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Using Kindergarten Number Sense to Predict Calculation Fluency in Second Grade

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Abstract

Children's number sense in kindergarten was used to predict their calculation fluency in second grade (*N* = 198). Using block entry regression, usual predictors of age, reading, memory, and verbal and spatial cognition were entered in the first block and number sense measures were added in the second block. Number sense measures contributed a significant amount of variance over and above the more general predictors (26%–42%). Uniquely predictive subareas were active memory for numbers, number knowledge, and number combinations, with number combinations standing out as the strongest single predictor. Number sense screening in kindergarten, using "at-risk" versus "not-at-risk" criteria, successfully ruled out 84% of the children who did not go on to have calculation fluency difficulties and positively identified 52% of the children who later showed fluency difficulties. The relation of early number skills to later calculation fluency has important implications for math screening and intervention.

Keywords

early identification/intervention; mathematics; mathematics computation

Fluency refers to the ease and accuracy with which a skill is carried out. Fluency in basic calculation is an important tool for solving most math problems. Performing operations with multidigit numbers—whole or rationale, positive or negative—depends on fluent knowledge of number combinations. Weak consolidation of number facts reduces cognitive and attentional resources that are necessary for higher-level problem solving (Goldman & Pellegrino, 1987; Hasselbring, Goin, & Bransford, 1988). Poor fact retrieval creates particular challenges for instruction in algebraic reasoning (Gersten, Jordan, & Flojo, 2005).

Dysfluent calculation is a distinguishing characteristic of children with math difficulties (MD). In a series of studies, Jordan and colleagues (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2003; Jordan, Hanich, & Kaplan, 2003a; Jordan & Montani, 1997) found that children with MD, regardless of whether the math difficulties are specific (MD-only) or accompanied by reading difficulties (MD/RD), perform below children with normal math achievement on timed calculation tasks. Similarly, Barnes et al. (2006) reported that children with MD had deficits in speed and accuracy in single-digit addition, independent of reading status. In contrast, children with MD only had an advantage over children with MD/RD on untimed arithmetic word problems, which depend on language (Jordan et al., 2003a). Children with MD who are competent readers appear to use their verbal strengths to compensate for their weaknesses in number. In fact, children identified with MD only in

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second grade achieved at a faster rate in math between second and third grades than did children with "double-deficit" MD/RD, even when IQ was considered in analyses (Jordan, Kaplan, & Hanich, 2002).

Jordan, Hanich, and Kaplan (2003b) identified third graders with good and poor mastery of number combinations in addition and subtraction. Using longitudinal data, they were able to look at development of numerical and cognitive competencies across second and third grades. Children with poor fact mastery showed remarkably flat growth on timed fact-retrieval tasks. Moreover, on untimed tasks, they consistently relied on their fingers for calculation support. In contrast, children with good fact mastery showed incremental growth in fact retrieval, along with a gradual decrease in finger counting. The transition from physical (counting-based) to mental (memory-based) representation is important for developing fact fluency (Geary & Hoard, 2005; Gersten et al., 2005). Jordan et al.'s (2003b) mastery groups performed at the same level in reading decoding and verbal ability, suggesting that mastery of facts is relatively independent of reading and language facility. Instead, development of fluency on number combinations seems to be constrained by weaknesses in accessing, comparing, and mentally manipulating number representations. Landerl, Bevan, and Butterworth (2004) define developmental dyscalculia, a clinically diagnosed severe calculation disability, as having a "highly selective" deficit in representing or processing numerical information.

