

High-seas fish wars generate marine reserves

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Edited by Alan Hastings, University of California, Davis, CA, and approved January 29, 2016 (received for review September 17, 2015)

The effective management of marine fisheries is an ongoing challenge at the intersection of biology, economics, and policy. One way in which fish stocks—and their habitats—can be protected is through the establishment of marine reserves, areas that are closed to fishing. Although the potential economic benefits of such reserves have been shown for single-owner fisheries, their implementation quickly becomes complicated when more than one noncooperating harvester is involved in fishery management, which is the case on the high seas. How do multiple self-interested actors distribute their fishing effort to maximize their individual economic gains in the presence of others? Here, we use a game theoretic model to compare the effort distributions of multiple noncooperating harvesters with the effort distributions in the benchmark sole owner and open access cases. In addition to comparing aggregate rent, stock size, and fishing effort, we focus on the occurrence, size, and location of marine reserves. We show that marine reserves are a component of many noncooperative Cournot-Nash equilibria. Furthermore, as the number of harvesters increases, (i) both total unfished area and the size of binding reserves (those that actually constrain behavior) may increase, although the latter eventually asymptotically decreases; (ii) total rents and stock size both decline; and (iii) aggregate effort used (i.e., employment) can either increase or decrease, perhaps nonmonotonically.

fisheries management | game theory | marine protected areas | marine reserves | spatial bioeconomic model

Any harvested biological resources, including fish, are mobile. Organisms in different locations experience different ecological conditions (e.g., food availability or predation rates), and harvesters experience different economic conditions (e.g., costs, price, or efficacy of the harvest technology). The dynamic value of a unit of biomass (its in situ, or "shadow," value) is, therefore, location-dependent. Because it is only economically efficient to remove units of biomass for which instantaneous net benefits exceed in situ value, it matters where harvest is done in addition to harvest intensity (1). Fishery analysts and managers have, thus, become increasingly aware of the spatial dynamics of exploited resources, the associated response by harvesters, and the portfolio of regulatory instruments that can be used to mitigate market failure along this dimension (2).

Of particular interest is the role that closed areas (also known as marine protected areas or marine reserves) should play in the regulation of fishing (3, 4). There are compelling reasons to impose no-take zones—for example, as a safeguard against environmental volatility or for the preservation of existence values and non-extractive ecosystem services—but we focus here on the potential economic impacts of marine reserves on the harvesting sector. Bioeconomic models have shown that reserves can, depending on context, leave yield unchanged (5), increase sustainable yield (6), or increase economic rents (7–10). However, arbitrarily or randomly placed marine reserves can impose opportunity costs exceeding their benefits (7, 11), and reserves located solely on the basis of biological criteria can engender fierce political opposition because of their short-run welfare costs (12).

Most bioeconomic models of exploited renewable resources assume that exploitation rights fall at one of two opposite ends of a spectrum. At one end is the sole owner case, in which a single harvester fully internalizes all dynamic benefits and costs of harvest activity and thus maximizes economic net benefits in what is often

referred to as an efficient or "first-best" outcome (13). At the other end of the property rights spectrum is open access, in which a large number of harvesters have the nonexclusive right to extract the resource. The lack of property rights leads harvesters to withdraw units of the resource without regard to the dynamic impacts on the stock: they ignore the "stock externality" or the reduction in marginal productivity of effort (or increase in marginal harvest costs) that they impose on each other by depleting the stock. In the extreme, unregulated open access leads to a complete dissipation of economic rents or the so-called "tragedy of the commons" (14), in which society is no better off for the existence of the resource.

In reality, a range of scenarios falls between sole ownership and open access. For example, on the high seas, multiple states interact in a bioeconomic fishing game. These states or "players" can either cooperate in the regulation of their fishing sectors—in which case the result is something much like the sole owner outcome (15, 16)—or interact noncooperatively. In the latter case, each of the states optimizes effort as a function of harvest by the other states. Simultaneous solution of the resulting response functions defines a mutual best response [i.e., a Cournot– Nash equilibrium (17)]. The classic analysis by Levhari and Mirman (18) of a two-player "fishing war" (i.e., a duopoly) shows that the equilibrium outcome falls between the sole owner and open access extremes. Subsequent studies have extended their results in a variety of ways [e.g., to more than two players (19, 20), a system in which the players harvest interacting species (21), and a case where the players are myopic (22; reviews of the subject are in refs. 23 and 24)] but have generally avoided an explicit treatment of spatial dynamics.

