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ORIGINAL ARTICLE

Artificial root foraging optimizer algorithm with hybrid strategies

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Abstract In this work, a new plant-inspired optimization algorithm namely the hybrid artificial root foraging optimizion (HARFO) is proposed, which mimics the iterative root foraging behaviors for complex optimization. In HARFO model, two innovative strategies were developed: one is the root-to-root communication strategy, which enables the individual exchange information with each other in different efficient topologies that can essentially improve the exploration ability; the other is co-evolution strategy, which can structure the hierarchical spatial population driven by evolutionary pressure of multiple sub-populations that ensure the diversity of root population to be well maintained. The proposed algorithm is benchmarked against four classical evolutionary algorithms on well-designed test function suites including both classical and composition test functions. Through the rigorous performance analysis that of all these tests highlight the significant performance improvement, and the comparative results show the superiority of the proposed algorithm.

1. Introduction

Nature-inspired methods which mimic the intelligent behaviors of certain creatures have the excellent abilities of tackling complex NP-hard problems. For examples, the particle swarm optimization (PSO) (Gao et al., 2013), the ant colony optimization (ACO) (Huang et al., 2008), the artificial bee colony (ABC) (Bhandari et al., 2015), and the bacterial foraging algorithm (BFA) (Bouaziz et al., 2015) have been found to perform better than the classical heuristic methods and already come to be widely used in many areas.

Recently, many top researchers are paying more interest investigation how to improve the accurate of the result and greatly reduce the computation time. The computational models of artificial root foraging behavior have attracted more and more attention, due to their remarkable adaptive growth processes can provide novel insights into new computing paradigm for global optimization (Karaboga and Basturk, 2008; McNickle et al., 2009). In this paper, we implement a novel hybrid artificial root foraging optimizion (HARFO) by incorporating a set of hybrid strategies in the following aspects:

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- The proposed HARFO adopts the root-to-root communication. The individual roots share more information from the elite roots through different effective topologies in the early exploration stage of the algorithm.
- By introducing a multi-population co-evolution mechanism. The hierarchical population of roots can be structured with enhanced interactions of individual behaviors from different sub-populations.

The rest of this paper is organized as follows: in Section 2, we outline the classical Artificial Root Foraging Model in sufficient details. Section 3 gives a brief overview of the model and algorithm of the proposed HARFO. In Section 4, the experiment studies of the proposed HARFO and the other algorithms are presented with descriptions of a set of benchmark functions. In Section 5, the conclusions are drawn, the results show that the proposed HARFO is feasible and have a powerful search ability.

2. Classical Artificial Root Foraging Model (ARFO)

2.1. Biological basis of plant root growth optimization

Plant roots are generally composed of the main root and lateral roots. The main root is developed by radicle, main root growth model mainly shows geotropism, means vertical growth down to the ground. Lateral roots are many branches which are grown from the main root edges. The lateral roots can grow another lateral root, these lateral roots can be graded to be first degree, second degree, third degree and so on. Lateral root growth direction has an angle from the main root. The lateral root growth pattern is mainly reflected in hydrotropism. Based on the plant growth mechanisms and biological model, and Optimal foraging theory and adaptive optimization model of plant growth, the classical Artificial Root Foraging Model (ARFO) is presented, The optimization process of ARFO is shown in Fig. 1.

2.2. Basic concepts

We regard the soil environments for plant root growth as a minimization problem and the root population as a homogeneous biomass, so each root apex represents a feasible solution of the specific problem, and each of its search for the optimal area through adjusting the directions, elongation length and

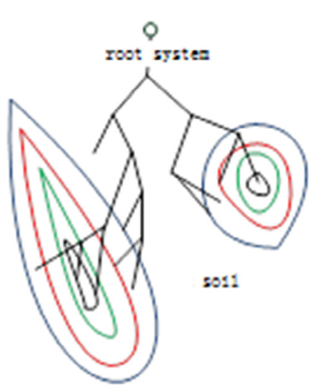


Figure 1 The optimization process of ARFO.

