Accuracy of Individual Trabecula Segmentation Based Plate and Rod Finite Element Models in Idealized Trabecular Bone Microstructure

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Currently, specimen-specific micro finite element (μ FE) analysis based micro computed tomography (μCT) images have become a major computational tool for the assessment of the mechanical properties of human trabecular bone. Despite the fine characterization of the three-dimensional (3D) trabecular microstructure based on high-resolution μ CT images, conventional μ FE models with each voxel converted to an element are not efficient in predicting the nonlinear failure behavior of bone due to a prohibitive computational cost. Recently, a highly efficient individual trabecula segmentation (ITS)-based plate and rod (PR) modeling technique has been developed by substituting individual plates and rods with shell and beam elements, respectively. In this technical brief, the accuracy of novel PR μ FE models was examined in idealized microstructure models over a broad range of trabecular thicknesses. The Young's modulus and yield strength predicted by simplified PR models strongly correlated with those of voxel models at various voxel sizes. The conversion from voxel models to PR models resulted in an \sim 762-fold reduction in the largest model size and significantly accelerated the nonlinear FE analysis. The excellent predictive power of the PR µFE models, demonstrated in an idealized trabecular microstructure, provided a quantitative mechanical basis for this promising tool for an accurate and efficient assessment of trabecular bone mechanics and fracture risk. [DOI: 10.1115/1.4023983]

Introduction

Osteoporosis is characterized by low bone mass and microarchitectural deterioration of trabecular bone [[1](#page-4-0)] leading to bone fragility and high fracture risk [[2](#page-4-0)–[5](#page-4-0)]. Based on high resolution micro computed tomography (μ CT), micro finite element (μ FE) models have been widely utilized to assess trabecular bone mechanical properties and have shown excellent prediction power compared with experimental measurements [[6](#page-4-0),[7](#page-4-0)]. However, voxel-based μ FE models are not efficient in clinical applications because of its prohibitive computational cost. Besides, the micromechanics of trabecular bone is not easily accessible by the cumbersome voxelbased μ FE models, since it is difficult to navigate through millions of elements in order to examine individual trabecular failure. Therefore, there is a strong need for an alternative approach that advances in efficiency, while maintaining the fine threedimensional (3D) architectural details of trabecular bone.

Recently, a novel plate and rod (PR) μ FE modeling technique has been developed based on individual trabecula segmentation (ITS), which significantly simplifies trabecular microarchitecture as a combination of individual trabecular plates and rods [[8](#page-4-0)–[10\]](#page-4-0). An ITS-based morphological analysis allows for the separate assessment of trabecular plates and rods regarding volume, orientation, thickness, and connectivity, which has added tremendous insight about the distinct roles of plates and rods in governing the mechanical properties and failure behaviors of trabecular bone [[2,11,12\]](#page-4-0). The ITS measurements of clinically available high resolution peripheral quantitative CT (HR-pQCT) images have furthermore demonstrated its capacity to detect subtle but important microstructural changes in osteoporosis and its treatment [[13–15](#page-4-0)] and to discriminate fracture status independent of areal bone mineral density [[16\]](#page-4-0). Computationally, one can model individual rods/plates by beam/shell elements, respectively. These specimenspecific PR μ FE models show great promise in efficiently and accurately predicting the mechanical properties of trabecular bone.

The objective of this technical brief was to examine the accuracy of PR μ FE models in predicting Young's moduli and yield strengths of idealize trabecular bone microstructural models, which allows fundamental accuracy testing independent of biological variations and image noise, etc. Human trabecular bone varies greatly in density and microarchitecture across anatomic sites [\[8\]](#page-4-0). Trabecular bone microstructure captured by μ CT depends on the image resolution and signal-to-noise ratio [\[14](#page-4-0)]. A variety of

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trabecular bone mechanical behaviors have been simulated by such idealized models as elastic deformation, fatigue damage accumulation, and trabecular bone loss in osteoporosis [\[17–22\]](#page-4-0). In this study, idealized plate-rod and rod-rod microstructures of various thicknesses were constructed at typical resolutions of μ CT or HR-pQCT images. Voxel-based and PR μ FE models were generated and subjected to FE analysis for predicting Young's moduli and yield strengths.

