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Transformation optics for antennas: why limit the bandwidth with metamaterials?

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In the last decade, a technique termed transformation optics has been developed for the design of novel electromagnetic devices. This method defines the exact modification of magnetic and dielectric constants required, so that the electromagnetic behaviour remains invariant after a transformation to a new coordinate system. Despite the apparently infinite possibilities that this mathematical tool introduces, one restriction has repeatedly recurred since its conception: limited frequency bands of operation. Here we circumvent this problem with the proposal of a full dielectric implementation of a transformed planar hyperbolic lens which retains the same focusing properties of an original curved lens. The redesigned lens demonstrates operation with high directivity and low side lobe levels for an ultra-wide band of frequencies, spanning over three octaves. The methodology proposed in this paper can be applied to revolutionise the design of many electromagnetic devices overcoming bandwidth limitations.

Since 2006, in which two pioneering papers were published in *Science* defining the concept of transformation optics[1–3], many scientists, inspired by the incredible possibilities of this tool, have proposed innovative alternatives to classic devices[4]. According to the theory, any given electromagnetic device can be transformed into an infinite number of new ones with same electromagnetic responses. For instance, this tool was proposed to redesign traditional devices such as polarization rotators[5], wave collimators and beam benders[6]; or to reduce the size of conventional lenses[7,8], to enhance transmission through sub-wavelength apertures[9,10] and generally to simplify the complex geometry of certain antennas[11–15]. This research promised great technological advances and scientists began to wonder if they should continue to still be governed by the classic rules of electromagnetic design.

However, the limitations of this tool soon started to become apparent. When the original coordinate space is transformed into a new arbitrary one, the dielectric and magnetic constants can become anisotropic diagonal tensors with components that depend upon position and can take either extremely high, or low values[11,14]. The anisotropy can be achieved for homogeneous media using either commonly found crystals[16,17], or multi-layered media[18,19], and have both been used to realise macroscopic invisibility cloaks in the optical frequency range. Obviously, the necessity of having spatially varying permittivities and permeabilities introduces enormous difficulties in the manufacturing process. But the limitation of the initial proposals was the need for magnetic or dielectric constants lower than unity. Therefore, dispersive metamaterials are necessarily required which will significantly reduce the bandwidth of operation and are lossy[15]. To further complicate matters, some authors have proposed designs in which the coordinate transformation requires anisotropic materials which are characterized by non-diagonal tensors[20]. Although a technique to diagonalise the new electromagnetic components has been proposed[20], it is only an approximation and its applicability is only valid for particular cases. Therefore, the promise of transformation optics to revolutionise future devices was brought into doubt.

In order to avoid the limitation in bandwidth, some authors proposed simplifications of the electromagnetic distributions in the new coordinate space. These simplifications consist of an elimination of sub-unity regions in the refractive index[11] or to only apply this approximation to the individual dielectric or magnetic tensor elements which are dispersive[7]. However, some of this work propose the use of meta-surfaces[11,21] or periodic structures[22,23] for their experimental validations, which are inherently limited in bandwidth.

Despite the common use of periodic structures for the development of future devices, in this paper, a full dielectric solution based on discrete coordinate transformation[13] that can provide practically unlimited bandwidth devices based on transformation optics is given. As a particular example, we show here how a hyperbolic lens can be transformed to be a planar equivalent, although it should be noted that the results presented here are not restricted to this specific example. The technique can easily be extended to other electromagnetic devices, provided that the spatial transformation does not involve significant anisotropy or large regions that require permittivities less than unity.