the propagation strategies. Some criteria should be obeyed to obtain the ideal plant root growth behaviors, which is shown as follows:

a. *Criterion-1:* The auxin concentration regulating the roots' spatial structure reallocated dynamically instead of static.
b. *Criterion-2:* One root apex elongates forward in the substrate and produces daughter root apices.
c. *Criterion-3:* According to the size of auxin concentration, there are three categories in the whole root system, main roots, the lateral-roots and the dead-roots.
d. *Criterion-4:* The hydrotropism influences the growth trajectory of plant roots, and makes the growing direction of the root tips toward the optimal individual position.

2.3. Auxin regulation

Various growth operations and the branching number should be controlled by the auxin. In artificial soil environment, the nutrient distribution should be formulated as the following expression (He, 2015; Wang et al., 2006):

$$f_i = \frac{fitness_i - f_{min}}{f_{max} - f_{min}} \tag{1}$$

Then the auxin concentration can be stated mathematically as below:

$$A_i = \frac{f_i}{\sum_{j=1}^{S} f_i} \tag{2}$$

where $fitness_i$ is the functional fitness value, f_i is the normalization fitness value of the root i, f_{min} and f_{max} are the maximum and minimum of the current population, respectively, S is the size of current population.

2.4. The growth strategy of main root

According to the criteria, a main root regrows including regrowing operator and branching operator (He, 2015; Ma et al., 2015a,b).

2.4.1. Regrowing operator

A main root makes the growing direction toward the best individual of current population, the operator is formulated as the following expression (He, 2015; Ma et al., 2015a,b).

$$x_i^t = x_i^{t-1} + l \cdot rand \cdot (x_{lbest} - x_i^{t-1}) \tag{3}$$

where x_i^t is the new position, x_i^{t-1} is the original position of root i, l is a local learning inertia, $rand$ is a random coefficient varying within [0, 1], x_{lbest} is the local best individual in current population.

2.4.2. Branching operator

Once some specific conditions are met, branching operator means a root apex generates new individuals. When auxin concentration value is more than a branching threshold T_Branch, it will start generating a certain number of new individuals as follows.

$$\begin{cases} branch\ w_i\ individuals & if\ A_i > T\_Branch \\ nobranching & otherelse \end{cases} \tag{4}$$

The number of newborn root apices is calculated as follows (He, 2015; Wang et al., 2013):

$$w_i = R_1 A_i(s_{max} - s_{min}) + s_{min} \tag{5}$$

where R_1 is a random coefficient within the range [0, 1], A_i is the auxin concentration of root i, S_{max} and S_{min} are the pre-set maximal branching number and the minimal branching number.

The position of a newly branching root is initialized from the parent main root with Gauss distribution $N(x_i^t, \sigma^2)$, where σ can be calculated as follows (He, 2015; Ma et al., 2015a,b):

$$\sigma_i = ((i_{max} - i)/i_{max})^n \cdot (\sigma_{ini} - \sigma_{fin}) + \sigma_{fin} \tag{6}$$

where i_{max} is the maximum of iterations, i is the current iteration index, σ_{ini} is the initial standard deviation which is determined by the range of searching and σ_{fin} is the final standard deviation.

2.5. Lateral roots growth: random walking

All lateral roots will conduct random searches at each feeding process according to the random search strategy. Each lateral root generates a random elongated length and random growth angle, which is given by Eqs. (7) and (8):

$$x_i^t = x_i^{t-1} + rand \cdot l_{max} D_i(\varphi) \tag{7}$$

$$\varphi = \delta_i / \sqrt{\delta_i^T \cdot \delta_i} \tag{8}$$

where $rand$ is a random number with uniform distribution in [0, 1], l_{max} is the maximum elongate-length unit, φ is a growth angle computed by a random vector δ_i.

2.6. Dead roots growth: shrinkage

The root does not get enough nutrients from soil, which would cease to function. When the sustained growth probability will be stagnated, the corresponding root should be simply removed from the current population, which is decided by Auxin distribution.

