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Reader Reaction to "A Robust Method for Estimating Optimal Treatment Regimes" by Zhang et al (2012)

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Summary

A recent paper (Zhang et al., 2012) compares regression based and inverse probability based methods of estimating an optimal treatment regime and shows for a small number of covariates that inverse probability weighted methods are more robust to model misspecification than regression methods. We demonstrate that using models that fit the data better reduces the concern about non-robustness for the regression methods. We extend the simulation study of Zhang et al (2012), also considering the situation of a larger number of covariates, and show that incorporating random forests into both regression and inverse probability weighted based methods improves their properties.

Keywords

Optimal Treatment Regime; Random Forests

1. Introduction

In an excellent article (Zhang et al., 2012), on estimating an optimal treatment regime, the authors consider the following situation: n subjects in a study, who are either in the treatment ($A = 1$) or the control ($A = 0$) group. Each subject has p baseline covariates $\boldsymbol{X} = (X_1, \ldots, X_p)$ and higher values of the continuous outcome measure (Y) are better. A treatment regime $g(\boldsymbol{X})$ is a function from $\boldsymbol{X}$ to $\{0, 1\}$, such that patients should receive $A = 1$ if $g(\boldsymbol{X}) = 1$ and $A = 0$ if $g(\boldsymbol{X}) = 0$. The value of $g(\boldsymbol{X})$ is determined by whether $\eta_0 + \sum_{j=1}^{p} \eta_j X_j$ is positive or not. The goal is to find the optimal treatment regime. Both a randomized trial and an observational study setting were considered. The authors develop and compare different approaches. One is a regression approach (*RG*), which requires a model for $\mu(A, \boldsymbol{X}) = E(Y|A, \boldsymbol{X})$. Other approaches are based on inverse probability weighted estimators (*IPWE*). The standard *IPWE* does not require a model for $\mu(A, \boldsymbol{X})$, but does requires a model for $P(A = 1|\boldsymbol{X})$. The authors extend the *IPWE* to an Augmented Inverse Probability Weighted estimator (*AIPWE*), which requires models for both $\mu(A, \boldsymbol{X})$ and $P(A = 1|\boldsymbol{X})$. The *AIPWE* results is a gain in efficiency relative to *IPWE* and has a double robustness property. In a simulation study, the *RG* method was the best if the model for $\mu(A, \boldsymbol{X})$ was correctly specified, but was

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not robust to misspecification of $\mu(A, \boldsymbol{X})$. With correct specification of $\mu(A, \boldsymbol{X})$, the *AIPWE* method was not quite as efficient as *RG*.

For the misspecified model for $\mu(A, \boldsymbol{X})$, residual plots would immediately recognize the model as providing a poor fit to the data. In this paper we examine the relative merits of *RG*, *IPWE* and *AIPWE* when one uses a model for $\mu(A, \boldsymbol{X})$ which better fits the data.

2. Review Of Methods

Let $Y(g)$ be the response for a patient who follows regime g. For a randomly chosen patient from a population the expected response if regime $g(\boldsymbol{X})$ is followed is given by $E(Y(g)) = E_X[\mu(1, \boldsymbol{X})g(\boldsymbol{X})+\mu(0, \boldsymbol{X})\{1-g(\boldsymbol{X})\}]$. The optimal treatment regime is $g^{opt}(\boldsymbol{X}) = I\{\mu(1, \boldsymbol{X}) > \mu(0, \boldsymbol{X})\}$. Let $\hat{g}$ denote an estimated regime that is derived from a dataset.

Denote by $Q(g)$ the average value of the expected response for subjects in a future population of very large size N if regime g were to be used, where $Q(g)$ is given by

$$Q(g)=\frac{1}{N}\sum_{i=1}^{N}[\mu(1, \boldsymbol{X}_i)g(\boldsymbol{X}_i)+\mu(0, \boldsymbol{X}_i)\{1 - g(\boldsymbol{X}_i)\}] \quad (1)$$

Larger values of $Q(g)$ are better. Thus, when $\mu(A, \boldsymbol{X})$ is known, the success of different methods for estimating g can be based on $Q(\hat{g})$ and also compared to $Q(g^{opt})$.

