



Numerical Simulation for the Unsteady MHD Flow and Heat Transfer of Couple Stress Fluid over a Rotating Disk

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Abstract

The present work is devoted to study the numerical simulation for unsteady MHD flow and heat transfer of a couple stress fluid over a rotating disk. A similarity transformation is employed to reduce the time dependent system of nonlinear partial differential equations (PDEs) to ordinary differential equations (ODEs). The Runge-Kutta method and shooting technique are employed for finding the numerical solution of the governing system. The influences of governing parameters viz. unsteadiness parameter, couple stress and various physical parameters on velocity, temperature and pressure profiles are analyzed graphically and discussed in detail.

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Introduction

The flow problems of non-Newtonian fluids are attracted the interest of many researchers because of its significance in modern technology and industries. The natural and industrial applications of such fluids are volcanic lava, extrusion of polymer fluids, drilling mud, suspension solutions, cosmetic and food products, solidification of liquid crystals, cooling of metallic plates in a bath, exotic lubricants, colloidal and many others. In recent times, among various non-Newtonian fluid models, the couple stress fluid model has got the special status because of the spin field in the fluid which sets up an anti-symmetric stresses, known as couple stresses. The basic theory and constitutive equations for couple stress fluid initially developed by Stokes [1], is one amongst the polar fluid theories which consider couple stresses in addition to the classical Cauchy stresses. This type of fluid encloses inflexible and random-oriented particles in a viscous medium. Examples of couple stress fluids are liquid crystals, colloidal fluids, liquids containing long-chain molecules such as polymer suspensions, blood, lubrications and additive electro-rheological fluids etc. These fluids are considered by many investigators because of the simplest model of couple stress fluid amongst others. (See recent attempts [2–4]).

Magnetohydrodynamics (MHD) occurs when a conducting fluid flows in the presence of an electromagnetic field, such that the induced electromagnetic force affects the motion of the fluid. Some significant technological applications of MHD are, in experiments of controlled thermonuclear fusion where a strong magnetic field is used to confine rings or columns of hot plasma, to generate electricity where liquid metals are driven through a magnetic field, etc. Moreover, MHD principles are used in plasma accelerators for ion thrusters, for spacecraft propulsion and for light-ion-beam powered inertial confinement. Some remarkable attempts have been made to study the influence of MHD on various flow situations. The influences of MHD stagnation point

flow was analyzed by Hayat et al. [5], MHD flow in a circular magnetic field was considered by Kumar et al. [6], viscous dissipation and Joule heating on unsteady mixed convection MHD flow was discussed by Osalusi et al. [7]. Investigation of fractional MHD Oldroyd-B fluid over an oscillating plate was the subject of Jamil et al. [8], and the influence of MHD in the porous medium and heat transfer analysis of couple stress fluid between two parallel plates with variable viscosity was investigated by Sreenadh et al. [9]. Numerical methods to obtain the solutions of some complicated MHD flow problems were presented in [10–12].

Rotating-disk flow of non-Newtonian fluids is intriguingly growing interest, as it is momentous in the study of engineering flows on rotating machineries, spin-coating, centrifugal pumps, rotational viscometers, computer storage devices and some aerodynamic related problems in fluid mechanics. Taking into account both industrial and technological applications, there have been a number of studies presented in literature on steady flow of various types of fluid over a rotating disks and sphere with and without considering magnetic effect [13–25]. It is worth mentioning that the real world applications encounter the unsteadiness in the flow and temperature fields of almost all processes. Osalusi et al. [26] presented the effects of Ohmic heating and viscous dissipation on unsteady MHD and slip flow over a porous rotating disk. The series solutions for unsteady laminar MHD flow near forward stagnation point of an impulsively rotating and translating sphere in presence of buoyancy forces are found by Dinarvand et al. [27]. Chamkha et al. [28] studied the unsteady MHD flow of mixed convection fluid with heat and mass transfer. Ram et al. [29] analyzed the influence of phase difference between highly oscillating magnetic field and magnetization on the unsteady ferro fluid flow due to a rotating disk. Nadeem et al. [30] worked on the analytical treatment of unsteady mixed convection MHD flow on a rotating cone in a rotating frame.

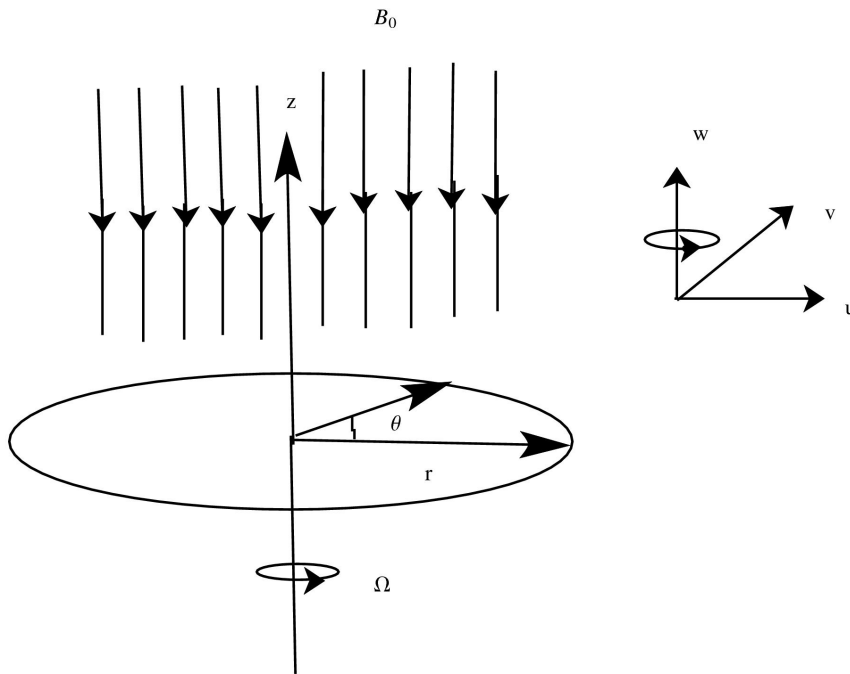


Figure 1. Coordinates system and flow configuration.
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Takhar et al. [31] examined the unsteady MHD flow of an ambient fluid with heat transfer over a rotating disk, present study aims at extending aforesaid work to incorporate the couple stress effects with magnetic field and heat transfer analysis is also taken into account. However, to the best of our knowledge, no attempt is available in the literature regarding unsteady MHD flow of couple stress fluid over a rotating disk. In this note, an investigation has been carried out to analyze the unsteady flow and heat transfer of couple stress fluid with the effect of transverse magnetic field on an infinite rotating disk along z -axis. The governing time dependent momentum and energy partial differential equations are transformed into a system of ordinary differential equations by using suitable similarity transformations. These ODEs are then solved numerically by employing a Runge-Kutta technique coupled with shooting method. The variations in the flow field, heat transfer and pressure of the fluid due to pertinent parameters are obtained through graphs and discussed thoroughly.