A point source (or electrically small antenna) generates a spherical (or quasi omnidirectional) radiation pattern since the electromagnetic waves are equally spread in all directions[24]. This means that the majority of the energy is potentially wasted, since the receiver of the signal is in one particular direction. From the beginning of the history of antennas, engineers have strived to obtain directive beams, and the most common ways to achieve this are: arrays[25], reflectors[26,27] and lenses[28]. Arrays have the drawback of requiring a complex feeding network[25], and on the other hand, reflectors are bulky[26,27]. Lenses are attractive devices for increasing the directivity of omnidirectional sources, while maintaining low side lobe levels and low cross-polarization[29,30]. For instance, Fresnel lenses were traditionally used to produce very directive beams of light to guide boats from lighthouses (an example of which is illustrated in Figure 1.a). One of the advantages of Fresnel lenses is that they are planar in shape, but their bandwidth of operation is very restricted[15]. Other lenses, such as hyperbolic and elliptical lenses, can also be used to produce directive beams without limitations in bandwidth[29]. However, as with reflectors, they are conventionally bulky and rely upon curved geometry which represents a drawback for certain applications. In this paper, we propose the use of transformation optics to modify the shape of an original hyperbolic lens into a planar one, obtaining a radiation system which is very directive, exhibits low side lobe levels, and is compact, without bandwidth limitations.

Results

Design of a planar hyperbolic lens using transformation optics. Let us consider the hyperbolic lens illustrated in Figure 1.b. As previously stated, this curved lens can potentially be electromagnetically transformed. That means it is possible to change the coordinate system of our problem, in order to obtain a new refractive index distribution which will present the same electromagnetic properties[1,3]. Mathematically, this transformation can be seen as a modification of the dielectric and magnetic constants as follows[8].

$$\varepsilon' = \frac{J\varepsilon J^T}{|J|} \tag{1}$$

$$\mu' = \frac{J\mu J^T}{|J|} \tag{2}$$

where J represents the Jacobian transformation tensor; ε and μ represent the dielectric and magnetic constants in the original space; and ε' and μ' are the same constants in the transformed one. We assume that $\mu' = \mu = 1$ for simplicity of the subsequent manufacturing. Since this approximation was applied, in order to maintain the required refractive index, the permittivity was altered accordingly[13]. Therefore, the original space of coordinates (in which the original hyperbolic lens was placed) can be mapped as shown in Figure 1.c. Since all the lines are orthogonal to the boundaries, these lines will define a second space in which the lens will be completely flat[11,13]. In this new coordinate space, the dielectric constant distribution is the one illustrated in Figure 1.d. In order to obtain the three-dimensional lens, the two-dimensional map was rotated about the centre to create a rotationally symmetric configuration[22,23]. This process involves a slight approximation[31], but it ensures a purely isotropic dielectric material requirement. This new lens will have exactly the same electromagnetic properties as the original one, with the exception of the matching in the boundaries due to the approximation applied to the magnetic constant, as indicated before.

Full dielectric realization of the planar hyperbolic lens. After this design, the lens was manufactured making use of nano- and micro-sized titanates dispersed into a polymeric matrix material. During this process, it was necessary to implement a discretization defined by nine spherical and linear boundaries which separate permittivity regions varying between $\varepsilon_r = 3$ and $\varepsilon_r = 12.6$. A half-cut prototype of the lens is illustrated in Figure 2.a in which the different regions can be appreciated by the different colours; and in Figure 2.b the final lens is shown.