2.1 Regression Method

The *RG* method is to posit a parametric regression model for $\mu(A, \boldsymbol{X}) = \mu(A, \boldsymbol{X}; \boldsymbol{\beta})$, estimate $\boldsymbol{\beta}$ from the data, then $\hat{g}_{reg}^{opt}(\boldsymbol{X}) = I\left\{\mu\left(1, \boldsymbol{X}; \hat{\boldsymbol{\beta}}\right) > \mu\left(0, \boldsymbol{X}; \hat{\boldsymbol{\beta}}\right)\right\}$ Below we will also consider alternative nonparametric regression models for $\mu(A, \boldsymbol{X})$.

2.2 Inverse Probability Weighted Estimators

For the *IPWE* method, a parametric form for $g(\boldsymbol{X}) = g(\boldsymbol{X}; \boldsymbol{\eta})$ is specified. For fixed $\boldsymbol{\eta}$, define $C_{\eta,i} = A_i g(\boldsymbol{X}_i; \boldsymbol{\eta}) + (1-A_i)\{1-g(\boldsymbol{X}_i; \boldsymbol{\eta})\}$ and $\pi(\boldsymbol{X}) = P(A = 1|\boldsymbol{X})$. Then the expected population average outcome is $\frac{1}{n}\sum_{i=1}^{n} C_{\eta,i}Y_i/\pi_C(\boldsymbol{X}_i)$ which is maximized over $\boldsymbol{\eta}$ to give $\hat{g}_{IPW}^{opt}(\boldsymbol{X}) = g(\boldsymbol{X}; \hat{\boldsymbol{\eta}})$ where $\pi_C(\boldsymbol{X}_i) = \pi(\boldsymbol{X}_i)^{A_i}\{1 - \pi(\boldsymbol{X}_i)\}^{1-A_i}$ For a randomized study the propensity score $\pi(\boldsymbol{X})$ is estimated by the sample proportion assigned to treatment 1, which will be close to 0.5. For a non-randomized study logistic regression is used to estimate $\pi(\boldsymbol{X})$.

For the *AIPWE* method, $\boldsymbol{\eta}$ is obtained by maximizing

$$AIPWE(\eta) = \frac{1}{n}\sum_{i=1}^{n}\frac{C_{\eta,i}Y_i}{\pi_C(\boldsymbol{X}_i)} - \frac{C_{\eta,i} - \pi_C(\boldsymbol{X}_i)}{\pi_C(\boldsymbol{X}_i)}m\left(X_i; \eta, \hat{\boldsymbol{\beta}}\right) \quad (2)$$

over $\boldsymbol{\eta}$, where $m(\boldsymbol{X}; \boldsymbol{\eta}, \boldsymbol{\beta}) = \mu(1, \boldsymbol{X}; \boldsymbol{\beta})g(\boldsymbol{X}; \boldsymbol{\eta}) + \mu(0, \boldsymbol{X}; \boldsymbol{\beta})\{1-g(\boldsymbol{X}; \boldsymbol{\eta})\}$

The Zhang et al (2012) also considered the consistency properties and calculation of standard errors for $\hat{\boldsymbol{\eta}}$ we will not consider these in the current paper.

3. Simulation Study

In the simulation study in Zhang et al (2012), in Table 8 of the Supplementary Materials, data were generated from a true model $Y_i = \mu(A_i, \boldsymbol{X}_i) + e_i$, where $e_i \sim N(0, 1)$ and

$$\mu(A, \boldsymbol{X}) = exp\left\{2.0 - 1.5X_1^2 - 1.5X_2^2 + 3.0X_1X_2 + A(-0.1 - X_1 + X_2 + 0.2X_3)\right\},$$

