THE DYNAMICS OF CROWD INFECTION.*

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YOU and I are members of a community of human beings. We act and interact upon one another. We move about in a rather haphazard manner amongst our neighbours. All sorts of things happen to us as the result of these movements and interactions. We meet and exchange ideas. We meet and contract measles or some other contagious complaint. Our lives are a train of such incidents, a succession of events, one following the other.

This is the line of thought which I propose to follow this afternoon. It is the basic principle which lies at the root of every epidemiological problem, and on the proper understanding of it depends the solution of these problems.

Think of yourselves as little molecules and each of your lives as a train of events of one sort or another. Think of yourselves as moving in all sorts of dimensions, perhaps only forwards, perhaps forwards at one time and backwards at another. Your behaviour will then not greatly differ from that of a molecule of a gas, and I hope to show you that you obey in general much the same sort of laws as molecules of a gas obey.

You may raise the objection that the movements of a human being are not haphazard or random—that they are not predictable but are purposeful and subject to individual choice. That is no doubt true. But you must remember on the one hand that according to the most recent physical view the same unpredictableness holds for individual atoms, and on the other that, in the absence of some particular law which may generally govern the movements of the members of a crowd, the movements of the individuals within that crowd, each member of which may be moving purposefully, are in the aggregate hardly distinguishable from random motion. An illustration will make this obvious. We can make fairly accurate predictions as to the number of road accidents which will occur in a particular period of time, though we cannot make statements as to the particular individuals who will be hurt. We can

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estimate with reasonable precision the number of births or deaths which will occur during the ensuing year, though we cannot say who will bear children or who will die. Thus if we fix our attention on the crowd or the aggregate of individuals, and not on the individuals themselves—if we get rid of the idea of wondering how one particular individual will behave, and consider how a community will behave—we find that laws emerge which are almost identical with those which apply to inanimate particles, and which are almost as exact.

It will be clear to you from these considerations that in many biological phenomena we meet with a principle of mass action very similar to the principle of mass action met with in chemistry. You will remember that this principle states that the rate at which any reaction takes place is proportional to the concentration or mass of the constituents which enter into the reaction. Before the molecules react they must meet, and it is because their chance of meeting is proportional to their concentration that the law of mass action holds. The situation is approximately the same in the biological field. The chance of an infected individual infecting a healthy one, or of a large fish in the sea eating a small one, or of a phagocyte engulfing a bacterium, is to a first approximation proportional to the number of individuals, fish, or phagocytes and bacteria which are present.

In order to fix our ideas, let us take a simple example which can be tested by experiment. Let us take the case of the phagocytes and the bacteria (3, 4, 9, 12, 13). Here we have a swarm of phagocytes wandering about in what appears to be a random manner in a substratum which contains micro-organisms. A phagocyte collides with an organism and, if conditions are favourable, ingests it. We stop the experiment after fifteen minutes or so and count the ingested bacteria. If a phagocyte is empty we write 0, if it contains one organism we write 1, and so on. We thus find the numbers of phagocytes containing 0, 1, 2, 3, etc., organisms, and from this the mean number of organisms ingested, which is the phagocytic index.

I have been in the habit of examining such phenomena by making use of a schema of this sort (Fig. 1).

All the phagocytes are at first empty, and may be considered as occupying the compartment marked zero. As time elapses some pick up an organism and pass towards the right into the compartment marked 1, some of these ingest a second organism and pass into the compartment marked 2, and so on. The

arrows indicate the probability that a phagocyte will pass from one compartment to the next, and by the principle of mass action the number which pass out of any compartment during unit time will depend upon the number of phagocytes which are already in that compartment. The advantage of this method of schematic representation is that it can be directly

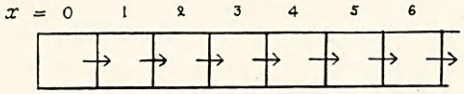

Fig. 1

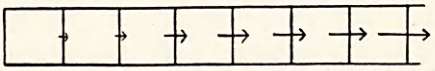

Fig 2

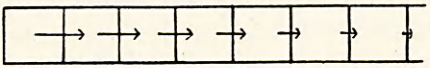

Fig. 3

transformed into mathematical language and the equations which result solved in such a way that the relative numbers in each compartment can at once be calculated. I shall not trouble you here with the working of the mathematical machine. It is sufficient for the present purpose that it can be operated, and that it yields the desired results.