3. Hybrid artificial root foraging optimizion

3.1. Root-to-root communication

As a new strategy, root-to-root communication is a process that allows a number of connected individuals exchange information in some specific topologies, which is the intrinsic property of the "population" in swarm intelligence. There are two important roles for the spatial topological structure, one is that individuals should enhance dynamic interaction through it, the other is that individuals should optimize information propagation path across the structured population.

In classical ARFO, it is seen from Eq. (3), an individual's candidate neighborhood termed x_{lbest} is selected from the entire population. In other words, one central node is influenced by all other members of the population. Accordingly, the population topological structure of ARFO essentially falls into the star topology, which is a fully connected neighborhood relation, as shown in Fig. 1(a).

Through an in-depth study of the relationship between algorithmic performances in and population topological structures (Kembel et al., 2005), the Von Neumann exhibits better convergence speed on a variety of test functions (Dannowski, 2005), which is shown in Fig. 2(b) and (c). It concludes that the Von Neumann has a lower connectivity while covering a larger search space than the star type, which tends to maintain better diversity of population and reduces the chances of falling into local optima (He, 2015; Ma et al., 2015a,b).

3.2. Co-evolution mechanism

In the development of plant root system, hierarchy is a common phenomenon. There is an important role for plant-plant positive interactions aggregated by lower root-root communication to determine the population dynamics and spatial topological structures (Kennedy and Mendes, 2002).

The hierarchical co-evolution mechanism is a process that decomposes large-scale problems into simple tasks optimized in parallel to improve algorithm efficiency. The flat ARFO is structured into two levels with different topologies in Fig. 3 (Ma et al., 2015a,b).

Suppose that population $P = \{S_1, S_2, \ldots, S_M\}$, and each swarm $S_k = \{x_1, x_2, \ldots, x_N\}$. In each growth phase, the new individual or agent in level 2 is defined as:

$$x_i^t = x_i^{t-1} + l_1 \cdot rand_1 \cdot (x_{ibest}^{t-1} - x_i^{t-1}) + l_2 \cdot rand_2 \cdot (x_{pbest}^{t-1} - x_i^{t-1}) \tag{9}$$

where x_{ibest}^t is the best individual within current population which donates cooperation in level 2 and x_{pbest}^t is the global best individual among all populations which exchanges information across populations in level 1. l_1 and l_2 is the random coefficient. $rand_1$ and $rand_2$ are random number with uniform distribution in [0, 1] respectively.

3.3. The proposed algorithm

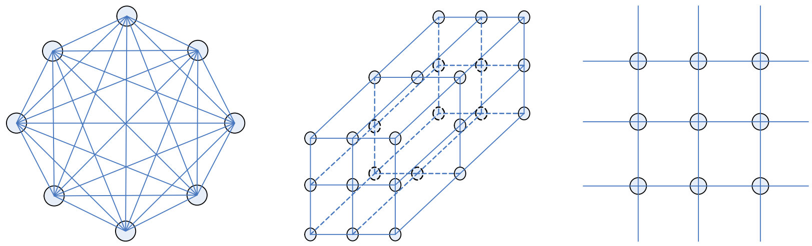
The HARFO hybridize the strategies of root-to-root communication mechanism and co-evolution mechanism, which can regulate the trajectory of each root through the specific topology. In the proposed method, the evolution of population is guided by global best information in level 1 and historical experience in level 2 to ensure the amply diversity of population. The flowchart of HARFO algorithm is presented in Fig. 4.

We can see the main procedures of the proposed HARFO as followed:

The HARFO algorithm:

Begin

(1) **Initialization**:
  a. Initialize M root populations, each consists of N individuals. And set the maximum iteration number $MaxT$,
  b. Set t = 0
  c. Calculate auxin concentration values of all populations by Eq. (2).



(a) Star (b) Three-dimensional Von Neumann (c) Two-dimensional Von Neumann

Figure 2 Population topology.

