the four parameters are obtained by fitting the aggregated relative dispersion plotted versus the aggregation number on log-log graph paper. Note that the mod-

Table I. The four coefficients from the least square fitting formula for the relative dispersion are listed in the table for each of the ten experimental stride interval time series

Walker	Interval time series				
	a_1	a_2	a_3	a_4	χ^2
1	–1.73	–0.23	–0.64	0.39	0.02
2	–2.04	–0.19	–0.13	1.07	0.004
3	–2.00	–0.22	–0.54	0.41	0.01
4	–3.17	–0.24	–0.52	0.26	0.02
5	–2.06	–0.36	–2.29	0.06	0.02
6	–3.28	–0.33	–0.60	0.48	0.02
7	–3.28	–0.17	–0.45	0.60	0.01
8	–3.28	–0.38	–1.09	0.14	0.04
9	–2.87	–0.28	–0.67	0.40	0.01
10	–2.34	–0.36	–1.19	0.12	0.05
Averg.	-2.60 ± 0.63	-0.27 ± 0.08	-0.81 ± 0.60	0.39 ± 0.29	

Table II. Here we list the fractal dimension, Hurst exponent and nearest neighbor correlation coefficient for the ten experimental stride interval time series

Walker	D-fratal dimension	H-Hurst exponent	r_1-correlation coefficient
1	1.23	0.65	45%
2	1.19	0.86	54%
3	1.22	0.66	47%
4	1.24	0.66	43%
5	1.36	0.56	21%
6	1.33	0.41	27%
7	1.17	0.83	58%
8	1.38	0.62	18%
9	1.28	0.72	36%
10	1.36	0.64	21%
Average	1.27 ± 0.08	0.73 ± 0.08	$45\% \pm 14.4\%$

ulation term in (16) is a particular form for the general function $A(n)$ discussed in the last section. The identification is made complete if we associate the parameters as follows: $a_1 = a$, $a_2 = -\alpha$ and $a_4 = 2\pi/\log b$. The regression of (16) on the aggregated relative dispersion is shown in Figure 3 to be remarkably good for all ten data sets. The relative dispersion for the seventh walker is seen in Figure 2 to deviate markedly from the inverse power law. In Figure 3 we see that (16) fits these data to the form

$$RD^{(n)} = \frac{0.13}{n^{0.17}} e^{-0.45 \sin[0.6 \ln n]} \tag{17}$$

and the constants obtained by fitting the other nine data sets are recorded in Table I. We introduced (16) into the nonlinear fitting technique in *Mathematica* to fit the data, and record the parameter values in Table I including a χ^2 value for the fit. Note that the modulation function in (17) is periodic, but not harmonic. The fractal dimension for the gait time series of the seventh walker is now $D = 1.17$, a value quite different from 1.3 that we obtained earlier, using a simple inverse power law to fit the data. Alternatively, we can use the relation $D = 2 - H$ to determine the Hurst exponent to be $H = 0.83$. These values of the parameters imply a 58% correlation coefficient as recorded in Table II.

The range of correlations in the stride interval time series was determined to be $0.21 \leq r_1 \leq 0.58$, that is, from very strongly correlated to nearly uncorrelated. From Table II we see that an uncorrected random process is one having a fractal dimension close to $D = 1.5$. In the statistical physics literature this is called Brownian motion as we observed above. It is clear that seven out of the ten participants have a correlation coefficient greater than 25% and would therefore qualify as having long-term memory. We discuss this more completely in Section 7.

5.1. SIGNIFICANCE OF RESULTS

One way to determine if the gait time series corresponds to a linear, additive, uncorrelated random process (white noise), or to a nonlinear dynamical process with long-time memory, is through the use of surrogate data. Theiler et al. [12] point out that, in the language of statistical hypothesis testing, we require two components: a null hypotheses against which observations are tested, and a discriminating statistic with which to test the hypothesis. In this method the null hypothesis is a potential explanation that the discriminating statistic is used to show is inadequate for explaining the data. Surrogate data are obtained by shuffling the data points, which is to say, randomly changing their relative positions in the data set, much like shuffling a deck of cards. This shuffling suppresses any long-time correlations that may be present in the time series and which resides in the order of the data points. The null hypothesis we formulate is that the difference between the original and surrogate data sets may be explained by a linear, additive, uncorrelated normal statistical process, that is, white noise. We emphasize these properties of white noise, because these are precisely the properties of the statistics being tested by the surrogate data technique. Thus, we randomly shuffle the points in the gait data sets to obtain new time series containing all the original data points, but in a randomly changed order. The aggregation procedure is applied to ten realizations of the surrogate data and the resulting relative dispersion versus aggregation number are plotted in Figure 4 for the seventh walker time series. A larger number