In the above we have assumed that all the arrows were of equal value. But this might not be so. With each ingestion the appetite of the phagocyte might become greater, as depicted in Fig. 2, or less, as shown in Fig. 3.

Here is an example showing the agreement between theory according to the schema, and the results of an actual experiment.

	Observed.	Calculated.
0 bacteria	19	19
1 ,, ...	59	58
2 ,, ...	98	88
3 ,, ...	88	90
4 ,, ...	65	68
5 ,, ...	37	42
6 ,, ...	17	21
7 ,, ...	8	9
8 ,, ...	5	3·5
9 ,, or more . . .	4	1·5

You will see that the agreement between calculated and observed figures is very close (in fact, on the assumption that

the process operates in the manner which I have described, a fit as bad as or worse than that observed would be expected to occur six times out of ten).

I give you this example as a very simple illustration of the type of method and reasoning which we are discussing. It is, as a matter of fact, the very simplest case of a phagocytic experiment. Complexities arise owing to the fact that during the process of phagocytosis the organisms are themselves becoming agglutinated into masses by the action of the serum. I have worked out this problem (12), but its discussion would deflect us from the line of thought which I propose to follow in this lecture.

The above example, based upon the results from an actual experiment in the laboratory, may be considered as a study of a certain type of epidemiological problem with which medical men are often confronted (4 to 11, 13). The epidemiologist is frequently furnished with data giving the number of houses in a certain locality which have harboured 0 case, 1 case, 2 cases, 3 cases, etc., of a particular disease, and he is asked if, from the figures, he can tell whether infection has been distributed from a randomly acting source such as a water supply, or whether there is evidence of contagion within the houses. If the source be a random one, then the process is exactly the same as that depicted in Fig. 1. If contagion is operating, then the chance of occurrence of a second case within a house will be greater than the chance of occurrence of the first case, and so on, and the diagram will be that of Fig. 2. The approximate rate of increase of the probability of infection (*i.e.* of the arrow) can be determined from the figures by the mathematical machine.

Here are some figures relating to house infection in cancer.

	Observed.	Calculated.
Houses with 0 case . . .	64	65
,, ,, 1 ,, . . .	43	40
,, ,, 2 cases . . .	10	12
,, ,, 3 ,, . . .	2	2·5
,, ,, 4 ,, . . .	1	0·4

The fit is a very good one, and as the rate of increase of the probability with the number of cases within the house was found to be negligible, it may be concluded that there is no evidence of contagion within the houses. In other words, the distribution was a random one, a conclusion which is just the opposite of that arrived at by Behla, who collected the data.

Here is another example referring to an epidemic of cholera in a village in India.

	Observed.	Calculated.
Houses with 0 case . . .	168	37
,, ,, 1 ,, . . .	32	34
,, ,, 2 cases . . .	16	16
,, ,, 3 ,, . . .	6	5
,, ,, 4 ,, . . .	1	1
	223	93

You will see that apart from the first row of figures referring to houses which remained throughout free from the disease, the fit is excellent. The first observation requires some comment. It is obvious that the determination by observation in the field of the number of houses in which no cases have occurred is a matter of great difficulty; the epidemic may have affected only an indeterminable portion of the locality. Fortunately, the mathematic machine is able to cope with this difficulty.

The total number of houses which were concerned in the epidemic may be calculated from the observed figures apart from that relating to houses with 0 case, and the rate of increase of the arrows may then be found. In the instance of the cholera village, the rate of increase of the arrows was found to be negligible, and so this case also is of the type represented in Fig. 1. In other words, there was no evidence of contagion within the houses, and the suggestion was that the disease was water-borne, that there were a number of wells, and that the inhabitants of some 93 out of the 223 houses drank from certain wells which were infected. On further local investigation it was found that one out of a number of wells in the village was infected.