Methods

Construction of Idealized Microstructure. In an idealized plate-rod microstructural model, two square plates 1000 \times 1000 μ m² were connected by four 1000 μ m-long rods in a unit cell (see Fig. $1(a)$). An idealized rod-rod model consisted of eight interconnected 1000 μ m-long rods in a unit cell (see Fig. 1(b)). The thickness of the plates and the diameter of the rods varied in 80, 160, 240, and 320 μ m increments. Then, simulated μ CT images were constructed for each model at three isotropic voxel

Fig. 1 Schematics of the experimental design for a comparison of the voxel-based FE model and the PR model for (a) platerod, and (b) rod-rod structures. The generation of the PR model and the voxel model were illustrated only in a unit cell.

sizes: 20, 40 (typical μ CT images of human samples), and 80 μ m (typical clinical HR-pQCT images). The simulated μ CT images were binary matrices representing the 3D trabecular microstructure in the same manner as an actual μ CT scan.

ITS-Based PR Model. The trabecular network was completely decomposed into individual plates and rods and modeled. In brief, digital topological analyses based skeletonization [\[23](#page-4-0)] was applied first to erode the image into a representation of the surfaces and curves skeleton while preserving the topology and morphology of the plate/rod structure. Then, a digital topological classification was performed in which each voxel on the surfaces and curves skeleton was uniquely classified as belonging to plate, rod, platerod junction, or rod-rod junction [[24\]](#page-4-0). Based on the segmented microstructure, each square plate was modeled by multiple triangular shell elements and each rod was modeled by a two-node beam element with circular cross-section. The thickness of the shell and the diameter of the beam elements were evaluated after reconstructing the 3D volume of individual plates and rods [\[8\]](#page-4-0).

Voxel-Based Model. A voxel-based μ FE model was directly generated from each simulated μ CT image by converting each voxel to an eight-node brick element [[25\]](#page-4-0).

lFE Analyses. Both the voxel-based and PR models were interfaced with a commercial software Abaqus (SIMULIA, Dassault Systemes, Providence, RI) to perform the μ FE analyses. Given the relatively uniform yield strain of trabecular bone across anatomic sites [\[2,3](#page-4-0)], this study adopted a new constitutive model recently proposed by Gupta [[26\]](#page-4-0) in which the yield surface is generated from the principle strain space to eliminate the effect of heterogeneity in trabecular bone failure. The new constitutive law was implemented via UMAT, a material model interface in Abaqus. Trabecular bone was modeled as an isotropic linear elastic material with a Young's modulus (E_s) of 15 GPa and a Poisson's ratio of 0.3. The tissue-level yield strains are 0.33% in tension and 0.81% in compression. Approximately 5% post-yield hardening was used. For each trabecular microstructural model, a uniaxial compression test was performed to calculate the apparent Young's modulus (E^*) and apparent yield strength of trabecular bone (σ_v^*).

Statistical Analysis. The normalized Young's modulus (E^*/E_s) and yield strength (σ_y^*/E_s) predicted by the PR FE models were correlated with those of the voxel-based models by linear regression using the statistical software NCSS 2007 (NCSS LLC, Utah). Then, R^2 was calculated for each linear regression which showed the strength of the linear correlation between the two data sets.

Results

The stress-strain curves of the PR models under uniaxial compression up to 1.5% strain was highly consistent with those of voxel-based models for the plate-rod structure with elements of varying thickness and rod-rod structures with thin elements, as shown in Fig. [2](#page-2-0). However, deviations appeared in the rod-rod structure with thick elements with trabecular thicknesses of 240 μ m and 320 μ m (paired t-test $p < 0.05$), which suggested the PR models to be less stiff than voxel-based models when trabecular rods are relatively thick. For both plate-rod and rod-rod structures, the normalized Young's modulus and yield strength predicted by the PR models strongly correlated with those of the voxel-based models ($R^2 \ge 0.99$) at any voxel size: 20, 40, and $80 \, \mu m$ (see Fig. [3](#page-2-0)). At each voxel size, the correlation was calculated between the PR and voxel-based models with four different trabecular thicknesses. Both the PR and voxel-based models for the plate-rod structure were highly robust to the resolution of images and the thickness of the trabeculae and, furthermore, provided nearly identical predictions of Young's modulus and the yield strength at any voxel size for a trabecular network of any

