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On the entropy function in sociotechnical systems

(Sears Roebuck catalogues/stimulus–response/traffic flow/fluctuations)

ELLIOTT W. MONTROLL

Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742

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ABSTRACT The entropy function $H = -\Sigma p_j \log p_j$ (p_j being the probability of a system being in state j) and its continuum analogue $H = \int p(x) \log p(x)\, dx$ are fundamental in Shannon's theory of information transfer in communication systems. It is here shown that the discrete form of H also appears naturally in single-lane traffic flow theory. In merchandising, goods flow from a wholesaler through a retailer to a customer. Certain features of the process may be deduced from price distribution functions derived from Sears Roebuck and Company catalogues. It is found that the dispersion in logarithm of catalogue prices of a given year has remained about constant, independently of the year, for over 75 years. From this it may be inferred that the continuum entropy function for the variable logarithm of price had inadvertently, through Sears Roebuck policies, been maximized for that firm subject to the observed dispersion.

In his retiring address as President of the National Academy of Sciences, Philip Handler pleaded ". . . what I would particularly like to direct to your attention is the pressing need to develop sophisticated analytic approaches to large sociotechnical systems. . . ." Similarly concerned, I have sporadically explored the possible application of quantitative approaches of the physical sciences to the characterization of social dynamics and sociotechnical systems (1–3). In response to his request, I present this essay on yet one more approach for consideration, the entropy style pioneered by Ludwig Boltzmann in his investigations of systems of many molecules.

Carved into Boltzmann's tombstone in the central cemetery of Vienna is his entropy formula $S = k \log W$; W is the number of equivalent ways a molecular system may be constructed in its equilibrium state, and k is Boltzmann's constant. He postulated a molecular system to be as randomized as possible within the constraints implied by various conservation laws. Those associated with children, adults, or organizations witness a similar pervasive relaxation into a random state as controls weaken. Hence, as a grand general theory of sociotechnical systems emerges, perhaps it too will enjoy an entropy principle.

Shannon's theory of information (4, 5) transmitted in communication channels marked the first appearance of Boltzmann's entropy function (see Eq. 1 below) in a sociotechnical system. This report shows how that function also appears as a consequence of the interplay between data analysis and model construction in examples taken from two other sociotechnical systems, highway transportation and marketing.

THE ENTROPY FUNCTION

Boltzmann measured disorder through the entropy function

$$H = -\sum_{j=1}^{N} p_j \log p_j \quad \text{with all } p_j \geq 0. \qquad [1]$$

In a system that may exist in states $j = 1, 2, \ldots, N$, with p_j representing the probability that it is in the jth state, the p_j are normalized and bounded.

$$\sum_{j=1}^{N} p_j = 1 \quad \text{and} \quad 0 \leq p_j \leq 1. \qquad [2]$$

From the possible sets $[p_j]$, H is maximized by

$$p_j = 1/N \text{ for all } j. \text{ Then, } H = \log N.$$

It is easy to show (2) that any deviation of p_j from $1/N$ reduces the value of H from $\log N$. When a single $p_j = 1$, with all others vanishing, $H = 0$.

The continuous state entropy function ($-\infty < x < \infty$) is

$$H = -\int_{-\infty}^{\infty} p(x) \log p(x) dx. \qquad [3]$$

$p(x)dx$ represents the probability that x lies in the interval $(x, x + dx)$. It is well known (6) that when $p(x)$ also satisfies

$$\langle x^2 \rangle \equiv \int_{-\infty}^{\infty} x^2 p(x) dx = \sigma^2, \qquad [4]$$

the Gauss function maximizes [3] with normalization and [4].

$$\text{If} \quad p(x) = (2\pi\sigma^2)^{-1/2} \exp -\frac{1}{2}(x^2/\sigma^2),\ H = \log \sigma(2\pi e)^{1/2}. \qquad [5]$$

Let us now examine briefly the manner of appearance of the entropy function in Shannon's communication theory.

