PROFILE

Profile of Keith Moffatt

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Science Writer

Keith Moffatt was born in 1935 and raised in Edinburgh, Scotland. He was four years old when his father left home to serve in the Second World War. "At that age, you accept things," Moffatt recalls. "It was a time of great shortage. Everything in the UK was rationed—food, clothes, fuel, even sweets!"

The elder Moffatt, an accountant, had introduced his son to mathematical games and arithmetic puzzles, and, when he returned at the war's end in 1945, recreational mathematics recommenced.

The early tutelage had a lasting effect: Moffatt's facility with numbers would transform into an aptitude for applied mathematics and its use in fluid mechanics and astrophysics. His contributions to our understanding of magnetic and spin fields as well as his advocacy for the study of mathematics around the world have earned him fellowships in the Royal Societies of both London and Edinburgh. A 2005 recipient of the Royal Society of London's Hughes Medal, he serves as an emeritus professor of mathematical physics at the University of Cambridge. He was elected as a foreign associate of the National Academy of Sciences in 2008.

Magnetic Fields in a Turbulent Medium

Moffatt completed his undergraduate studies in mathematics at the University of Edinburgh. He left Scotland for Trinity College, Cambridge, in 1957, where he came under the influence of George Batchelor, an Australian fluid dynamicist. Batchelor set Moffatt to work on his first research project and played a decisive role in his career. "George was very sympathetic to students who came from outside Cambridge, as it had been his own career path," Moffatt explains.

In 1959 Batchelor founded the Department of Applied Mathematics and Theoretical Physics at Cambridge—a department that counts in its ranks some of the world's most influential fluid dynamicists and theoretical physicists. He gave Moffatt a prepublication draft of his paper on how turbulence interacts with temperature or a contaminant that does not influence the turbulence (1).

Two years later, Moffatt's first paper in 1961 extended Batchelor's ideas to the interaction of turbulence with a weak magnetic field and obtained the spectrum of the magnetic fluctuations (2). The result would be verified experimentally nearly three decades later by French researchers, using liquid gallium and an applied magnetic field (3).

"The application that I had most in mind at that time," Moffatt notes, "was in astrophysics." In the 1950s, dynamo theory, which is the study of how the earth, stars, and other heavenly bodies generate and maintain magnetic fields, was beset by conflicting ideas. Furthermore, how these magnetic fields interacted with the interstellar medium and its turbulent ionized gases was poorly understood.

Knotted Vortices

Toward the end of the 1960s, while teaching a course on magnetohydrodynamics, Moffatt identified an integral characteristic of fluid flow that he named "helicity" (4), building on the observations of two scientists working nearly a century apart.

"In 1869 Lord Kelvin recognized the frozen property of vortex lines in fluid flows, and in 1958 Lodewijk Woltjer, an astrophysicist then at the University of Chicago, demonstrated the invariance of a puzzling integral quantity in the flow of a perfectly conducting or Euler fluid. In struggling to find a physical interpretation of Woltjer's invariant, it dawned on me that there must be an analogous result for the nonlinear Euler equations."

Helicity, as Moffatt explains, represents the degree of linkage or entanglement in a field of vorticity, which can be thought of as the spin associated with a velocity field. Helicity is an invariant of the Euler equations, which govern ideal, nonviscous flows. This concept established a bridge between classical fluid mechanics and topology, the study of shapes and their continuous deformation.

Moffatt gives credit for the finding to Jean-Jacques Moreau, now an emeritus professor in Montpellier, France, who had found the same invariant several years before his publication (5). "As so often happens in our subject," Moffatt explains, "someone else proved the concept in a paper that was completely buried. Moreau didn't call it helicity, but the result is there. He and I discovered it quite independently."

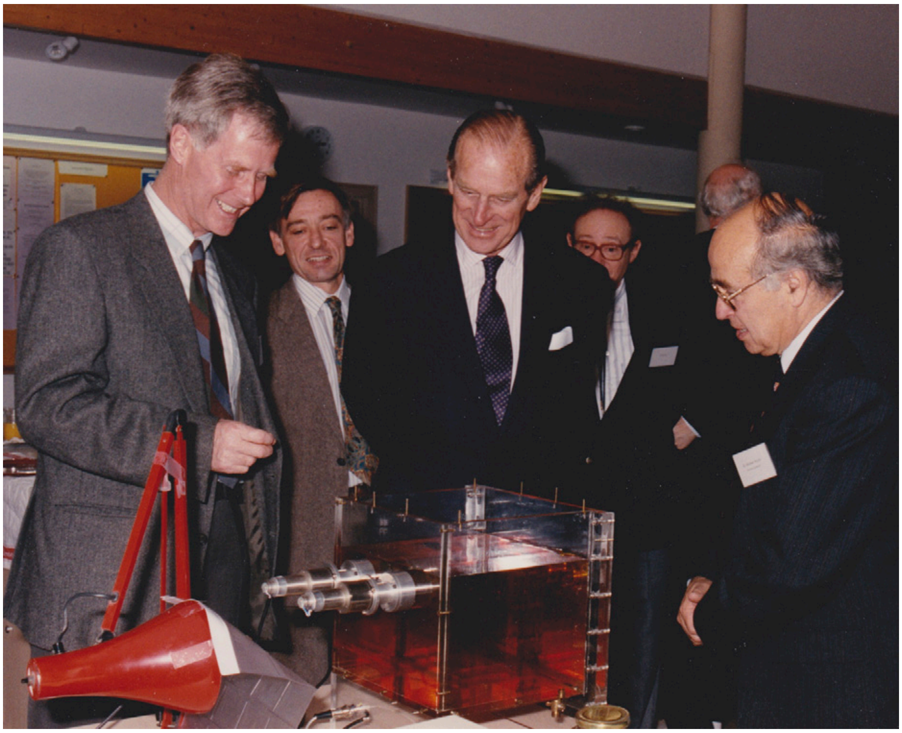
Keith Moffatt. Photo by Jill Paton-Walsh.