Problem Formulation

Consider the three dimensional, unsteady, laminar, incompressible, MHD flow and heat transfer of couple stress fluid over a rotating disk which rotates about the z -axis with angular velocity Ω in a cylindrical polar coordinate system r, θ and z . The r -axis is in the direction of radius of disk, and the applied uniform magnetic field B_0 is in the direction of z -axis, normal to the surface of disk. As the flow is axisymmetric therefore radial, tangential and axial components of velocity u, v, w , temperature T and pressure P are independent of θ . The induced magnetic field can be neglected in comparison with the applied magnetic field by considering the magnetic Reynolds number Re_m much less than 1. Fig. 1 represents the flow description and geometrical configuration of the physical model.

The governing partial differential equations are

$$\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial}{\partial z} (w) = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu (\nabla^2 u - \frac{u}{r^2}) \\ + \frac{\kappa}{\rho} \left(\frac{\partial^2 K}{\partial r^2} + \frac{1}{r} \frac{\partial K}{\partial r} + \frac{\partial^2 K}{\partial z^2} - \frac{K}{r^2} \right) - \frac{\sigma B_0^2 u}{\rho}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \\ = \nu (\nabla^2 v - \frac{v}{r^2}) + \frac{\kappa}{\rho} \left(\frac{\partial^2 L}{\partial r^2} + \frac{1}{r} \frac{\partial L}{\partial r} + \frac{\partial^2 L}{\partial z^2} - \frac{L}{r^2} \right) - \frac{\sigma B_0^2 v}{\rho}, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \\ = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu (\nabla^2 w) + \frac{\kappa}{\rho} \left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} + \frac{\partial^2 N}{\partial z^2} \right), \end{aligned} \quad (4)$$

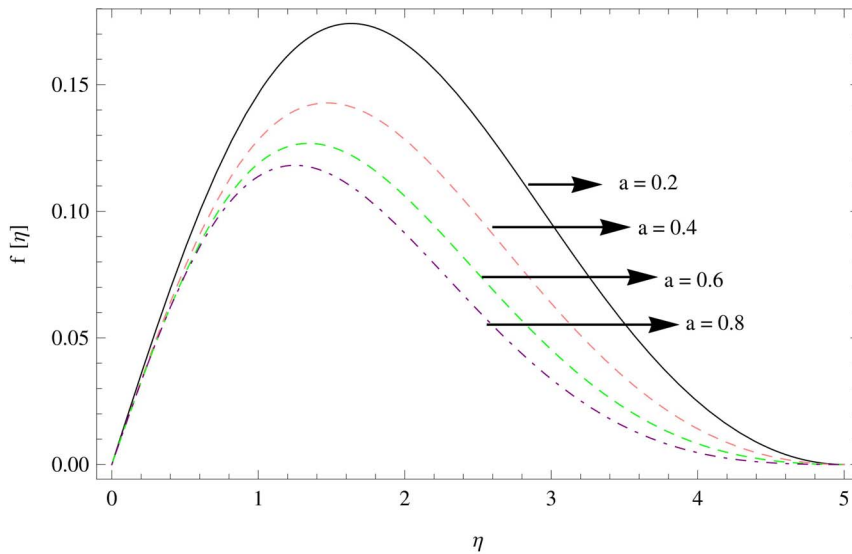


Figure 2. The radial velocity field for $M=1.5, \zeta=0.8$.
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$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha(\nabla^2 T), \quad (5) \quad T(r,z,t) = T_w, \text{ as } z=0, \quad (6)$$

where $K = -\nabla^2 u - \frac{u}{r^2}, \quad L = -\nabla^2 v - \frac{v}{r^2}, \quad N = -\nabla^2 w,$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

The boundary conditions are

$$u(r,z,t)=0, \quad w(r,z,t)=0, \quad u_z(r,z,t)=v_z(r,z,t)=0, \quad (7) \\ T(r,z,t)=T_\infty, \text{ as } z \rightarrow \infty.$$

$$u(r,z,t)=0, \quad v(r,z,t)=r\Omega, \quad w(r,z,t)=0, \\ u_{zz}(r,z,t)=v_z(r,z,t)=0,$$

Here, u, v and w are the components of velocity along the r, θ and z directions respectively, P and T are the pressure and temperature, respectively. ρ, ν and α are density, viscosity and