Fig. 2 Stress–strain curves under a uniaxial compression test of the idealized (a) plate-rod structure, and (b) rod-rod structure with varying trabecular thicknesses: 80, 160, 240, and 320 μ m (voxel size: 20 μ m)

Fig. 3 Linear correlation of the normalized Young's modulus E^*/E_s between the PR and voxel models for the idealized (a) plate-rod structure, and (b) rod-rod structure; correlation of normalized yield strength σ_y */E_s between the PR and voxel models for the idealized (c) plate-rod structure, and (d) rod-rod structure

Table 1 Comparison of axial mechanical property of the PR model and the voxel model for the idealized plate-rod and rod-rod microstructure

thickness (see Table 1). Such a consensus remained but weakened in the rod-rod structure, with the PR model predictions generally lower in magnitude than those of the voxel-based models. In addition, the PR models achieved an \sim 762-fold major reduction in the largest model size from \sim 167,680 elements in a voxel-based model to \sim 220 elements in a PR model, which markedly increased the computational efficiency of the μ FE analysis.

Discussion

In this study, a simplified PR model of human trabecular bone was quantified in nonlinear FE analysis for the determination of the elastic and yield properties of trabecular bone. The accuracy of the PR model was examined on idealized plate-rod and rod-rod trabecular microstructures, which demonstrated its excellent predictive power with reference to corresponding voxel-based models. The wide applicability of novel PR models was evident from its superior accuracy at both μ CT resolution and HR-pQCT resolution, over a broad range of trabecular thickness. Human trabecular bone is mostly either plate-predominant or a combination of plates and rods at the femur neck, proximal tibia, and vertebral body [[8](#page-4-0)], where osteoporotic fractures occur most frequently. While the rod-predominant structure exists in extremely osteoporotic trabecular bone, the trabecular thickness is approximately

100 μ m on average at the distal tibia and radius [27], in which case the PR model provided an accurate estimation of the Young's modulus and yield strength, similar to those for plate-rod structures. The deviation of the PR model predictions from the voxel model in the thick rod-rod structure was partly caused by the limitation of the beam theory, such as the shear deflection effects at a low beam length-to-thickness ratio [28]. Although we have examined the elastic-plastic behaviors of the trabecular bone model, this approach could be applicable to other nonlinear phenomena such as viscoelastic simulations.

Moreover, a tremendous reduction in the element number and CPU time was achieved by the conversion from voxel models to PR models. Nonlinear FE analysis for a $5.6 \times 5.6 \times 5.6$ mm³ trabecular bone cube can be done within 14 s by a PC using the HR-pQCT-based PR model, which requires 5h using the HRpQCT-based voxel model and 246h using μ CT-based voxel model (unpublished work). There have been other groups trying to assess the mechanical properties of trabecular bone using beam-shell or beam FE models [9,10]. They have demonstrated the excellent predictive power of the beam-shell model in estimating the elastic modulus by linear FE analysis with an average 33 fold reduction in CPU time. Our ITS-based PR modeling technique has multiple advantages compared to their approach in terms of higher efficiency, an average 900-fold reduction in CPU time for a nonlinear FE analysis, accurate prediction of both the elastic modulus and yield strength by nonlinear FE analysis, and similarly excellent performance on clinical HR-pQCT images.

There are several limitations in this newly developed technique. First, the low resolution of the source images, e.g., an $82 \mu m$ voxel size in the HR-pQCT scans, remains a major challenge for the PR models to maintain predictive accuracy despite the loss of structural details. Second, the moduli and yield strengths predicted by the PR models were generally underestimated to some degree, which possibly resulted from the neglect of the thickening effect near trabecular joints that positively contributes to the mechanical competence of trabecular bone [29]. Lastly, given the altered details in the microstructure of trabecular bone by the PR assumption, this approach may not be applicable for microcrack development of damage accumulation.

This technical brief provides fundamental evidence for the accuracy of PR μ FE models based on idealized trabecular microstructure. In future studies, the PR model will be tested on μ CT and HR-pQCT images and validated against experimental measurements and gold standard μ CT-based voxel μ FE models. With accuracy combined with efficiency, the specimen-specific PR model shows great promise as a powerful tool for the future assessment of trabecular bone failure, damage accumulation, and fracture risks.

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