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Unifying Complexity and Information

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Complex systems, arising in many contexts in the computer, life, social, and physical sciences, have not shared a generally-accepted complexity measure playing a fundamental role as the Shannon entropy *H* in statistical mechanics. Superficially-conflicting criteria of complexity measurement, i.e. complexity-randomness (C-R) relations, have given rise to a special measure intrinsically adaptable to more than one criterion. However, deep causes of the conflict and the adaptability are not much clear. Here I trace the root of each representative or adaptable measure to its particular *universal data-generating or -regenerating model* (UDGM or UDRM). A representative measure for deterministic dynamical systems is found as a counterpart of the *H* for random process, clearly redefining the boundary of different criteria. And a specific UDRM achieving the intrinsic adaptability enables a general information measure that ultimately solves all major disputes. This work encourages a single framework coving deterministic systems, statistical mechanics and real-world living organisms.

A widely-appropriate complexity measure is needed in numerous areas[1–4], for directly applications or for a unified theoretical framework like statistical mechanics[5]. To date, except some special cases[6–8], most calculable complexity measures are superficially classified into three types (I, II, and III) depending on three C-R relations: a monotonically ascending curve, a convex curve, and a monotonically descending curve, respectively[8]. Competition is mainly between the type-I and the type-II. According to many type-II supporters, highly complex systems like human brains evidently exist at a critical transition point between randomness (or deterministic chaos) and regularity called *the edge of chaos* or *weak chaos*[9–13], and then an ideal type-II measure should regard an object of *weak chaos* as the most complex and low-periodic objects and completely-chaotic objects the simplest, despite most type-II measures, e.g. the logical depth[14], do not suffice.

There is another deeper but perhaps not strictly accurate classification. *Deterministic measures* are usually estimations of the incomputable concept *Kolmogorov complexity*[3] (*KC*), the uncompressible information account of individual object defined as the length of the minimal computer program that regenerates the object; while *statistical measures* are chiefly derived from *H* for systems describable in probabilistic language[9,10]. A quantity is called *extensive* if it scales (asymptotically) with the size r_w of the random word (string) $X^{r_w} = X_1X_2...X_{r_w}$ that describes the system under consideration[15]. In an isolated system, *H* is extensive, whereas even some statistical measures appear not[9]. Then disputes arise.

Amazingly, a recently-introduced *deterministic* measure *lattice complexity* C_L exhibits *intrinsic adaptability* to various C-R relations[6,7], even a degree of sensitivity to *weak chaos*, implying an ultimate solution of all above-mentioned disputes. For a deterministic α-nary symbol string *s*, *intrinsic adaptability* with a parameter *r* needs only treating all (overlapping) length-*r* words in *s* as α^r-nary symbols for specific measures' calculation; while *extrinsic adaptability* needs extra variables and operations to show two existing different-type measures' behavior alone or jointly[8]. Traditionally, *r* is related to r_w, because *s* is often considered a collection of outcomes of a length-*r* random word, and a statistical form of *intrinsic* adaptability may help to find the proper theoretic fundamental.

In this article, C_2, a statistical measure previously known as of type II[16,17], is found of somewhat *intrinsic* C-R adaptability. Further analysis reveals a contradiction between the adaptability and the *random UDGM* (r-UDGM), i.e. *random process*, in which entropies are exclusively rooted. With the nonlinear deterministic iterative system being identified as the *deterministic UDGM* (d-UDGM) that can generate any arbitrary symbol string as the traditional r-UDGM can, the C-R competition is clarified. A particular UDRM containing both r- and d-UDGM is shown to unit major competing ideas of complexity measurement naturally in an estimation of *KC*.

Results

Deterministic adaptable complexity. With its widely-used type-I estimation Lempel-Ziv complexity[18] (C_{LZ}), *KC* is traditionally considered a measure of randomness[3,10,19]. Although this judgment is valid for random objects, the adaptable estimation C_L reveals more aspects of *KC*.

Both C_{LZ} and C_L simulate a machine reading the given string s over a finite alphabet Σ continuously into an unlimited memory. Alongside the reading procedure, both algorithms virtually separate s into *uncompressible units* and count the unit number as the complexity value of s. The *present unit* is one such that has a *present symbol* just being read. It can still be compressible and then extendable.