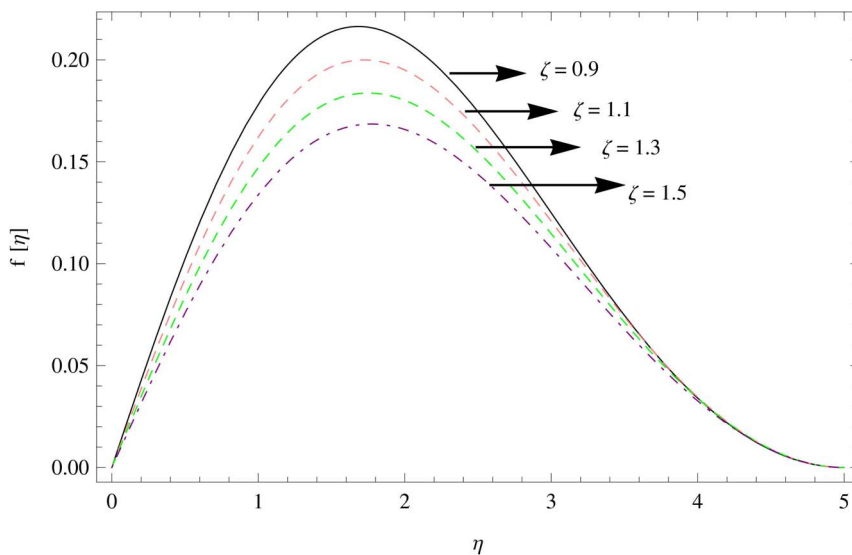


Figure 3. The radial velocity field for $M=1, a=0.2$.
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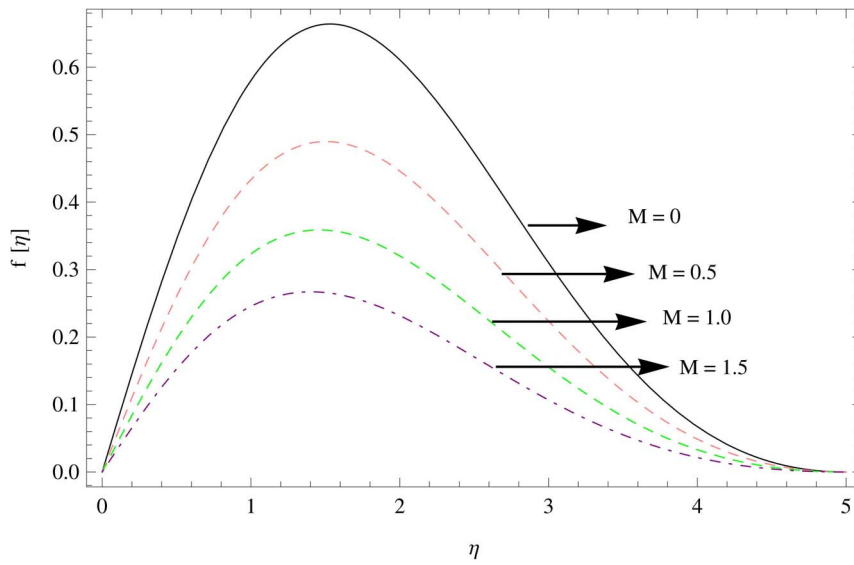


Figure 4. The radial velocity field for $a=0.5, \zeta=0.8$.
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thermal diffusivity, respectively. B_0 is the applied magnetic field, t is the fixed time and κ is the couple stress parameter respectively.

Using similarity transformations

$$u = \frac{ar}{t}f(\eta), v = \frac{ar}{t}g(\eta), w = a\sqrt{\frac{v}{t}}h(\eta), p = \mu\frac{a}{t}P(\eta),$$

$$\eta = \frac{z}{\sqrt{vt}}, \Omega = \frac{a}{t},$$

$$T = T_w + (T_w - T_\infty)\phi(\eta), M = \frac{(\text{Ha})^2}{\text{Re}_r}, \text{Pr} = \frac{\nu}{\alpha},$$

$$\text{Re}_r = \frac{\Omega r^2}{\nu}, (\text{Ha})^2 = \frac{\sigma B_0^2 r^2}{\mu}.$$
(8)

The equation of continuity yields the relation

$$2f + h' = 0,$$
(9)

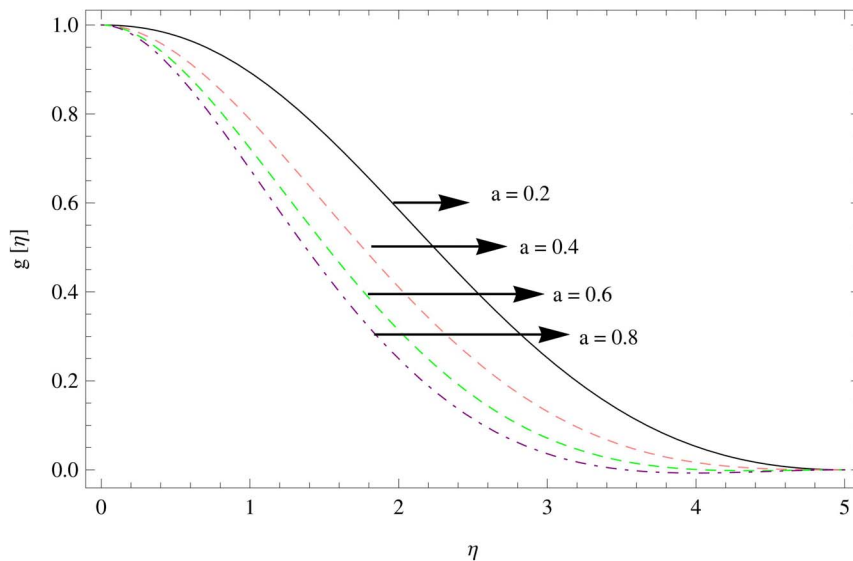


Figure 5. The azimuthal velocity field for $M=1.5, \zeta=0.8$.
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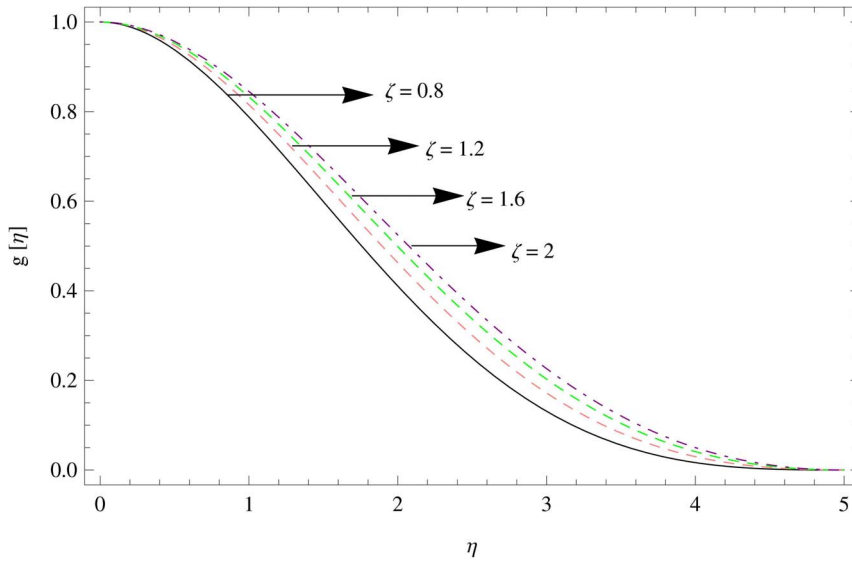