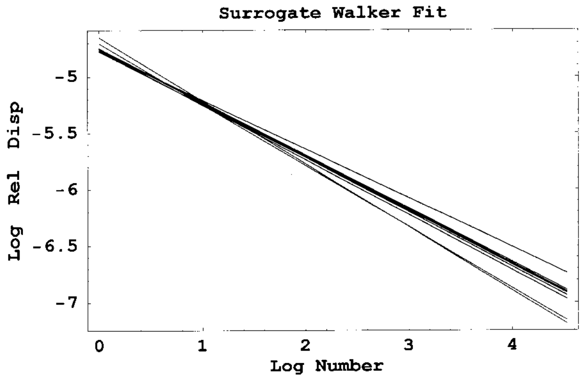

Figure 4. The solid lines are the best mean-square regressions to the ten realizations to the surrogate data obtained by shuffling the data of the seventh walker time series. The average slope of these ten curves is –0.49 with a standard deviation of 0.04 so the average fractal dimension is 1.49 ± 0.04 in agreement with Brownian motion.

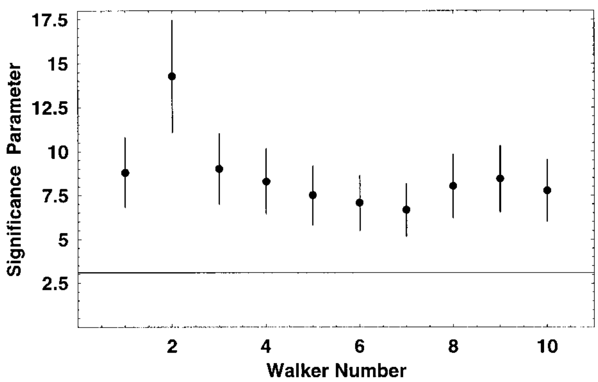

Figure 5. Testing the statistical significance of the fractal dimension using the relative dispersion. For each of the walkers, the mean D_S from ten data sets obtained by shuffling the data points, was compared with D of the original data set using (18). S is the number of standard deviations separating the original and shuffled fractal dimension values. Error bars are computed using (19).

of realizations of the surrogate data is often employed, but this is generally done to improve the statistics of the measures of statistical significance. In the present case, however, the statistics are so good with only these few realizations that more was considered to be a waste of computer time. The average slopes of the curves in Figure 4 is given by -0.49 ± 0.04 (SD), in agreement with an uncorrelated Brownian motion process. We can see from the figure that the ten realizations form a tightly interwoven set of straight lines, indicating that the various surrogates are very much like one another. They are not identical, however.

6. Measure of significance

Due to the variability in the slopes of the realizations of the surrogate data, seen in Figure 4, we need a quantitative measure of how statistically different the original gait variability data and surrogate time series are from one another. We use the fractal dimension as the discriminating statistic, that is, as an indicator of the nonlinear dynamical properties of the time series and determine the level of statistical significance using a t-test:

$$S = \frac{|D - D_s|}{\sigma_s} \tag{18}$$

where D is the fractal dimension for the gait variability data, D_s is the average fractal dimension for the ensemble of surrogate data using that particular gait time series, and σ_s is the standard deviation in the values of the fractal dimensions in the surrogate ensemble. As discussed by Theiler et al. [12] if the distribution of the statistic is normal (and numerical experiments indicate that a linear, additive, uncorrelated normal distribution is often a reasonable approximation) the probability of observing a significance S or larger is $p = erfc[S/\sqrt{2}]$, where $erfc$ is the complementary error function. Thus, the surrogate gait data cannot be distinguished from the uncorrelated random time series with normal statistics and the probability that the difference between the original data and the surrogate data can be described by a linear, additive, uncorrelated random time series is p. Said differently, in comparing the fractal dimension of the gait with that of the average of the corresponding surrogate ensemble we require a significance level greater than 0.001. This level of significance is achieved with $S > 3.29$. The significance level for the seventh walker is determined to be substantially greater than this value, in fact, $S = 8.03$, which might be called a eight sigma point. All ten of the stride interval time series are well above the required level of significance, cf. Figure 5.