Compressibility is reduced to duplicability in C_{LZ}. If the *present unit* can be duplicated from any section of the *exhaustive* memory including the already-read part of the unit itself, the duplication operation extends the *unit* simply symbol by symbol until no section of the memory equals the *unit*. At that time the *present symbol* is regarded as an *insertion* making the *present unit* uncompressible and the next unit, with its first symbol, will become the *present* (see the example below).

In C_L, a present-unit-extending mechanism prior to the duplication is the deterministic *iterative map* on Σ following either chaotic (no-symbol-repeating) rule or periodic rule. Iterations of such map are regarded as compressible as duplications, inspired by the fact that short-program-described iterative systems, e.g. logistic maps[20], can produce any symbolic sequence out of chaotic or periodic orbits.

For example, let s = 0010001100 11010111 and the dot and the sign " $\vee$ " denote the insertion in C_{LZ} and C_L, respectively. The results are as follows:

$$s=0\cdot 01\cdot 000\cdot 11\cdot 001101\cdot 0111\cdot,$$

$$s=001\vee 0001\vee 1001\vee 101011\vee 1.$$

According to C_{LZ}, the first symbol is always an insertion without prefix. The second has a duplicable prototype in the memory, but the third makes the unit 01 does not match the exhaustive memory 00. The fourth and fifth symbols 00 together can be duplicated from the previous symbols 0010, while the sixth can not, because 000 has no prototype in 00100. Three following uncompressible units are 11, 001101 and 0111. Since there are 6 separated units, $C_L(s) = 6$.

According to C_L, the first two symbols 00 are generated by a 1-period iteration, but the third symbol 1 interrupts the iteration. Because 001 is not duplicable, it is an uncompressible unit. The next unit 0001 is identified similarly. Concerning the third unit 1001, since the first two symbols 1 and 0 are different from each other, they should be assumed following chaotic rule; the third symbol 0 implies that a periodic rule is employed with an initial state 1. After a periodic rule is broken, neither 1001 nor its follower unit 101011 is found duplicable. With the last unit 1 being separated, we see $C_L(s) = 5$.

Let parameter $r = 2$, any two-symbol word in s compose a refined symbol and $C_L(s^2) = 3$. Let $r \geq 6$, $C_L(s^r) = 1$. Indeed, as has been shown[6,7], for a finite s, there is a critical order r^* such that once $r \geq r^*$, s^r can be regarded as a single iteration and then $C_L = 1$. When r reaches the particular r^* of a given "completely chaotic" object, with both the "completely chaotic" and the low-periodic object obtaining the minimal C_L, C_L achieves the transition from a type-I measure to a type-II. The objects of highest r^*, with the most difficulty to obtain $C_L = 1$, are strings of the period-doubling accumulating points known as *weak chaos*[13].

Adaptable entropy. In classical information theory[21], when the probability p_i of any event x_i is obtainable, with a random variable X representing all α possible events, the α-nary Shannon entropy is the mean of the *information content* $-\log_\alpha p_i$ of X

$$S=-\sum_i^{\alpha} p_i \log_\alpha p_i. \tag{1}$$

If $\alpha = 2$, the unit of S is just bit; and if not, since $\log_2 \alpha \cdot \log_\alpha x = \log_2 x$, one may time S by $\log_2 \alpha$ to get the entropy H of bit. For an binary independent and identically distributed random word $X^{r_w} = X_1 X_2 \ldots X_{r_w}$ with 2^{r_w} elemental events and any event $X_i^{r_w}$ having the probability $p_i^{(r_w)}$, 2^{r_w}-nary S becomes the *entropy rate (entropy per symbol) of order* r_w h_{r_w},

$$\begin{aligned} S(X^{r_w}) &= -\frac{\log_2 2}{\log_2 2^{r_w}} \sum_i^{2^{r_w}} p_i^{(r_w)} \log_2 p_i^{(r_w)} \\ &= -\frac{1}{r_w} \sum_i^{2^{r_w}} p_i^{(r_w)} \log_2 p_i^{(r_w)} = \frac{1}{r_w} H(X^{r_w}) = h_{r_w} \end{aligned} \tag{2}$$

It is well-known that $S(X^{r_w})=1$ for uniformly distributed X^{r_w}. Measuring the mean of bits needed for the shortest description of the random word's experimental outcome, $H(X^{r_w})$ is extensive because in the limit $r_w \to \infty$ the *entropy rate*[21] h is a constant.