Figure 6. The azimuthal velocity field for $M=1, a=0.2$.
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And Eqs. (2)–(5), after using Eqs. (8)–(9) reduce to

$$\zeta h'''' - h''' + a \left(hh'' - \frac{h^2}{2} + 2g^2 + Mh' \right) - \left(h' + \frac{\eta}{2} h'' \right) = 0, \quad (10)$$

$$P' + \zeta h'''' - h'' + ah'h' - \frac{\eta}{2} h' - \frac{h}{2} = 0, \quad (12)$$

$$\zeta g'''' - g'' + a(hg' - gh' + Mg) - \left(g + \frac{\eta}{2} g' \right) = 0, \quad (11)$$

$$\frac{\phi''}{Pr} + \frac{\eta}{2} \phi' - ah\phi' = 0, \quad (13)$$

with boundary conditions

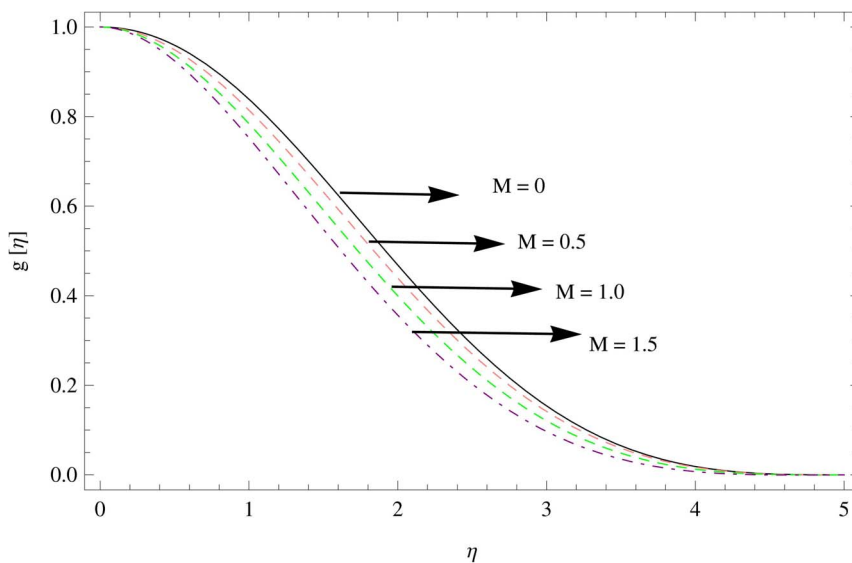


Figure 7. The azimuthal velocity field for $a=0.5, \zeta=0.8$.
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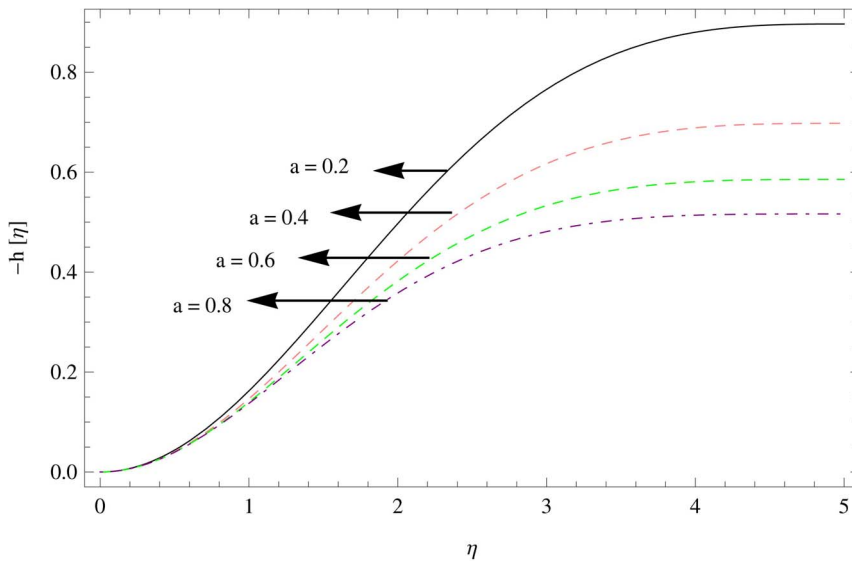


Figure 8. The axial velocity field for $M = 1.5, \zeta = 0.8$.
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$$\begin{aligned} h = h' = 0, \quad h''' = 0, \quad g = \phi = 1, \quad g' = 0, \quad P = 0, \quad \text{at } \eta = 0, \\ h = 0, \quad h'' = 0, \quad g = 0, \quad g' = 0, \quad \text{at } \eta \rightarrow \infty. \end{aligned} \tag{14}$$

Here h' , h and g are the radial, axial and tangential components of the dimensionless velocity, ϕ and P are the dimensionless temperature and pressure of the flow, a is the dimensionless unsteadiness parameter which measures the decrease or increase in the angular velocity Ω , μ is the coefficient of viscosity, Pr is the Prandtl number, Ha is the Hartmann number, M is the magnetic parameter, Re_r is the Reynolds number with respect to radius respectively. The dimensionless couple stress parameter ζ depends on time t , the Eqs.(10)–(13) are not pure similarity equations, to avoid ambiguity, t has been fixed which led the above equation to

approximate similarity. Therefore $\zeta \left(= \frac{\kappa}{t_0 v^2 \rho} \right)$ is valid approximation. And the solutions obtained here are approximations to solutions of the original nonlinear PDEs.

The shear stress of the fluid in the radial and axial directions can be found as

$$\tau_{rz} = \left(\mu(u_z + w_r) + \kappa \left(\frac{\partial^2(u_z - w_r)}{\partial r^2} + \frac{\partial^2(u_z - w_r)}{\partial z^2} \right) \right) \Big|_{z=0}, \tag{15}$$

and

$$\tau_{\theta z} = \left(\mu(v_z) - \kappa \left(\frac{\partial^2(-v_z)}{\partial r^2} + \frac{\partial^2(-v_z)}{\partial z^2} \right) \right) \Big|_{z=0}, \tag{16}$$