where X_{i1} and X_{i2} were $U(-1.5, 1.5)$ and X_{i3} and A_i were Bern(0.5). For this model $g^{opt}(\boldsymbol{X}) = I(-0.1 - X_1 + X_2 + 0.2X_3 > 0)$. They considered two parametric regression models for $\mu(A, \boldsymbol{X})$, a correctly specified model of the form

$$\mu_t(A, \boldsymbol{X}; \boldsymbol{\beta}) = exp\left\{\beta_0 + \beta_1 X_1^2 + \beta_2 X_2^2 + \beta_3 X_1 X_2 + A(\beta_4 + \beta_5 X_1 + \beta_6 X_2 + \beta_7 X_3)\right\} \quad (3)$$

and a misspecified simple linear model of the form

$$\mu_{msl}(A, \boldsymbol{X}; \boldsymbol{\beta}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + A(\beta_4 + \beta_5 X_1 + \beta_6 X_2 + \beta_7 X_3). \quad (4)$$

From standard residual plots it is obvious that the misspecified model gives a very bad fit to the data, and wouldn't be seriously entertained, particularly for the *RG* method. Inspection of the data suggests that some transformation of the response *Y* may lead to an improved fit. Although log(*Y*) might appear to be a natural choice, it is not possible because a small fraction of the *Y*'s are negative, thus we choose $Y^{1/3}$ as an approximation. Thus the question is, if one used a better fitting model for *Y* in both the *RG* and *AIPWE* methods would the results improve? We consider two parametric models, as well as a non-parametric estimator. The first misspecified parametric model recognizes the benefit of a transformation, and the second also recognizes the need for quadratic terms and interactions. In these models we develop predictions for $Z = Y^{1/3}$, and then cube these predictions of *Z* to obtain predictions of *Y*. The simple misspecified cube root model is given by

$$E(Z) = \mu_{ms33}(A, \boldsymbol{X}; \boldsymbol{\beta}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + A(\beta_4 + \beta_5 X_1 + \beta_6 X_2 + \beta_7 X_3). \quad (5)$$

and the misspecified complex cube root model is given by

$$\mu_{mc33}(A, \boldsymbol{X}; \boldsymbol{\beta}) = \mu_{ms33}(A, \boldsymbol{X}; \boldsymbol{\beta}) + \sum_{j=1}^{2} \beta_{j+7} X_j^2 + \beta_{10} X_1 \times X_2 + \beta_{11} X_1 \times X_3 + \beta_{12} X_2 \times X_3. \quad (6)$$

Standard model assessment methods would recognize some lack of fit for μ_{mc33}, although it is a noticeable improvement over μ_{msl} and μ_{mc33}.

In the other approach, we used random forests as a non-parametric estimator of $\mu(A, \boldsymbol{X})$. and denote the estimate by $\hat{\mu}(A, \boldsymbol{X})$. The RG_{rf} method consists of maximizing

$$\frac{1}{n}\sum_{i=1}^{n}\left[\hat{\mu}(1, \boldsymbol{X}_i)\, g(\boldsymbol{X}_i; \boldsymbol{\eta}) + \hat{\mu}(0, \boldsymbol{X}_i)\{1 - g(\boldsymbol{X}_i; \boldsymbol{\eta})\}\right] \quad (7)$$

with respect to η. While we present results for random forests, other non-parametric estimators could be considered. To implement random forests, with $Y^{1/3}$ as the response, we used the function *randomforest* in R, using default settings except that the number of trees was 1000. Similar to previous work (Foster et al., 2011), we found that the performance of random forests was improved by using $A, X_k, X_k^2, X_k I(A=1)$ and $X_k I(A = 0)$ for $k = 1$ to p as input covariates. We note that random forests with $Y^{1/3}$ as the response gave a very mild improvement over random forests with Y as the response.

To fit the linear model in equations 4, 5 and 6, the R function *lm* was used. To fit the non-linear model in equation 3, the R function *nlsLM* was used. To maximize the criteria in equations 2 and 7, we used the R function *genoud*, as described in Zhang et al. (2012).