Accompanied by a type-I measure C_1, C_2[16,17] is derived from "a hierarchical approach to complexity of infinite stationary strings[22]." By *stationary*, we see that statistical properties of the strings in consideration are fixed with time or space changing, a precondition for all statistical measures' application.

Given a binary string s of length n, there are totally 2^r distinct words of length r (or r-words for short). Let $F_a(r)$ denote the frequency of a distinct *allowed r-words* that really emerges in s and $F_f(r)$ the frequency of a distinct *forbidden r-words* in s counted as follows: if in s (except its end) a $(r-1)$ -word $s_1 s_2 \ldots s_{r-1}$ emerges x times, but no r-word $s_1 s_2 \ldots s_{r-1} s_r$ emerges, we account $s_1 s_2 \ldots s_{r-1} s_r$ a forbidden r-word emerging x times. For example, when basic alphabet $\Sigma = [0,1]$, if $s_r = 0$, then we regard the frequency of the allowed r-word $s_1 s_2 \ldots s_{r-1} 1$ as that of the forbidden r-word $s_1 s_2 \ldots s_{r-1} 0$.

With the probability being replaced by relative frequency in s, C_1 is the entropy rate of allowed r-words and C_2 of forbidden r-words:

$$C_1 = -\sum_i P_{ai} \frac{\log_2 P_{ai}}{r}, \tag{3}$$

$$C_2 = -\sum_i P_{fi} \frac{\log_2 P_{fi}}{r}. \tag{4}$$

Here, i denotes each distinct r-word; $P_{ai} = \frac{F_{ai}(r)}{\sum_i F_{ai}(r)}$ and $P_{fi} = \frac{F_{fi}(r)}{\sum_i F_{fi}(r)}$ are relative frequencies.

Given a finite r, if s containing all possible (including overlapping) 2^r r-words, no forbidden r-word occurs. For a completely chaotic (random) case, as $n \to \infty$, the number of distinct forbidden r-words $N_f(r) \to 0$, then $C_2 \to 0$.

Since s is finite, C_1 is a real-world estimate of h_r, while C_2 is roughly adaptable to type I and type II. First, the critical order r^* still works. If s is of a level of randomness, every r^*-word in s is distinct as well as every $(r^* + 1)$ -word is. So the number of distinct allowed $(r^* + 1)$-word $N_a(r+1) = N_f(r+1)$, thus $C_1(r^*+1) = C_2(r^*+1)$. Second, there may exist another critical word length r_w^* such that for $r \leq r_w^*$, $N_f(r) = 0$ and then $C_2 = 0$. By increasing r from the r_w^* of a given completely chaotic object to the r^* of the same object, C_2 roughly achieves a transformation from a type-II measure to a type-I.

If s is a sequence of minimum period m, when $r \geq m$, there are m different r-words of equal frequency, and then $C_2(r)$ can simply be predicted.

Functionally, C_2 is composed of a type-I measure C_1 and a type-II measure $N_f(r)$. As $N_a(r)$ is a low-precision version of C_2 and $N_a(r) \geq N_f(r)$, $N_f(r)$ may cause a precision problem of C_2 in showing type-II behavior.

What is really involved in C_2 calculation is the frequency of every distinct allowed r-word sharing length-$(r-1)$ prefix with a forbidden r-word. This means that C_2 is actually applicable to α-nary strings. For convenience and without loss of generality, let us assume that the strings under consideration are binary hereafter.

Intrinsic adaptability. Let us use the logistic map $x_t = \mu x_{t-1}(1 - x_{t-1})$ to exhibit the intrinsic adaptability. The case $\mu = 4$ is known as the completely chaotic (pseudo-random) object and the case $\mu = 3.57$ a representative sample of *weak chaos*. After 25000 times iteration deleted as transient, from a trajectory of x_t we got a binary symbolic sequence of length 8204 by the partition 0.5. With different r, C_{LZ}, C_L, C_1, and C_2 for both $\mu = 4$ and $\mu = 3.57$ are calculated and shown in Fig. 1.