Let us consider an example of a somewhat different sort (1). An organism is grown in a fluid nutrient medium. Multiplication takes place by simple fission. The organism absorbs food-stuff and increases in volume. When it is approximately double its initial size it splits into two. The period of a generation is very short, and the amount of food-stuff required for upkeep is negligible as compared with the amount which goes to swell the bulk of the organism prior to division. Here we have a collisional phenomenon similar to that of phagocytosis, but which differs from phagocytosis in two respects. The first is that the organism divides. The second is that whereas in the case of the phagocytic experiment the duration of the experi-

ment was limited to fifteen minutes, in the present instance the process is allowed to go on indefinitely. Thus we have to take into account the fact that the food-stuff becomes used up. Here again the process takes place according to the law of mass action, and the rate at which it takes place depends on the one hand on the number of parent organisms at the moment, and on the other, on the limited amount of food-stuff available. The whole process may be considered as a conversion of food-stuff into organisms.

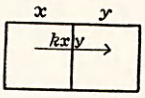

FIG. 4.—x = unconverted stuff.
y = converted stuff.
$x+y$ = constant.

The diagrammatic representation is simple (Fig. 4).

There are in this case only two compartments; the first refers to unconverted food-stuff, the second to converted food-stuff, that is to say, to bacteria.

Fig. 5 shows the agreement between observed figures and

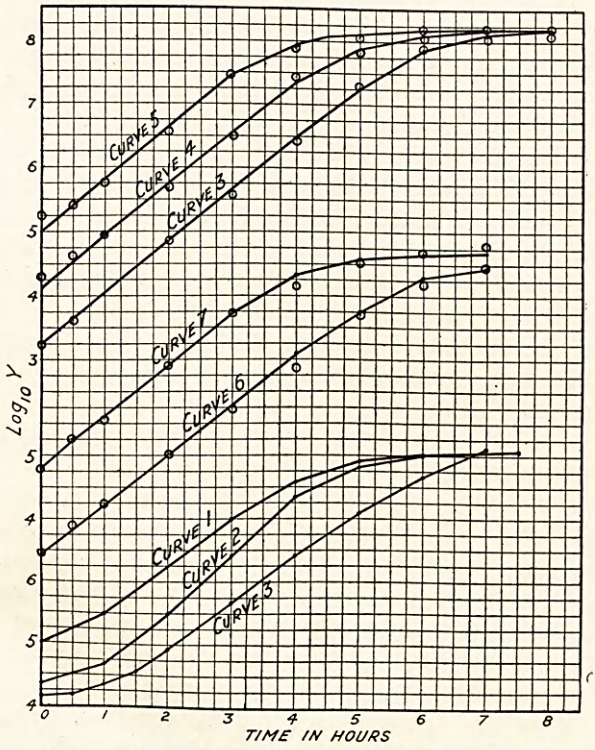

FIG. 5.—Rate of growth of bacteria, logarithmic curve.
rings = observed; line = calculated.

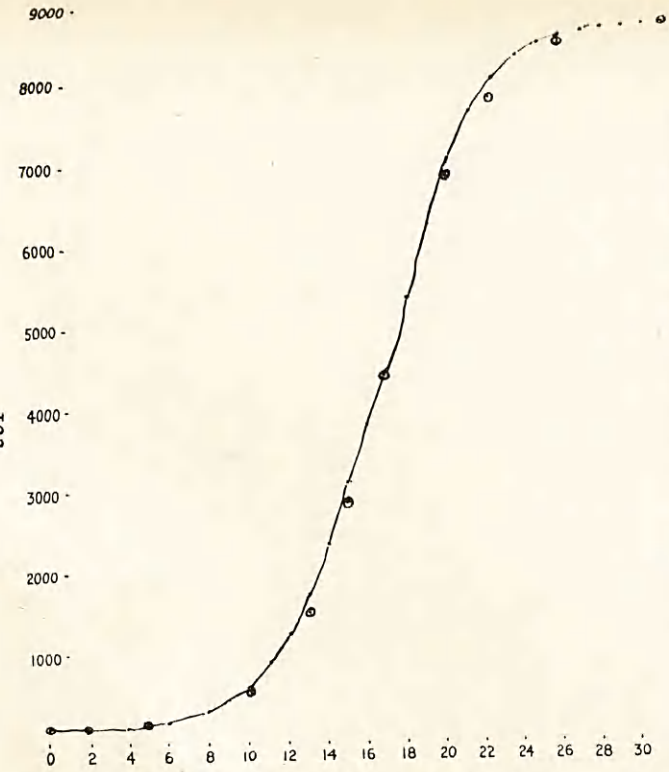

FIG. 6.—Invasion of a community by plague.
line = observed; rings = calculated.