Thus, the chances that the difference between the gait time series and the surrogate data could be explained by a linear, additive, uncorrelated normal process are less than one in a million and we can feel confidant in rejecting the null hypothesis. From the analysis here and in the preceding section we can associate the long-time memory in the gait time series with an underlying nonlinear dynamical process. This does not mean, however, that we can associate the source of the fluctuations with chaos.

6.1. VARIABILITY OF THE SIGNIFICANCE

As a final point we calculatie the error bars on the measure of significance, S, for a typical data set. Assuming the statistics in the measure of significance given by (18) are normally distributed, the error ΔS is determined using the usual propagation of error methodology to be [12]

$$\Delta S = \sqrt{\frac{1}{N_s}\left(1 + \frac{S^2}{2}\right) + \frac{1}{N}\left(\frac{\sigma_D}{\sigma_s}\right)^2}. \tag{19}$$

Since there is only a single realization of the gait time series in each test, we have for the variance of the fractal dimension $\sigma_D = 0$ in (19). In this way the error bars in the present analysis, for ten realizations of the surrogate ensemble ($N_s = 10$) for walker number seven, becomes $\Delta S = 1.82$. Thus, we have for the measure of significance $S = 8.03 \pm 1.82$, so that even with these few realizations of the surrogate ensemble the rejection of the null hypothesis is statistically significant. The measure of the significance parameter along with the associated error bar for each of the walker data sets is graphed in Figure 5. The horizontal line is the value of the $p = 0.001$ significance parameter. The least significant of the data sets has $p \approx 10^{-11}$.

7. The random mode-switching model

One way in which random time series are modeled in physics is by means of random walks, in which case the second moment of the random walk variable, $X(n)$, is, after n steps [9],

$$\langle X(n)^2 \rangle \propto n^{2H} \tag{20}$$

so that the mean-square value of the random variable is a power law in time (the number of steps). In the language of random walks for $1/2 \leq H \leq 1$ the walker has a tendency to continue in the direction she is going, so there is *persistence* to the process; given a step in a particular direction that step is remembered and the likelihood of the next step being in the same direction is greater than that of changing directions. Analogously, for $1/2 \geq H \geq 0$ the random walker prefers to change her mind with each step, so there is an *anti-persistence*; given a step in a particular direction that step is remembered and the likelihood of the next step being in the same direction is less than that of reversing directions. Finally, for $H = 1/2$ there is no bias and the random walker is equally likely to step in either direction no matter what the last step. The last is a normal diffusion process where the second moment grows linearly in time. The former two walks lead to anomalous diffusion.

We use a slightly modified version of the argument of Hausdorff et al. [2] and introduce the random variable $I(n)$ for the stride interval of the nth step of the walker. The total length of the time series is the sum of N steps in an experiment

$$X(N) = \sum_{j=1}^{N} I(n_j). \tag{21}$$

We can use this quantity to obtain a theoretical fractal dimension for the locomotion process. The second moment of this time series is given by

$$\langle X(N)^2 \rangle = \langle \sum_{j=1}^{N} I(n_j) \sum_{k=1}^{N} I(n_k) \rangle, \tag{22}$$

where the brackets denote an average over the steps in the time series. We now assume that the stride interval is related to an underlying mode (frequency) in which the locomotor system is operating [2]. If κ indexes the mode, then $\kappa(n)$ denotes the mode in operation at the step n. Now since the stride intervals are demonstrably random, the locomotor system must make random transitions from mode to mode. In this way we can reduce (22) to

$$\langle X(N)^2\rangle \propto \sum_{j=1}^{N}\sum_{k=1}^{N}\langle \delta[\kappa(n_j) - \kappa(n_k)]\rangle \tag{23}$$

so that irrespective of the functional form for the stride interval mode, the modes are statistically independent of one another. Here we interpret the brackets on the right-hand side of (23) as being an average over all configurations of the modes in the locomotor system.

The Dirac delta function in (23) can be replaced by its integral representation

$$\langle X(N)^2\rangle \propto \sum_{j=1}^{N}\sum_{k=1}^{N}\int \frac{dz}{2\pi}\langle e^{iz[\kappa(n_j)-\kappa(n_k)]}\rangle \tag{24}$$

where z is a dummy variable. If the distribution of modes is Gaussian, with zero mean, the average term under the integral can be replaced with

$$\langle X(N)^2\rangle \propto \sum_{j=1}^{N}\sum_{k=1}^{N}\int \frac{dz}{2\pi}\exp\left[-\frac{z^2}{2}\langle\{\kappa(n_j) - \kappa(n_k)\}^2\rangle\right]. \tag{25}$$

If the mode index itself is represented by a simple random walk process, that is, delta correlated steps, then for $n_j > n_k$

$$\langle\{\kappa(n_j) - \kappa(n_k)\}^2\rangle = n_j - n_k \tag{26}$$

and the integration can be carried out to yield [2]

$$\langle X(N)^2\rangle \propto \sum_{j=1}^{N}\sum_{k=1}^{N}(n_j - n_k)^{-1/2} \propto N^{3/2}. \tag{27}$$

Note that this result is a consequence of the statistics of the random transitions between modes, so that with each step we randomly access a different frequeney in the locomotor system.