To our knowledge, most spatial game theoretic models of fisheries (25–29) are implicitly spatial (30, 31). That is, they either allude to space in the context of interactions between neighboring countries or assume that a given percentage of habitat is equivalent to the same percentage of the stock. This latter assumption, however, is rarely, if ever, correct. Fish habitat is highly

Significance

Marine reserves—areas where fishing is prohibited—have been implemented to conserve fish stocks and their habitats. They have been established in near-shore fisheries, where a single state (or "sole owner") regulates the distribution of fishing effort. Modeling has shown that, under some conditions, the sole owner may also use closed areas to maximize sustainable profit. Here, we show that reserves may also play a role in fisheries management on the high seas, where a limited number of states compete in a noncooperative fishing game. Our theoretical analysis complements recent empirical studies of high-seas protected areas and is relevant in other management contexts characterized by a limited number of harvesters.

Author contributions: G.E.H., H.V.M., and M.G.N. designed research, performed research, and wrote the paper.

The authors declare no conflict of interest

This article is a PNAS Direct Submission

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This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1518509113/-/DCSupplemental.

heterogeneous: the rates of reproduction, survival, and movement vary tremendously from place to place. In addition, because fish move, fishing not only reduces the stock size but also changes its spatial distribution. We hypothesize that the endogenous location of the stock, combined with rational (profitmaximizing) reallocations of harvest inputs in space, can have important implications for the outcomes of such systems. Furthermore, the action space of players in a spatially explicit game takes on another dimension, physical space, which adds richness and realism to the problem while increasing its complexity.

In this analysis, we consider a fishery with explicit spatial dynamics governed by a reaction-diffusion process (6, 8). This resource is harvested by a number, h, of identical states, each of which regulates the spatial distribution of its own fishing fleet's effort. We consider a range of participating states in this fishing game from a single sole owner state (h=1) to open access (h large). Our main objective is to better understand the impact of the number of states on the spatial distribution of effort. In particular, we wish to see whether marine reserves are part of the noncooperative effort distribution and, if so, how much habitat is set aside from harvest compared with the sole owner and open access extremes. We also show how important metrics of system performance (equilibrium economic rents, stock levels, and aggregate effort) are affected by the number of participants.

We begin by considering a stock that lives within a one-dimensional habitat. A simple model for the dynamics of such a stock is the so-called Fisher-KPP equation (32, 33):

$$\frac{\partial N}{\partial t} = rN\left(1 - \frac{N}{K}\right) + D\frac{\partial^2 N}{\partial x^2}.$$
 [1]

In this equation, the variable N(x,t) represents the stock density (i.e., the number of individuals per unit length) at location x and time t. The stock density changes as a result of three processes: birth, death, and dispersal. The parameter r is the maximum per capita population growth rate. The realized growth rate at a given location decreases linearly with population density. If the population density is equal to the carrying capacity K, the population growth rate is zero. The second term on the right side of Eq. 1 describes the population-level effect of the movement of individuals as diffusion, where D is the diffusion coefficient.

In general, the environment in which fish live is heterogeneous (i.e., r and K are both functions of x). The ability to account for such heterogeneity is one of the primary reasons to use a spatial model. We assume the simplest form of biological spatial heterogeneity: that fish cannot survive outside of a finite stretch of suitable habitat of length L. Dirichlet boundary conditions of the form

$$N(0,t) = N(L,t) = 0$$
 [2]

capture this heterogeneity.

Together, model (Eq. 1) and boundary conditions (Eq. 2) have played a central role in theoretical population ecology, and much is known about the dynamics of this model (34, 35). In particular, if the habitat size L is less than a critical size $L_c = \pi \sqrt{D/r}$, the population will not persist (36, 37). For a small habitat (but larger than L_c), the equilibrium population density reaches its maximum at the center of the habitat (at x = L/2). For very large habitat sizes, the equilibrium population density at locations away from the boundaries is approximately equal to the carrying capacity. For the remainder of this paper, we choose $L > L_c$.