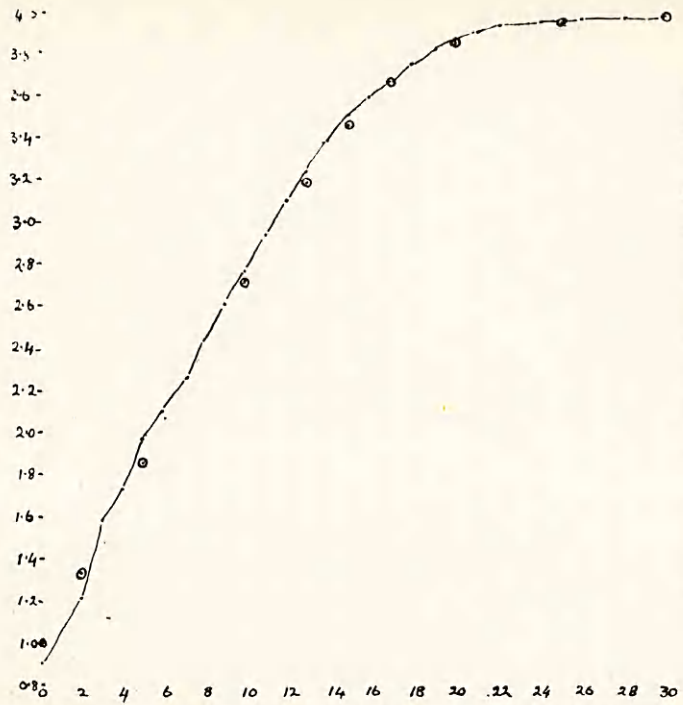

FIG. 7.—Invasion of a community by plague, logarithmic curve.
line = observed; rings = calculated.

those calculated from the above schema (McKendrick and Pai, 1911). The logarithms of the numbers of bacteria and not the actual numbers have been charted. (This method of representation is more convenient because it brings out the fact that the initial rise of the logarithmic curve should, according to the theory represented in Fig. 4, be a straight line.) The agreement is very close.

But let us pause and consider for a moment. The growth of bacteria in broth is in all respects similar to the growth of leaven (*i.e.* yeast) in a lump of dough, and this is stated in the Scriptures to be analogous to the growth of the kingdom of heaven. In other words, the parable enunciates the law which should govern the spread of an idea or an influence or anything contagious or communicable amongst the members of a population. It enunciates a law which is inherently mathematical. Let us examine a few applications of the parable.

1. Take first the spread of a contagious disease.

In Fig. 6 is shown the number of persons who had contracted plague during an epidemic in India, and in Fig. 7 the logarithms of the same numbers are plotted.

2. In Fig. 8 are shown the numbers of persons treated for hydrophobia at the Pasteur Institute of Kasauli from 1900-1928, and in Fig. 9 the logarithms of the same numbers are plotted. These figures show the spread of knowledge regarding the value of Pasteurian treatment amongst a population, the members of which were at first ignorant of its benefits.

I have examined many other cases to which the law applies, and so also in later years has Raymond Pearl. But I have shown you enough to illustrate my point. There can be no doubt that the axiom stated in the parable is borne out by facts. We have proved it to be true regarding the illustration of the rate of growth of organisms, on the one hand, and have given instances of its application to the spread of contagious influences on the other.