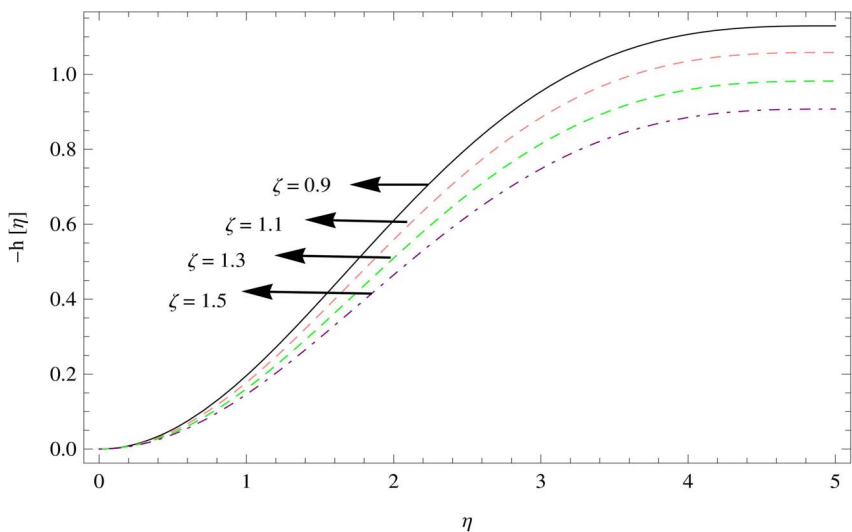


Figure 9. The axial velocity field for $M = 1, a = 0.2$.
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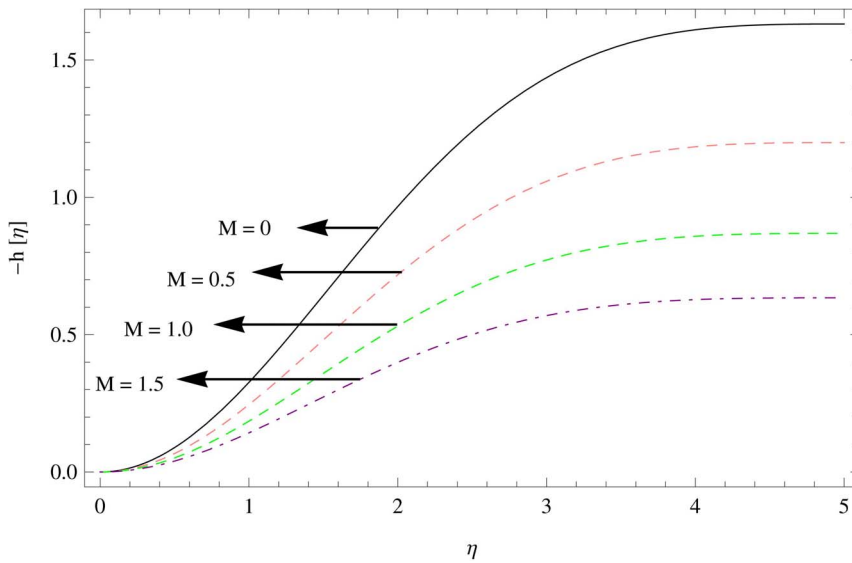


Figure 10. The axial velocity field for $a=0.5, \zeta=0.8$.
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Hence the local skin friction coefficients are given by

$$C_f \text{Re}_r^{\frac{1}{2}} = a^{-\frac{1}{2}} \left(-h''(0) - \frac{\zeta}{2} h'''(0) \right), \tag{17}$$

$$C_g \text{Re}_r^{\frac{1}{2}} = a^{-\frac{1}{2}} (g'(0) + \zeta g'''(0)).$$

$$\text{Nu} = - \frac{r \left(\frac{\partial T}{\partial z} \right) \Big|_{z=0}}{T_w - T_\infty} = - \text{Re}_r^{\frac{1}{2}} a^{-\frac{1}{2}} \phi'(0), \tag{18}$$

We use the result $\tau_{\theta z}$ with a radial strip integration to find the total torque required to turn a disk of radius b .

The heat transfer coefficient in terms of the Nusselt Nu number can be expressed as

$$Q = 2 \pi \int_0^b \tau_{\theta z} \Big|_{z=0} r^2 dr = \frac{\pi a \mu b^4}{2 t (tv)^{\frac{1}{2}}} (g'(0) + \zeta g'''(0)), \tag{19}$$

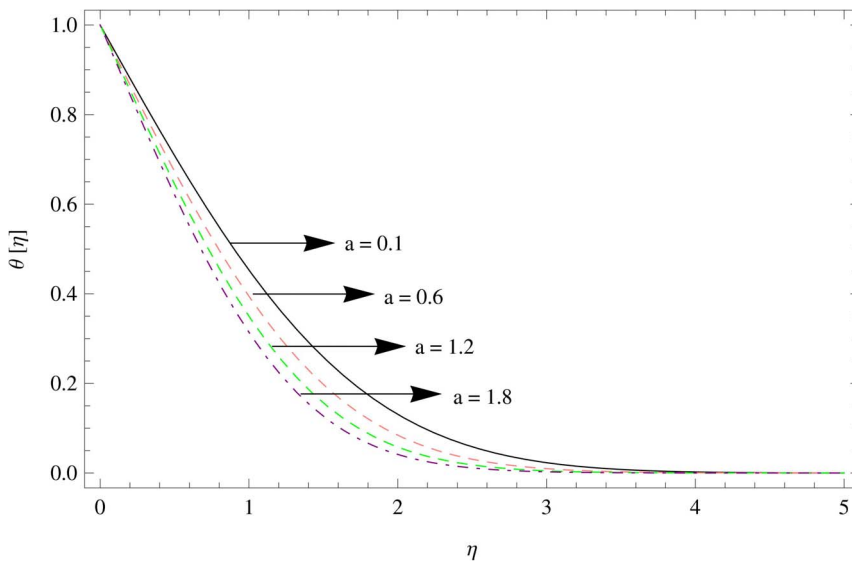


Figure 11. The temperature gradient for $M=1, Pr=1, \zeta=1$.
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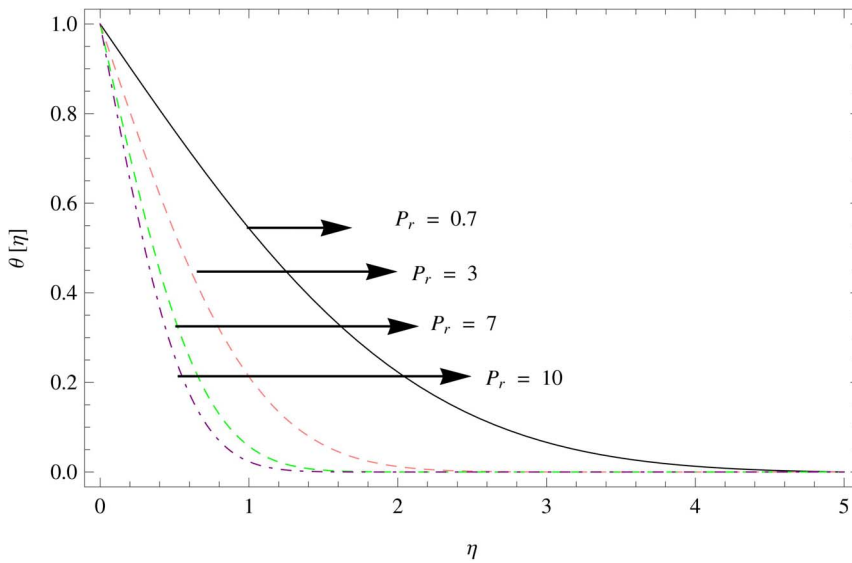


