Statistical modeling for selecting housekeeper genes

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Abstract

There is a need for statistical methods to identify genes that have minimal variation in expression across a variety of experimental conditions. These 'housekeeper' genes are widely employed as controls for quantification of test genes using gel analysis and real-time RT-PCR. Using real-time quantitative RT-PCR, we analyzed 80 primary breast tumors for variation in expression of six putative housekeeper genes (*MRPL19* (mitochondrial ribosomal protein L19), *PSMC4* (proteasome (prosome, macropain) 26S subunit, ATPase, 4), *SF3A1* (splicing factor 3a, subunit 1, 120 kDa), *PUM1* (pumilio homolog 1 (*Drosophila*)), *ACTB* (actin, beta) and *GAPD* (glyceraldehyde-3-phosphate dehydrogenase)). We present appropriate models for selecting the best housekeepers to normalize quantitative data within a given tissue type (for example, breast cancer) and across different types of tissue samples.

Background

Genes that exhibit minimal variation in messenger RNA (mRNA) quantity across a variety of cell types and biological conditions provide valuable controls for relative quantification. Normalizing quantitative data with housekeeper genes has many applications, from identifying genes regulated during embryogenesis to developing new cancer diagnostics. Although finding biological significance in gene-expression data can rely heavily on the performance of the housekeeper genes, there is a paucity of information on testing these genes for their suitability for this role.

The copy number of a housekeeper gene should be proportional to the amount of poly(A) RNA present in the sample and this proportion should be maintained across a variety of experimental conditions. As nucleic acids show high absorbance at 260 nm (A_{260}), spectrophotometers provide approximate amounts of total DNA/RNA present in a sample. Using absorbance methods alone, however, gives no information about the type of nucleic acid (for example DNA versus RNA) or contributions from different nucleic acid fractions (for example rRNA versus mRNA). It is assumed that mRNA comprises approximately 1-3% of the total RNA. However, this contribution may change depending on the extraction method used. For instance, column extraction methods provide better exclusion of ribosomal RNA than solvent extraction methods [1]. By combining capillary electrophoresis with absorbance, it is possible to accurately quantify these different fractions [2].

Traditionally, housekeepers have been used in Northern blot analysis to represent the amount of mRNA in the sample and to control for sample loading, blot transfer and probe hybridization. Highly expressed genes serving fundamental roles in the cell are commonly used for this purpose but may not be optimal under certain experimental conditions [3-5]. For

example, the sensitivity and accuracy of northern blot analysis with densitometry may be decreased using a highly expressed housekeeper gene that can saturate the autoradiographic signal [6]. To resolve this problem and compensate for limitations in dynamic range, control genes may be chosen to have a level of gene expression similar to the gene(s) of interest (that is, the test genes).

Microarrays are more practical for genome-wide expression analysis than northern blots [7]. With cDNA microarrays, a common reference sample is usually used to compare the expression of each gene across many experimental sample(s) [8,9]. Because each gene in the experimental sample is directly compared to the same gene in the common reference, housekeeper genes are not necessary for normalization. Microarrays are commonly applied to finding genes with differential expression across experimental conditions, however the data may also be used to identify stably expressed genes that can serve as important controls for northern blot analysis, ribonuclease protection assays and quantitative reverse transcription PCR (RT-PCR). In turn, these other quantitative methods are often used to verify differentially expressed genes identified by microarray [10-12].

Housekeeper genes are often adopted from the literature and used across a variety of experimental conditions, some of which may induce differences in their expression. If unrecognized, unexpected changes in housekeeper expression could result in erroneous conclusions about real biological effects such as responses to drugs. In addition, this type of change would be difficult to detect because most experiments only include a single housekeeper gene. It is difficult to determine whether a given gene has the constitutive property of a housekeeper when the true amount of mRNA in a sample is unknown. As a way round this dilemma, Vandesompele *et al.* postulate that gene pairs that have stable expression patterns relative to each other are proper control genes [13]. An alternative method for quantitative analysis of RT-PCR data that does not require housekeeper genes for normalization is to use global pattern recognition (GPR). For instance, Akilesh *et al.* used a GPR algorithm to search for eligible normalizing genes within an assay plate and then used those genes as controls to identify differentially expressed genes [14].

Although relative quantification using housekeeper genes is a practical method of estimating the expression level of a test gene, the transcript amount in the sample is a summation and the method does not consider transcript differences on a cell-to-cell basis. Fluorescence *in situ* hybridization (FISH) is clinically used to determine absolute DNA copy number (for example, *HER2* amplification) in a cell, but these methods still average the copy number after counting many cells and the technique is expensive and laborious [15]. *In situ* methods for detecting RNA transcripts have been developed but the assays are semiquantitative and subjective [16].

In the work presented here, we applied several models to selecting the best housekeeper genes for breast cancer and give algorithms that can be generalized to find housekeeper genes that are appropriate for normalizing quantitative data within and between tissue types.