From Fig. 1, with r increasing remarkable symmetry can be found in behavior of the two pairs of measures, C_{LZ} and C_L, and C_1 and C_2. When $\mu = 4$, $r_w^* = 10$, and if we scale the ordinate logarithmically, we will find $r^* = 26$. Although C_{LZ} and C_L of low r are almost equal, the difference between them increases with r until $r \geq r^*$. On the other hand, with $r \leq r_w^*$, $C_1 = 1$ and $C_2 = 0$. They rapidly converge when r approaching to r^* and stay equal when $r \geq r^* + 1$.

When $\mu = 3.57$, $r_w^* = 3$ [Fig. 1(d)], and $r^* = 834$ (not shown in Fig. 1). Actually, in a long range of r between about 26 and 834, $C_1 \approx C_2$. To let C_2 act as a type-II measure, r must be within the range from 4 to 10.

With $r \leq 3$, the $\mu = 3.57$ and the $\mu = 4$ cases are not distinguishable by C_2. When $26 > r > 10$, C_L and C_2 are both in a type-transition state. The transition range is near the word length $r = 13$ [Fig. 1(b)], the solution of the equation $n = 2^r - r + 1 = 8204$. It is easy to see that $r_w^* \leq 13$ and $r^* \geq 13$ are valid for any length-n string and $r_w^* = r^* = 13$ is valid only if in s each overlapping distinct possible r-word just appears once[6].

Fixing r but letting μ vary with $\Delta\mu = 0.0001$ from 3.5 to 4, results are as shown in Fig. 2. When $r = 3$, almost all cases in the region about $\mu > 3.555$ obtain $C_2 = 0$ (not shown in Fig. 2); when $r = 4$, the zero-C_2 region reduces to about $\mu > 3.907$ [Fig. 2(b)]; and when $r = 4$ to about $\mu > 3.978$ (not shown in Fig. 2). Since r_w^* is either 3 or 4 for most of chaotic cases, we see a serious precision problem of C_2 of low r [Fig. 2(b)].

As $C_2(10) = 0$ for $\mu = 4$ [Fig. 2(c)], $C_2(10)$ is roughly type-II. For any chaotic s, C_2 always rapidly converge to C_1 once the r_w^* of s has been exceeded [Fig. 1]. Since 10 is certainly larger than 3 or 4, within most chaotic area except a small region very close to the point $\mu = 4$, C_1 and C_2 act similarly.

Highest C_2 [as shown in Fig. 2 (c)] is not close to the edge of chaos as highest C_L is [see Fig. 8 in Ref. 6]. When r is only a little higher than the r_w^* of $\mu = 4$ case, e.g. $r = 12$, C_2 becomes definitely a type-I measure [Fig. 2 (d)]. Thus, C_2 has a significantly smaller range of choice of r for roughly type-II behavior than C_L.

Symbolic dynamical analysis and UDGM. Symbolic dynamics for one-dimensional nonlinear iterative systems including logistic maps provides a one-to-one correspondence between any semi-infinite symbol string and the initial point of the trajectory producing the string[20]. A finite r-word represents a deterministic segment enclosing the initial point. Increments of r will rescale the segment into a shorter one. Therefore, the parameter r in C_L is also called *fine-graining order*, while in entropy this name may not be appropriate, as discussed below.

Table 1 shows the distribution of all possible 4-bit words in $\mu = 3.645$ case. If two adjacent 3-bit-prefix-sharing words both emerge or not emerge, they are ignored. Hence we get only three distinct 4-bit forbidden words **0110**, **0100**, and **1100**, but ignore **1001**, which is also adjacent to an allowed word. Moreover, since one r-word creates two prefix-sharing $(r+1)$ -words, when $r = 5$, all (virtual) segments corresponding to 4-bit forbidden words are ignored. It makes $N_f(r)$ fluctuate irrelevantly to the real spatial structure in phase space. For instance, there always exist mid-position adjacent segments not being visited, but when $r = 4$, 5, and 6, the number of distinct forbidden words corresponding to such segments equals 2, 0, and 2, respectively. Thus the curves of C_2 versus r can hardly be smooth except some cases of almost complete chaos [Fig. 1 (b) and (d)].