THE ENTROPY FUNCTION IN COMMUNICATION THEORY

Shannon's famous memoirs (4, 5) expanded and systematized pioneering work of Hartley and Nyquist on quantification of information transfer rate in communication devices by exploiting Boltzmann's entropy function. If, after long experience with message transfer through a communication channel incorporating a code that employs a set of N symbols identified by $j = 1, 2, \ldots, N$, it is found that the jth symbol appears with probability p_j, then the maximum information transfer rate becomes

$$H = -c \sum_{j=1}^{N} p_j \log p_j. \qquad [6]$$

The constant c depends on the logarithm base and the rate at which the device emits symbols. Generally, the maximum rate is not quite achieved, but a code due to Huffman (6) yields a rate that deviates by no more than one bit from the theoretical bound. Unfortunately, that code is without redundancy so that a single coding or noise error may turn the message beyond the first error into nonsense.

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Abbreviation: SR, Sears Roebuck and Company.

The information transmission rate may also be estimated from the continuum entropy function [3] when the message is propagated as a continuous wave form. Let the transmitter have the bandwidth W. Then the message wave form may be Fourier analyzed as a linear combination of W harmonics. If an ensemble of continuous messages is coded so that the Fourier coefficients all have a Gauss distribution with a common dispersion, then the information transmission rate is proportional to [3]. This follows from assertion **5** concerning the Gauss distribution.

Noisy circuits carry less information. If P is the signal power and N is that of interfering Gaussian noise, then, as Shannon (5) showed, the information transfer rate is proportional to $H = W\log[1 + (P/N)] \simeq WP/N$. The asymptotic form is valid when $N \gg P$, conforming to engineers' rule of thumb that broad bandwidth circuits carry information at a higher rate and that a simple way to overcome noise is to enhance the signal.

This identification of information transfer rates with the entropy function makes one speculate whether the transport of people or goods along transport routes might also enjoy such a relationship. At high vehicle densities that tax road systems, a system is effectively a collection of single lanes with little interlane exchange. On this basis, let us survey the flow of interacting carriers on a long single-lane road. We start with a general discussion of systems of interacting individuals and then specialize to the process of interest.

The Entropy Function in a Sociotechnical System Driven by Personal Interactions. Personal interactions progress through a succession of stimuli and responses. The level of group productivity in the performance of an assigned task is a compromise between an orderly execution of responsibilities and a pervasive disorder caused by factors such as misunderstanding, miscalculation, accident, grievance, and the human frailty of relaxing into a state of negligence.

A feature of stimulus–response interactions that may lead to instabilities and decline in productivity is the existence of time lags between stimulus application and response execution. Typical lags have three components: perception time, decision-making time, and time for response execution; their sum is the total process time lag. Generally, the response may be characterized by a parameter (say λ), or several parameters that measure the strength of the response force to the stimulus.

A continuously applied stimulus of fluctuating magnitude and sign that excites a very strong response after a long time lag may induce violent instabilities. In that case the stimulus may have changed sign by the time the strong response occurs so that the response aggravates rather than smooths fluctuations. It has been observed in electrical and mechanical systems that stability is best maintained by making small corrections immediately as fluctuations occur. Management by negligence followed by strong shock responses violates that style.

A stimulus–response process may be exhibited as a "graph" of points, representing individuals, connected by lines, representing their interaction. Fig. 1*A* represents a typical four-tier organization in which crossing of lines is forbidden, direction comes from above, and queries originating below are channeled upward with no authority bypassing. Fig. 1*B* characterizes a line of individuals who interact only with their neighbors. This very simple graph is appropriate for a platoon of cars on a single-lane highway. In the simplest systems of interacting individuals, each has but one mode of response. More modes exist in more complicated systems.

Our example of a single lane of traffic as an interactive sociotechnical system fortunately corresponds to the simple graph of Fig. 1*B* and requires the interacting individuals to have but a single form of response, to accelerate or to decelerate. This system was investigated (7–11) by exploiting experimental data derived from instrumented cars driven on the General Motors test track, on city streets, and in tunnels.