3.1 Results For Three Covariates

In our simulation study 1000 datasets, each of size 500 was generated. We report in table 1 two quantities: the average of the ratio $Q(\hat{g})/Q\left(g^{opt}\right)$ and the average fraction who would be treated if, following each trial, $\hat{g}$ were to be used. For each of the 1000 datasets $Q(\hat{g})$ and $Q(g^{opt})$ were calculated from equation 1, with N=1,000,000.

The 2nd and 3rd columns show the results for three covariate, labeled as Case A, matching the situation considered in (Zhang et al., 2012). For this setting $Q(g = 0) = 3.02$, $Q(g = 1) = 3.48$, and $Q(g^{opt}) = 3.95$. The first two rows are for ideal, but not applicable in practice, methods in which the structure of the true model for $\mu(A,\boldsymbol{X})$ is known. Amongst these two, RG_t is slightly better than $AIPWE_t$, achieving the desired values of 1 and 0.5 for the Ratio to optimal and the Fraction treated respectively. Amongst the applicable methods, RG_{mc33} generally improves on RG_{msl} and RG_{ms33}, and RG_{rf} is the much better than both. Amongst the *AIPWE* methods they are all preferable to *IPWE*, with $AIPWE_{rf}$ being the best.

Also of note is that the inverse probability methods tend to recommend treating closer to the true 50% fraction of patients, than the regression methods. The regression methods tend to include too many subjects in the region $\hat{g}(\boldsymbol{X})=1$.

In the 4th and 5th columns of the table we show results for Case B, a situation in which the optimal regime is that 90% of the subjects should be treated. The data were generated from the model

$\mu(A,\boldsymbol{X})=exp\left\{2.0-1.5X_1^2-1.5X_2^2+3.0X_1X_2+A(-0.1-X_1+X_2+0.2X_3)\right\}$, where X_{i1} was $U(-1.5,1.5)$, X_{i2} was $U(0.2,3)$, X_{i3} was Bern(0.6) and A_i was Bern(0.5). For this setting $Q(g = 0) = 1.66$, $Q(g = 1) = 2.51$, and $Q(g^{opt}) = 2.64$.

We considered four parametric outcome regression models. The first one is $\mu_t(A, \boldsymbol{X}; \boldsymbol{\beta})$, the correct nonlinear regression model, as given in equation 3; the other three were the misspecified models $\mu_{msl}(\boldsymbol{A, X; \beta})$, $\mu_{ms33}(\boldsymbol{A, X; \beta})$ and $\mu_{mc33}(\boldsymbol{A,X; \beta})$, as given in equations 4, 5 and 6. The results again show the benefit of using random forests to estimate $\mu(A, \boldsymbol{X})$ in both *RG* and *AIPWE* methods and that again RG_{rf} has similar performance as $AIPWE_{rf}$.

We also considered a case similar to case A, but in which the covariates were correlated. The results were very similar to the uncorrrelated case and are not presented here.

In the 6th and 7th columns of the table we show results for Case C, a situation where the optimal g is not determined by a linear combination of the covariates. The data were generated from the model

$$\mu(A,\boldsymbol{X}) = exp\left\{2.0 - 1.5X_1^2 - 1.5X_2^2 + 3.0X_1X_2 + A(-0.1 - X_1 + X_2 + 0.2X_3 + 0.5X_1X_3)\right\}$$

where X_{i1} and X_{i2} were $U(-1.5,1.5)$, X_{i3} and A_i were Bernoulli(0.5). For this setting $Q(g = 0) = 3.02$, $Q(g = 1) = 3.49$, and $Q(g^{opt}) = 3.99$. The optimal $g(\boldsymbol{X})$ is $I(-0.1 - X_1 + X_2 + 0.2X_3 + 0.5X_1X_3 > 0)$.