So far I have been dealing with rather simple problems in which most of the essential facts can be relatively easily ascertained. I shall next deal with much more complicated problems, the study of which has occupied Dr Kermack and myself for many years (8, 13 to 19). The spread of a disease throughout a community depends in its detail on all kinds of elaborate circumstances, on particular acts of this or that individual, on the chance meetings of people, on their

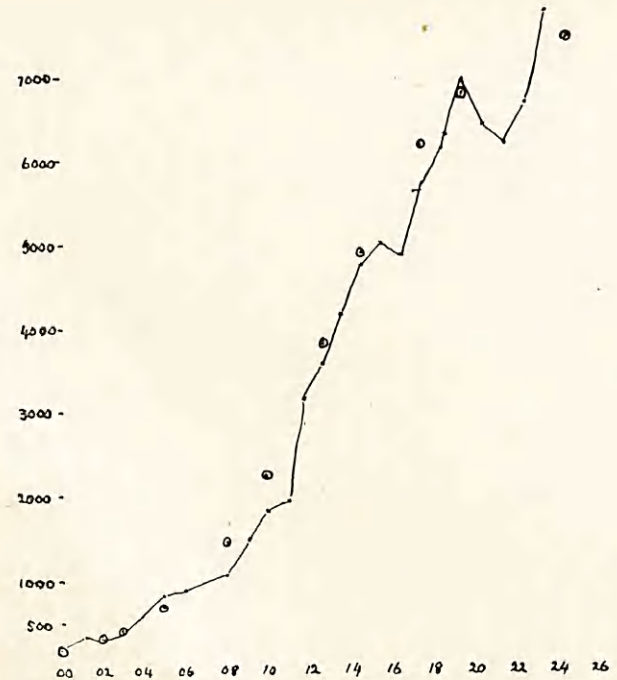

FIG. 8.—Patients treated at Pasteur Institute.
line = observed; rings = calculated.

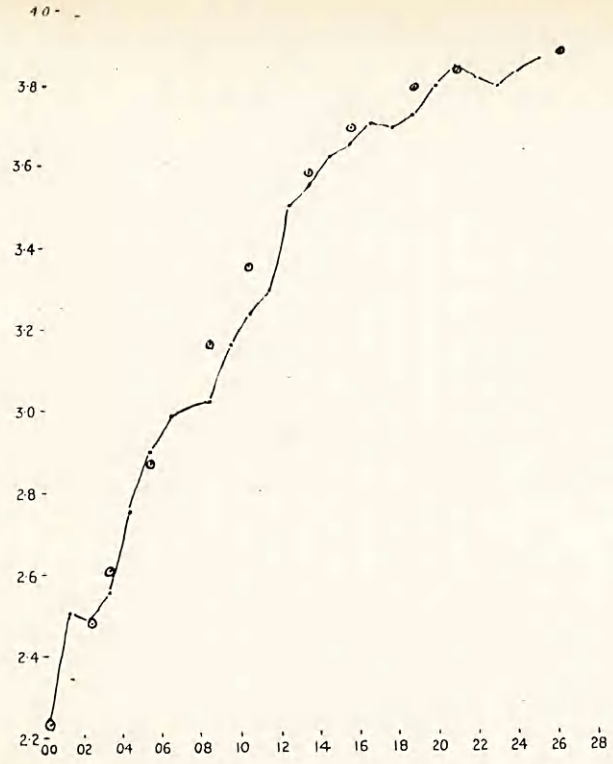

FIG. 9.—Patients treated at Pasteur Institute, logarithmic curve.
line = observed; rings = calculated.

congregation in streets, schools, and so on. It is quite impossible to take all these circumstances into account in any workable theory. This type of difficulty, however, is not one peculiar to problems of epidemiology. It is indeed one met with in all branches of scientific theory. Even the physicist cheerfully neglects all kinds of obvious facts and works out his problems on the basis of purely diagrammatic schemes.

The important condition for any such simplification is that the essential features of the problem should be retained and the incidental ones disregarded. The results obtained are of course only approximate, but the answer gives us the substance of what we really want, if it does not give us the trimmings. Indeed, it may be asserted that it is this ability of abstracting the essential from a mass of detail which makes human thought possible, and it is the ability to do this with confidence and with success which distinguishes the genius from the pedant.

In attempting then to "understand" the course of an epidemic we have to try to abstract the essential features into some sort of diagrammatic form which is simple enough to be dealt with by the mathematical tools at our disposal, and at the same time is sufficiently near to reality to avoid gross misrepresentation. It is here that our law of mass action stands us in good stead as a first approximation. We can assume that the rate at which new cases of disease are produced in a community is proportional to the number who are ill and to the number who are susceptible. We can assume, too, that the number who die or recover is proportional to the number who are ill.