Figure 12. The temperature gradient for $M = 1, a = 0.1, \zeta = 1$.
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The dimensionless moment coefficient for a disk of radius b can be defined as

$$C_Q = -\frac{4Q}{\rho \Omega^2 b^5} = -2\pi \text{Re}_r^{-\frac{1}{2}} a^{-\frac{1}{2}} (g'(0) + \zeta g'''(0)). \quad (20)$$

The system of Eqs. (10)–(12) have been modeled the unsteady flow of a couple stress fluid over a rotating disk. The case when there is no body couples or rotation in the fluid, we set the couple stress parameter ζ equal to zero in the governing system, which results in the following form

$$h''' - a \left(hh'' - \frac{h^2}{2} + 2g^2 + Mh' \right) + \left(h' + \frac{\eta}{2} h'' \right) = 0, \quad (21)$$

$$g'' - a(hg' - gh' + Mg) + \left(g + \frac{\eta}{2} g' \right) = 0, \quad (22)$$

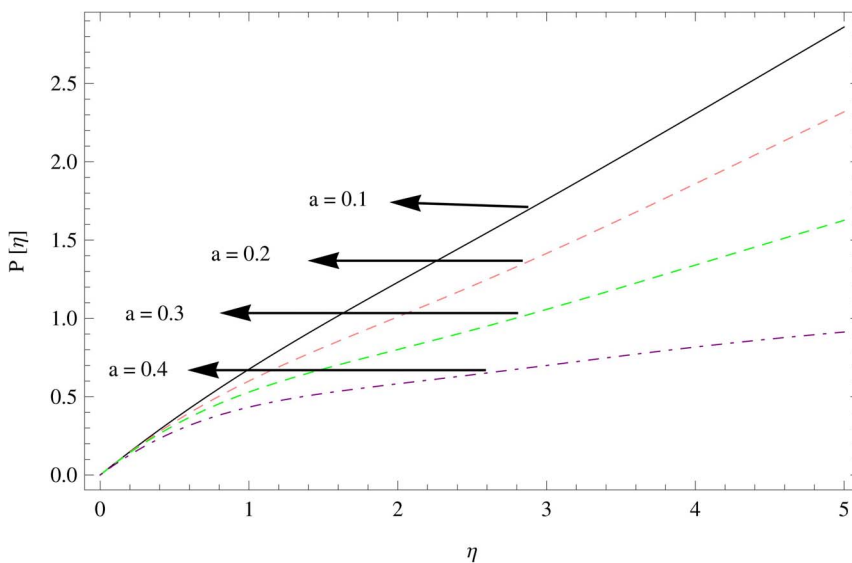


Figure 13. The pressure gradient for $M = 1.5, \zeta = 0.6$.
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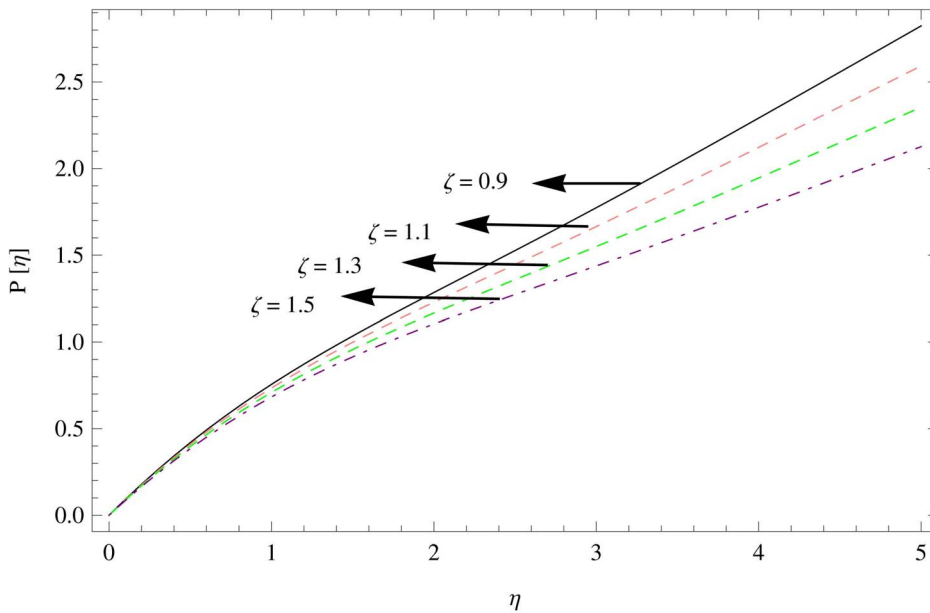


Figure 14. The pressure gradient for $M=1, a=0.2$.
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$$P' - h'' + ah'h' - \frac{\eta}{2}h' - \frac{h}{2} = 0. \tag{23}$$

It can be noticed that the Eqs. (21)–(23) represents the model of unsteady flow of an electrically conducting viscous fluid which was considered and studied by Takhar et. al. [31].

Numerical Procedure

The resulting highly nonlinear system of ODEs (10)–(13) with boundary conditions (14) are solved numerically using fourth-order Runge-Kutta integration scheme with shooting technique for fixed time t_0 . In shooting method, the resulting system of nonlinear ordinary differential equations is transformed to a set of simultaneous first order differential equations. A systematic guessing of initial conditions and numerical integration as an initial value problem has been done until the desired degree of accuracy has obtained. We have relied on the computational software MATHEMATICA package to solve the considered