We considered four parametric outcome regression models. The first one is $\mu_t(A,\boldsymbol{X};\boldsymbol{\beta})$

$$\mu_t(A,\boldsymbol{X};\boldsymbol{\beta}) = exp\left\{\beta_0 + \beta_1X_1^2 + \beta_2X_2^2 + \beta_3X_1X_2 + A(\beta_4 + \beta_5X_1 + \beta_6X_2 + \beta_7X_3 + \beta_8X_1X_3)\right\} \quad (8)$$

corresponding to the correct nonlinear regression model; the other three were $\mu_{msl}(A,\boldsymbol{X};\boldsymbol{\beta})$ and $\mu_{ms33}(A,\boldsymbol{X};\boldsymbol{\beta})$, and $\mu_{mc33}(A,\boldsymbol{X};\boldsymbol{\beta})$ as given in 4, 5 and 6.

The results are similar to those for Case A, with for the *RG* methods a mild improvement by using the complex parametric model and substantial improvement by using random forests. The results again show the benefit of using random forests to estimate $\mu(A, \boldsymbol{X})$ in the *AIPWE* methods. The fact that the optimal g is not within the class of models being estimated does not seem to have negatively impacted the performance of the methods.

3.2 Results For Fifteen Covariates

The above results are for a small number of three covariates. With a larger number of covariates, the task of building models for $\mu(A,\boldsymbol{X})$ is more challenging. Fitting parametric models with many quadratic terms and interactions is not feasible, or would require variable selection. The ability of non-parametric regression methods, such as random forests, to give reliable predictions decreases with increasing p. The performance of the *AIPWE* methods is also likely to deteriorate with larger p, because the maximization in equation 2 will give poorer estimates of $\boldsymbol{\eta}$. To investigate this, we considered a situation of 15 covariates, where the true model for $\mu(A, \boldsymbol{X})$ was

$$\mu(A,\boldsymbol{X}) = exp\left\{2.0 - 1.5X_1^2 - 1.5X_2^2 + 3.0X_1X_2 + A(-0.1 - X_1 + X_2 + 0.2X_3)\right\},$$

with corresponding $g^{opt}(\boldsymbol{X}) = I(X_2 > X_1 - 0.2X_3 + 0.1)$, and where X_1 and $X_2 \sim U(-1.5, 1.5)$, $X_3 \sim$ Bern(0.5), X_4, X_5, X_7, X_8, X_{10}, X_{11}, X_{13}, $X_{14} \sim U(-1.5,1.5)$ and X_6, X_9, X_{12}, $X_{15} \sim$ Bern(0.5). For this setting $Q(g = 0) = 3.02$, $Q(g = 1) = 3.48$, and $Q(g^{opt}) = 3.95$. The linear combination that determines the estimated g could include 15 variables.

We considered three possible parametric outcome regression models. The first one was $\mu_t(A,\boldsymbol{X};\boldsymbol{\beta}) = exp\left\{\beta_0 + \beta_1X_1^2 + \beta_2X_2^2 + \beta_3X_1X_2 + A(\beta_4 + \beta_5X_1 + \beta_6X_2 + \beta_7X_3)\right\}$, which corresponds to the correct nonlinear regression model; the second one is

$\mu_{msl}(A, \boldsymbol{X};\boldsymbol{\beta}) = \beta_0 + \sum_{j=1}^{15}\beta_j X_j + A\left(\beta_{16} + \sum_{j=1}^{15}\beta_{j+16}X_j\right)$. The third one is $\mu_{ms33}(A, \boldsymbol{X};\boldsymbol{\beta})$, which is the same as $\mu_{msl}(A,\boldsymbol{X};\boldsymbol{\beta})$ except that the response is $Y^{1/3}$. Fitting $\mu_{mc33}(A,\boldsymbol{X};\boldsymbol{\beta})$ was not feasible in this case. The RG_{rf} and *AIPWE* methods were also implemented.

The results for Case D, given in the 8th and 9th columns, differ from those of Case A. Here the *RG* method with a simple misspecified linear model has properties as good as those from *AIPWE* using this misspecified model and better than the *IPWE* method. Again we see that both *RG* and *AIPWE* methods are improved by the use of random forests. The general performance of all the methods, is clearly worse when there are more covariates.

3.3 Results For Non-randomized Trial Setting

For this situation the *RG* methods are unchanged, but the *IPWE* and *AIPWE* require formulating and fitting an additional model for $P(A=1|\boldsymbol{X})$.