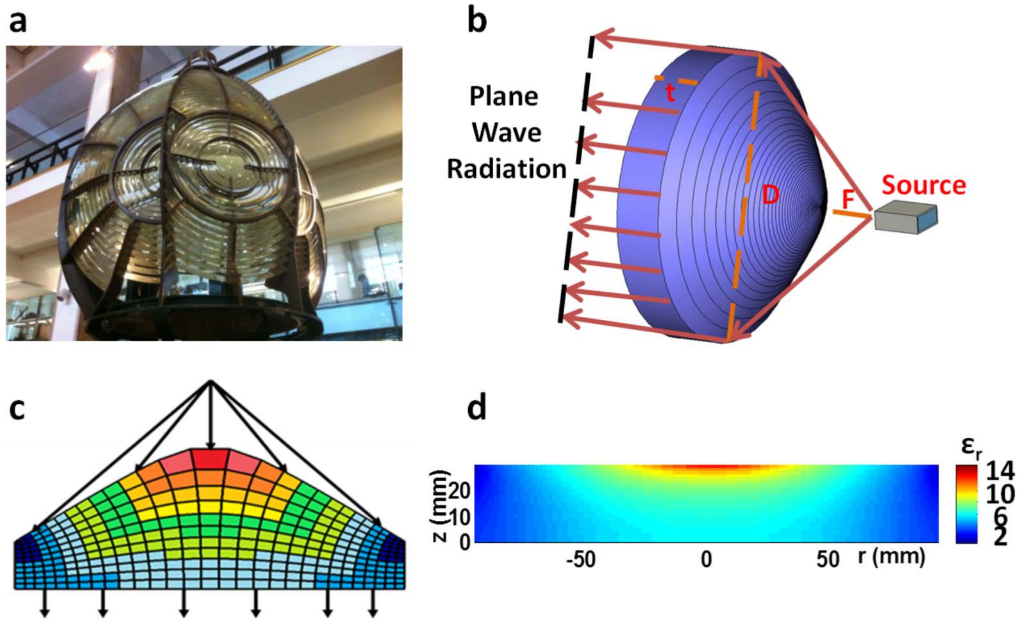

Figure 1 | **Flat hyperbolic lens generation.** (a) The Eilean Glas Light (1907): Lighthouse lens which makes use of the Fresnel Principle to focus the beam. Photograph taken by Dr. Oscar Quevedo-Teruel at the Science Museum of London; (b) Original Hyperbolic lens: D = 190 mm, F = D/4 = 47.5 mm and t = 29.5 mm; (c) Discrete coordinate transformation of the original shape; (d) Dielectric constant distribution for the new coordinate space.

The fabrication process was a liquid phase vacuum casting route that enabled the manufacture of multiple layers in a sequential manner to build up the alternative permittivity layers with controlled thickness profiles, and with the required hemispherical geometry of each layer. The thickness of each layer of the lens was controlled by using a closed cavity mould technique that enabled injection of the matrix-particulate mix with accurate dimensional tolerances.

As a range of alternative permittivity values was required, extensive studies to understand how to tailor the dielectric behaviour were undertaken. The dielectric properties are dependent upon a range of factors including the particle sizes, the homogeneity of dispersion of the particles within the host matrix, the amount of particulates in the composite material and the particle shape among others. In this particular design, the alternative permittivity regions have been achieved through a combination of tailoring the particle size, dispersion and volume fraction of materials. Figures 2.c and 2.d show examples of the micro sized and nanosized powders used in this design by processing of titanates to give the required micro-nanosized ratios that enable appropriate dispersion and the dielectric behaviour necessary for the planar hyperbolic lens. A combination of particle sizes is necessary to achieve homogeneity and the range of permittivity values required.

This manufacturing technique is inexpensive and highly reproducible, but the main advantage of this fabrication process when compared to metamaterial approaches recently reported in the literature[11,22] is the frequency independence of the electromagnetic properties of the dielectric materials. In Fig. 2.e, the measured permittivity of these materials versus the frequency (8–12.5 GHz) is illustrated, demonstrating their stable response across the X-band range. Therefore, the final planar transformed lens will retain this frequency independence of their operation.

Simulated and experimental results. Once the lens was manufactured, measurements in an anechoic chamber were developed to test its operation. The lens was excited with pyramidal horns, operating at different frequencies, placed at the focal point. In Figure 3, the comparison between simulations and measurements of the near-field distribution (of amplitude and phase) after the lens are shown (more details can be found in Methods). The simulations were obtained with a commercial FDTD simulator and the measurements were taken with a near-field scanner. The lens is placed in the plane $z = 0$ and it is centred at $x= 0, y = 0$. In this case, the plane $y = 0$ represents the H-plane of the transmitter, and $x = 0$ the E-plane. The agreement between simulations and measurements is excellent. After the lens, the fields exhibit a practically constant phase in the transversal direction and the energy is mainly concentrated in the region defined by the lens (between $y = -50$ mm and $y = 50$ mm). Therefore, we can predict that the radiation pattern of the combination of horn and lens will be very directive.