Essentially, any entropy is only applicable for a random string emitting r-words with measureable stationary distribution, or the r-UDGM of arbitrary given deterministic string s. To apply the r-UDGM exclusively, one has to ignore all temporal or spatial information of s unrelated to the distribution, let alone nonstationary objects of no stable distribution. For instance, given $r = 2$, the 2-periodic infinite string $(01)^\infty$ has $C_2 = 0.5$ and be regarded as a medium complex case despite its simple temporal structure.

In contrast to the entropy, KC estimations C_{LZ} and C_L are in itself designed for single deterministic strings. The successive process in

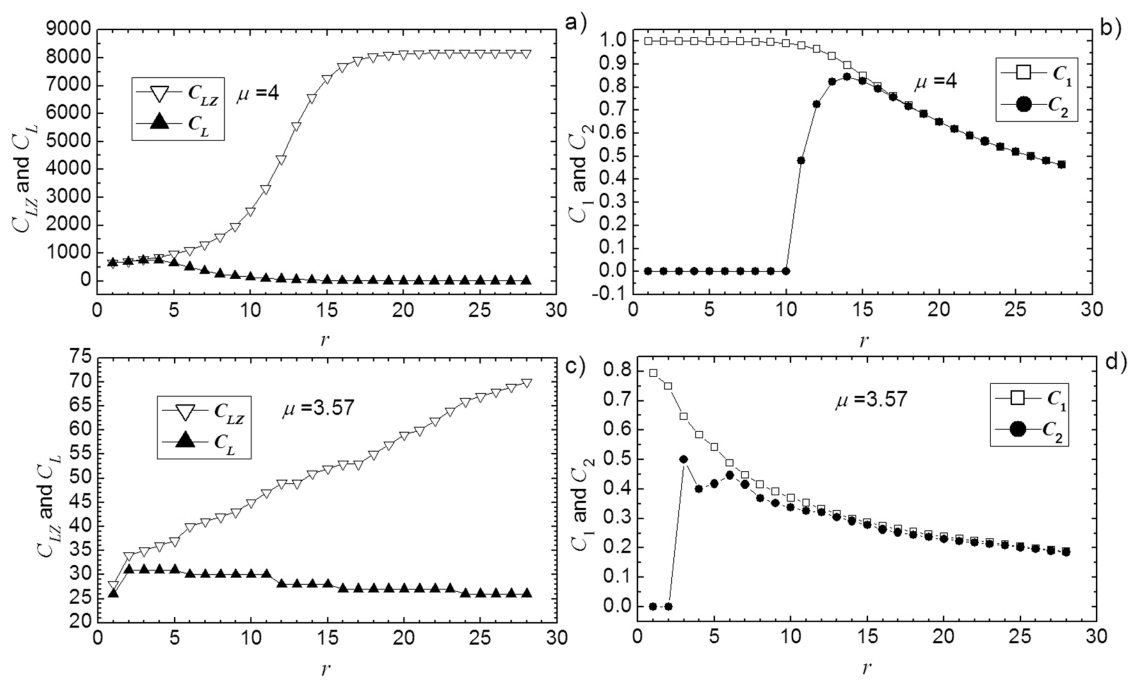

Figure 1 | Complexity of logistic map. (a) C_{LZ} and C_L for $\mu = 4$; (b) C_1 and C_2 for $\mu = 4$; (c) C_{LZ} and C_L for $\mu = 3.57$; (d) C_1 and C_2 for $\mu = 3.57$.