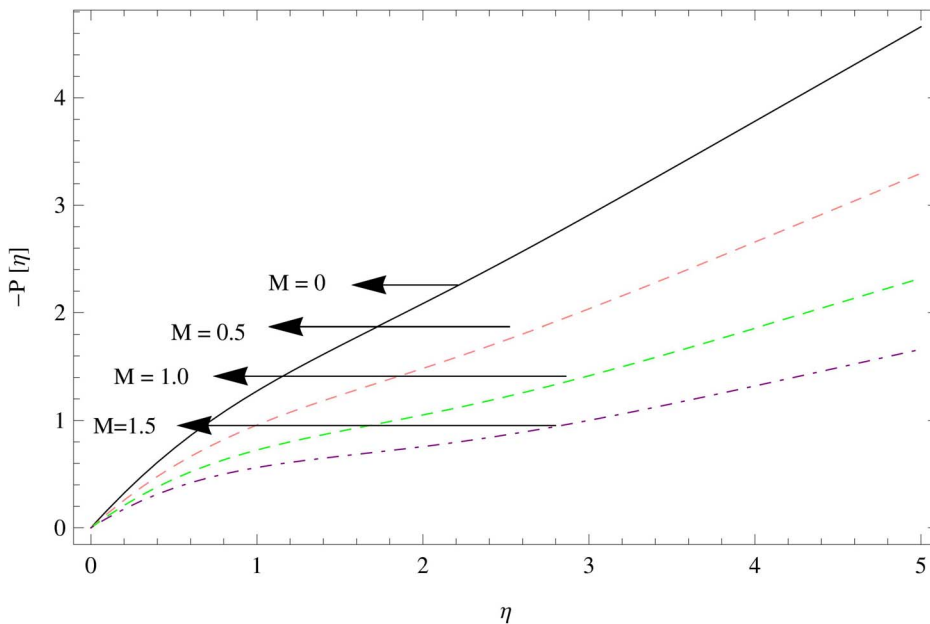


Figure 15. The pressure gradient for $a=0.5, \zeta=0.8$.
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problem. Influences of all relevant parameters are presented graphically.

Results and Discussion

The numerical solution of the problem has been computed via fourth order Runge-Kutta method based on shooting technique. The effect of various physical parameters namely the unsteadiness parameters a , magnetic parameter M , couple stress parameter ζ and Prandtl number Pr have been examined to measure the variations in the angular velocity, in the velocity field, in the temperature gradient and in the pressure distribution of the fluid (see Figs. 2–15). The changes in radial velocity f for the variations in the parameters a , M and ζ are displayed in Figs. 2–4. It can be seen from Fig. 2 that the unsteadiness parameter a decreases the radial velocity f , the stress due to the micro rotation particles in the fluid also weakens the motion of the couple stress fluid as increase in the couple stress parameter ζ decreases the radial velocity (see Fig. 3), whereas Fig. 4 shows that the increase in magnetic parameter M results in the decline of velocity function in radial direction which indicates that the inhibiting Lorentz forces opposes the flow in this direction.

The effects of tangential velocity g as a function of η for a , M and ζ are shown in Figs. 5–7. The decline in the tangential velocity component can be observed with the increase in unsteadiness parameter a as shown in Fig. 5. An increase in the tangential flow over the disk is observed with the increment in the couple stress parameter ζ increasing the boundary wall thickness (see Fig. 6). Since the magnetic force is in the direction normal to the disk, it opposes the tangential flow velocity, therefore the tangential velocity is lowered everywhere. Thus the effect of the Lorentz forces leads to decelerate the azimuthal velocity with increase in M . (see Fig. 7).

Figs. 8–10 shows the effect of aforesaid parameters on axial velocity h as a function of η . The axial velocity field behaves in a similar way to the radial velocity field. Flow field is reduced in the axial direction with the enhancement in the unsteadiness parameter a (as shown in Fig. 8), Fig. 9 shows the effects of increasing couple stress parameter which brings unfavorable change in the axial velocity component. The effect of magnetic parameter can be observed in Fig. 10 which displays the fall in the axial velocity component of couple stress fluid flow with the increase in magnetic parameter M .

In Figs. 11–12, the effect of temperature gradient ϕ as a function of η on the different values of Pr and a are observed. Increasing a results in decline in the temperature gradient (Fig. 11), while the similar behavior is observed with increasing Pr (Fig. 12).

Figs. 13–15 describes the outcome of the pressure distribution against a , M and ζ . These figures depicts the inverse relation between pressure distribution and the unsteadiness parameter, magnetic parameter and couple stress parameter, indicating an increase in above mentioned parameters led the decreasing behavior of the pressure distribution.

Limitations of the Study and Future Recommendations

It is imperative to enlighten diverse limitations of this research. This discussion aims at assisting readers to comprehend this study, which ultimately, provide an opportunity to extend the current research. The following assumptions and limitations are considered.

- Flow is unsteady, incompressible and laminar.

- Flow is three dimensional and rotating over a disk about the z – axis.
- A uniform magnetic field is applied in the direction normal to the surface of the disk which is assumed unchanging with a small magnetic Reynolds number $Re_m \ll 1$.
- It is assumed that the effect of dissipation in the energy equation is negligible.

In this study we attempt to solve governing equations numerically for a fixed value of t_0 (i.e., at a fixed time) since $\zeta \left(= \frac{\kappa}{t_0 \nu^2 \rho} \right)$. It is interesting to note that for large time the higher derivatives in equations (10)–(11) are vanished as ζ tends to zero. This leads the case of degenerate limit and difficult to solve with the presented technique, so a different approach should be sought such as singular perturbation theory to solve the problem for the large-time limit. It is worth observing that, on considering the aforesaid limitations for future research, it will turn out to be a demanding task for the researchers regardless of their particular interests in this area. For the model like couple stress fluid, an extensive range of problems can be defined for both theoretical and experimental aspects, i.e. flow over an off centered disk, between two coaxial disk, due to rotating sphere, over an inclined disk etc. So, fascinated by the diverse applications of non-Newtonian fluids, the present work can be extended to other non-Newtonian fluids model by employing a variety of numerical methods.

Conclusions

In this paper, the response of unsteady MHD flow of couple stress fluid over a rotating disk is discussed. The nonlinear differential equations have been solved numerically using Runge-Kutta method and shooting technique. The numerical formulas for radial and tangential shear stress, heat transfer coefficient in term of Nusselt number and dimensionless moment coefficient are presented. The influence of various physical parameters and couple stress parameter on the velocity field, temperature and pressure distributions are all examined numerically and discussed graphically. The following results can be drawn:

- The unsteadiness parameter decelerates the flow and temperature field over the rotating disk.
- The presence of couple stresses in the fluid enhances the fluid motion in tangential direction and weakens in radial and axial directions.
- The magnetic field opposes the fluid velocity in all directions i.e. radial, tangential and axial.
- There is a significant reduction in the temperature field due to an increase in Prandtl number and unsteadiness parameter.
- Pressure gradient decline while unsteadiness parameter, magnetic parameter and couple stress parameter increases.