It is well-known that by using the measurements in the near-field, it is possible to accurately approximate the far-field radiation pattern. This far-field pattern at 7, 10, 12 and 14 GHz (obtained as indicated in the Methods section) is illustrated in Figure 4.a–d respectively, which shows the variation of the normalized co-polarized directivity in dB as a function of the elevation and azimuthal angles (vertical and horizontal, respectively). In this case, the values of elevation equal to 0° corresponds to the H-plane of the transmitting horn. These measurements demonstrate the correct operation of the lens which has a very directive beam and very low side lobe level. To verify the lower frequency band of operation of the lens, direct far-field measurements were carried from 1 GHz to 7 GHz (with a larger horn feed in the anechoic chamber, as indicated in the Methods section). These results are summarized in Figure 4.e with the representation of the gain and side lobe level in the H-plane. They demonstrate that the lens contributes to the increase of the directivity (and therefore the gain) of the original horn from the 2–7 GHz, however, at 1 GHz the lens is not operating due to its small size in terms of wavelengths. Therefore, we can affirm that 2 GHz is the lowest frequency of operation of the lens. This limitation is intrinsic to any lens and it is not related to the design method, although the approximations taken in the implementation of the transformed space would slightly increase this cut-off frequency.

The proposed planar lens allows for a wider bandwidth of operation than other existing planar lenses in the literature, such as Fresnel lenses[32–34], whilst retaining the same volume and aperture size. In

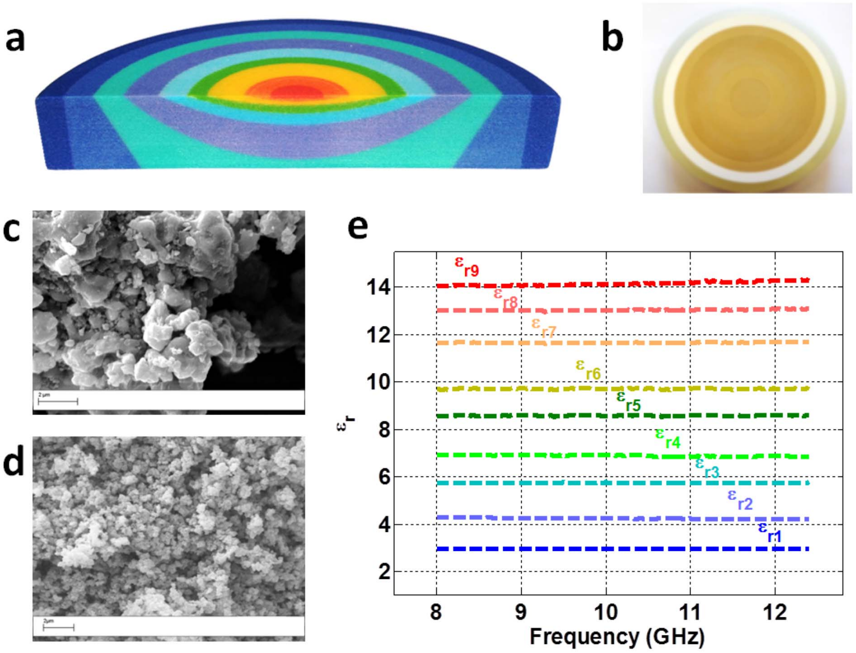

Figure 2 | **Planar hyperbolic lens.** (a) Photo of a half-cut manufactured lens; (b) Photo of the final prototype; (c, d) Pictures of powders illustrating alternative size distributions: (c) micron sized and (d) nanosized material; (e) Frequency dependence of the dielectric permittivities of the employed materials.