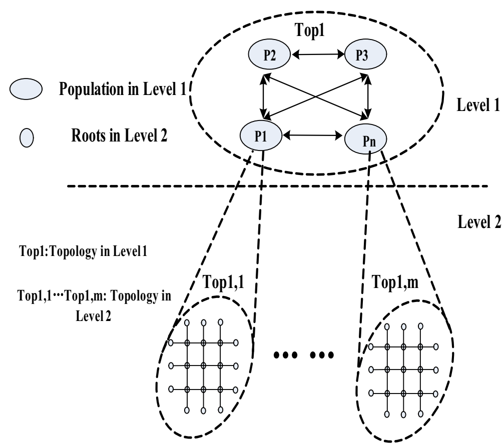


Figure 3 Multi-species coevolution mechanism.

(2) **While (terminal conditions not satisfied)**
  a. *Divide each population into main root and lateral root groups according to auxin concentration*
  b. *For each population P_i, Construct Von Neumann topology as follows.*
  Split population of P roots into M rows and N cols, and P = M * N.

Begin

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(2) **While (terminal conditions not satisfied)**
  a. *Divide each population into main root and lateral root groups according to auxin concentration*
  b. *For each population P_i, Construct Von Neumann topology as follows.*
  Split population of P roots into M rows and N cols, and P = M * N.
  Begin
    For i = 1:N
      xi4(i,1) = (i – Cols) mod N;
    If xi4(i,1) = = 0 xi4 (i,1) = N;
      xi4(i,2) = i-1;
    If (i – 1) mod Cols = = 0
      xi4(i,2)) = i – 1 + Cols;
      xi4(i,3) = i + 1;
    If i mod Cols = = 0
      xi4(i,3) = i + 1 – Cols;
      xi4 (i,4) = (i + Cols) mod N;
    If xi4(i,4) = = 0
      xi4 (i,4) = N;
    End
  End
  c. *For each main roots group*
  Implement regrowing operator by Eq. (9).
  Evaluate auxin concentration values of renewal main roots, and apply greedy selection.
  If the condition of branching determined by Eq. (4) is met, continue; otherwise, go to lateral-root loop.
  Calculate the branching number by Eq. (5), and branching new roots by Eq. (6).
  Adjust the population size.
  End for
  Loop over each root tip of lateral-roots
  d. *For each Lateral-root group.*
  Lateral-root take regrowing operator by Eqs. (7) and (8).
  Evaluate the auxin concentration values of the renewal lateral roots, apply greedy selection.
  Adjust the corresponding nutrient concentration value by Eq. (2);
  End for
  Loop over each root tip of lateral-roots
  e. *Remove the dead individuals from each population*
  Loop over each population
  t = t + 1;

End while
Output the best result

4. Benchmark test

4.1. Classical test functions

The static test suite includes basic benchmarks and CEC 2005 test beds, which are commonly used in other state-of-the-art