Let us assume that fish living in this habitat are at risk for being caught by the fishing fleets of h identical states. If $E_i(x,t)$ is the spatial distribution of the ith state's fishing effort, we can include the effects of harvesting on the stock by amending Eq. 1 as

$$\frac{\partial N}{\partial t} = rN\left(1 - \frac{N}{K}\right) + D\frac{\partial^2 N}{\partial x^2} - qN\sum_{i=1}^h E_i(x, t).$$
 [3]

The costs and benefits that accrue to each state depend on the state's own effort distribution as well as the effort distributions of all of the other states. We assume that states catch fish at a rate that is proportional to their effort and the stock density. The proportionality constant q is called the "catchability coefficient" (13). Given a fixed price p, the revenue generated by the fleet of state i in the interval dx is, thus,

$$R_i(E_i(x,t), N(x,t)) dx = p q E_i(x,t) N(x,t) dx.$$
 [4]

Using the notation $\mathbf{E}(x,t) = [E_1(x,t), E_2(x,t), \dots, E_h(x,t)]'$, we can write the cost of fishing to state i as

$$C_i(\mathbf{E}(x,t)) dx = (w_0 + w_1 || \mathbf{E}(x,t) ||_1) E_i(x,t) dx,$$
 [5]

where w_0 is the cost of effort, and w_1 is the additional cost per unit effort associated with interference between harvesters when they try to fish in the same location. We define the difference between R_i and C_i as the rent density for state i:

$$\rho_i(\mathbf{E}(x,t),N(x,t)) = R_i(E_i(x,t),N(x,t)) - C_i(\mathbf{E}(x,t)).$$
 [6]

The rent density and its derivatives play an important role in our understanding of marine reserves.

If every state has complete information about the status of the stock as well as the costs and effort levels of the other states, then each state can regulate the spatial and temporal distribution of its own fleet's effort to maximize the present value of its private rent given the effort distributions of its competitors' fleets. In particular, state i maximizes

$$\Pi_i(\mathbf{E}, N) = \int_0^\infty \int_0^L \rho_i(\mathbf{E}(x, t), N(x, t)) e^{-at} dx dt$$
 [7]

by choosing the effort distribution $E_i(x,t)$ subject to the constraint $E_i(x,t) \ge 0$. The positive constant a is the discount rate.

The outcome of these simultaneous maximizations is a Cournot-Nash equilibrium, at which no state can do better by unilaterally changing its distribution in space or time. After a sufficient period, we expect this noncooperative equilibrium to be characterized by a steady state, at which neither the spatial distribution of the stock nor the effort distributions of the various states change in time. We derive this steady state numerically (Materials and Methods), and report on its properties.

Results

The steady-state behavior of our model depends on the values of 10 parameters $(r, K, D, q, h, L, p, c_0, c_1, and a)$. This number is significantly reduced and our analysis is correspondingly simplified by rescaling the variables by

$$u = \frac{N}{K}$$
, $\tau = rt$, $\xi = x\sqrt{\frac{r}{D}}$, $f_i = \left(\frac{q}{r}\right)E_i$, and $\pi = \frac{\Pi}{(pK\sqrt{rD})}$ [8]

and introducing the four (dimensionless) parameters $\ell = L\sqrt{r/D}$, $c_0 = w_0/(pqK)$, $c_1 = rw_1/(pq^2K)$, and $\delta = a/r$ (Materials and Methods). These four parameters summarize the bioeconomic setting. Small values of ℓ correspond to a species living in a relatively small habitat with a large diffusion coefficient or small population growth rate; c_0 is the cost per unit effort when effort is small relative to the revenue per unit effort when the stock is at its carrying capacity. All else equal, large values of c_0 represent a less profitable resource. For the results that follow, c_1 , which measures the negative effects of interference between harvesters, is held constant at 0.01, and the discount rate is held at $\delta = 0.03$. Our results are qualitatively insensitive to modest changes of these parameters (SI Text).

Because each of the h states is identical, the effort distributions of their fleets are also identical. To illustrate how outcomes vary with cost, habitat size, and number of participating states, we found the steady state of the Cournot–Nash equilibrium for a set of representative parameter choices (*Materials and Methods*). For each of two habitat sizes (one small and one large), we considered both low- and high-cost cases. We analyzed outcomes for 1 (sole owner) to 100 states as well as the infinite player (open access) case. As the number of players increases, the spatial distributions of fish and fishing effort change qualitatively (Fig. 1). The aggregate (integrated across space) effort, stock, rent, and reserve area also change, sometimes nonmonotonically, from sole owner to open access values (Fig. 2).

