Casimir light: Field pressure

(pulse/conservation)

JULIAN SCHWINGER

University of California, Los Angeles, CA 90024-1547

ABSTRACT The electromagnetic field is assigned a self-consistent role in which abrupt slowing of the collapse produces radiation and the pressure of the radiation produces abrupt slowing. A simple expression is introduced for the photon spectrum. Conditions for light emission are proposed that imply a high degree of spatial localization. Some numerical checks are satisfied. A study of the mechanical equations of motion suggests an explanation of the very short time scale in terms of oppositely directed field pressures and the speed of light.

The remarkable phenomenon of coherent sonoluminescence (1) has posed the following question: How does a macroscopic, classical, hydromechanical system, driven by a macroscopic acoustical force, generate an astonishingly short time scale and an accompanying high electromagnetic frequency, one that is at the atomic level? Bubble collapse in itself [Lord Rayleigh had already described it in 1917 (2)] gives no hint of light-creation-capability.

To the latter point, I offer the hypothesis that light plays a fundamental role in the mechanism. Provocatively put:

The collapse of the cavity is slowed abruptly by the pressure of the light that is created by the abrupt slowing of the collapse.

A more manageable approach would separate this into two parts: finding the radiation pulse emitted by an abrupt slowing, and finding the abrupt slowing produced by the emission of a radiation pulse; to be followed by a self-consistent choice of parameters.

I begin with the weak, single pair emission probability of ref. 3, but with instantaneous collapse replaced by a highly localized, but finite loss of dielectric kinetic energy. The initial steps are those of ref. 3 that lead to the pair emission probability amplitude

$$\langle \vec{k} | \vec{k}' | 0 \rangle = -i \left[\frac{(d\vec{k})}{(2\pi)^3} \frac{(d\vec{k}')}{(2\pi)^3} \frac{\omega\omega'}{4} \right]^{1/2}$$

$$\times \int (d\vec{r}) dt \, \delta\epsilon(x) \exp[it(\omega + \omega') - i\vec{r}\cdot(\vec{k} + \vec{k}')].$$

What is different is the structure of $\delta\epsilon(x)$, now

$$\delta\epsilon(x) = -(\epsilon - 1)\, \eta(r(t) - r).$$

The situation thus described, for $|\epsilon - 1| \ll 1$, has the uniform dielectric constant ϵ for all radii $r > r(t)$ but is the vacuum of unit dielectric constant for $r < r(t)$.

The differential probability for pair emission, the absolute square of the probability amplitude, is given by the product of the factor

$$\frac{(d\vec{k})}{(2\pi)^3} \frac{(d\vec{k}')}{(2\pi)^3} \frac{\omega\omega'}{4} (\epsilon - 1)^2,$$

with the multiple integrals

$$\int\int d^*t dt \, \exp[-i(^*t - t)(\omega + \omega')]$$

$$\times (d^*\vec{r})(d\vec{r}) \exp[i(^*\vec{r} - \vec{r})\cdot(\vec{k} + \vec{k}')],$$

where the latter coordinate integrals are restricted by $^*r < {}^*r(^*t)$, $r < r(t)$.

It helps to introduce the new variables

$$\vec{\rho} = \frac{1}{2}(^*\vec{r} + \vec{r}), \qquad \vec{\sigma} = {}^*\vec{r} - \vec{r},$$

$$(d^*\vec{r})(d\vec{r}) = (d\vec{\rho})(d\vec{\sigma}),$$

so that the coordinate integral reads

$$\int (d\vec{\rho}) \int (d\vec{\sigma}) \exp[i\vec{\sigma}\cdot(\vec{k} + \vec{k}')].$$

Then, in the spirit of ref. 3, for the circumstance $rk \gg 1$, the $\vec{\sigma}$ integral is approximated by $(2\pi)^3 \delta(\vec{k} + \vec{k}')$, and the $\vec{\rho}$ integral becomes the volume of a sphere with radius $\frac{1}{2}(r(^*t) + r(t))$.

On eliminating explicit reference to $\vec{k}' = -\vec{k}$, the differential probability for the emission of a photon pair with equal and opposite momenta emerges as

$$\frac{(d\vec{k})}{(2\pi)^3} (\epsilon - 1)^2 \frac{\omega^2}{4} \frac{4\pi}{3} \int d^*t dt \, e^{-2i\omega(^*t - t)} \left[\frac{1}{2}(r(^*t) + r(t)) \right]^3.$$

There should be no significant contributions from end points in carrying out a partial integration on $^*t$ and on t. That transforms the integrand of the double time integral into

$$e^{-2i\omega(^*t - t)} \left(\frac{1}{2\omega} \right)^2 \frac{3}{2} v(^*t)v(t) \left[\frac{1}{2}(r(^*t) + r(t)) \right],$$

in which the respective vs are the time derivatives of the corresponding rs.