In the first simulation scenario presented by Zhang et al (2012), they generated data from a true model of the form $Y_i = \mu(A_i, \boldsymbol{X}_i) + \varepsilon_i$, where $\varepsilon_i \sim N(0, 1)$ and

$$\mu(A, \boldsymbol{X}) = exp\left\{2.0 - 1.5X_1^2 - 1.5X_2^2 + 3.0X_1X_2 + A(-0.1 - X_1 + X_2)\right\} \quad (9)$$

where X_1 and X_2 were independent $U(-1.5, 1.5)$. The treatment group indicator A_i was determined by the model logit $\{P(A=1|\boldsymbol{X})\} = -0.1 + 0.8X_1^2 + 0.8X_2^2$.

For model (9) $g^{opt}(\boldsymbol{X}) = I(-0.1 - X_1 + X_2 > 0)$, and $E\{Y(g^{opt})\} = 3.71$. Two regression models for $\mu(A, \boldsymbol{X})$ were considered, a correctly specified model of the form

$$\mu_t(A, \boldsymbol{X};\boldsymbol{\beta}) = exp\left\{\beta_0 + \beta_1X_1^2 + \beta_2X_2^2 + \beta_3X_1X_2 + A(\beta_4 + \beta_5X_1 + \beta_6X_2)\right\}$$

and a misspecified simple linear model of the form

$$\mu_{msl}(A, \boldsymbol{X};\boldsymbol{\beta}) = \beta_0 + \beta_1X_1 + \beta_2X_2 + A(\beta_3 + \beta_4X_1 + \beta_5X_2).$$

The misspecified model μ_{ms33} is $E(Z) = \beta_0 + \beta_1X_1 + \beta_2X_2 + A(\beta_3 + \beta_4X_1 + \beta_5X_2)$ and misspecified model μ_{mc33} is

$$E(Z) = \beta_0 + \beta_1X_1 + \beta_2X_2 + A(\beta_3 + \beta_4X_1 + \beta_5X_2) + \sum_{j=1}^{2}\beta_{j+5}X_j^2 + \beta_8X_1 \times X_2$$

where $Z = Y^{1/3}$. Method rf applies the random forest approach. The input covariates used were $A, X_k, X_k^2, X_kI(A=1)$ and $X_kI(A = 0)$ for $k = 1$ to 2, and the response variable is Z.

The propensity score model for $P(A=1|\boldsymbol{X})$, required for *IPWE* and *AIPWE*, is either correctly specified as logit $\{P(A=1|\boldsymbol{X})\} = \gamma_0 + \gamma_1X_1^2 + \gamma_2X_2^2$ or incorrectly specified as logit $\{P(A = 1|\boldsymbol{X})\} = \gamma_0 + \gamma_1X_1 + \gamma_2X_2$.

The results show that the use of random forests in the *RG* methods is as good as any other method, and that poorly fitting parametric models for both *RG* and for *AIPWE* when the propensity score model is incorrect can lead to regimes with noticeably worse properties.

4. Discussion

The Zhang et al (2012) paper illustrates that regression methods may not be robust to model misspecification, and that *AIPWE* methods do have an appealing robustness property. However, this robustness property shouldn't be an excuse for not seeking reasonably fitting models for the data. We demonstrate in a small simulation study, that modeling the response for the regression method with a better fitting parametric model leads to some improvement, while using a readily available non-parametric method removes concerns about non-robustness of the regression method. Furthermore, the properties of both regression and augmented inverse probability weighted methods are improved by using the non-parametric method for the response compared to parametric models, and are quite similar. Thus the extra modeling needed for the AIPWE is not doing any harm, but also may not be necessary.