Dynamos

Helicity had immediate applications in dynamo theory, and especially for the explanation of the earth's magnetic field. About halfway to the center of the earth, the mantle gives way to a liquid metal core, "a conducting fluid in random motion superposed on differential rotation," Moffatt explains. This combination generates a magnetic field. He showed that dynamo action occurs even in a weakly conducting fluid, provided the fluid domain is large enough and the turbulence has an average nonzero helicity (6).

While on a sabbatical at the Université Pierre et Marie Curie in Paris, Moffatt wrote the first monograph on dynamo theory, incorporating helicity and its implications for planetary and astrophysical dynamos (7). "By the late 1970s," he notes, "the basic principles were well established, so the time was ripe for such a book, which is still a standard introductory text." The publication led to an explosion of activity in the field of dynamo theory, with three other books on the subject produced across the world within the next five years. A massive computational effort throughout the 1980s and 1990s followed. Moffatt is currently working on an updated version of the monograph.

This is a Profile of a recently elected member of the National Academy of Sciences to accompany the member's Inaugural Article on page 3663.



His Royal Highness Prince Philip visits the Newton Institute, October 1992. Keith Moffatt (*Left*) demonstrates the production of a cusp singularity at a free surface. Sir Michael Atiyah, Founder Director of the Institute (*Right*). Photo courtesy of MM Photographic.

Nearly thirty years elapsed before a major experiment carried out at the International Thermonuclear Experimental Reactor site in Cadarache, France confirmed the basic ideas in Moffatt's book (8). In the experiment, a turbulent flow of liquid sodium driven by counter-rotating propellers produced dynamo action (9). More recently, physicists at the University of Chicago succeeded in creating knotted vortices in water using carefully fabricated hydrofoils (10). In his Inaugural Article, Moffatt recaps his investigations into helicity and dynamo generation. He remarks that "it's a good moment to talk about this work when theory meets experiment!" (11).

Applying Mathematics

After three years as professor of applied mathematics at the University of Bristol, Moffatt returned in 1980 to Cambridge, where he has remained ever since as a professor of mathematical physics and a fellow of Trinity College. There, he focused on developing the topological approach to fluid dynamics, leading to work on magnetic relaxation, which has implications for thermonuclear fusion devices. Similar research describes the evolution of the magnetic field in the solar corona (12, 13).

These efforts led Moffatt to consider what happens when a knotted magnetic flux tube relaxes to a minimum energy configuration while conserving its knot topology—a problem closely related to the pure mathematical question: What is the smallest length of rope of a fixed diameter needed to tie a given knot? (14). "The theory of tight knots," Moffatt explains, "has been considerably developed with applications in polymer physics and molecular biology" (15). "The beauty of working in applied mathematics," he continues, "is that your results sometimes turn out to have application in fields far from your initial interests."

Newton Institute

In 1991 the British mathematician Sir Michael Atiyah and others founded the Isaac Newton Institute for Mathematical Sciences at Cambridge as a national visitor research center for the United Kingdom. Moffatt ran one of the first programs, on dynamo theory, and succeeded Atiyah as director of the institute for a five-year term beginning in 1996. Moffatt says that "the major problem for any director is fundraising, and this was a major preoccupation for me during these years."

While director, he became involved in the early planning of the African Institute for Mathematical Sciences (AIMS), which was eventually founded in 2003 by former Cambridge mathematical physicist Neil Turok. Located in a suburb of Cape Town, the institute trains graduate students from across the continent. "The concept has worked extremely well," notes Moffatt, "and has now expanded through the AIMS Next Einstein Initiative to three other locations across Africa in Senegal, Ghana, and Cameroon."

Mathematics of Toys and Soap Films

In recent years, Moffatt has worked with Japanese mathematician Yutaka Shimomura, Cambridge's Tadashi Tokieda, and others on mechanical toys that exhibit puzzling behaviors: the Euler disk, which has a perceived finite-time singularity (16), the spinning hard-boiled egg with a friction-driven instability (17), and the rattleback, with a chiral instability and asymmetric behavior (18). Moffatt says the real interest in these problems is that they illustrate behavior that appears in complex fluid situations like the helicity-driven dynamo, or in the friction-driven instability of pressure-driven flow in a 2D channel.

With Raymond Goldstein and Adriana Pesci at Cambridge, Moffatt worked on the problem of topological jumps in soap films that span twisted wire loops. The researchers formed a one-sided Mobius-strip soap film on a doubled-over loop of wire and found that it jumps to a two-sided surface in milliseconds at a critical moment when the wire is slowly untwisted (19). With the aid of high-speed photography, they described the detailed mechanism of this jump, and in the process opened up a new branch of topological fluid dynamics.

Topologists, Moffatt notes, have been studying minimal-surface area problems for more than a century, and "this new work provides welcome interaction between pure mathematics and fluid dynamics."

Musings

Although he still performs research, Moffatt retired from teaching in 2002. He recently built a personal Web site that chronicles his life and career in applied mathematics. The site details his early life in Edinburgh during the war, and catalogs major contributions, periods of sabbatical, and graduate students he has mentored. The Web site also serves as a repository for his poetry, much of it science themed. "I don't call myself a poet," he says, "but when occasion demands or when the Muse inspires, I don't hesitate to put pen to paper!"

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