Results and discussion
One tissue type
The genes *MRPL19*, *PSMC4*, *SF3A1*, *PUM1*, *ACTB* and *GAPD* were analyzed by real-time quantitative RT-PCR. Starting copy numbers for the six candidate housekeeping genes were measured across 80 primary breast tumor samples. The data are available as Additional data file 1 with the online version of this article. Plots of the raw and log-scaled expression levels (all logarithms in this paper are natural base (e) logarithms) are shown in Figure 1. The breast tumor samples are ordered according to the mean of the log-expression levels of all the genes. It is evident from the plot that for the raw data the variability of within-sample measurements increases with the mean expression, whereas the variability stays approximately the same for all the samples with the log-transformation. In addition, the log-transformation allows us to model fold changes in expression levels in an additive way.

To select the best housekeepers for normalizing data across a single tissue type, we tested three variations of a model (Model 1, a-c) with real-time quantitative RT-PCR data generated from primary breast samples (see Materials and methods for details).

Model 1a
We model the expression y_{ij} of gene j in sample I by

$$\log y_{ij} = \mu + T_i + G_j + \varepsilon_{ij}, \text{ where } \varepsilon_{ij} \sim N(0, \sigma_j^2)$$

where μ is the overall mean (log-) expression, T_i is the difference of the ith tissue sample from the overall average and G_j is the difference of the jth gene from the overall average. The key feature of this model that makes it different from a traditional ANOVA model is that it allows heteroscedastic errors to account for different variability in the genes [17]. The variability around the gene-specific mean log-expression $\mu + T_i + G_j$ is quantified by the error standard deviation σ_j. The Bayesian information criterion (BIC) was used to avoid overfitting the data [18]. Model 1a had the best BIC value and was selected from a range of competing models that included a method with equal error variances (Model 1b in Materials and methods) and a more complex method with correlated errors (Model 1c in Materials and methods).

Using Model 1a, standard deviations were determined to select the best control genes for breast cancer. Table 1 shows that *MRPL19* has the smallest variability across the breast cancer samples and would be the best choice for a single
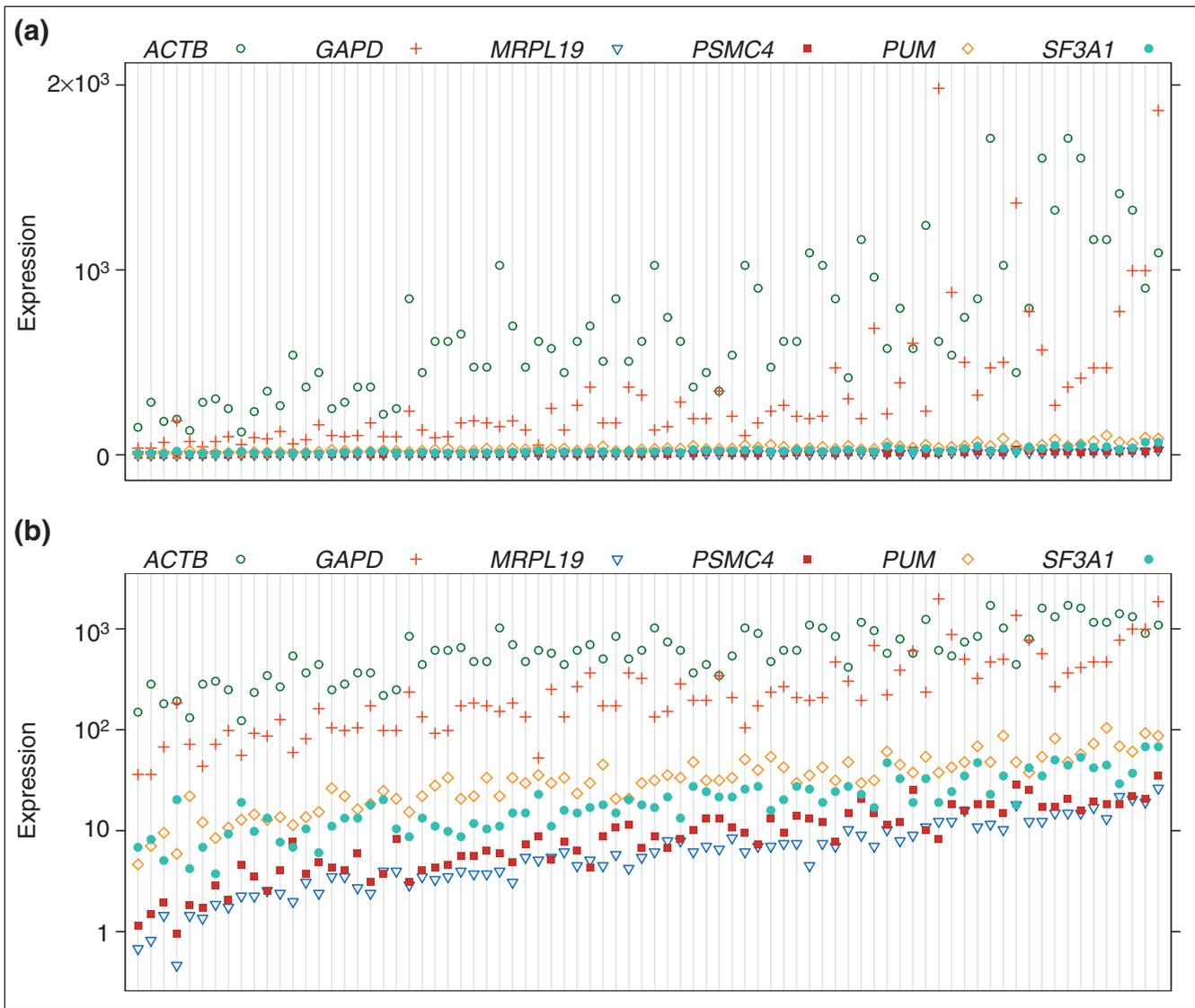
Figure 1
Relative levels of expression determined by real-time quantitative RT-PCR are shown for 6 housekeeper genes in 80 breast tumors. The top panel displays the raw data and the bottom panel displays a log-scale of the data.