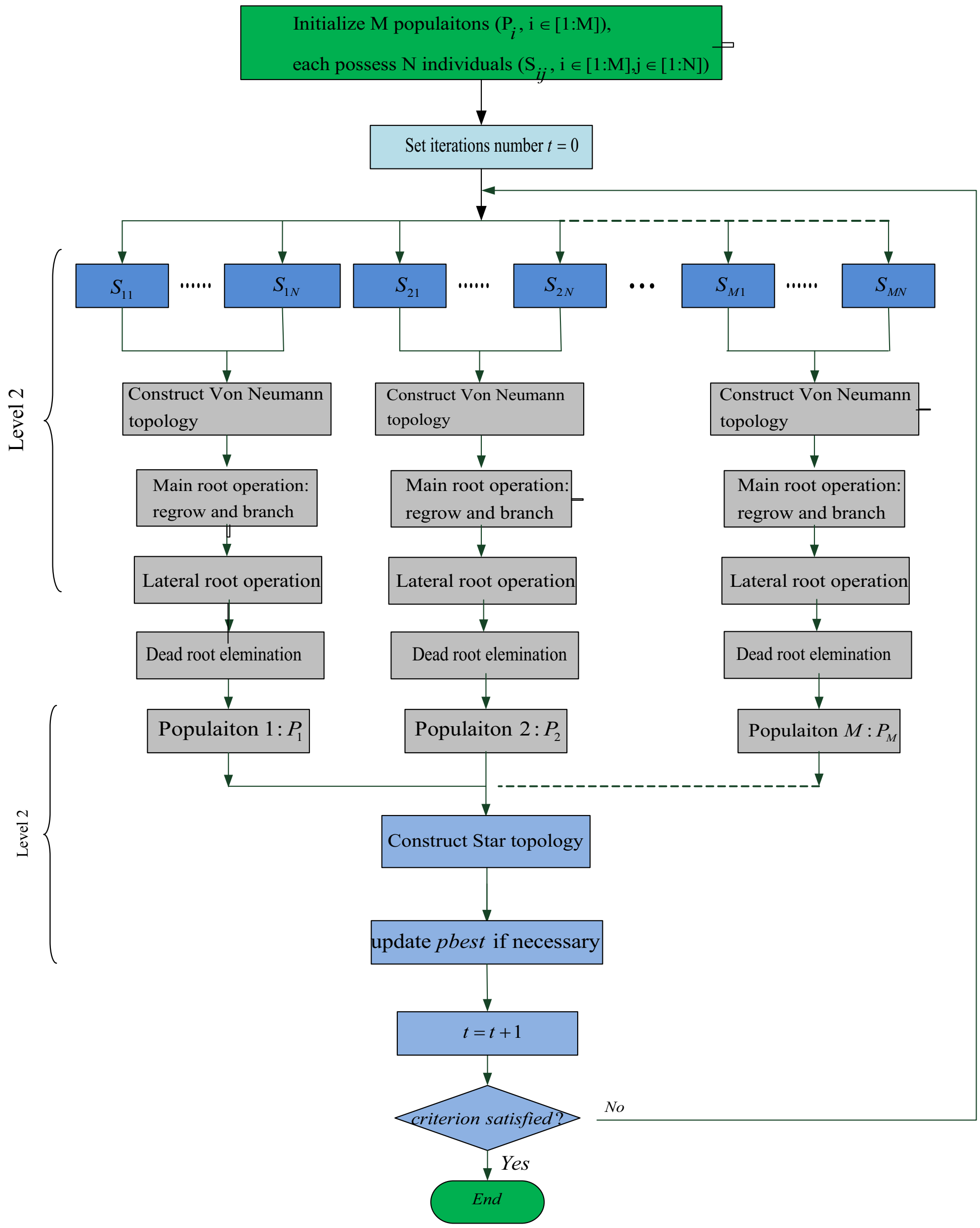

Figure 4 The flowchart of HARFO algorithm.

meta-heuristic algorithms. The definition and mathematical representation are shown in Tables 1 and 2, where f_1–f_5 is basic benchmark functions, f_6–f_{10} is taken from CEC 2005 test suit, which is a complex rotation and drift problem based on the basic test functions.

4.2. Experimental setting

To evaluate the performance of the HARFO, four classical evolutionary algorithms were used for comparison.

- Genetic algorithm (GA).
- Covariance Matrix Adaptation Evolution Strategy (CMA-ES).
- Classical artificial root foraging optimization algorithm (ARFO).
- Artificial bee colony algorithm (ABC).

In this section, we adopt ten test benchmarks which is given above. The values of the population size used in each algorithm is chosen to be 20. The other parameters are given as follows:

GA settings: Adopt the real-coded version with intermediate crossover and Gaussian mutation, its parameters including mutation rate and crossover rate, directly follow the default setting of (de Kroon et al., 2005).

CMA-ES settings: The parameter setting of CMA-ES is $\sigma_F = 0.2, \lambda = 10, \mu = 5$.

ABC settings: Set the *limit* to $SN \times D$, where SN is half of population size, *and* D is the dimension of the problem.