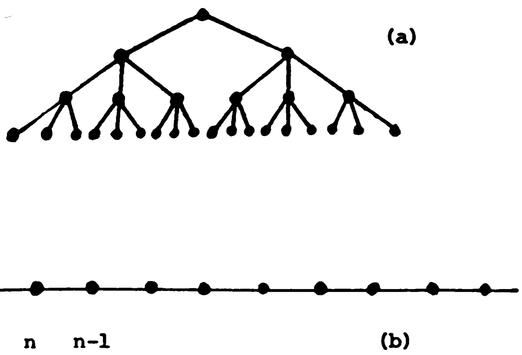

FIG. 1. Two classes of stimulus–response interaction. Class *B* corresponds to vehicular traffic in the single-lane no-passing mode.

An equation found to describe, with remarkable accuracy, the behavior of a second car (represented by $n + 1$) following a leader (represented by n) is

$$dv_{n+1}(t + \Delta)/dt = \lambda_0\left[\frac{v_n(t) - v_{n+1}(t)}{x_n(t) - x_{n+1}(t)}\right] \quad [7]$$

in which $v_n(t)$ is the velocity of car n at time t, $x_n(t)$ is the location of the front end of that car at time t, and Δ is the time lag between the stimulus provided by the lead car and the response by the follower. In our basic car-following experiments the second car was instrumented to record continuously in time the numerator and the denominator of the term in the brackets (as well as the ratio) and the acceleration of the second car. Δ and λ_0 were selected to make the left side of the equation best agree with the right. Refs. 7–11 contain certain theoretical justifications of Eq. 7. For our purpose, it is sufficient to consider Eq. 7 to be an accurate empirical formula. The time lag Δ, which varies from person to person, is about 1.5 sec.

Consider a platoon of identical cars, each interacting only with its predecessor according to Eq. 7 where the index n identifies the nth car in the platoon; $n = 1, 2, \ldots$ with 1 being leader of the platoon. One can further allow Δ and λ_0 to depend upon n to derive results similar to those discussed below, but with extra fluctuation terms.

High-density traffic stability is studied by approximating

$$x_n(t) - x_{n+1}(t) \simeq d, \quad [8]$$

where d is a constant, the average space available per car. If we set $\lambda = \lambda_0/d$, then the resulting equation (8, 9) is:

$$dv_{n+1}(t + \Delta)/dt = \lambda[v_n(t) - v_{n+1}(t)], \quad n = 1, 2, \ldots. \quad [9]$$

When a platoon leader executes an irregular pattern of acceleration and deceleration, the other cars respond. In stable traffic, fluctuations propagating down the platoon become damped; in an unstable situation, they become amplified so that a rear-end collision results or traffic stops to prevent an accident. The irregular pattern of the lead car may be Fourier analyzed and the propagation of each Fourier component investigated (7, 8). It can be shown that stabilization of all frequency components in platoon motion requires that

$$\lambda_0\,\Delta/d = \lambda\Delta < 1/2. \quad [10]$$

Strong responses in a platoon with a long time lag induce instabilities upon the violation of this inequality. An increase in the spacing d is a stabilizing influence.

Values of λ and Δ have been measured for numerous drivers

(7, 9). The average of the observed product of λ and Δ of those drivers is very close to 1/2, implying that even freely flowing traffic is on the verge of instability.

Having witnessed for years a diversity of activities generated by interacting individuals, I conjecture that, were we able to construct equations analogous to [7], derive stability conditions similar to [10], and make the appropriate measurements of interaction parameters, we would find that we ride close to the crest of instability in numerous group activities.

Integration of the stimulus–response Eq. 7 yields an equation of state for traffic, a relationship between vehicular flow rate and density in single-lane traffic. First we integrate Eq. 7 between times t_1 and t_2 to obtain, for all n,

$$v_{n+1}(t_2 + \Delta) - \lambda_0 \log[x_n(t_2) - x_{n+1}(t_2)] = v_{n+1}(t_1 + \Delta) - \lambda_0 \log[x_n(t_1) - x_{n+1}(t_1)] \quad [11]$$

$$\text{so that} \quad v_{n+1}(t + \Delta) - \lambda_0 \log d_{n+1}(t) = \text{constant} \quad [12]$$

$$\text{with} \quad d_{n+1}(t) \equiv x_n(t) - x_{n+1}(t) \quad [13]$$

being the space available per car at the location between the nth and $(n + 1)$th cars at time t. The constancy of [12] is a consequence of the left-hand side of [11] being a function of only t_2 and the right-hand side, only of t_1.