housekeeper control. Although some of the confidence intervals overlap, a direct comparison between the genes selected from the microarray (*MRPL19*, *PSMC4*, *PUM1*, *SF3A1*) to the classical housekeepers (*GAPD* and *ACTB*) shows significant difference (*p* = 0.0014).

As the biological function of many genes is still unknown, it is difficult to predict how different experimental conditions may affect the expression of putative housekeeper genes. Thus, a safer approach is to use an average expression of several genes that show small variance across conditions. On the basis of the selected model, the estimate of the variance of the log-average of the expression of several genes can be calculated (see Materials and methods for details). Table 2 shows the standard deviations of the log-average of the best gene set

for each possible set size (that is, 1-6). These standard deviation values are approximately equal to the coefficient of variation in the original scale. From the estimates, the four-gene set of *PSMC4*, *MRPL19*, *PUM1* and *SF3A1* provides the lowest overall variability when choosing a combination of genes. However, this four-gene set is barely different from the three-gene combination of *MRPL19*, *PUM1* and *PSMC4*, which in turn is far better than the best two-gene combination. For economy, and because *SF3A1* had a relatively high individual variability compared to others in the set, our choice for the normalizing set is the geometric mean of the expressions of *MRPL19*, *PUM1* and *PSMC4*.

These findings illustrate the importance of performing an unbiased and genome-wide search for housekeepers rather

Table 1

Standard deviation estimates of log expression using Model 1a for selecting the single best housekeeper gene for breast cancer

Gene	Estimated standard deviation	95% confidence interval
MRPL19	0.218	(0.168, 0.284)
PUM1	0.265	(0.215, 0.328)
PSMC4	0.288	(0.235, 0.352)
SF3A1	0.393	(0.327, 0.472)
ACTB	0.448	(0.376, 0.533)
GAPD	0.519	(0.439, 0.613)

Table 2

Standard deviation estimates of log expression using Model 1a for selecting the best housekeeper gene(s) for breast cancer

| Set size	Gene set	Standard deviation
1	MRPL19	0.2182
2	PUM1, MRPL19	0.1718
3	PSMC4, MRPL19, PUM1	0.1494
4	PSMC4, MRPL19, PUM1, SP3A1	0.1490
5	PSMC4, MRPL19, SF3A1, PUM1, ACTB	0.1491
6	PSMC4, MRPL19, SF3A1, PUM1, GAPD, ACTB	0.1513

than relying on traditional housekeeper genes. We used microarray data to select genes with low variability in expression across breast tumors and cell lines. Because the quantitative differences between the microarray and RT-PCR platforms are relative, genes with low variability in expression across tumors by microarray should also show low variability in expression by RT-PCR. Although the quantitative data from microarray tends to have an overall smaller dynamic range compared to RT-PCR, this is primarily due to loss of information from genes expressed at low levels. Our microarray dataset was filtered to remove genes with signals near background noise.

The result is very similar using Vandesompele *et al.*'s *M* value method, with only the positions of *PUM1* and *PSMC4* changing in stability rank. It should be noted that the *M*-value method does not order the two best genes (*MRPL19* and *PSMC4*). Their best gene-set selection approach would suggest using the (log-scale) average of these two best genes as a control. Such a concordance is not surprising given the close relationship between the *M* value and our model using the variability of the average of several genes (see Materials and methods for details). A benefit of our approach is the ability to compare the variability of individual genes to that of an average of several genes.

Multiple tissue types

Gene(s) with minimal variation in expression across different cell types serve as good 'universal' housekeepers. A universal control may be a single gene or combination of genes. While the former should display both low variability within a given tissue type and consistent basal levels of expression across tissue types, the latter may comprise a gene set with individually different, but complementary, basal expression levels across tissue types.