Now in comparing (27) with the general expression for anomalous diffusion (20) we observe that $H = 0.75$ in the random mode-switching model. This value of the Hurst exponent is completely consistent with the experimental stride interval data, so that the stride interval phenomenon is persistent in terms of the random walk model.

8. Conclusion

The observation that the stride-interval time series is a random fractal is consistent with the results of Hausdorff et al. [2], who used a detrending fluctuation analysis in processing their data, and whose interpretation of the gait time series as a random fractal was elaborated on by Liebovitch and Todorov [13]. In fact we find from our data that the average Hurst exponent we obtain, $H = 0.73 \pm 0.08$ (SD), is in essential agreement with these earlier findings and with the random mode-switching model. In terms of the random walk model the phenomenon of fluctuations in human gait is persistent. The stride interval fluctuations are not completely random, like Brownian motion, nor are they the result of processes with short term correlations. Instead, the inverse power-law form of the correlation function reveals that the stride intervals at any given time is influenced by fluctuations that occurred hundreds of strides earlier, see also [2]. This behavior is a consequence of the fractal nature of the stride interval time series.

The principle of allometry, as it is called in the biological literature, has long been an expression of the interdependence, organization and concinnity of physiological processes. We have extended the allometry idea to irregular allometric time series in terms of the properties of feedback control. An allometric system achieves its purpose through scaling, enabling a complex system such as that of gait to be adaptive, and accomplish concinnity of the many interacting subsystems [4, 5]. If each of these subsystems has its own characteristic set of frequencies then the mode-switching model shifts the burden of control randomly from one subsystem to another, thereby achieving stability. This random switching provides the overall persistence that is observed in the stride interval time series.

The control process required for human gait therefore manifests scaling through the long-time correlations of the fluctuations in the stride interval. However, there is the additional mechanism of the modulation of this monofractal behavior. The tying together of the long and short time scales is necessary in order for the feedback to adaptively regulate the complex gait process in a changing environment. The log-periodic modulation of the inverse power law was shown by Shlesinger and West [14] to be a consequence of the correlation function satisfying a renormalization group relation and having a complex fractal dimension. This theme was expanded on by Sornette [15] who argues that the log-periodicity is a result of what he calls discrete scale invariance (DSI), that is also a consequence of renormalization group properties of the system. In Section 4 we used such an argument to show that the relative dispersion should have such a log-periodic modulation in general. In the present context this implies that the motor control system for gait does have a discrete characteristic scale that is an invariant of the random mode switching.

Jörgi et al. [16] explain that DSI is qualitatively similar to the idea of lacunarity, which we interpret to be the multiplicative spacing between frequencies in the random-switching model. This is also the multiplicative preferentical scale developed through the renormalization group ideas in Section 4. The physiological implication of this observation is that the frequency spectrum of the human gait control system is not continuous, but has gaps due to the spacing between frequencies of the constituent subsystems. A more detailed interpretation of this mechanism requires additional experiments.

One final comment regarding processing. We choose the relative dispersion technique because, while it is not new, it does highlight the modulation of the inverse power law. Other techniques, that smooth the data, suppress this effect [16]. In addition it is straight forward and because it does not involve higher order moments of the time series, is readily understood by scientists with minimal training in statistics and mathematics.

Failure to accommodate changes through an inability to regulate high frequency motor control in a coordinated way with low frequency motor control occurs in certain pathologies and in the elderly [3]. The suppression of high frequency response is indicated through an decrease in the value of the power-law index, H, below the range of that of healthy individuals. Note that as H decreases the fractal dimension increases towards the Brownian motion value of 1.5 and the memory in the gait fluctuations process disappears. The implication is that the control vanishes with the memory. This is consistent with a hypothesis made by West and Goldberger [17] concerning the use of inverse power laws in physiology as diagnostics concerning the health of physiological system.

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