The traffic density at the location of car n, $\rho_n(t)$, is the reciprocal of the space available per car: $\rho_n = 1/d_n$ = number of cars per unit length.

In a freely moving stable stream of traffic, $v_n(t + \Delta)$ with $\Delta \simeq 1.5$ sec is practically the same as $v_n(t)$ and [12] becomes

$$v_n(t) = -\lambda_0 \log \rho_n(t)/\rho_c. \quad [14]$$

The local traffic flow rate (dropping the explicit dependence upon time) is then

$$q_n = \rho_n v_n = -\lambda_0 \rho_c (\rho_n/\rho_c) \log(\rho_n/\rho_c). \quad [15]$$

Notice that $0 < \rho_n/\rho_c \leq 1$ and that the dimensions of our variables might be cars per hr for q, cars per mile for ρ, and miles/hr for v. By averaging over N cars in a line of traffic, the mean flow rate becomes

$$q = \lambda_0 \rho_c \left(-\frac{1}{N} \sum_{n=0}^{N} \frac{\rho_n}{\rho_c} \log \frac{\rho_n}{\rho_c} \right). \quad [16]$$

The term in parentheses has the form of the Boltzmann entropy function [1] without the normalization condition [2] if one sets $p_n \equiv \rho_n/\rho_c$. Hence our model yields a traffic flow rate proportional to the Boltzmann entropy function in a form analagous to that found in Shannon's information transfer rate formula [6].

Let us now compare the expected flow rate in a line of traffic of uniform density (all ρ_n being equal) with one that includes fluctuations in ρ_n about a mean value ρ. For this purpose we define $\Delta\rho_n$ as the local fluctuation through

$$\rho_n \equiv \rho + \Delta\rho_n \quad \text{with} \quad \Sigma \Delta\rho_n = 0. \quad [17]$$

Then, from Eqs. 16 and 17,

$$q = \rho\lambda_0 \log(\rho_c/\rho) - \frac{\rho\lambda_0}{N} \sum_{n=1}^{N} \left(1 + \frac{\Delta\rho_n}{\rho}\right) \log\left(1 + \frac{\Delta\rho_n}{\rho}\right) \quad [18a]$$

$$= \rho\lambda_0 \log(\rho_c/\rho) - \frac{\rho\lambda_0}{2N} \sum_{1}^{N} \left(\frac{\Delta\rho_n}{\rho}\right)^2 \quad [18b]$$

in the regime $(\Delta\rho_n/\rho)^2$ small. This flow rate formula shows certain features common to Shannon's information theory. For a given set of parameters λ_0, ρ_n, and ρ_c, the throughput is maximal when all drivers are synchronized with $\rho_1 = \rho_2 = \ldots = \rho$. Then all $\Delta\rho_n$ vanish, leaving only the positive term in [18]. The close packing density ρ_c increases as car lengths decrease, thus increasing the flow rate. This is the analogue of shortening symbol lengths in communication channels. Finally, by increasing the response sensitivity, λ_0, the throughput is increased. This is done, however, at the expense of violating the stability condition [10]. If λ_0 is increased to the instability level, traffic stops with throughput vanishing. This is analogous to the employment of the Huffman code in a system with noise, for then nonsense appears in the decoding process. A more prudent response to a trend toward instability would compensate for the increase in λ_0. Finally, because it is unlikely for each driver to respond in the same manner to an onset of instability, the various $(\Delta\rho_n)^2$ would increase, increasing the negative fluctuation terms in [18].

In summary we note that, through years of driving experience, the public has inadvertently fallen into a pattern that relates traffic flow rate directly to the entropy function. In smoothly flowing traffic the negative noise terms in [18] are quite small as is evident from the excellent fit of the equation of state $q = \rho\lambda_0 \log(\rho/\rho_c)$ to observational data (11, 12).