Underpinnings of Calculation Fluency

Number Sense

It has been suggested that poorly consolidated number sense contributes to calculation deficits and MD (Gersten et al., 2005; Mazzocco & Thompson, 2005). Most broadly, number sense refers to understanding of numbers and number relationships (Malofeeva, Day, Saco, Young, & Ciancio, 2004). Operational definitions include the ability to subitize small quantities, to compare numerical magnitudes, to count, and to perform simple arithmetic calculations (Berch, 2005). Based on these principles, Jordan, Kaplan, Olah, and Locuniak (2006) developed a number sense battery to screen kindergartners at risk for learning difficulties in math. The "core" battery (Jordan, Kaplan, Locuniak, & Ramineni, 2007) included measures of counting, number knowledge (e.g., numerical magnitude comparisons), nonverbal calculations, story problems, and number combinations. Jordan et al. (2006) found that high-risk children from low-income families had weak number sense in general, with particular difficulties on story problems. Low-income children were overrepresented in an empirically defined group of children with low performance and flat growth in number sense over four time points during kindergarten. Moreover, number sense at the beginning as well as the end of kindergarten was highly correlated with first-grade math achievement (Jordan et al., 2007). Although calculation skill is a component of math achievement, most tests are untimed and it is not clear how number sense relates in particular to fluency or speed—a marker for MD in second grade and beyond—and whether some components of number sense are more predictive of calculation fluency than others. Knowledge of counting schemas, awareness of numerical magnitudes, and understanding of addition and subtraction operations all should help children internalize and master basic combinations (Baroody, 1985). We suspected that basic skills related to counting, number knowledge, and simple arithmetic would be more predictive of calculation fluency than more general cognitive competencies (Mazzocco & Thompson, 2005).

Memory Span

Simple arithmetic involves general memory processes, although strong number sense (e.g., counting knowledge) fortifies representations of basic facts in long-term memory (Geary &

Hoard, 2005; Jordan, Levine, & Huttenlocher, 1994). Of course, application of counting procedures during calculation relies on working or active memory (e.g., on the combination, 7 + 9, a counting-on strategy requires the child to hold the larger number 9 in memory while counting 7 more to make 16; LeFevre, DeStefano, Coleman, & Shanahan, 2005). Working memory (as measured, for example, by a backward digit span test, which requires individuals to hold a digit string in short-term memory and then to repeat the string in reverse order) has a strong, positive correlation with number knowledge (Chard et al., 2005). Children with MD tend to have weaker working memory capacity than children with normal math achievement (Geary, Brown, & Samaranayake, 1991; Koontz & Berch, 1996; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001). As children master number combinations, the demand for working memory involved in strategic counting and problem solving is reduced (Geary & Hoard, 2005).

Reading/Language

It has been argued that number facts are stored in verbal form and that word reading and calculation deficits may result from a core weakness in representing and accessing semantic information (Geary, 1993; Geary, Hamson, & Hoard, 2000; Robinson, Menchetti, & Torgesen, 2002). As noted previously in this article, however, studies of children with MD do not always support this assertion. Jordan et al. (2003a) found that children with RD-only perform as well as children with normal reading in fact retrieval. Nevertheless, verbal ability is a significant predictor of both reading and arithmetic skills (Durand, Hulme, Larkin, & Snowling, 2005). Although language-representation deficits may not be a primary source of fact-mastery weaknesses, they often coexist with MD and are likely to aggravate number-skill development on language-dependent tasks (Jordan et al., 2002).

Spatial Ability

Geary and Hoard (2005) suggest that children with MD have difficulties interpreting spatial information. Although general spatial skills appear important for written calculation and understanding the base-10 system (Geary, 1993), it is not obvious how spatial skill relates to the development of calculation fluency in particular (Butterworth, 2005). Jordan et al. (2003b) report that children with fact-mastery deficits perform worse in nonverbal IQ than children with good fact mastery, but not on verbal IQ. Spatial facility might underlie representation of numbers and relations between numbers (Dehaene & Cohen, 1997; Geary & Hoard, 2005), which in turn could affect the development of procedures that lead to mastery of number combinations.

The Present Study

The goal of the present study was to examine number-specific and more general predictors of computational fluency in second grade, using multiple regression procedures. Our regression model included number sense in kindergarten as well as skill on related tasks assessing reading, oral language, memory, and spatial skill. We also considered children's age, because of the relatively wide age variation at kindergarten entry. Number sense in the spring of kindergarten was used because some children start kindergarten with delayed number sense but catch up by the middle of the school year (Jordan et al., 2006). Two memory measures allowed us to compare the relative contributions of simple short-term memory (forward number recall) versus working memory (backward number recall; Schofield & Ashman, 1986). It has been suggested that forward recall of digits strings demands auditory attention whereas backward recall invokes central executive and visual–spatial systems to perform a transformation (Reynolds, 1997).