To test our models for selecting universal housekeepers, we used published data from Vandesompele *et al.* [13]. They measured the expression level of 10 genes in neuroblastoma cell lines (NB), cultured normal fibroblasts (FIB), normal leukocytes (LEU) and cells from normal bone marrow (BM). In addition, normal tissues from pooled organs (breast, brain, fetal brain, heart, kidney, uterus, lung, trachea and small intestine) were also profiled. A plot of these housekeepers across the different tissues is shown in Figure 2. It is notable that a gene can have stable expression within a given tissue type but can change rank position compared to other housekeepers across tissues. For example, *GAPD* has relatively high expression in fibroblasts compared to other housekeepers but low expression in leukocytes. Thus, *GAPD* may be a good single housekeeper within certain tissue types but may not be
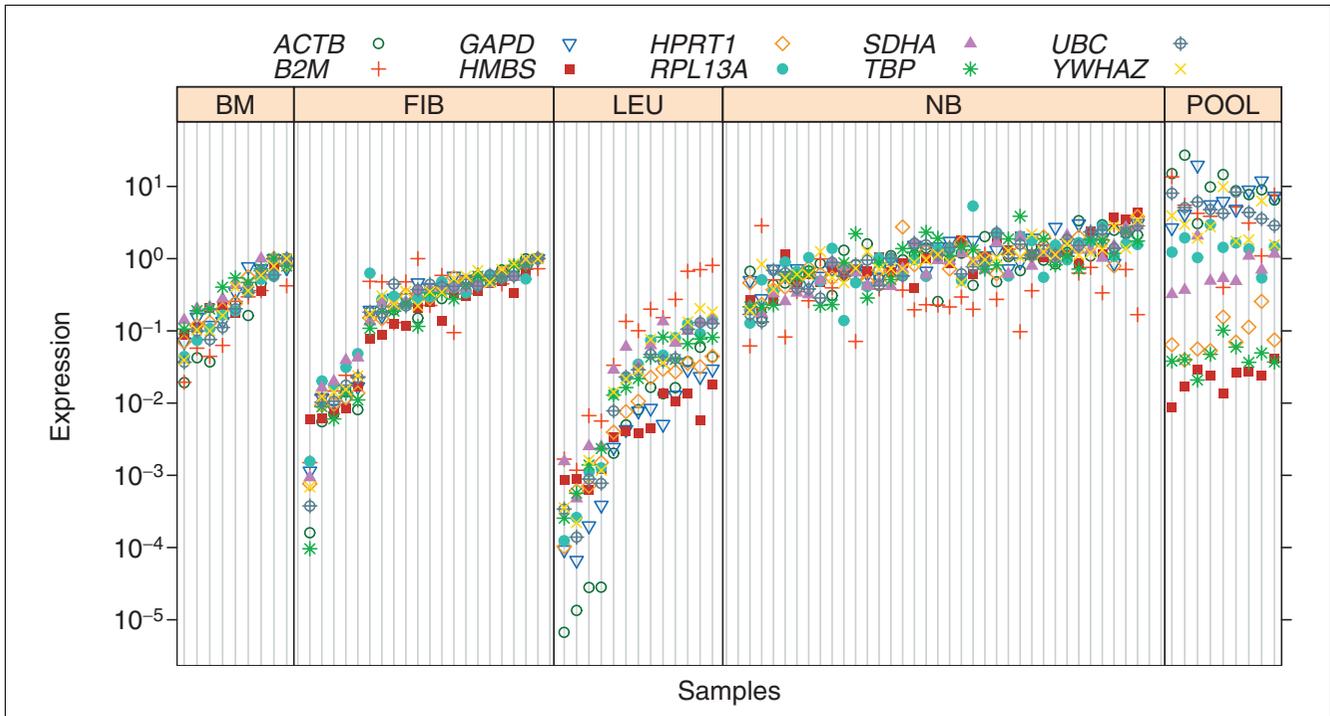
Figure 2
Dataset from Vandesompele et al. [13] in log-scale showing expression levels of 10 putative housekeeper genes by sample and tissue type. Tissue types analyzed included normal bone marrow (BM), cultured normal fibroblasts (FIB), normal leukocytes (LEU), neuroblastoma cell lines (NB), and pooled normal tissue from breast, brain, fetal brain, heart, kidney, uterus, lung, trachea and small intestine (POOL).

an optimal universal housekeeper unless it is used within a complementary gene set.

Model 2
To compare the performance of housekeepers within and between different tissues, we made a Model 2 (see Materials and methods for further details) that models the expression of gene *j* in the *i*th sample of tissue-type *k* by

$$\log y_{i(k)j} = \mu + C_k + T_{i(k)} + G_j + (CG)_{kj} + \varepsilon_{i(k)j}, \text{ where } \varepsilon_{i(k)j} \sim N(0, \sigma_j^2 \varsigma_k^2)$$

where μ denotes the overall mean (log-) expression, C_k is the difference of the *k*th tissue type from the overall average, $T_{i(k)}$ is the specific effect of the *i*th sample of tissue-type *k*, G_j is the difference of the *j*th gene from the overall average and $(CG)_{kj}$ is the tissue-type specific effect of gene *j*. Variability in calculation comes from two sources: the specific gene (σ_j) and the tissue-type (ς_k). The estimates of these parameters are given in Table 3. The single gene with the overall lowest variability within each tissue type is *GAPD*, followed closely by *UBC* (ubiquitin C), *HPRT1* (hypoxanthine phosphoribosyltransferase 1) and *YWHAZ* (tyrosine 3-monooxygenase/tryptophan 5-monooxygenase activation protein, zeta polypeptide). This result correlates closely with Vandesompele *et al.*'s approach. That is, the top five genes have exactly

the same order when we rank the genes within each tissue type according to their *M*-value. Here we assign a rank of 1.5 to the unordered best pair and then average the ranks to obtain an overall ordering of the genes.