Marine reserves (areas that are unfished at steady state) arise in our analysis for two reasons. First, they may occupy areas that are unprofitable to fish [where $u(x) - c_0 < 0$]. States do not need to regulate the effort of their fleets in these areas. Alternatively, reserves may arise when a local closure has the benefit of increasing equilibrium harvest elsewhere. We term these latter closures "binding reserves"; because of local positive marginal profit density at steady state, they require regulatory oversight by the states of their own fleets. These binding reserves can further be divided into central reserves that protect especially valuable units of stock and peripheral "flux inhibiting" reserves that have the effect of slowing the net diffusion of stock from profitable

areas into unprofitable ones by reducing the stock density gradient. The size of reserves depends on the interference cost c_1 . Specifically, the lower this interference cost, the smaller the penalty for concentrating effort in a single location; thus, for smaller interference costs, distributions of effort are more concentrated in space, and as a consequence, reserves are larger (compare Fig. 1 with Fig. S1). Binding closures persist even when more than one noncooperating state fishes the same habitat (Fig. 1).

Equilibrium outcomes differ substantially between the two property rights extremes (8) (compare Fig. 1 A, E, F, J, K, O, P, and T with Fig. 2, left and right plot edges). Whereas a sole owner maintains binding reserves as part of a rent-maximizing strategy, these reserves disappear in the open access case. Under open access conditions, harvesters tend to pile up their effort toward the center of the habitat, and the size of unprofitable, nonbinding reserves at the habitat edges increases. Furthermore, under open access conditions, total effort and stock size are lower than in the sole owner case, and profits are completely dissipated.

One might expect a smooth transition in effort and stock distribution, as well as in aggregate metrics, as the number of harvesting states increases from sole owner to open access extremes. Indeed, for some parameter choices, smooth transitions occur (e.g., Fig. 1 F–J). In this case, as the number of participants increases, stock and rent decline (Fig. 2 D and E). Total effort increases (Fig. 2D) and becomes increasingly centralized (Fig. 1 F–J), and although declines in profitability accompany an increase in the total unfished area, the size of binding reserves decreases (Fig. 2F).

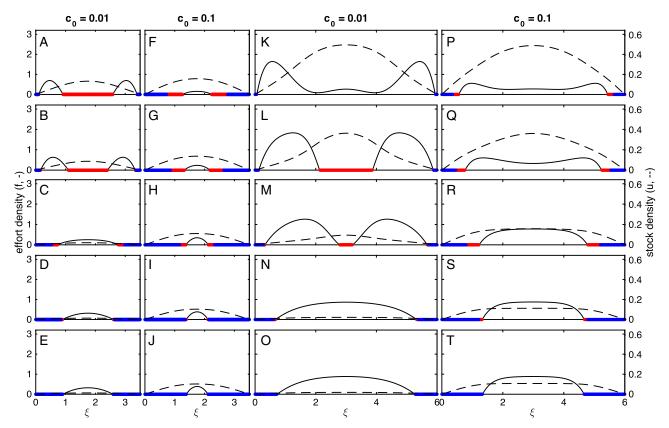


Fig. 1. Distribution of stock $u(\xi)$ and fishing effort $\sum_i f_i(\xi)$ over space (ξ) . In A–J, ℓ = 3.5; in K–T, ℓ = 6. For A–E and K–O, c_0 = 0.01; for F–J and P–T, c_0 = 0.1. The numbers of states are (A, F, K, and P) 1, (B, G, L, and Q) 2, (C, H, M, and R) 10, (D, I, N, and S) 100, and (E, J, O, and T) infinite (open access). Locations of binding reserves are highlighted in red, and nonbinding reserves are in blue. For all panels, c_1 = 0.01, and δ = 0.03.