We believed that number sense in kindergarten would be a strong predictor of later calculation fluency, even when related predictor variables were considered in our regression model. However, we also expected working memory to be a key cognitive predictor because it facilitates strategy application. The present study, a longitudinal extension of the work by our research team (Jordan et al., 2006; Jordan et al., 2007), uses validated number sense measures that are closely aligned with the National Council for Teachers of Mathematics (2006) kindergarten focal points. The varied number sense tasks (i.e., counting, number knowledge, nonverbal calculation, story problems, and number combinations) allowed us to look at the relative importance of different but related quantitative skills (as well as overall number sense) to calculation fluency. We expected the findings to have potential significance for the screening of MD and for providing directions for intervention.

Method

Participants

The 198 participants were originally recruited in kindergarten from six schools in a school district in Northern Delaware. All of the schools used the same math curriculum (Teaching Integrated Math and Science Curriculum, 2004). Descriptive information for the children is shown in Table 1.

Procedure

Children's number sense and early reading skills were assessed in the spring of kindergarten (2004). Cognitive measures were given in the winter of first grade (2005) and the calculation fluency measure in the winter of second grade (2006). Female undergraduate or graduate students who were trained fully in the testing procedures did all assessments with individual children. A bilingual tester who was fluent both in Spanish and in English administered the assessments to English language learners (n = 13). Although all assessments were administered in English, on the number sense battery English language learners were permitted to ask that directions be read in Spanish and/or to respond in Spanish.

Number Sense—The number sense battery (Jordan et al., 2006) assessed counting, number knowledge, nonverbal calculation, story problems, and number combinations. None of the tasks was timed. There were 50 items in total with a coefficient alpha of .90.

Counting: “Counting” measured children's abilities to enumerate sets, to count, to identify violations of counting principles, and to name numbers.

- *Enumeration.* Children were shown a series of five stars on a piece of paper and were asked to count them. They were given 1 point if they did this successfully. The children were then asked to state how many stars were on the paper. The children were given another point if they did this successfully. The same procedure was followed with seven stars.
- *Count sequence.* Children were asked to count as high as they could but were stopped if they reached 50. They were given 1 point if they could count to 10 and 1 point if they could count to 50. Self-correcting was allowed.
- *Counting principles.* Adapted from Geary, Hoard, and Hamson (1999), each child was presented eight sets of alternating yellow and blue dots, one at a time. The sets consisted of either five or nine dots. The children were shown a finger puppet and told that the puppet was learning to count. They were instructed to tell the puppet whether he counted “OK” or “not OK” for each of the eight trials. There were two clearly correct; four “unusual,” but correct; and two incorrect trials. In the clearly

correct trials, the puppet counted all the dots in the conventional left to right order. In the unusual but correct trials, the puppet counted the blue dots first followed by the yellow dots, the yellow dots first followed by the blue dots, and from right to left. The incorrect trials involved counting from left to right with the first dot being double counted.

- *Number recognition.* Children were asked to name the following numbers, which were shown individually: 2, 8, 9, and 13.

The number of possible points on counting was 18.

Number knowledge: Adapted from Griffin (2002), children were first shown a number (i.e., 7) and asked what number comes after that number and what number comes two numbers after that number. Given two numbers, children were asked to identify which number was bigger (i.e., 5 or 4; 7 or 9) or which number was smaller (i.e., 8 or 6; 5 or 7). Shown three numbers, each placed in a corner of an equilateral triangle, children were asked to identify which number was closest to the target number that was placed at the top of the triangle (i.e., **5**: 6 or 2; **7**: 4 or 9). The number of possible points on number knowledge was 8.