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Author Contributions

Conceived and designed the experiments: Najeeb Khan SA Nadeem Khan. Performed the experiments: Najeeb Khan SA. Analyzed the data: Najeeb Khan SA. Contributed reagents/materials/analysis tools: Najeeb Khan SA. Wrote the paper: Najeeb Khan SA.

References

1. Stokes VK (1966) Couple stresses in fluids. *Phy Fluids* 9: 1710–1715.
2. Khan NA, Mahmood A, Ara A (2013) Approximate solution of couple stress fluid with expanding or contracting porous channel. *Eng Comput* 30: 399–408.
3. Farooq M, Islam S, Haroon T (2011) Heat transfer analysis of the couple stress fluid between two parallel plates. *Heat Trans Res* 42: 737–780.
4. Ramzan M, Farooq M, Alsaedi A, Hayat T (2013) MHD three-dimensional flow of couple stress fluid with Newtonian heating. *Europe Phy J* 128: 1–15.
5. Hayat T, Abbas Z, Sajid M (2009) MHD stagnation point flow of an upper convected Maxwell fluid over a stretching sheet. *Chaos Solitons and Fractals* 39: 840–848.
6. Kumar SK, Thacker WI, Watson LT (1987) Rotating magnetohydrodynamic flow in a circular magnetic field. Dept. of computer Science, Winthrop College, Rock Hill, S.C. Tech. Rept. 321.
7. Osalusi E, Side J, Harris R, Clark P (2008) The effect of combined viscous dissipation and Joule heating on unsteady mixed convection MHD flow on a rotating cone in a rotating fluid. *Int Commun Heat Mass Transf* 35: 413–429.
8. Jamil M, Khan NA, Shahid N (2013) Fractional MHD Oldroyd-B fluid over an oscillating plate. *Therm Sc* 17: 997–1011.
9. Sreemadh S, Kishore SN, Srinivas ANS, Hemadri R (2011) The influence of MHD on the couple stress fluid in the porous medium. *Adv Appl Sc* 2: 215–222.
10. Lorrain P, Lorrain F, Houle S (2006) *Magneto-fluid dynamics: fundamentals and case studies of natural phenomena*. Springer, New York.
11. Dehghan M, Mirzaei D (2009) Meshless local Petrov-Galerkin (MLPG) method for the unsteady magnetohydrodynamic flow through pipe with arbitrary wall conductivity. *Appl Numer Math* 59: 1043–1058.
12. Dehghan M, Mirzaei D (2009) Meshless local boundary integral equation (LBIE) method for the unsteady magnetohydrodynamic flow in rectangular and circular pipes. *Comput Phys Commun* 180: 1458–1466.
13. Maleque AK, Sattar AM (2005) Steady laminar convective flow with variable properties due to a porous rotating disk. *J Heat Transf* 127: 1406–1409.
14. Wiesche SAD (2007) Heat transfer from a rotating disk in a parallel air cross-flow. *Int J Therm Sc* 46: 745–754.
15. Ariel PD (2001) On computation of MHD flow near a rotating-disk. *Z Angew Math Mech* 82: 235–246.
16. Fang T (2007) Flow over a stretchable disk. *Phy Fluids* 19: 128105.
17. Rashidi MM, Dinarvand S (2009) Purely analytic approximate solutions for steady three-dimensional problem of condensation film on inclined rotating disk by homotopy analysis method. *Nonlinear Anal RWA* 10: 2346–2356.
18. Khan I, Fakhar K, Sharidan S (2012) Magnetohydrodynamic rotating flow of a generalized Burgers' fluid in a porous medium with hall current. *Transport in Porous Media* 91: 49–58.
19. Rashidi MM, Mohimaniyan SAP, Hayat T, Obaidat S (2012) Analytic approximate solutions for steady flow over a rotating disk in porous medium with heat transfer by homotopy analysis method. *Comp and Fluids* 54: 1–9.
20. Khan I, Fakhar K, Anwar MI (2012) Hydromagnetic rotating flows of an Oldroyd-B fluid in a porous medium. *Special Topics and Review in Porous Media* 3: 89–95.
21. Ming C, Zheng L, Zhang X (2011) Steady flow and heat transfer of the power-law fluid over a rotating disk. *Int Commun Heat and Mass Transf* 38: 280–284.
22. Turkyilmazoglu M (2012) MHD fluid flow and heat transfer with varying Prandtl number due to a rotating disk subject to a uniform radial electric field. *Appl Therm Eng* 35: 127–133.
23. Rashidi MM, Mehr NF (2012) Series solutions for the flow in the vicinity of the equator of an MHD boundary-layer over a porous rotating sphere with heat transfer. *Therm Sc* DOI:10.2298/TSCI120301155R.
24. Turkyilmazoglu M (2012) Three dimensional MHD stagnation flow due to a stretchable rotating disk. *Int J Heat Mass Transf* 55: 6959–65.
25. Bellout H, Vajravelu K, Van Gorder RA (2013) Existence results for coupled nonlinear systems approximating the rotating MHD flow over a rotating sphere near the equator. *Z Angew Math Phy* 64: 83–100.
26. Osalusi E, Side J, Harris R (2007) The effects of Ohmic heating and viscous dissipation on unsteady MHD and slip flow over a porous rotating disk. *Int Comm Heat Mass Transf* 34: 1017–1029.
27. Dinarvand S, Doosthoseini A, Doosthoseini E, Rashidi MM (2010) Series solutions for unsteady laminar MHD flow near forward stagnation point of an impulsively rotating and translating sphere in presence of buoyancy forces. *Nonlinear Anal, RWA* 11: 1159–1169.
28. Chamkha AJ, Ahmed SE (2012) Unsteady MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere at different wall conditions. *Chem Eng Comm* 199: 122–141.
29. Ram P, Bhandari A (2013) Effect of phase difference between highly oscillating magnetic field and magnetization on the unsteady ferrofluid flow due to a rotating disk. *Results in Phy* 3: 55–60.
30. Nadeem S, Saleem S (2013) Analytical treatment of unsteady mixed convection MHD flow on a rotating cone in a rotating frame. *J Taiwan Int Chem Eng* 44: 596–604.
31. Takhar HS, Singh AK, Nath G (2002) Unsteady MHD flow and heat transfer on a rotating disk in an ambient fluid. *Int J Therm Sc* 41: 147–155.