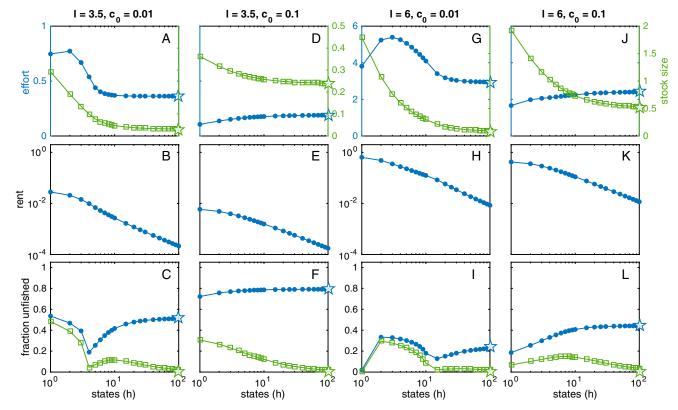


Fig. 2. (A, D, G, and J) Total effort (blue) and stock size (green), (B, E, H, and K) rent, and the fraction of habitat that is (C, F, I, and L) unfished (blue) and in binding reserves (green) as functions of the number of states. Values under open access conditions are indicated by stars. Note that, by definition under open access, rents are completely dissipated (i.e., equal to zero), and therefore, no stars are shown in B, E, H, and K. Each column corresponds to parameter choices (habitat size and cost per unit effort) in Fig. 1. For all panels, c_1 = 0.01, and δ = 0.03.

A similarly smooth transition in effort and stock density distributions from sole owner to open access is shown in Fig. 1 A-E and P-T. For large habitats in which fishing is costly (Fig. 1 P-T), binding reserve size is relatively larger for an intermediate number of states but ultimately declines as the number of states increases (Fig. 2L). In the smaller habitat, the centralization of effort during this transition results in the disappearance of the central marine reserve (Fig. 1 A-E) and a sharp change in the relationship between binding reserve size and the number of fishing states (Fig. 2C).

Interestingly, in some cases where a sole owner does not maintain a binding central reserve, two or more noncooperating states may maintain one—for example, in the large-habitat, lowcost case (Fig. 1 K-O). Binding reserves in this case persist even as dozens of independent states enter the system, although if the number of states is large enough (and certainly in the open access equilibrium), these reserves ultimately disappear (Fig. 2 C, F, I, and L). An increase in the discount rate has a similar effect as an increase in the number of states on the distribution of effort (Fig. S2).

The intuition behind this result rests on density-dependent dynamics. In the large-habitat, low-cost case, the sole owner does not use a central closure; if this area was closed, the central stock density would equilibrate close to the carrying capacity, and too much productivity would be lost to density-dependent effects. A reserve only appears as part of the steady-state Cournot-Nash equilibrium as the number of harvesting states increases, the stock density is correspondingly depleted, and the density dependence is reduced. This central reserve ultimately disappears when h is sufficiently large, resulting in a sharp change in the relationship between binding reserve size and the number of states (green lines in Fig. 2 C and I).

Discussion

By incorporating diffusive spatial dynamics into a logistic growth model, we extend the noncooperative fishing game by Levhari and Mirman (18) into a spatially explicit context (38). This extension allows us to show that marine reserves are maintained, even when multiple self-interested harvesters compete for use of the same fishing grounds. This characterization of strategic behavior in space constitutes a significant theoretical advance, linking together the theory of commons games with that of spatial bioeconomics. The results improve our understanding of the circumstances under which protected areas can be justified on grounds of efficiency and incorporate space as another dimension (i.e., in addition to the quantity of effort) of the strategic interaction between participants.

In our spatial version of the "fish war," changes in the habitat size, the cost of effort, and the number of participants affect the equilibrium levels of total harvest, stock abundance, and rents, which one observes in a nonspatial model (18). However, these parameters also affect the spatial distribution of fishing effort and stock. Each state's response function implicitly accounts for the impact of its competitor's effort allocation on the spatial dynamics of the resource.

Marine reserves are part of the noncooperative equilibrium over a wide range of scenarios. Their size, location, and need for enforcement (i.e., binding vs. nonbinding) vary with the number of states. We identified three distinct kinds of "closed areas": central reserves that protect "sources" of biomass to be harvested later in adjacent fished "sinks," flux inhibiting reserves that mitigate costly dispersal out of profitable fishing habitat, and economic reserves that are unfished because of lack of interest on the part of the harvesters because the marginal rent per unit effort is negative in these locations. The first two types of closure

would require regulatory oversight by each state of their own fleet, whereas the third would not. Regardless, all three forms of marine reserves are noteworthy because of the many benefits of reserves for which we do not explicitly account here. For example, reserves serve as a guard against environmental volatility, a means of augmenting the stock dynamics of other species that might be disturbed by fishing activity, and a means of supporting other economically valuable activities, such as recreational fishing and other forms of tourism (39, 40).