Nonverbal calculation: The tester and child sat facing each other with 45 cm × 30 cm mats in front of each of them and a box of 20 chips placed off to the side. Four addition and four subtraction calculations were presented: 2 + 1; 4 + 3; 2 + 4; 3 + 2; 3 − 1; 7 − 3; 5 − 2; 6 − 4. The tester placed chips on her mat (in a horizontal line), told the child how many chips were on the mat, and then covered the chips with the box lid. Chips were added or removed (through the side opening) one at a time. For each trial, the children were to indicate how many chips were left "hiding" under the box either verbally or by placing the appropriate number of chips on the mat. Children were corrected for the first addition and subtraction problem if their answer was incorrect. To avoid confusion, addition problems were presented first. The number of possible points on nonverbal calculation was 8.

Story problems: Four addition and four subtraction story problems were presented orally, one at a time. The calculations were the same as those used on nonverbal calculation. The addition problems were phrased as "Jill has *m* pennies. Jim gives her *n* more pennies. How many pennies does Jill have now?" The subtraction problems were phrased as "Mark has *m* cookies. Colleen takes away *n* of his cookies. How many cookies does Mark have now?" The number of possible points on story problems was 8.

Number combinations: Four addition and four subtraction number combinations were presented orally, one at a time. The calculations were the same as those used on nonverbal calculation. The items were phrased as "How much is *m* and *n*?" and "How much is *m* take away *n*?" The number of possible points on number combinations was 8.

Reading—The sixth edition of the *Dynamic Indicators of Basic Early Literacy Skills* (DIBELS; Good & Kaminski, 2002) included measures of letter-naming fluency, phoneme-segmentation fluency, and nonsense-word fluency. The raw score for each measure was the number of letters, phonemes, and nonsense words identified in 1 minute. Scores from the three related measures were totaled for each child and used for the analysis. Average test–retest reliability for the end of kindergarten was .91 (Good, Simmons, Kame'enui, Kaminski, & Wallin, 2002).

Cognitive Measures—Memory span was assessed with the fourth edition of the *Wechsler Intelligence Scale for Children* (WISC-IV) Digit Span subtest (Wechsler, 2003). The measure requires children to repeat digits verbatim (digit span forward) and to repeat the digits in reverse sequence (digit span backward). Digit span forward is a measure of short-term recall whereas digit span backward adds an active memory component. Thus we analyzed the subareas separately (Reynolds, 1997). Internal reliability at age 7 is .79 for digit span forward and .69 for digit span backward.

Children also were given the *Wechsler Abbreviated Scale of Intelligence* (WASI; Wechsler, 1999), which involves oral Vocabulary and nonverbal Matrix Reasoning subtests. Internal reliability at age 7 was .86 for Vocabulary and .94 for Matrix Reasoning. The Vocabulary subtest is highly correlated with overall verbal IQ (.93) and Matrices with overall performance IQ (.87).

Calculation Fluency—Calculation fluency was measured with the “Assessment of Math Fact Fluency” (Fuchs, Hamlett, & Powell, 2003). Children were presented first with an addition fluency measure in which 25 problems were presented horizontally on a page (sums up to 18). They had 1 minute to answer as many problems as they could with a pencil. Children were next presented with a comparable subtraction test (minuends up to 18). The calculation fluency score was the total number of correct addition and subtraction problems. The coefficient alpha for calculation fluency in third grade is .92.

Results

Raw scores were used for all analyses. The mean raw scores and standard deviation for all tasks are presented in Table 2. Simple correlations among the number sense, reading, cognitive, and calculation fluency measures are presented in Table 3. As expected, all of the correlations were positive and significant. The three kindergarten addition/subtraction tasks had the strongest relation to second-grade calculation fluency: number combinations (r = .57), story problems (r = .51), and nonverbal calculation (r = .51).

Block entry regression analysis allowed us to predict calculation fluency in second grade, taking into account general and number sense variables separately. The results are presented in Table 4. General predictors of age, reading skill, vocabulary, matrix reasoning, digit span forward, and digit span backward were entered in the first block (Model 1). Number sense measures (i.e., counting, number knowledge, nonverbal calculation, story problems, and number combinations) were added in the second block (Model 2) to see if they made a unique contribution to children's calculation fluency, over and above the general predictors in the first block. The effect of number sense on calculation fluency can be evaluated through change in total variance from Model 1 to Model 2.