ENTROPY IN THE CATALOGUES OF SEARS ROEBUCK AND COMPANY

A communication system is composed of a message input element, a channel for message propagation, and a message output element. A highway transportation system has an input provision for a traveler and a road providing the channel for travel to an exit point. We have seen that the flow-through rate in each of these systems may be related to an entropy function. Another important sociotechnical system is the merchandising system. Goods flow into the warehouse or distribution center of the retailing firm, remain temporarily as an inventory, and finally are carried out by or delivered to the customer. Hence, a company's profit depends upon the flow-through rate of goods and the prices associated with the goods. This similarity between merchandising and the previous two examples suggests that the entropy function may appear in an analysis of that process.

A further consideration of this idea requires merchandising data. By good fortune, the firm of Sears Roebuck and Company (SR) has given us a rich legacy of information on this subject in its annual catalogues, which form a magnificent data base of Americana of the past 80 years. The prices listed in the catalogues were generally right for their times and the items listed reflect the public taste of the time. At first, through its mail-order operation, the firm made available to the farm family products normally found in cities of medium to large size; then it tried to compete with city merchants for the urban trade. The catalogues may be regarded as a merchandise model of a medium-sized city, listing available goods at reasonable prices.

The preparation of the catalogues was a major concern of SR. Basically, each page was audited to produce its share of the profit. For example (14), in 1930 the goals set ranged from \$5000 to \$20,000 per page, depending upon the responsible merchandising department. Since the profit that year (13) was \$14,300,000 and the catalogues ran 1000–1500 pages, the profit per page averaged about \$10,000. Expensive goods, beautifully illustrated, often attracted attention to pages containing cheaper bargain items. Many pages reserved a small space for the tentative introduction of new products. If the public responded favorably, the allocation increased the next year. As annual sales of an item declined, its space allocation decreased: sometimes it even disappeared completely from the catalogue. Various department heads, anxious for raises and promotions, were very competitive in the production of pages that listed items that were hoped to outsell those of their colleagues.

Although the SR catalogues have been woven into the lives of millions, Herman and I (15) may have been the first to regard the lists of prices as a statistician's delight, to be exploited as a microcosm of the merchandising world. Motivated by reasons expressed in ref. 15, we found the distribution function of prices by year listed in many of the catalogues. Because prices range from a few cents to hundreds of dollars, we "expanded" the scale of low-cost items and "contracted" that of higher priced ones by recording the data as the logarithm of the price (to the base 2), $\log_2 P$. Of course, we were aware (as many before us dating back to D. Bernoulli) that $\log P$ is psychologically a more important variable than the price itself because one is especially sensitive to relative price changes, $(\Delta P)/P \simeq \Delta \log P$.

Examination of the price distribution from many catalogues indicates that in a given catalogue the distribution of $\log_2 P_i$ (P_i being the price of the ith item) is very close to the normal distribution [**5**]. Three examples are shown in Fig. 2. We also investigated the mean $\log_2 P$ and the dispersion of $\log_2 P_i$. If N is the number of prices sampled we define

$$\log \bar{P} \equiv \langle \log_2 P \rangle \equiv \frac{1}{N} \sum_{i=1}^{N} \log_2 P_i \qquad [\mathbf{19}]$$

$$\sigma^2_{\log P} \equiv \frac{1}{N} \sum_{i=1}^{N} (\log_2 P_i - \langle \log_2 P \rangle)^2 \qquad [\mathbf{20}]$$

The findings for these quantities for 18 years appear in Table 1.

The variation in $\langle \log_2 P \rangle$ over the years reflects changes in cost of living through the 20th century. Catalogue prices changed in two manners: (*i*) by the change in price of an invariant item such as a clothespin, a 1910 specimen being indistinguishable from one of 1940, and (*ii*) by the change in the nature of the item listed to reflect an evolving technology and a varying public taste. The 1910 bicycle was quite different from a 1970 model. The 1910 buggy whip had disappeared from the catalogue and the CB transmitter was known only to science fiction writers in 1925. Many interesting deductions follow from changes in $\langle \log_2 P \rangle$ (15), but it is upon the third column, $\sigma^2_{\log P}$, of Table 1 that I wish to direct attention.