The absence of property rights appears not only in high-seas fisheries but also at smaller scales, such as in limited entry coastal fisheries within a nation's territorial waters. If the governing body in such a case were to limit the number of harvesters (i.e., to fix h) but not directly regulate the amount or location of fishing effort, the harvesters might find it in their interests to leave an area unfished-even when a sole owner might fish there. Similarly, marine reserves may emerge locally within territorial use rights fisheries (TURFs) (reviewed in ref. 41) when the costs of cooperation among TURF owners are low (30); in such cases, finite allocation of spatial property rights effectively restricts entry to the fishery. However, because stock movement across TURF boundaries erodes property rights, generally topdown imposition of reserves is necessary to improve stock health and profits (30, 31). Our results are distinct from such analyses in that reserves emerge even in the complete absence of property rights.

Finally, our theoretical results also corroborate recent empirical analyses (31, 42) that suggest that the imposition of a closure of the high seas to fishing could be beneficial or at least not that costly to a collection of self-interested states exploiting a shared mobile stock. We show here that, for a large habitat, a closure might even emerge in the absence of any coordinated regulatory action (such as a treaty). Although our model assumes that all states are equal (e.g., have identical production functions, which is likely not the case in real world systems), it nevertheless suggests that convincing a collection of states to agree to such a closure might be easier than otherwise expected. The fact that marine reserves can prove economically beneficial even under noncooperative circumstances should strengthen motivation for accords on high-seas closures (31).

Materials and Methods

Rescaling. Applying the substitutions in Eq. 8 transforms Eq. 3 to

$$\frac{\partial u}{\partial \tau} = u \left[1 - u - \| \mathbf{f}(\xi, \tau) \|_1 \right] + \frac{\partial^2 u}{\partial \xi^2},$$
 [9]

with the boundary conditions

$$u(0,\tau) = u(\ell,\tau) = 0.$$
 [10]

The rent density (Eq. 6) becomes

$$\varrho(\mathbf{f}(\xi,\tau), u(\xi,\tau)) = [u(\xi,\tau) - c_0 - c_1 || \mathbf{f}(\xi,\tau) ||_1] f_i(\xi,\tau),$$
 [11]

and the discounted present value of the stream of profits earned by state i becomes

$$\pi_{i}[\mathbf{f}(\xi,\tau),u(\xi,\tau)] = \int\limits_{0}^{\infty}\int\limits_{0}^{\ell}\varrho(\mathbf{f},u)e^{-\delta\tau}d\xi\,d\tau. \tag{12}$$

Given the other states' effort distributions, state i attempts to maximize π_i by choosing $f_i(\xi,\tau)$ for $0 < \xi < \ell$ and $\tau > 0$.

Analysis. To solve the *i*th state's profit maximization problem, we first discretize the spatial coordinate, so that $\xi_j = (j-1)\Delta\xi$, and we approximate the partial differential equation (Eq. 9) with the system of ordinary differential equations:

$$\frac{du_j}{d\tau} = u_j \left[1 - u_j - \sum_{i=1}^h f_{ij} \right] + \mathbf{Q}[u_j]$$
 [13]

for $1 \le j \le m$ and $m = \ell/(\Delta \xi)$. Here, $u_j(\tau) = u(\xi_j, \tau)$, $f_{ij}(\tau) = f_i(\xi_j, \tau)$, and $\mathbf{Q}[u_j] = (u_{j-1} - 2u_j + u_{j+1})/(\Delta \xi)^2$. Under this approximation, Eq. 12 is

$$\pi_{i} = \int_{0}^{\infty} \sum_{j=1}^{m} \left[u_{j} - c_{0} - c_{1} \sum_{k=1}^{h} f_{kj} \right] f_{ij} e^{-\delta \tau} \Delta \xi d\tau.$$
 [14]

The maximization problem for state i now consists of choosing $f_{ij}(\tau)$ at every j subject to the state equations (Eq. 13). This problem is a standard problem in optimal control theory, to which we may apply Pontryagin's maximum principle (43). Because $\Delta \xi$ is a constant, we maximize $\pi_i/\Delta \xi$, which is computationally convenient.