Model 1 (age, reading, and general cognitive measures) accounted for 26% of the variance in calculation fluency in second grade, with vocabulary, matrix reasoning, and digit span backward reaching significance. Model 2 (included variables in Model 1 with the addition of the number sense tasks) accounted for 42% of the variance in predicting performance on calculation fluency. With related cognitive variables in the model, number sense was a significant and unique predictor of calculation fluency ($p < .01$).

In Model 2, digit span backward, number knowledge, and number combinations made unique contributions. For every 1-point increase in digit span backward, there was a .70 increase in calculation fluency. For every 1-point increase in number knowledge, there was a .86 increase in calculation fluency, and for number combinations there was a .76 increase in calculation fluency. The effects of vocabulary and matrix reasoning, which were significant in Model 1, were not significant in Model 2.

Hit-Rate Analysis

To provide practical perspective on our results, we compared children's screening status (at risk, not at risk) to their outcome status (fluency difficulty, no difficulty; Ritchey & Speece, 2004). For screening status we used number knowledge and number combinations—the uniquely significant number predictors in Model 2 of the regression analysis. Although working memory also was predictive, this measure is more general and thus has less relevance for teaching math. “At risk” was defined as performing at or below the 25th percentile on the two kindergarten number measures. Likewise, “difficulty” was at or below the 25th percentile on calculation fluency in second grade. The 25th percentile cutoff is considered an appropriate cutoff criterion for identifying children with learning difficulties (Fletcher et al., 1994; Swanson & Beebe-Frankenberger, 2004).

The classification results are summarized in Table 5. Of the 150 children who were not identified by our number sense predictors as being at risk, 126 also did not meet our calculation difficulties criterion in second grade, with a true negative rate of 84%. Of the 48 children who were positively identified by the number sense predictors, 25 went on to have fluency difficulties in second grade with a true positive rate of 52%. Although the 48% false-positive rate was high, it is noteworthy that half of these children performed between the 25th and 50th percentile in second-grade fluency.

Discussion

As expected, all of the areas assessed in the present study were positively correlated with each other. However, our regression analysis showed that kindergarten number sense predicted calculation fluency over and above general predictors of age, reading, oral vocabulary, memory, and spatial reasoning. Number combinations and number knowledge along with working memory were uniquely predictive, with number combinations accounting for most of the unique variance. Basically, children who had a better grasp of basic addition and subtraction in kindergarten were more likely to achieve better fluency by second grade. This finding is in keeping with Mazzocco and Thompson (2005), who found that kindergarten addition is predictive of math learning disabilities in second and third grades.

In our initial model—that was concerned with general variables—language (oral vocabulary), spatial skill (matrix reasoning), and working memory (digit span backward) were significant predictors of calculation fluency. When we added number sense in the subsequent model, however, the significant contributions of these variables were suppressed, with the notable exception of digit span backward. Forward digit span, a measure of passive short-term recall for number sequences, was not a significant predictor in either model. Although Swanson and Beebe-Frankenberger (2004) found that digit span forward and digit span backward loaded on a short-term memory factor rather than a working memory factor (that included a task that required subjects to remember numerical information embedded in sentences), there nevertheless seems to be some kind of control process involved in digit span backward that is associated with calculation fluency. As noted earlier, active sequential memory is necessary for carrying out counting procedures, such as counting on from the larger addend to solve an addition problem, which in turn supports and facilitates fact acquisition (Geary, 1994; Siegler, 1986). Although our data suggest that working memory is more important for developing calculation fluency than general attention, Fuchs et al. (2005) found that distractibility, as rated by classroom teachers, accounted for unique variance in predicting first-grade calculation fluency, in addition to working memory. However, it is difficult to determine what the relevant cognitive underpinnings are in the relatively broad construct of distractibility, particularly as judged by teachers.