The phrase "abrupt slowing of the collapse" is more usefully presented as follows: during the short time interval in which the $|v|$s decrease significantly, the rs undergo a small relative change. Thus the latter bracket is effectively the constant r. Then the double time integral emerges as the absolute square of a single integral, which, through an additional partial integration, introduces the acceleration: $d/dt \, v(t)$. The outcome for the differential probability is

$$\frac{(d\vec{k})}{(2\pi)^3}(\epsilon - 1)^2 \frac{\pi}{8} r \left| \int dt \frac{1}{2\omega} e^{2i\omega t} \frac{d}{dt} v(t) \right|^2.$$

Perhaps the simplest model for the acceleration is the Gaussian

$$\frac{d}{dt} v(t) = |v|\gamma\pi^{-1/2} \exp[-\gamma^2(t - t_0)^2],$$

where v is (approximately) the velocity at the onset of slowing, just prior to time t_0. The implied Fourier transform is

$$\int dt e^{2i\omega(t-t_0)} \frac{d}{dt} v(t) = |v|\exp\left[-\frac{\omega^2}{\gamma^2}\right].$$

The somewhat remote aspect of these considerations ($|\epsilon - 1| \ll 1$) makes them—at best—qualitative. Accordingly, I omit all fixed factors and focus on the shape of the energy spectrum (energy per wavelength):

$$\sim \frac{1}{\lambda^3} \exp\left[-\frac{3}{2}\left(\frac{\lambda_o}{\lambda}\right)^2\right],$$

where λ_o locates the peak intensity. For long wavelengths ($\lambda \gg \lambda_o$), the intensity varies as $1/\lambda^3$, whereas the Gaussian structure dominates for $\lambda \ll \lambda_o$. For $\lambda = \frac{1}{2}\lambda_o$, the intensity drops, relative to the peak value, by the factor $1/11.3$; the analogous factor for $\lambda = 2\lambda_o$ is $1/2.6$.

What is so remarkable about coherent sonoluminescence is not that macroscopic bodies can produce lights—our early ancestors could manage that—but that the light emission is confined to a very short time interval. In order to be quantitative, I submit this more specific hypothesis.

The conditions for light emission are at hand when the fluid kinetic energy becomes independent of t, for a short time interval, and similar remarks apply immediately after the emission act. In effect, one is picking out the circumstances for spontaneous radiation, from a coherent state of definite energy, to another such state of definite, lower energy.

Outside regions of electromagnetic activity, the conserved mechanical energy is (4)

$$E = 2\pi\rho_0 \left[r^3v^2 + \Omega^2 r^3 + \frac{p_0}{\rho_0^3}\frac{2}{3}r_0^3\left(\frac{r_0}{r}\right)^{3(\gamma-1)}\right]$$

$$\equiv T + V_C + V_p.$$

For a conserved E, and a locally constant in time T, so must $V_C + V_p$ be locally constant in time, which means that $V_C + V_p$ have a vanishing r derivative. This singles out an equilibrium point,[†] where the attractive force of V_C balances the repulsive force of V_p. Notice that the statements for the two circumstances are the same; there is a unique radial distance, r_ℓ, where the light is emitted. It is given by

$$1 = \frac{p_0}{\rho_0\Omega^2}\frac{2}{3}(\gamma - 1)\left(\frac{r_0}{r_\ell}\right)^{3\gamma},$$

or

$$\frac{r_0}{r_\ell} = \left[\frac{3}{2}\frac{\rho_0\Omega^2}{p_0}\frac{1}{\gamma - 1}\right]^{1/3\gamma}.$$

†In ref. 4, the term "equilibrium radius" was misleadingly applied to r_0. It is more appropriately assigned to r_ℓ.

For $\gamma = 1.4$, and $p_0 = \rho_0\Omega^2$, $r_0/r_\ell = 1.37$. Thus, in the example of ref. 5, where $r_0 \simeq 4 \times 10^{-4}$ cm, one has $r_\ell \simeq 3 \times 10^{-4}$ cm.