The risk of normalizing data to a housekeeper gene with variable overall expression level across different tissues can be represented mathematically as bias error. A housekeeper that has low bias for a particular tissue has an expression level that is near its mean expression across tissues. In our second model, the term $(CG)_{kj}$ represents this tissue-type specific bias. The measure of variability around an intended value when bias is present is called the mean squared error (MSE): MSE = bias² + variance. Thus, to find a set of genes for normalization across the various tissue types we use a minimax MSE criterion: minimizing the largest MSE of the combination. Table 4 provides a list for the best gene set of each size along with the minimax-MSE value. Although *GAPD* has relatively low overall variability within each tissue type, its basal expression changes across tissue types making it a poor choice for a single universal control. The data shows that *RPL13A* (ribosomal protein L13a) is the best single universal housekeeper, but it is clear that no single gene is optimal for a universal housekeeper. Actually, choosing all the candidates provides the smallest MSE, which is not surprising as the set of all 10 genes is unbiased by definition. For routine application it is reasonable to limit the number of control

Table 3

Components of the standard deviation estimates of the log-expression of the data of Vandesompele *et al.* [13]

Standard deviation of genes (σ_j)

GAPD	UBC	HPRT1	YWHAZ	SDHA	RPL13A	TBP	HMBS	ACTB	B2M
0.211	0.226	0.227	0.232	0.255	0.339	0.339	0.431	0.460	0.562

Tissue-type specific multipliers (ς_k)

Bone marrow	Cultured normal fibroblasts	Neuroblastoma cell lines	Normal leukocytes	Pooled*
1.000	1.204	1.582	1.879	2.014

*Pooled normal tissue from breast, brain, fetal brain, heart, kidney, uterus, lung, trachea and small intestine.

Table 4

Minimax MSE optimal gene sets for each set size

Maximum number of members	Gene set	Maximal MSE
1	*RPL13A*	0.544
2	*HPRT1, UBC*	0.328
3	*HPRT1, RPL13A, UBC*	0.136
4	*HPRT1, RPL13A, UBC*	0.136
5	*HPRT1, RPL13A, UBC*	0.136
6	*ACTB, HPRT1, SDHA, TBP, UBC, YWHAZ*	0.131
7	*ACTB, HPRT1, RPL13A, SDHA, TBP, UBC, YWHAZ*	0.064
8	*ACTB, HPRT1, RPL13A, SDHA, TBP, UBC, YWHAZ*	0.064
9	*ACTB, HPRT1, RPL13A, SDHA, TBP, UBC, YWHAZ*	0.064
10	*ACTB, B2M, GAPD, HMBS, HPRT1, RPL13A, SDHA, TBP, UBC, YWHAZ*	0.049