As one superficially scans successive catalogues, one is impressed with the tremendous variety of articles available and the steady change from year to year. We have been as much impressed by the existence of an almost invariant statistical quantity—an "economic constant of the motion"—for the marketing operation. It is remarkable that for more than 75 years the dispersion $\sigma_{\log P}$ (defined by Eq. **20**) has hardly changed. The average value of $\sigma_{\log P}$ is 2.26 with $\langle (\sigma - \bar{\sigma})^2 \rangle^{1/2} = 0.17$. Table 1 shows the largest observed deviation of $\sigma_{\log P}$ from 2.26 to be 1.91, in the 1932–1933 winter catalogue, at the depth of the Great Depression. That catalogue contained a statement to the effect that, because of the high cost of catalogue production and somewhat reduced demand for high-priced furniture, the furniture listing is meager. A separate furniture catalogue was available upon request. The combination of the regular 1932–1933 catalogue with the furniture catalogue would lead to a $\sigma_{\log P}$ value closer to 2.26.

Having observed the constantcy of σ, we can construct a simple mathematical model to "explain" it. Let us suppose that, in a given year, all prices are inflated (or deflated) by the same factor, α. Then the transition experienced by the price of the nth catalogue item in that year would be $P_i \rightarrow \alpha P_i$ so that the transition of $\log_2 P_i$ would be $\log_2 P_i \rightarrow \log_2 P_i + \log_2 \alpha$ and the difference $[\log_2 P_i - \langle \log_2 P \rangle]$ would remain invariant because

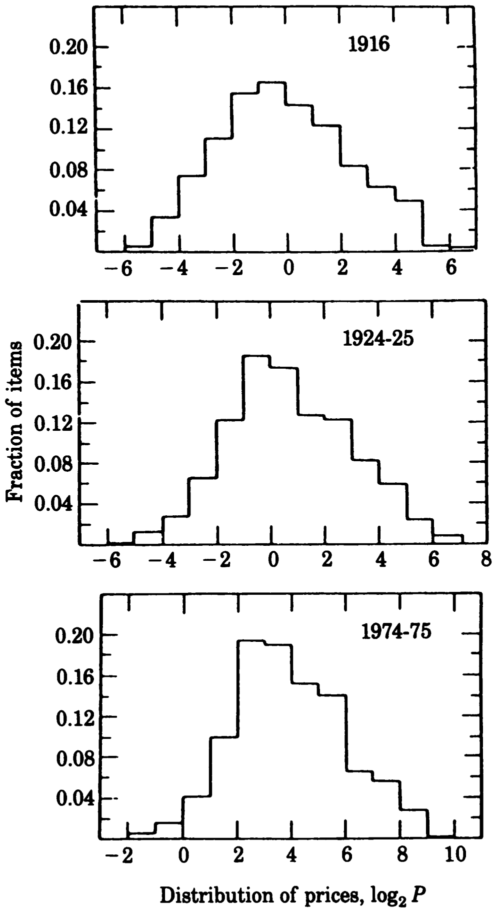

FIG. 2. Histogram of distribution of prices in SR catalogues for years 1916, 1924–1925, and 1974–1975. The fraction of items in each price range in each catalogue is plotted as a function of $\log_2 P$, P being the price. (From ref. 17, by permission.)

the α-dependent contributions of each term cancel. On this basis, $\sigma_{\log P}$ defined by Eq. **20** also remains invariant under the constant-inflation-factor postulate.

The inflation model may be made more realistic by assuming that the ith item has its own inflation factor α_i expressed as an average inflation factor plus a small correction $\Delta\alpha$; $\alpha_i = \alpha + \Delta\alpha_i$ with $\langle \Delta\alpha_i \rangle = 0$. Then, to first order in $(\Delta\alpha_i/\alpha)$, $\log P_i$ in one year is transformed in the next to

$$\log \alpha_i P_i \simeq \log P_i + \log \alpha + (\Delta\alpha_i/\alpha).$$

Hence, to first order,

$$\frac{1}{N} \sum_{i=1}^{N} \log P_i \rightarrow \frac{1}{N} \sum_{i} \log P_i + \log \alpha \quad \text{and}$$

$$\sigma^2 \log P \rightarrow \sigma^2 \log P + \frac{2}{N} \sum_{i=1}^{N} (\log_2 P_i - \log_2 \bar{P})(\Delta\alpha_i/\alpha) + \frac{1}{N} \sum_{i=1}^{N} (\Delta\alpha_i/\alpha)^2. \qquad [\mathbf{21}]$$