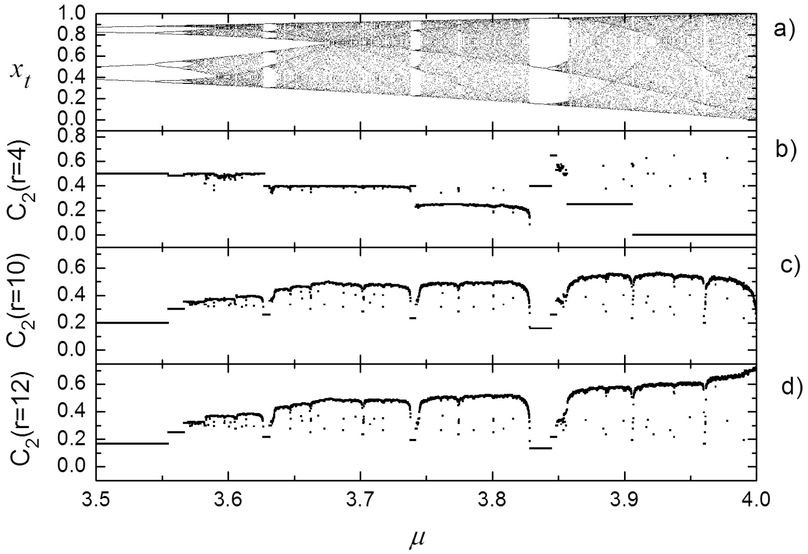

Figure 2 | The C_2 of logistic map. (a) Bifurcation diagram; (b) $r = 4$; (c) $r = 10$; (d) $r = 12$.

searching duplicable section ensures the low complexity value for simple regular strings, e.g. $(01)^\infty$ of $C_{LZ} = 3$ and $C_L = 1$.

Due to the absence of a perspective of deterministic chaos in its process of dealing with irregular strings, C_{LZ} is still type-I. Using the terminology of C_1 and C_2, we may say in s C_{LZ} successively counts up non-overlapping allowed words in assorted lengths, each of which is adaptively increased from 1 to such a value that the word is distinct from any part of the *exhaustive* memory. The word length adjusting mechanics makes C_{LZ} be much more fine-grained than $N_a(r)$ and let the parameter r become meaningless.

Deterministic iterative systems are considered in C_L a sort of simple data-generating models. They can be regarded as logistic maps, in which *fine-graining order r* represents the size of the segments, or as equivalent one-bit-output binary recurrence equations, in which r represents the bit number of input. For any given s, when $r < r^*$, this sort of models is *non-universal data-generating model* (NUDGM) , and when $r \geq r^*$, become universal, i.e. the d-UDGM, and leave no space for others.

The d-UDGM can be used alone by regard s as a single trajectory of a simplest equation, whose operation assignment must be defined as regular as possible. Determining the exact optimal set of arithmetic operations and their assignment in the equation may need countless tentative calculations. What can absolutely not be reduced is the smallest bit number of input, the r^* of s, representing the system's uncompressible information.

Henceforth let r_w denote also the length of assumed random word in the r-UDGM of s to distinguish it from the fine-graining order r. For a length-n s, let $C_L(r, n)$ replace $C_L(s^r)$ and let $r = 0$ mean that no segment involves iterative mappings, C_{LZ} can be viewed as a special case of C_L denoted by $C_L(0, n)$.

Discussion

Logically, a quantity designed for single deterministic strings is unconditionally suitable for stationary random strings because one can calculate the quantity's probabilistic mean, whereas the mean may not fit any individual string. The quantity's mean of a length-r_w random string can be interpreted as the average of k individual results each computed from a length-r_w string emitted by the random string with $k \to \infty$. To arrange such k emitted strings in a single *sample time series*, we must assume that the random string, or equivalently the time series, is *ergodic*, i.e. the relative frequency of any distinct length-r_w deterministic string in the time series equals the string's probability: $P_{ai}(r_w) = p_i^{(r_w)}$, $i = 1,2,\ldots,2^{r_w}$.

If every $P_{ai}(r_w)$ is determined by previously known $p_i^{(r_w)}$, in an *ergodic* time series the arrangement of all length-r_w emitted strings can be overlapping or non-overlapping, in a certain order or disorder, which is pointless for calculating a quantity's mean: the mean can be theoretically obtained without numerical computation. A theorem of Brudno[23] states that the *KC per symbol* of almost all emitted strings of infinite length is equal to the *entropy rate h*. Likewise, when fine-graining effect is absent or negligible (i.e. r = 0 or 1), maximum C_L relates to emitted strings of maximum randomness and one can prove that[6,18,21]

$$\limsup_{r_w \to \infty} \frac{C_L(0,r_w)\log_2 r_w}{r_w} = \limsup_{r_w \to \infty} \frac{C_L(1,r_w)\log_2 r_w}{r_w} = \lim_{r_w \to \infty} \frac{H(r_w)}{r_w} = h. \quad (5)$$

with probability 1. Hence, in traditional statistical-mechanics language $C_L(0,r_w)\log r_w$ or $C_L(1,r_w)\log r_w$ are not only extensive but also asymptotically equal to H.