genes, as the cost of assaying additional genes needs to balance the extra precision obtained. With this in mind, it is instructive to note that the three-member set of *HPRT1, RPL13A* and *UBC* is an excellent choice because it maintains a priority ranking even when selection is open to including four- or five-element sets. The housekeeper genes we tested by RT-PCR on breast tumor samples were not assayed across other tissue types and thus could not be evaluated as universal controls. Nevertheless, it is likely that our results in breast tissue would hold up across other tissue types as our genes were initially selected from microarray data that included 17 different and diverse cell lines as well as primary breast tumors [19].
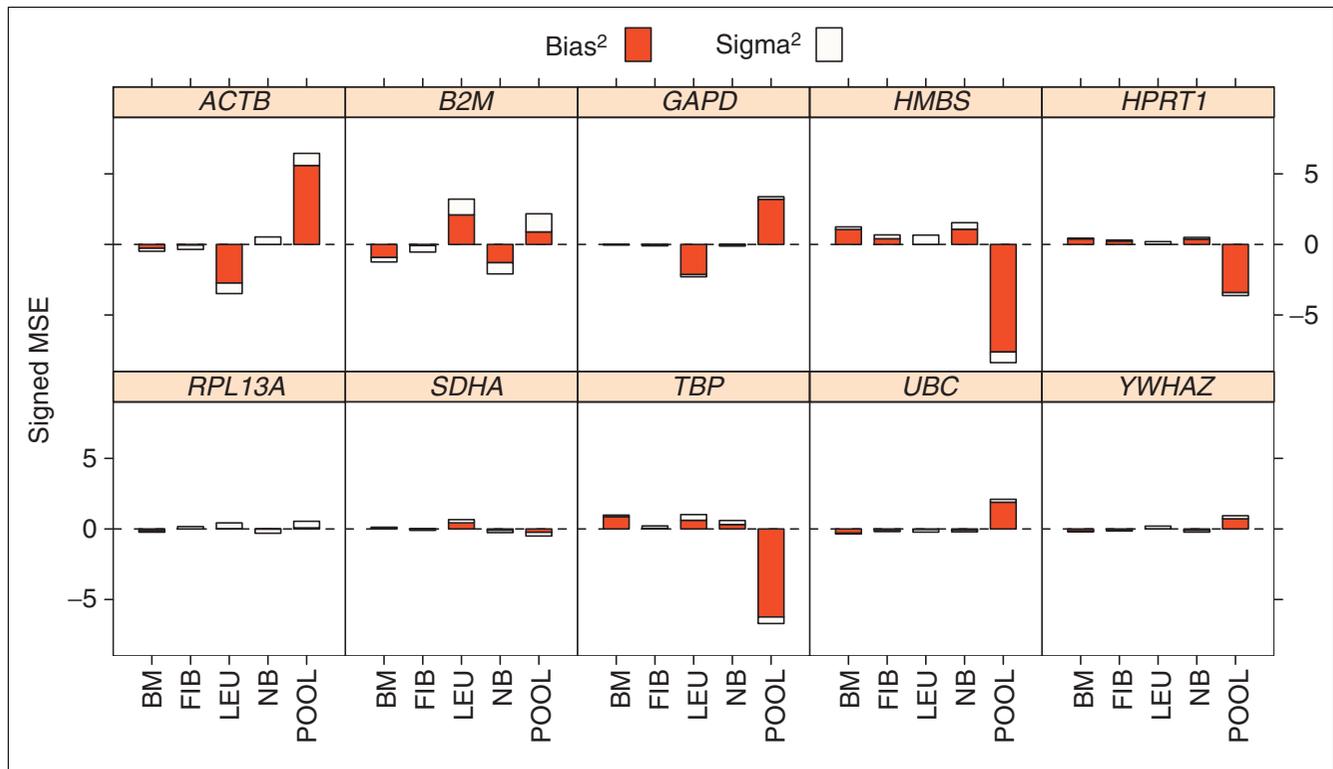
Figure 3 shows the MSE of each gene broken down into the squared-bias and variance components. The direction of each bar shows the sign of the bias. It is apparent that the large bias dominates the large values of MSE. The use of the (log-) average of several genes tends to reduce the variance, due to the effect of bias reduction where opposite biases cancel each other out. For example, both *ACTB* and *TBP* (TATA box binding protein) have a large bias in the pooled normal samples, but in opposing directions. The mean squared error of the

(log-) average of *ACTB* and *TBP* in these samples is only 0.35, which is much lower than their individual MSEs above 6.

In summary, we have modeled the performance of putative housekeepers to test their goodness-of-fit in serving as normalization controls for relative insert quantification. A major advantage of a model approach is that the terms are placed within a solid statistical framework and are not *ad hoc*, which allows the algorithm to be generalized to a variety of different experimental conditions. The genes and algorithms that we have selected for normalization should have broad utility for diagnostics and research.

Materials and methods
Pre-selection of assayed genes from microarray experiments

Four candidate housekeepers (*PSMC4, MRPL19, PUM1* and *SF3A1*) were selected from a microarray dataset containing 40 different breast tumors, three normal breast samples and 19 cell lines representing 17 different cell lines of diverse nature including lymphocytes, fibroblasts and epithelial cells [8]. All experiments were done using a common reference

Figure 3
Mean squared error (MSE) of each gene by tissue-type. The sign is determined by the direction of the bias. The MSE is broken down into the contributing components of the squared bias (Bias^2) and the variance (Sigma^2). Dataset from Vandesompele *et al.* [13]. Tissue types analyzed included normal bone marrow (BM), cultured normal fibroblasts (FIB), normal leukocytes (LEU), neuroblastoma cell lines (NB), and pooled normal tissue from breast, brain, fetal brain, heart, kidney, uterus, lung, trachea and small intestine (POOL).

strategy in which all experimental samples are compared to the same reference comprised of a pool of RNAs isolated from 11 diverse human cell lines [19].

To select housekeepers, we first 'filtered' the microarray data to select genes with Cy3 and Cy5 signal intensities greater than 500 units across at least 75% of the experiments. This requirement ensures that the gene is well expressed not only in the experimental samples, but also in the common reference sample. Next, we used the SAS/STAT Analysis Package Version 8 (SAS Institute Inc., Cary, NC) to identify a set of genes that showed a small range of expression across sample types and the least variance of the array-mean normalized log-ratios. For real-time RT-PCR, we selected four of the top six genes - *PUM1*, *PSMC4*, *MRPL19* and *SF3A*. The two other low-variability genes identified in the data were *IER3* (immediate early response 3) and *SRY* ((sex determining region Y)-box 2). We did not select these genes because of their potential for being differentially regulated under other conditions. However, we did include *GAPD* and *ACTB*, which are commonly used reference genes [20], in the set of candidate genes for comparison to the microarray selection.