In a year with a mean inflation rate of 10%, $\alpha = 1.1$. A reasonable range for $\Delta\alpha_i$ might be $-0.1 \langle \Delta\alpha_i < 0.1$, yielding the range $-0.09 < \Delta\alpha_i/\alpha < 0.09$ so that typically $(\Delta\alpha_i/\alpha)^2 \simeq 0.01$. When the inflation rate is independent of the price of the item the cross term of first order in $\Delta\alpha_i/\alpha$ in [**21**] vanishes. However, when the inflation rate for low-priced items is generally higher than that for higher-priced ones (a common situation), the mid-

Table 1. Standard deviation of $\log_2 P$ from mean $\langle \log_2 P \rangle$ for various years in the period 1900–1976

Year*	$\langle \log_2 P \rangle$	$\sigma_{\log P}$	Year	$\langle \log_2 P \rangle$	$\sigma_{\log P}$
1900	0.150	2.43	1939–40	0.627	2.62
1902	0.212	2.34	1946–47	0.532	2.15
1908	−0.228	2.29	1948–49	1.336	2.37
1916	−0.068	2.38	1951–52	1.785	2.34
1924–25	0.422	2.32	1962	2.403	2.24
1929–30	0.998	2.26	1972–73	3.030	2.27
1932–33	0.691	1.91	1973–74	3.322	2.05
1934–35	0.673	2.22	1974–75	3.870	2.12
1935–36	0.537	2.39	1975–76	4.060	2.03

$\bar{\sigma} = 2.26$; $\langle(\sigma - \bar{\sigma})^2\rangle^{1/2} = 0.17$.

* An entry identified by a single year corresponds to a "spring–summer" catalogue; an entry identified by a number such as "1924–25" corresponds to a "winter" catalogue.

dle term in [21] becomes negative and cancels the positive last term. Without that influence, σ^2 grows each year.

The constancy of σ^2 combined with the discussion at the end of the section *Entropy Function* implies that the normal distribution of $\log P_i$ maximizes the entropy function associated with that variable. Hence, in their marketing wisdom, Sears, Rosenwald, their staff, and their successors, created catalogues with goods priced so that year after year the price distribution maximized the entropy function associated with $\log P_i$.

The entropy function itself, defined by [3] for a log normal distribution function, has the form

$$H = -\int_0^\infty [\log P/\bar{P}]^2 \, p(\log P/\bar{P}) d \log P/\bar{P}$$

where $p(x)$ is the normal distribution function defined by [5]. $\log P/\bar{P}$ is similar to the utility function of classical economics, originally used by Bernoulli in his analysis of the St. Petersburg gambling paradox. Then H is the weighted average of the square of the function resembling the utility function.

Many other quantitative conclusions may be drawn from SR catalogues. One of interest to academics was especially noted by Cohn (14). In 1905, guitars enjoyed as great a popularity as they again did in the 1960s. Among the many styles available that year was the college name group. Cohn wrote ". . . one wonders whether the head of Sears' music department, when he priced and named his guitars, was not at the same time passing judgment upon the merits of the universities according to some secret or unconscious criteria of his own. Note the valuations:

The Stanford . . . $4.25	The Princeton . . $13.75
The Cambridge. . $8.95	The Yale $16.95
The Cornell . . . $11.35	The Harvard . . . $21.45

The traditional catalogue competitor of SR was Montgomery Ward & Co. It would be interesting to compare the price distributions in their catalogues with those of SR and to examine the annual variations in dispersion in $\log P_i$. Circa 1910, several of the great department stores, including Macy's, Wanamaker's, and Filene's, entered the catalogue marketing field in competition with SR but returned to their traditional marketing style after only a few years. It has been observed (13) that, for given classes of merchandise, SR stocked models that were cheaper and others more expensive (a broader distribution) than those listed in catalogues of the more traditional department stores.