Table 1 Formulas and initialization range of test functions.

Sphere function (f_1)	$f_1(x) = \sum_{i=1}^{n} x_i^2$
Rosenbrock function (f_2)	$f_2(x) = \sum_{i=1}^{n} 100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2$
Rastrigrin function (f_3)	$f_3(x) = \sum_{i=1}^{n} 100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2$
Schwefel function (f_4)	$f_4(x) = D * 418.9829 + \sum_{i=1}^{D} - x_i \sin(\sqrt{\lvert x_i \rvert})$
Griewank function (f_5)	$f_5(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$
Shifted sphere function (f_6)	$f_6(x) = \sum_{i=1}^{D} z_i^2 + f_{bias1}, z = x - o$
Shifted Rosenbrock's function (f_7)	$f_7(x) = \sum_{i=1}^{D-1} \left(100(z_i^2 - z_{i+1})^2 + (z_i^2 - 1)^2\right) + f_{bias}$
Shifted Schwefel's problem (f_8)	$f_8(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j\right)^2 + f_{bias}, z = x - o$
Shifted rotated Griewank's function without bounds (f_9)	$f_9(x) = \sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f_{bias}$
Shifted Rastrigin's function (f_{10})	$f_{10}(x) = \sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10) + f_{bias}, z = x - o$

Table 2 Parameters of the test functions.

f	Dimensions	Initial range	x^*	$f(x^*)$
f_1	20	$[-100, 100]^D$	[0, 0, ..., 0]	0
f_2	20	$[-30, 30]^D$	[1, 1, ..., 1]	0
f_3	20	$[-5.12, 5.12]^D$	[0, 0, ..., 0]	0
f_4	20	$[-500, 500]^D$	[420.9867, ..., 420.9867]	0
f_5	20	$[-600, 600]^D$	[0, 0, ..., 0]	0
f_6	20	$[-100, 100]^D$	[0, 0, ..., 0]	−450
f_7	20	$[-100, 100]^D$	[0, 0, ..., 0]	390
f_8	20	$[-100, 100]^D$	[0, 0, ..., 0]	−450
f_9	20	No bounds	[0, 0, ..., 0]	−180
f_{10}	20	$[-5, 5]^D$	[0, 0, ..., 0]	−330

ARFO and HARFO settings: The parameter setting of HARFO and ARFO can be empirically summarized in Table 3.

4.4. Computational results

Table 4 presents the best, worst, mean and standard deviation of the five algorithms on ten 20 dimensional benchmarks f_1–f_{10}. Every algorithm is independently implemented 20 times and terminated when the number of function evaluations reaches 100,000 for each run, the best value within five algorithms was shown in bold. From Table 4, the proposed HARFO achieved significantly better results and relatively outperformance for solving most test benchmarks than the other algorithms (He, 2015; Ma et al., 2015a,b).

It viewed that HARFO and ABC can obtain relatively satisfactory results on the functions of f_1 (Sphere) and f_2 (Rosenbrock), which are variable-separable and non-separable types of unimodal benchmark problems respectively. It is worthy to notice that the ARFO remarkably can solve the more complex unimodal Rosenbrock problem. HARFO performs slightly better than ABC, furthermore showing significant improvement over other three algorithms on the complex multimodal functions including variable-separable f_3 (Rastrigin), variable-separable f_4 (Schwefel) and non-separable f_5 (Griewank). ABC indeed exhibits better performance than HARFO on f_4. HARFO can obtain best performance on most of five benchmarks including f_6, f_8, f_9, and f_{10}. ARFO also outperforms other algorithms in terms of mean and standard deviation occasionally on f_7.

We compare the performance of HARFO, ARFO, ABC, CMA-ES, and GA on 20 dimension test functions. It is shown that the proposed algorithm appeared the superiority over these well-known benchmarks, which can include that the HARFO can utilize the root-to-root information change mechanism immediately to overstep the local extremum when other algorithms suffer being lost in the local optima dilemma. The complex task can be decomposed into smaller-scale sub-problems by employing the hierarchical multi-population co-evolution. This indicates that multi-level individuals in HARFO can span a larger search space and maintain a better diversity of population.