Samples and cDNA preparation

Breast samples were acquired under informed consent and received at the Huntsman Cancer Institute (Salt Lake City, UT) for gene expression analysis (University of Utah, IRB #8533). All specimens were expediently processed in pathology upon arrival from surgery. Samples were grossly dissected, procured by flash freezing in liquid nitrogen, and stored at -80°C until RNA extraction. Approximately 50-100 mg cancer tissue was homogenized from each sample, and total RNA was prepared using the RNeasy midi kit (Qiagen). The integrity of RNA was determined using the RNA 6000 Nano LabChip kit (Agilent Technologies) and an Agilent 2100 Bioanalyzer. Two microliters of total RNA (50 ng/μl) were heated to 70°C and 1 μl was loaded on the column. Degradation was evaluated using the signal of the 18S and 28S ribosomal peaks [21].

First-strand cDNA was synthesized from 1 μg total RNA using oligo(dT) primers and Superscript III reverse transcriptase following manufacturer's instructions (Superscript III First-Strand Synthesis System, Invitrogen Life Technologies). Briefly, the reaction was held at 48°C for 50 min, followed by

a 15 min step at 70°C. The cDNA was washed on QIAquick PCR purification column (Qiagen) and eluted in 2 × 50 μl of elution buffer. The cDNA was then diluted in TE' (10 mM Tris, 0.1 mM EDTA, pH 8.0), aliquoted and stored at -80°C for further use.

Real-time quantitative PCR

All PCR reactions were performed on the LightCycler. Each 20 μl reaction included 1 × PCR buffer with 3 mM $MgCl_2$ (Idaho Technology), 0.2 mM each of dATP, dCTP, and dGTP (Roche), 0.1 mM dTTP (Roche), 0.3 mM dUTP (Roche), 1 U of Platinum taq (Invitrogen Life Technologies), 1/40000 SYBR Green I (Molecular Probes), approximately 5 ng cDNA,

and 0.4 μM of each primer. The primers used for the RNA control genes are shown in Table 5.

PCR was done using the following protocol: initial denaturation 95°C for 1 min 30 sec, then 50 cycles at 94°C for 1 sec for denaturation, 60°C for 5 sec (20°C/sec transition) for annealing, 72°C for 8 sec (2°C/sec transition) for extension. Fluorescence emission of SYBR Green I (channel 1, 530 nm) was acquired each cycle after the extension step. A melting step was performed after PCR to determine product purity. For melting curve analysis, the reactions were rapidly (20°C/sec) cooled from 95°C to 60°C and then slowly heated (0.1°C/sec) back to 95°C while continuously monitoring fluorescence.

Table 5

Primers for housekeeper genes

	Length (bases)	GC(%)	*Tm (°C)
PSMC4 (UniGene reference Hs.211594 - Gene ID: 5704)			
GGCATGGACATCCAGAAG	18	55.6	60
CCACGACCCGGATGAAT	17	58.8	61
Amplified fragment	190	61	90
MRPL19 (UniGene reference Hs.44024 - Gene ID: 9801)			
GGGATTTGCATTCAGAGATCAG	22	45	62
GGAAGGGCATCTCGTAAG	18	56	61
Amplified fragment	182	44	83
PUM1 (UniGene reference Hs.153834 - Gene ID:9698)			
TGAGGTGTGCACCATGAAC	19	53	61
CAGAATGTGCTTGCCATAGG	20	50	61
Amplified fragment	187	53	87
SF3A1 (UniGene reference Hs.406277 -Gene ID:10291)			
GGAGGATTCTGCACCTTCTAA	21	47	61
GCGGTAGTAGGCATGGTAA	19	52	60
Amplified fragment	196	48	85
ACTB (UniGene reference Hs.426930 - Gene ID:60)			
TTCCTGGGCATGGAGTC	17	59	60
CAGGTCTTTGCGGATGTC	18	55	60
Amplified fragment	84	55	84
†*GAPD* (UniGene reference Hs.169476 - Gene ID:2597)			
AACAGCCTCAAGATCATCAGC	21	48	63
GGATGATGTTCTGGAGAGCC	20	55	62
Amplified fragment	198	56	89

*Primers Tm determined using Tm Utility with algorithms adapted from Santa Lucia [23]. The Tm for the amplified fragment is the empirical Tm. †Primers for *GAPD* [20].

