LETTER

No time for waiting: Statistical structure reflects subjective complexity

Apparent inability of human observers to understand and mimic randomness has occupied researchers for a long time. According to a recent report, the special status of streaks in randomness perception reflects sensitivity to subtle statistical properties of pattern structure. In PNAS, Sun et al. (1) have highlighted one such property, namely, waiting time, as possibly responsible for the biases in randomness perception. Although the probability of any outcome in a truly random sequence is equal, inherent structural differences between patterns affect the probabilities of their occurrence at different points of the tossing process. Specifically, patterns that possess the maximum degree of self-overlap (e.g., HHHH) are easily disrupted by individual tosses unlike nonoverlapping patterns (e.g., HHHT) and, as such, have the longest waiting time (time of first occurrence) because they tend to cluster together, accounting for the apparent inequality in distribution of different outcomes over time (2).

Post hoc examination of the structure of "random" sequences contradicts the accepted definition of randomness according to which a truly random process can produce any outcome at any time and successive outcomes are mutually independent and completely unpredictable (3). Focus on structural differences gives the mistaken impression that waiting time and related statistics squeeze new information out of a hitherto unknowable process. Knowing that a sequence of heads appeared on previous tosses tells us absolutely nothing about the outcome of the next toss. Waiting time is a subtle form of gambler's fallacy because it surreptitiously reintroduces the (erroneous) assumption of self-correction—self-overlapping patterns cluster and are consequently more widespread over a series of tosses—which is supposed to balance out the probabilities of different patterns and account for the "granularity" of random sequences.

Attempts at analyzing the structure of supposedly random sequences reveal the incompatibility of the mathematical concept of randomness with human cognition. A parsimonious explanation of Sun et al.'s results is offered in terms of structural complexity (1). Waiting time represents a rough measure (inverse) of pattern complexity. In addition to self-similarity, self-overlap index depends on first and last symbols being identical. In any set of patterns of reasonable length, simple patterns (e.g., HHHH. . .) represent extreme outliers that are highly unlikely to be selected through random sampling. Naturally, they arouse observers' attention and generate biases. A recently reported complexity model based not on probability but change (4) correlated significantly with the degree of self-overlap for strings of length 6 ($r = -0.72$, $P < 0.001$; $n = 20$) and performed almost as well as the Griffiths and Tenenbaum model (5) in accounting for subjective response to randomness despite making no theoretical assumptions. [Complements and mirror inversions were excluded from the analysis. Griffiths and Tenenbaum (5) tested strings of length eight. Their model accounted for 60% of the variance, whereas ours accounted for about 50%.] In conclusion, waiting time and similar statistics represent an implicit attempt to reintroduce structure into the probabilistic framework that is ultimately unsuited to describing and quantifying pattern perception and cognition.

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