The Hamiltonian for system equations (Eqs. 13 and 14) is

$$H_{i}[f_{i1}, \dots, f_{im}, u_{1}, \dots, u_{m}] = \sum_{j=1}^{m} \left\{ \left[u_{j} - c_{0} - c_{1} \sum_{k=1}^{h} f_{kj} \right] f_{ij} e^{-\delta \tau} + \lambda_{ij} \left[u_{j} \left(1 - u_{j} - \sum_{k=1}^{h} f_{kj} \right) + Q[u_{j}] \right] \right\}.$$
 [15]

Differentiating H_i with respect to the state variables, we find that the adjoint equations are given by $d\lambda_{ii}/d\tau = -\partial H_i/\partial u_i$ or

$$\frac{d\lambda_{ij}}{d\tau} = -\left(f_{ij}e^{-\delta\tau} + \lambda_{ij}\left[1 - 2u_j - \sum_{k=1}^h f_{kj}\right] + \mathbf{Q}\left[\lambda_{ij}\right]\right).$$
 [16]

The boundary conditions (Eq. 10) imply

$$u_1(\tau) = u_{m+1}(\tau) = \lambda_{i1}(\tau) = \lambda_{i,m+1}(\tau) = 0.$$
 [17]

The substitution of $\lambda_{ij} = e^{-\delta\tau} \mu_{ij}$ into Eq. 16 produces the equivalent autonomous system

$$\frac{d\mu_{ij}}{d\tau} = \delta\mu_{ij} - \left(f_{ij} + \mu_{ij} \left[1 - 2u_j - \sum_{k=1}^h f_{kj}\right] + \mathbf{Q}\left[\mu_{ij}\right]\right)$$
[18]

with

$$\mu_{i1}(\tau) = \mu_{i,m+1}(\tau) = 0.$$
 [19]

Finding nonnegative functions $f_{ij}(\tau)$ that together maximize π_i amounts to finding solutions to the state equations (Eq. 13) and the adjoint equations (Eq. 18) that satisfy the boundary conditions (Eqs. 10 and 19) and simultaneously maximize the Hamiltonian (Eq. 15). To maximize H_{ii} , the solutions must satisfy the necessary conditions:

$$\frac{\partial H_i}{\partial f_{ij}} = u_j \left(1 - \mu_{ij} \right) - c_0 - c_1 \left(f_{ij} + \sum_{k=1}^h f_{kj} \right) = 0.$$
 [20]

Conditions (Eq. 20) hold for each of the h states (that is, for i = 1, 2, ..., h and for j = 1, 2, ..., m).

We would seem to have a system of 2hm equations to solve. Note, however, that, because the states are identical, the shadow price μ_{ij} depends on the location ξ_j but not on the identity of the state. That is, $\mu_{ij}(\tau) = \mu_{kj}$ for all i and k values. It then follows from Eq. 20 that $\partial H_i/\partial f_{ij} - \partial H_s/\partial f_{sj} = c_1(f_{kj} - f_{ij}) = 0$. Thus, $f_{ij} = f_{kj}$ for all i and k values. All states use the same effort distribution at the Cournot–Nash equilibrium. Using this fact simplifies Eq. 20, and we find that

$$f_{ij}(\tau) = \max \left\{ \frac{u_j(1-\mu_{ij}) - c_0}{(1+h)c_1}, 0 \right\}.$$
 [21]

We are interested in steady-state solutions (i.e., solutions with $d\mu_{ij}/d\tau$, $du_j/d\tau$, and $df_{ij}/d\tau$ all zero). To find these solutions, we substitute Eq. **21** into Eqs. **13** and **18** and solve them numerically.

ACKNOWLEDGMENTS. This work was partially supported by a US National Science Foundation Graduate Research Fellowship (to H.V.M.), a US National Science Foundation Postdoctoral Research Fellowship in Biology DBI-1401332 (to H.V.M.), a Thomas B. Wheeler Award for Ocean Science and Society from the Woods Hole Oceanographic Institution (to M.G.N.), and US National Science Foundation Grants CNH-0707961 (to G.E.H.), OCE-1031256 (to M.G.N.), DEB-1257545 (to M.G.N.), and DEB-1145017 (to M.G.N.).

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