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References

Foster JC, Taylor Jeremy MG, Ruberg Stephen J. Subgroup identification from randomized clinical trial data. Statistics in Medicine. 2011; 30:2867–2880. [PubMed: 21815180]

Zhang B, Tsiatis AA, Laber EB, Davidian M. A robust method for estimating optimal treatment regimes. Biometrics. 2012; 168:1010–1018. [PubMed: 22550953]

Table 1

Simulation results, randomized studies. Ratio to optimal is $Q\left(\hat{g}\right)/Q\left(g^{opt}\right)$. Fraction treated denotes the fraction that would be treated in a future population if regime $\hat{g}$ was followed. Case A: 3 independent covariates. Optimal fraction treated = 0.5. Case B: 3 independent covariates. Optimal fraction treated = 0.902. Case C: 3 independent covariates. Optimal *g* includes interaction. Optimal fraction treated = 0.5. Case D: 15 independent covariates. Optimal fraction treated = 0.500. The subscripts denote the model that was used for estimating $\mu(A,\boldsymbol{X})$, t=true, msl=misspecified simple linear, ms33=misspecified simple with $Y^{1/3}$, mc33=misspecified complex with $Y^{1/3}$, rf=random forest

Method	**Case A p=3, indep**		**Case B p=3, indep**		**Case C p=3, interaction**		**Case D p=15, indep**	
	Ratio to optimal	**Fraction treated**	**Ratio to optimal**	**Fraction treated**	**Ratio to optimal**	**Fraction treated**	**Ratio to optimal**	**Fraction treated**
Assuming form of true model is known								
RG_t	1.000	0.50	0.999	0.90	1.000	0.50	1.000	0.50
$AIPWE_t$	0.997	0.50	0.995	0.90	0.996	0.51	0.997	0.50
Form of model is unknown								
RG_{msl}	0.925	0.67	0.946	0.93	0.922	0.63	0.893	0.62
RG_{ms33}	0.927	0.58	0.936	0.91	0.924	0.56	0.883	0.55
RG_{mc33}	0.948	0.58	0.936	0.91	0.942	0.57		
RG_{rf}	0.990	0.53	0.990	0.92	0.977	0.53	0.904	0.68
IPWE	0.971	0.49	0.970	0.89	0.956	0.50	0.879	0.58
$AIPWE_{msl}$	0.984	0.50	0.977	0.87	0.965	0.52	0.884	0.62
$AIPWE_{ms33}$	0.978	0.49	0.970	0.90	0.963	0.50	0.885	0.59
$AIPWE_{mc33}$	0.989	0.49	0.966	0.91	0.976	0.50		
$AIPWE_{rf}$	0.990	0.50	0.992	0.89	0.976	0.50	0.896	0.60

Table 2

Simulation results, nonrandomized studies. Ratio to optimal is $Q(\hat{g})/Q\left(g^{opt}\right)$. Fraction Treated denotes the fraction that would be treated in a future population if regime $\hat{g}$ was followed. Two independent covariates. Optimal fraction treated = 0.5. The subscripts denote the model that was used for estimating $\mu(A, \boldsymbol{X})$, t=true, msl=misspecified simple linear, ms33=misspecified simple with $Y^{1/3}$, mc33=misspecified complex with $Y^{1/3}$, rf=random forest

Method	Ratio to optimal	Fraction treated
$RG\mu_t$	1.000	0.47
$RG\mu_{msl}$	0.878	0.25
$RG\mu_{ms33}$	0.861	0.18
$RG\mu_{mc33}$	0.936	0.50
RG_{rf}	0.994	0.49
Propensity score model correct		
IPWE	0.979	0.47
$AIPWE\mu_t$	0.998	0.47
$AIPWE\mu_{msl}$	0.988	0.47
$AIPWE\mu_{ms33}$	0.984	0.47
$AIPWE\mu_{mc33}$	0.991	0.46
$AIPWE_{rf}$	0.995	0.47
Propensity score model incorrect		
IPWE	0.921	0.33
$AIPWE\mu_t$	0.998	0.47
$AIPWE\mu_{msl}$	0.961	0.39
$AIPWE\mu_{ms33}$	0.934	0.35
$AIPWE\mu_{mc33}$	0.982	0.42
$AIPWE_{rf}$	0.995	0.47