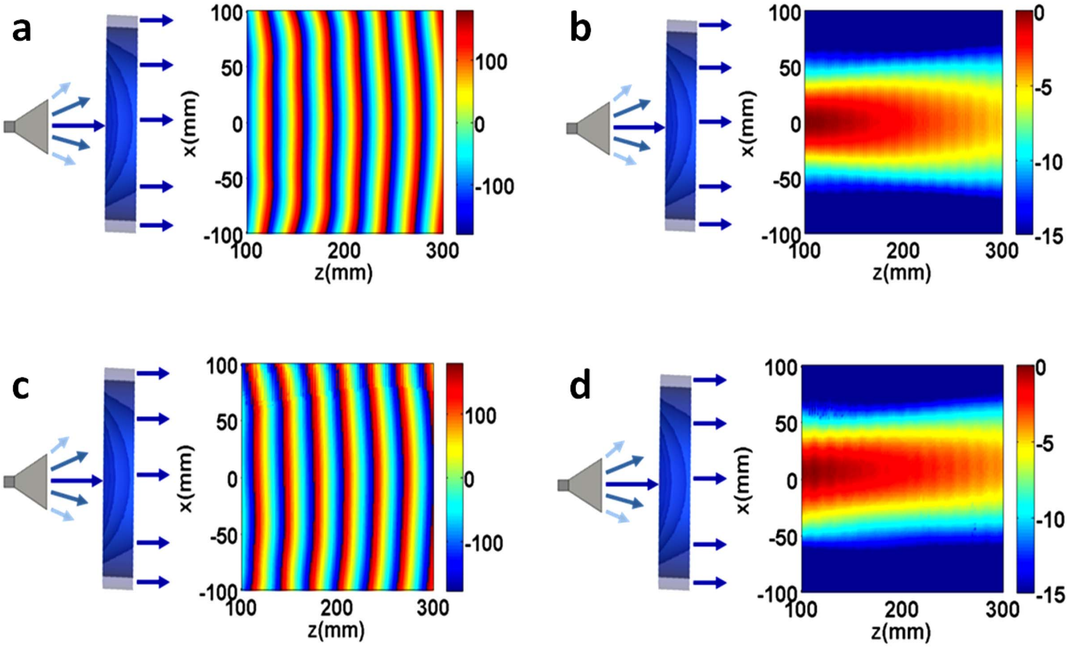

Figure 3 | **Simulated and measured field distribution after the lens.** (a, b) Simulated phase (°) and normalized amplitude distributions, respectively; (c, d) Measured phase (°) and normalized amplitude distributions, respectively.

Figure 4.f, the directivity of the original hyperbolic lens, its planar version after transformation, and a conventional Fresnel lens are illustrated for comparison (more detailed information about the radiation pattern can be found in the Supplementary Information). Although the Fresnel lens has a higher directivity at a specific frequency (11 GHz), deviation from this optimal frequency will produce a rapid degradation in its performance, since low directivity and high side lobe levels will develop, so that an adequate radiation pattern is only available across a narrow band. Furthermore, the transformed lens retains a very similar performance to the original hyperbolic lens.

Some reflections are present in the lens because of the mismatch at the boundary, however for this lens the introduced reflections are lower than -15 dB across the entire band (as can be found in the Supplementary Information). Furthermore, if these reflections are required to be reduced, they can be easily minimised utilising a thin multi-layer matching technique that operates over an ultrawide band of frequencies[29].

Discussion

To conclude, in this paper it has been demonstrated how the use of transformation optics can provide an equivalent planar version of a