Number knowledge, that involves magnitude judgments, was uniquely important for developing calculation fluency, but counting was not. Recall that our counting task required children to judge correct and incorrect counts made by a puppet. Using a similar task, LeFevre et al. (2006) found that kindergarten children with higher skill in math were less willing to accept unusual but correct counts (e.g., counting alternating colors) than lower skilled children. LeFevre et al. (2006) suggest that as young children acquire knowledge of counting, inessential features are first incorporated into their criteria for judging correctness of counts, but with experience and formal schooling they abandon these features. The ability to use counting strategically for comparing and manipulating sets (as reflected by performance on our number knowledge and number combinations tasks) may be more important for predicting fluency than simple counting tasks. It has shown, for example, that strategic counting (with fingers) on number combinations in kindergarten is highly related to accuracy (Jordan et al., 1994).

The unique influence of early knowledge of numbers and number combinations, as well as working memory facility, in predicting calculation fluency supports Baroody's (1985) assertion that fact mastery is not simply a process of associative memory for facts, one that is autonomous from fundamental number concepts and procedures. If the associative memory model were true, we would not expect number combinations in kindergarten—before formal instruction—to be very predictive. To the contrary, even at the beginning of kindergarten, accuracy on number combinations is predictive of first-grade math achievement (Jordan et al., 2007). Early in a child's life, a basic combination, such as 3 + 4, may be understood as a complex and potentially fascinating problem to be solved (Gersten et al., 2005; Siegler & Shrager, 1984), and children with better knowledge of numbers and numerical relationships have an advantage. Only with repeated experiences do combinations become internalized as routine facts. This perspective is referred to as the number sense view of expertise with addition and subtraction combinations, as opposed to a passive storage view (Baroody & Rosu, 2006; Gersten & Chard, 1999).

Early screening of target skills and knowledge has led to the development of effective, evidence-based interventions in reading (Gersten et al., 2005). As a result, kindergarten measures are widely used to screen students with potential reading problems. Research in early identification of math difficulties has lagged behind reading and few screening tools are available to educators. Our data suggest that screening for number sense in kindergarten would successfully rule out most of the children who are not at risk for fluency difficulties. Although some children (about 16%) slipped through the cracks, it is not clear whether this was because of error in our measure or because of outside factors that are hard to control (e.g., school absences, unusual life experiences, inadequate instruction, etc.). Among the children positively identified, a little more than a half went on to develop significant fluency problems. The high false-positive rate (about 48%) is of concern, however, and warrants further investigation. Some children who showed lags in kindergarten appeared to grow out of their problems, most likely through formal instruction in first and second grades. Still others showed relatively low fluency performance (between 25th and 50th percentiles), even though they were no longer classified as having “difficulties” by our cutoff criterion (<25%). Hit-rate accuracy might increase if risk variables (e.g., income level) along with teacher recommendations were considered. If a child performs poorly on the number sense measure but is doing well on other kindergarten indicators, educators might take a “watch and wait” approach to intervention.

Although fluency with basic operations is not sufficient for learning high-level mathematics, it is a necessary sub-skill. This “necessary but not sufficient” relationship is analogous to the connection between decoding fluency and comprehension in reading. Deficient calculation fluency is a defining characteristic of math difficulties and disabilities. The present findings,

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although based on a sample from only one school district with a 25% low-income and 42% minority population, should help shape the goals of early math intervention in kindergarten. Knowledge of the relative position and magnitude of numbers along with the ability to manipulate quantities through addition and subtraction are key kindergarten accomplishments (National Council of Teachers of Mathematics, 2006) and predict later fluency. Although we used a number sense measure at the end of kindergarten in the present study, prior work suggests that it can be reliably assessed by midyear (Jordan et al., 2006, 2007). Careful intervention research should reveal the extent to which early instruction in number sense relates to learning formal math and prevents or reduces fluency difficulties in particular.

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Biography

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Table 1

Demographic Information for Participants (*N* = 198)

Characteristic	Summary
Income	
Low income	49 (25%)
Middle income	149 (75%)
Gender	
Male	111 (56%)
Female	87 (44%)
Race	
Minority[a]	84 (42%)
Nonminority	114 (58%)
Mean kindergarten start age (*SD*)	66 months (4 months)
Mean IQ standard score (*SD*)	97.42 (13.21)

[a]Minority refers to Black (58%), Asian (13%), or Hispanic (29%).