Table 3 Parameters of HARFO and ARFO for optimization.

ARFO		HARFO	
Population number	8	The number of initial population	20
The number of initial population	4	The maximum number of population	100
The maximum number of single population	50	*T_Branch*	10
BranchG	10	*T_Nmority*	5
Nmority	5	S_{max}	4
S_{max}	4	S_{min}	1
S_{min}	1		

Table 4 Comparison of results with 20 dimensions obtained by HARFO, ARFO, ABC, CMA-ES, and GA.

Func		GA	ARFO	ABC	CMA-ES	HARFO
f_1	Mean	5.4316E+00	8.4537E−03	1.3230E−20	3.1100E−01	4.2817E−13
	Std	4.7588E+00	6.6176E−03	4.3300E−20	7.1920E−01	5.8831E−13
	Min	2.3610E+00	0	0	2.5140E−01	0
	Max	1.3334E+01	1.8978E−02	1.4100E−20	4.3560E−01	1.3123E−13
f_2	Mean	5.9087E+04	6.9380E+01	3.9681E+00	2.923 0E+00	7.2733E−01
	Std	6.6060E+04	1.3271E+00	3.7709E+00	5.2570E+00	1.5903E+00
	Min	1.6393E+04	2.0703E+01	1.9224E−01	1.4700E−00	0
	Max	1.6882E+05	1.7992E+02	9.2547E+00	1.2290E+00	3.6144E+00
f_3	Mean	2.7036E+02	7.5788E+01	4.9539E+01	3.1830E−01	2.1575E−13
	Std	1.1512E+02	1.1519E+00	1.3063E+01	6.6450E−01	1.2233E−12
	Min	1.3334E+02	5.4715E+01	2.8960E+01	3.1276E+01	1.1083E−13
	Max	4.4407E+02	1.3556E+02	6.2109E+01	1.5060E−01	3.6144E−13
f_4	Mean	2.9727E+03	4.4610E+03	2.8590E−04	7.6121E−00	7.6514E−04
	Std	6.0066E+02	2.2428E+02	2.7604E−04	7.0511E−04	6.3836E−04
	Min	2.1286E+03	4.1899E+03	2.7358E−04	2.6212E−04	2.3911E−04
	Max	3.6088E+03	4.8307E+03	3.0562E−04	1.2676E−03	1.5347E−03
f_5	Mean	3.6088E+01	9.2917E−01	8.3921E−02	3.0900E−01	5.0157E−03
	Std	2.6791E+01	2.8713E−01	7.3446E−02	4.5240E−01	4.9156E−03
	Min	9.8356E+00	6.4943E−01	1.4788E−02	3.3300E−01	0
	Max	5.9821E+01	1.2151E+00	2.1196E−01	1.0220E−01	1.1566E−02
f_6	Mean	6.4225E+03	6.3095E+02	7.5624E−14	6.8210E−01	4.0543E−14
	Std	3.9636E+03	7.8376E+02	3.0806E−14	2.5420E−01	3.4998E−14
	Min	1.0264E+03	2.4030E+02	5.3938E−14	5.6840E−01	1.8362E−14
	Max	1.5047E+04	1.5650E+03	1.1232E−13	1.1360E−03	6.2848E−14
f_7	Mean	2.8137E+09	1.4680E+00	2.4324E+01	1.1970+00	8.4784E+00
	Std	3.0583E+09	2.0352E+00	7.1995E+01	1.8820+00	1.2762E+00
	Min	3.1807E+08	5.3938E−04	3.8503E+01	1.9920E−02	4.8430E−02
	Max	4.4040E+09	3.7590E+00	3.7281E+01	4.5100E+00	1.4911E+01
f_8	Mean	1.4680E+04	1.9471E+02	9.0699E+02	7.9240E+02	1.9573E+02
	Std	4.4162E+04	1.5947E+01	5.8412E+02	3.189E+02	4.6811E+02
	Min	1.0655E+04	1.7129E+02	6.5190E+02	5.6720E+00	1.9573E+01
	Max	1.7861E+04	2.2675E+02	1.2693E+03	1.0180E+00	3.7701E+02
f_9	Mean	2.1408E+03	5.2497E+03	2.0949E+03	1.7780E+03	1.6015E+03
	Std	6.0800E+01	5.2374E+02	7.4802E−13	4.5470E−01	6.9952E−01
	Min	2.0063E+03	4.9416E+03	2.0210E+03	1.768 0E+00	1.5681E+03
	Max	2.2142E+03	5.5701E+03	2.1812E+03	1.7880E+03	1.6459E+03
f_{10}	Mean	2.0063E+02	3.4875E+02	5.9028E+01	2.350E+01	6.8062E+00
	Std	3.5721E+01	6.5806E+01	1.7745E−01	4.0820E−01	6.4614E−01
	Min	1.8105E+02	8.7248E+01	3.8818E+01	1.1360E+01	4.2261E+00
	Max	2.1898E+02	4.6335E+02	7.1598E+01	7.0720E+01	7.9405E+00