COMMENT

Log-normal distributions have been recorded for annual personal incomes in the United States in the range 5th to 97th percentile (16) and for common stock prices listed on The New York Stock Exchange (17). There is evidence that income distributions have dispersions that vary but slightly over the years.

In conclusion, I refer to some champions of the entropy style. Motivated by Shannon's work, Jaynes (18) coupled information theory with statistical mechanics. His ideas have been elaborated to the degree that a school identified as The Maximum Entropy Formalism School has emerged. Their work is well represented in the proceedings of a conference (19). Also, an interest has developed in applying these entropy/information ideas to economics and other social sciences. The book by Georgescu-Roegen (20) is prominent in this direction, as are the papers by Wilson (21).

The style here deviates somewhat from that of most of the cited authors. Rather than postulating an entropy principle, I started with data whose analysis led to entropy-like expressions.

By this paper I recognize Philip Handler, on the occasion of his retirement from the presidency of the National Academy of Sciences, for perceiving the need to explore more deeply the nature of our sociotechnical systems. I thank Prof. R. Herman for many discussions on traffic models and Sears Roebuck catalogues at the time the data that forms the basis for this paper were collected, and Prof. H. Riess for several interesting discussions on entropy models. This research was partially supported by the Advanced Research Projects Agency, Department of Defense, under contract with the Materials Research Council of the University of Michigan.

1. Montroll, E. W. & Badger, W. W. (1974) *Introduction to Quantitative Aspects of Social Phenomena* (Gordon & Breach, New York).
2. Montroll, E. W. (1978) *Proc. Natl. Acad. Sci. USA* **75**, 4633–4637.
3. Montroll, E. W. & Shuler, K. E. (1979) *Proc. Natl. Acad. Sci. USA* **76**, 6030–6034.
4. Shannon, G. E. (1948) *Bell Syst. Tech. J.* **27**, 379–423.
5. Shannon, G. E. (1948) *Bell Syst. Tech. J.* **27**, 623–656.
6. Huffman, D. A. (1952) *Proc. IRE* **40**, 1098–1101.
7. Chandler, R. E., Herman, R. & Montroll, E. W. (1958) *Oper. Res.* **6**, 165–184.
8. Herman, R., Montroll, E. W., Potts, R. B. & Rothery, R. W. (1959) *Oper. Res.* **7**, 86–106.
9. Gazis, D. C., Herman, R. & Potts, R. B. (1959) *Oper. Res.* **7**, 499–505.
10. Montroll, E. W. (1963) in *Proceedings of the First Symposium on Eng. Appl. of Random Function Theory & Problems* (Wiley, New York), pp. 231–248.
11. Edie, L. C., Foote, R. S., Herman, R. & Rothery, R. W. (1963) *Traffic Eng.* **33**, 21–25.
12. Greenberg, H. (1959) *Oper. Res.* **7**, 79–85.
13. Emmet, B. & Jeuck, T. E. (1950) *Catalogues and Counters* (Univ. Chicago Press, Chicago, IL).
14. Cohn, D. L. (1940) *The Good Old Days* (Simon & Schuster, New York).
15. Herman, R. & Montroll, E. W. (1977) in *Statistical Mechanics and Statistical Methods in Theory and Applications*, ed. Landman, U. (Plenum, New York), pp. 785–803.
16. Badger, W. W. (1980) in *Mathematical Models as a Tool for the Social Sciences*, ed. West, B. J. (Gordon & Breach, New York), pp. 87–120.
17. Osborne, M. F. M. (1959) *Oper. Res.* **7**, 145–155.
18. Jaynes, E. T. (1957) *Phys. Rev.* **106**, 620–630.
19. Levine, P. D. & Tribus, M. (1979) *The Maximum Entropy Formalism* (MIT Press, Cambridge, MA).
20. Georgescu-Roegen, N. (1971) *The Entropy Law and Economic Processes* (Oxford Univ. Press, London, England).
21. Wilson, A. G. (1970) *Oper. Res. Q.* **21**, 247–265.