I enquire again about the magnitude of $v_{a\ell}$, the radial velocity immediately *after* the blast of *light*. This time there is no appeal to information about the first bounce, and, of course, r_0 and r_ℓ are no longer equated. The assumption of energy conservation over the path between r_ℓ and r, the minimum radius, as expressed by

$$r_\ell^3\left[\frac{v_{a\ell}^2}{\Omega^2} + 1 + \frac{2}{3}\left(\frac{r_0}{r_\ell}\right)^{3\gamma}\right] = r^3\left[1 + \frac{2}{3}\left(\frac{r_0}{r}\right)^{3\gamma}\right]$$

with the experimental inputs $r_0 = 4 \times 10^{-4}$ cm, $r = 0.7 \times 10^{-4}$ cm, yields

$$\frac{v_{a\ell}^2}{\Omega^2} = 10.4, \qquad |v_{a\ell}| \simeq 3 \times 10^3 \text{ cm/s}.$$

One sees the possibility of calculating $|v_{a\ell}|$ either by introducing the first bounce parameter r_1, but not the minimum radius r (ref. 4, without the distinction between $r_{a\ell}$ and r_0), or, by using r, without reference to r_1. The two procedures are connected by the statement of energy conservation for the first bounce:

$$\left(\frac{r_1}{r_0}\right)^3 + \frac{2}{3}\left(\frac{r_0}{r_1}\right)^{3(\gamma-1)} = \left(\frac{r}{r_0}\right)^3 + \frac{2}{3}\left(\frac{r_0}{r}\right)^{3(\gamma-1)}.$$

The introduction of r = 0.7×10^{-4} cm yields $r_1 = 7 \times 10^{-4}$ cm, and conversely.

Finally, I look at the Lagrangian that includes both mechanical and electromagnetic effects:

$$L = T - V_C - V_p + L_e,$$

where the $\epsilon(\vec{r}t)$-dependent part of L_e is

$$L_\epsilon = \int (d\vec{r}) \frac{1}{2} \epsilon(\vec{r}t)\left[\frac{1}{c}\frac{\partial}{\partial t} A(\vec{r}t)\right]^2.$$

The contributions of the two regions, on opposite sides of the moving surface $r(t)$, are conveyed by

$$\epsilon(\vec{r}t) = \epsilon \times \eta[r - r(t) - 0] + 1 \times \eta[r(t) - r - 0],$$

where 0 is approached through positive values.

The stationary property of the actions, for the infinitesimal variations $\delta r(t)$, yields

$$[4\pi\rho_0 r^3]\frac{d}{dt} v(t) + \cdots$$

$$= \int (d\vec{r}) \frac{1}{2}\left[\frac{1}{c}\frac{\partial}{\partial t} A(\vec{r}t)\right]^2 \{\delta[r(t) - r - 0] - \epsilon\delta[r - r(t) - 0]\},$$

where $\delta[\ ]$ is a delta function, and $\cdots$ refers to terms of lesser degree in the time derivative, which will be of secondary importance for abrupt slowing.

Just before that begins, there is no significant field, and $v(t)$ is essentially constant. Then the field strength rises rapidly in the vacuum region, giving a positive value to the right side of the above equation. Accordingly, $v(t)$, which is negative, must move toward smaller magnitudes—the slowing has begun. That process will cease when the field, flowing at the speed of *light* toward the outer dielectric region, has produced the countering pressure.

For some specific results, it is useful to separate radial and transverse aspects,

$$(d\vec{r}) = dr\ dS,$$

and then perform the r integration in the respective volumes. On defining F_+, the outward surface force, and F_-, the inward force, which include the unwritten contributions of the other field type, one gets

$$[4\pi\rho_0 r^3]\frac{d}{dt}v(t) = F_+(t) - F_-(t).$$

First, multiply by the positive quantity $-v(t)$, and then integrate over the time span of the emitted pulse:

$$T_{b\ell} - T_{af} = \int dt(-v)(F_+ - F_-)$$

$$= \int |dr|(F_+ - F_-).$$

One sees that the kinetic energy released equals the work done on—the energy transferred to—the electromagnetic field.

Momentum, defined mechanically by

$$p = \frac{\partial T}{\partial V} = [4\pi\rho_0 r^3]v,$$

now enters as

$$\frac{d}{dt}p(t) = F_+(t) - F_-(t).$$

The integrated consequence here is

$$|p|_{b\ell} - |p|_{a\ell} = \int dt(F_+ - F_-):$$

the momentum removed from the mechanical system is transferred to the electromagnetic field.

The somewhat mysterious initial hypothesis has emerged clarified, as an unusual example of a familiar fact—spontaneous emission of radiation by an electrical system is a single, indivisible act that obeys the laws of energy and momentum conservation.

1. Barber, B. & Putterman, S. (1991) *Nature (London)* **352**, 318–320.
2. Rayleigh, J. (1917) *Philos. Mag.* **34**, 94.
3. Schwinger, J. (1993) *Proc. Natl. Acad. Sci. USA* **90**, 4505–4507.
4. Schwinger, J. (1993) *Proc. Natl. Acad. Sci. USA* **90**, 7285–7287.
5. Barber, B. & Putterman, S. (1992) *Phys. Rev. Lett.* **69**, 3839–3842.