In order to apply entropy to a given length-n time series, we have to assume that s is *ergodic* despite its real generating mechanism. Without previously-known $p_i^{(r_w)}$, from s only $C_1(r_w)$ rather than h can be obtained. To make $P_{ai}(r_w) \approx p_i^{(r_w)}$ and $C_1(r_w) \approx h$, we have to let $r_w \ll n$. For example, if s is a given completely random object without

Table 1 | Distribution of all possible 4-bit words in 8204-points $\mu = 3.645$ case

4-bit words	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
$F_a(4)$	0	0	0	0	0	1349	977	0	0	1349	852	1348	977	1349	0	0
Types	N	N	N	N	F	A	A	F	F	A	A	A	A	A	N	N

From left to right, all words are enumerated according to their corresponding segments' positions in the phase space of logistic maps. The frequency of a distinct allowed 4-bit word $F_a(4)$ equals the times the orbit visit the corresponding segment. In the third row, "F" means "forbidden word", "A" means "allowed word", and "N" means "neither forbidden word nor allowed word".

forbidden word, we must at least let $r_w < r_w^*$ to ensure that no distinct r_w-word has $P_{ai}(r_w) = 0 \neq p_i^{(r_w)}$.

Here we encounter an unsolvable paradox: to show C_2's type-I behavior, i.e. let $C_2(r_w) \approx C_1(r_w)$, it has to be valid that $r_w > r_w^*$, or even $r_w \to r^*$, for a given completely random object, thus the precondition $P_{ai}(r_w) \approx p_i^{(r_w)}$ for any entropy's applications cannot be satisfied. Moreover, neither an r-UDGM-rooted measure nor a d-UDGM-rooted can embody the intrinsic adaptability, since in the r-UDGM fine-graining process is not only meaningless but also harmful, and in the d-UDGM r^* of s is the ultimate point rather than an example of intrinsic adaptability.

Independent of any specific data-generating model, the *KC* estimation concerns lossless *regeneration* of given data. The insertion operation in C_L fills blanks left by any NUDGM with symbols already known from s and grants the UDRM containing this NUDGM universality. The duplication operation freely generates repeated words as the r-UDGM does, making the UDRM of $C_L(0, n)$ a sort of quasi-r-UDGM. Except for low-period objects, $C_L(0, n)$ shows no noticeable distinctness from C_1 in its C-R behavior. Without needing to previously set a $r_w \ll n$ for calculating the average $C_L(0,r_w) \log r_w$ over all allowed r_w-words in s, the whole s is treated as a single emitted object for C_L, and then $C_L(0,n) \log n$ *per symbol* appear to be estimations of h even better than C_1[24–26].

With r increasing, C_L appears a simulator not only of H but also of r^*. The intrinsic adaptability of C_L embodies indeed a general information/complexity measure presenting a smooth transition from the r-UDGM-rooted (superficially type-I) information concept to the d-UDGM-rooted (ideally type-II) complexity concept, all consistent with the principle of *KC*.

The r-UDGM and the d-UDGM identified here enable us to succinctly redefine the C-R conflict and avoid unnecessary confusions caused by misuse of each UDGM, e.g. about randomness and chaos, not only in complexity measurement. Besides many well-defined deterministic dynamical systems, living organisms[12,27–29], e.g. human brain, heart, and economic systems, appear nonstationary, *edge-of-chaos*, and, strictly speaking, beyond the scope of all types of r-UDGM-rooted statistical mechanics including generalised versions[5]. For these systems, a d-UDGM-rooted measure or framework is certainly an option and need further studies. However, in living organisms, randomness is not able to be excluded except that noise or *free will*[30] is. Thus, a UDRM-rooted framework that proceeds from a *KC*-based general information measure, C_L or its possible revised version, may have more adaptability to complicated real-world situations than a single-UDGM-rooted.

In brief, though we can separate *complexity* from *information* by using the d-UDGM and the r-UDGM alone, it would be more natural to accept a general measure encompassing H and its d-UDGM counterpart. The behavior of this measure should have been outlined by C_L, since the d-UDGM always becomes more and more *overwhelming* with r increasing.

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