Table 2

Mean Raw Scores on the Number Sense, Reading, Cognitive, and Fluency Tasks

Task	Mean Score (*SD*)
Number sense tasks (points)	
Counting (18)	15.20 (1.77)
Number knowledge (8)	6.44 (1.48)
Nonverbal calculation (8)	5.38 (1.78)
Story problems (8)	3.48 (2.36)
Number combinations (8)	4.20 (2.72)
Reading	100.43 (43.74)
Cognitive tasks	
Vocabulary	22.44 (6.54)
Matrix reasoning	10.87 (5.83)
Digit span forward	6.57 (1.69)
Digit span backward	4.85 (1.51)
Mean calculation fluency score	14.78 (7.46)

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Table 3

Correlations Among the General and Number Sense Measures

Measure	Reading	Vocabulary	Matrix Reasoning	Digit Span Forward	Digit Span Backward	Counting	Number Knowledge	Nonverbal Calculations	Story Problems	Number Combinations	Calculation Fluency
Reading	1	0.41	0.36	0.37	0.23	0.29	0.29	0.32	0.42	0.39	0.33
Vocabulary	0.41	1	0.32	0.44	0.28	0.28	0.38	0.35	0.46	0.44	0.35
Matrix reasoning	0.36	0.32	1	0.30	0.28	0.23	0.34	0.47	0.44	0.46	0.37
Digit span forward	0.37	0.44	0.30	1	0.31	0.24	0.27	0.31	0.41	0.34	0.29
Digit span backward	0.23	0.28	0.28	0.31	1	0.29	0.22	0.37	0.31	0.29	0.35
Counting	0.29	0.28	0.23	0.24	0.29	1	0.36	0.28	0.28	0.43	0.30
Number knowledge	0.29	0.38	0.34	0.27	0.22	0.36	1	0.43	0.46	0.49	0.45
Nonverbal calculations	0.32	0.35	0.47	0.31	0.37	0.28	0.43	1	0.61	0.61	0.51
Story problems	0.42	0.46	0.44	0.41	0.31	0.28	0.46	0.61	1	0.73	0.51
Number combinations	0.39	0.44	0.46	0.34	0.29	0.43	0.49	0.61	0.73	1	0.57
Calculation fluency	0.33	0.35	0.37	0.29	0.35	0.30	0.45	0.51	0.51	0.57	1

Note: All the correlations are significant, $p < .01$.

Table 4

Results of Block Entry Regression: Regression Coefficients and Variance Explained by Each Block of Variables

Coefficient	*B*	β	*t* Value	*p* Value
Model 1				
Age	–0.01	–0.01	–0.11	0.91
Reading	0.02	0.13	1.73	0.09
Vocabulary	0.17	0.15	2.04	0.04
Matrix reasoning	0.27	0.21	2.95	0.00
Digit span forward	0.23	0.05	0.73	0.47
Digit span backward	1.02	0.21	3.02	0.00
Model 2				
Age	–0.05	–0.03	–0.45	0.65
Reading	0.01	0.06	0.83	0.41
Vocabulary	0.01	0.01	0.17	0.86
Matrix reasoning	0.05	0.04	0.54	0.59
Digit span forward	0.03	0.01	0.10	0.92
Digit span backward	0.70	0.14	2.22	0.03
Counting	0.01	0.00	0.03	0.98
Number knowledge	0.86	0.17	2.47	0.01
Nonverbal calculation	0.57	0.14	1.74	0.08
Story problems	0.19	0.06	0.65	0.51
Number combinations	0.76	0.28	3.00	0.00

Model	R^2	R^2 Change	*F* Change	*df* 1	*df* 2
1	0.26		11.07**	6	191
2	0.42	0.16	10.26**	5	186

** $p < .01$.

Table 5

Classification by Number Sense (Number Knowledge and Number Combinations) Risk Status and Calculation Fluency Outcome Status (*N* = 198)

Risk Status	Fluency Difficulty	Fluency No Difficulty
At-risk number sense	25	23
Not-at-risk number sense	24	126