Relative quantification

Copy number was determined using the crossing point (Cp) value, which is automatically calculated using the LightCycler 3.5 software (Roche Molecular Biochemicals). The Cp value is reported as a fractional cycle number that is determined from the second derivative maximum (point of maximum acceleration) on the PCR amplification curve (fluorescence versus cycle number) [22]. A relative starting copy number was determined for each housekeeper using a calibration curve done with the same batch of master mix. Efficiency (*E*) of PCR was calculated from a plot of Cp versus log ng cDNA [22].

$$E = 10^{-1/slope}$$

Modeling expression data

As the effects of interest are fold changes, we modeled the log-transformed expression Model 1a.

$$\log y_{ij} = \mu + T_i + G_j + \varepsilon_{ij},$$

where $\sum_{i=1}^{n} T_i = 0, \sum_{j=1}^{g} G_j = 0, \varepsilon_{ij} \sim N\left(0, \sigma_j^2\right)$ independent

where μ denotes the overall mean (log) expression, T_i is the difference of the ith tissue sample from the overall average and G_j is the difference of the jth gene from the overall average. The key feature of this model that makes it different from a traditional ANOVA model is that it allows heteroscedastic errors: the variability of the genes is different.

We fitted the model using the gls routine of the nlme library for R, however other commonly available software such as PROC MIXED from SAS could have been used.

Based on the model, the variability of the logarithm of the geometric mean

$$\breve{y}_{iS} = (\prod_{j \in S} y_{ij})^{1/|S|}$$

of a gene-set S was estimated as

$$\mathrm{Var}\left(\log \breve{y}_{iS}\right) = \sum_{j \in S} \mathrm{Var}\left(\log y_{ij}\right)/|S| = \sum_{j \in S} \sigma_j^2/|S|.$$

Vandesompele *et al.*'s *M*-value is the average of relative standard deviations of the log-expression levels. Under Model 1, the *M*-value of the gene is closely related to its variance (under Models 2 and 3 below, the similar relationships can be derived):

$$V_{jk} = SD\left(\left\{\log\left(y_{ij}/y_{ik}\right)\right\}_{i=1}^{n}\right) = SD\left(\left\{\log\left(y_{ij}\right) - \log\left(y_{ik}\right)\right\}_{i=1}^{n}\right) = \sqrt{\sigma_j^2 + \sigma_k^2}$$

$$M_j = \sum_{\substack{k=1,\ldots,g \\ k \neq j}} V_{jk}/(g-1) = \sigma_j^2 \frac{\sum_{k \neq j} \sqrt{1 + \sigma_k^2/\sigma_j^2}}{g-1}$$

$$\sigma_j^2 \sqrt{1 + 1/R^2} \leq M_j \leq \sigma_j^2 \sqrt{1 + R^2}, \text{ where } R = \max_{i,k} \sigma_k/\sigma_i$$

We tested the assumption of unequal variances by fitting Model 1b that forces all the genes to have the same variability (this is the classical ANOVA model).

$$\log y_{ij} = \mu + T_i + G_j + \varepsilon_{ij},$$

where $\sum_{i=1}^{n} T_i = 0, \sum_{j=1}^{g} G_j = 0, \varepsilon_{ij} \sim N\left(0, \sigma^2\right)$ independent

Model 1c with a correlated error structure can be used to assess the assumption of (conditional) independence of the genes given the sample mean. If warranted, a more complicated correlation structure can be imposed.

$$\log y_{ij} = \mu + T_i + G_j + \varepsilon_{ij},$$

where $\sum_{i=1}^{n} T_i = 0, \sum_{j=1}^{g} G_j = 0, \varepsilon_i = \left(\varepsilon_{i1}, \ldots, \varepsilon_{ig}\right)' \sim N\left(\mathbf{0}, \Sigma\right)$

$$\text{and } \Sigma = \left(\sigma_1 \ \cdots \ \sigma_g\right) \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix} \cdot \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_g \end{pmatrix}$$

For the multiple tissue-type set-up the notation and the model need to be extended. We will denote the expression level of gene j of in the ith sample of type k by $y_{i(k)j}$, $i = 1, \ldots n_k$, $j = 1, \ldots, g$ and $k = 1, \ldots, m$. The best-fitting model for the data, which we call Model 2, had the form

$$\log y_{i(k)j} = \mu + C_k + T_{i(k)} + G_j + (CG)_{kj} + \varepsilon_{i(k)j},$$

where

$$\sum_{k=1}^{m} C_k = 0, \sum_{i=1}^{n_k} T_{i(k)} = 0, \sum_{j=1}^{g} G_j = 0, \sum_{j=1}^{g} (CG)_{kj} = \sum_{k=1}^{m} (CG)_{kj} = 0,$$

$$\varepsilon_{i(k)j} \sim N\left(0, \sigma_k^2 \varsigma_j^2\right) \text{ independent}, \quad \varsigma_1 = 1.$$

Thus the errors are independent and their variability is decomposed into a gene-specific and tissue-type specific multiplicative components. The last restriction ensures the uniqueness of the solution. Simpler models that we considered used uniform error variance, equal error variance for tissue types, and equal error variance for genes. We also considered more complex models that used exchangeable correlation structure for the errors and unstructured error variance (each gene-tissue-type combination has a variance parameter). The BIC was used as a basis for model selection.

Additional data files

Additional data available with this paper online is an Excel file with the relative copy numbers of six genes in the 80 breast cancer samples used in this study (Additional data file 1).

Acknowledgements

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