4.5. Timing complexity analysis

During optimization process, the population size of ARFO keeps evolving dynamically, which causes significant difficulties for an elaborate algorithmic complexity analysis for HARFO. By computing and comparing the average running time obtained by the algorithm on each objective functions, Fig. 5 gives the average computing time in 20 independent runs by all algorithms. We can observe that HARFO cannot get better performance on f_1 and f_2. However, HARFO and ARFO consume less computing time than other algorithms by handling complex shifted and rotated problems (e.g., f_5, f_7 and f_{10}), which is due to the fact that by the termed branch operator and dead-root elimination operator as described by Eq. (9), the population size of the HARFO can dynamically adaptive to the complexity of the objective functions, which can reduce the computational complexity of the optimization process.

5. Conclusions

In order to solve complex optimization problems effectively and efficiently, the HARFO optimization algorithm relies on the combination of the root-to-root communication and multi-population cooperative mechanism is proposed, which can improve the global search performance and keep diversity of population. The strategy of root-to-root communication can exchange information between individuals. The diversity of population should be well kept through the strategy of multi-population cooperative mechanism.

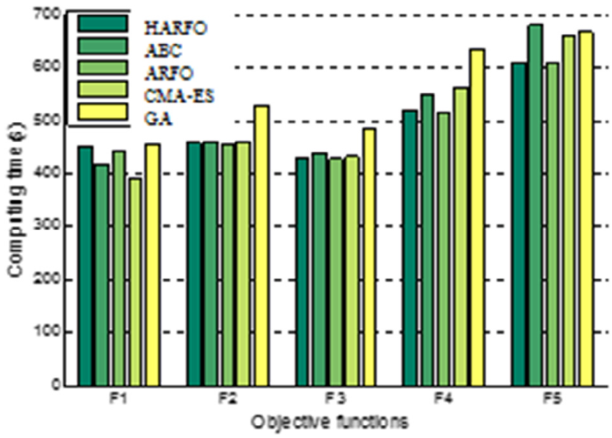

Figure 5 Computing time by all algorithms on selected benchmarks. $F1$ to $F5$ corresponds to f_1, f_2, f_5, f_7 and f_{10}, respectively.

In the comparative experiments, the experimental results validate the superiority of the proposed algorithm which has higher convergence speed and accuracy of the algorithm and has higher optimization efficiency compare to other mature intelligent optimization algorithm. All these show that the proposed algorithm has the potential to solve complex practical engineering optimization problems.

The future work should focus on perfecting this novel optimization framework from the perspective of relevance theory and how practically to use the HARFO algorithm for engineering optimization problems. The algorithm would provide a new way to solve the practical application of high-dimensional complex continuous optimization problems, such as the multilevel threshold image segmentation, large-scale sensor network scheduling problem, etc.

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