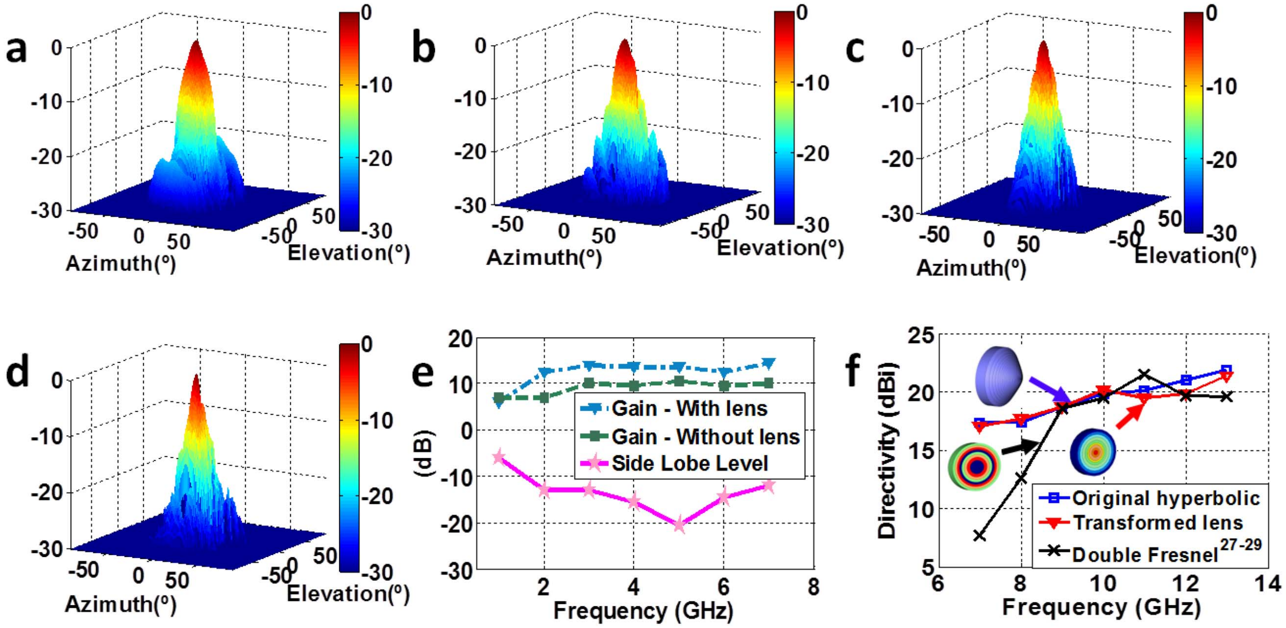

Figure 4 | **Measured radiation pattern.** (a–d) Measured 2D normalized far-field distribution (7, 10, 12 and 14 GHz, respectively); (e) Measured gain (with and without the transformed lens) and side lobe level at the lower band of operation (with the lens). (f) Simulated directivity for the original hyperbolic lens, transformed lens and Fresnel lens[32–34], all of them fed with an X-band waveguide at their focal point.

hyperbolic lens. This lens was implemented with pure dielectric materials which were generated with powders of titanate through tailoring the particle size, dispersion and volume fraction of materials. Since the permittivity of these dielectric materials is practically constant for an ultra-wide band (as are most of the commercial dielectric materials), the proposed lens has no bandwidth limitations. This is a clear advantage with respect to any other planar lenses proposed in the literature such as Fresnel lenses[32–34].

Furthermore, these results demonstrate the impressive possibilities that transformation optics can offer to the design of future electromagnetic devices; and moreover, we break the assumption that the outcome of transformation optics yields narrow-band devices, since the only bandwidth limitation was the assumed partnership between transformation optics and metamaterials.

Methods

Experimental Setup. The experimental results were developed using two measurement procedures in order to obtain the operation of the lens from 1–14 GHz. For the lower band of operation (1–7 GHz), conventional far field measurements were carried out in an anechoic chamber. For the measurements at the higher band (7–14 GHz), the measurements were developed with a near field scanner.

Configuration for the lower band (1GHz–7GHz). For the far field measurements at the lower band, two horns were employed. As a receiver, the model 3115 of ETS LINGREN[35], and for the transmitter, a model 3164-04 of ETS LINDGREN[36] was used. Although theoretically the manufacturer indicates that the transmitter operates only until 6 GHz; at 7 GHz the gain and matching are still adequate for the measurements as it is indicated in the sheet of specifications[36]. These measurements were carried out taking into account the location of the phase centre of the transmitting horn (which is shifted with the band of operation).

Configuration for the lower band (7GHz–14GHz). For the upper band under study, a near field scanner NSI-200V-3x3 was employed[37]. The scanner allows the electromagnetic field distribution after the lens to be obtained. In the transmitter, a conventional pyramidal horn working in the X-band was used; and in the receiver, a standard waveguide probe OEWG WR90 was used. Although the X-band is defined as 8–12 GHz, the measurements were carried out in the 7–14 GHz band, since the received signal was high enough to determine the electromagnetic field distribution. Using the near-field measurements, it is possible to approximate the far-field radiation pattern. Specifically, the far field was obtained using the software NSI2000[38].

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Author contributions

O.Q.-T. and W.T. carried out the discrete reshaping of the hyperbolic lens and its simulation. A.D. and H.D. contributed to the fabrication of the transformed lens and the characterization of the materials. This manufacturing process was led by S.H. O.Q.-T. and L.Z. carried out the experimental results. Y.H. initiated the idea and supervised the design and experimental characterisation. Finally, the manuscript was prepared by O.Q.-T. in collaboration with R.M.-T., Y.H. and S.H.

Additional information

Supplementary information accompanies this paper at http://www.nature.com/scientificreports

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