Let us be under no illusion. The degree of simplification which we have introduced here is a very great one. Not only have we neglected all the vicissitudes of contact, the variations in crowding of the individuals concerned, but we have also, tentatively at least, assumed that all the infected persons are equivalent, and that all the susceptible persons are equally liable to acquire infection. Neither have we taken into account variations of susceptibility during the different phases of the disease in the patient. We boldly simplify the problem and work out the consequences on the theory so simplified, and we compare these with the observed facts in order to see whether our diagrammatic scheme does substantially accord with reality.

We may then represent the system of an epidemic by a diagram of the type which we have already made use of (Fig. 10).

Let us confine our attention in the first place to the compartment y, that is, to the numbers who are sick of the disease in question. You will notice that the arrow $\bar{k}\bar{x}y$ denotes the chance of a virgin (*i.e.* a person who has never suffered from the disease in question) contracting the disease, whilst kxy denotes the chance of a recovered person becoming ill: and that ly and dy denote respectively the chances of a sick person recovering, and of dying of the specific disease. $\bar{\pi}\bar{x}$, ρy and πx denote respectively the chances that a virgin, one who is sick or one who is recovered, may die from some other cause. When a balance is struck between the number con-

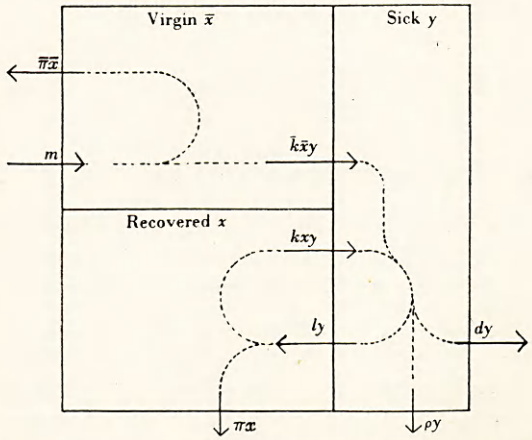

FIG. 10.—Schema illustrating invasion of a community by contagious disease.

tracting the disease and the number of sick who are removed either by death or recovery, the sick y will be in a state of equilibrium and their number will neither increase nor decrease. When the number contracting the disease is in excess, the number of sick will increase; when it is in defect, it will decrease. Thus following the arrows in the diagram,

when $\bar{k}\bar{x}y+kxy$ is greater than $(d+l+\rho)y$, y increases;
when $\bar{k}\bar{x}y+kxy$ is equal to $(d+l+\rho)y$, y is in equilibrium;
and when $\bar{k}\bar{x}y+kxy$ is less than $(d+l+\rho)y$, y decreases.

As y occurs on either side of the inequality, it may be eliminated, and so we arrive at the simpler relations:—

when $\bar{k}\bar{x}+kx>d+l+\rho$, y increases;
when $\bar{k}\bar{x}+kx=d+l+\rho$, y is in equilibrium;
and when $\bar{k}\bar{x}+kx<d+l+\rho$, y decreases;

and we note that these are independent of y, the number of sick at the moment.

As $\bar{k}$, k, d, l and ρ are constants, it follows that the behaviour of y depends only upon $\bar{x}$ and x, that is on the numbers of susceptible persons, whether they be in some degree immunised or not.

If we consider the case where complete immunity is conferred by the disease ($k = o$), we arrive at the still simpler relations :—

(1) when $\bar{x} > \frac{d+l+\rho}{\bar{k}}$, y increases ;

(2) when $\bar{x} = \frac{d+l+\rho}{\bar{k}}$, y is in equilibrium ;

and (3) when $\bar{x} < \frac{d+l+\rho}{\bar{k}}$, y decreases.

Thus if $\bar{x}$ is at first greater than $\frac{d+l+\rho}{\bar{k}}$ and thereafter decreases, as it must do during an epidemic, the curve which denotes the time variation of y will first rise, then reach a state of equilibrium and finally fall. The peak of the epidemic will occur when the number of susceptibles is exactly $\frac{d+l+\rho}{\bar{k}}$.

Let us now consider the case where a number of infected persons are introduced into a population which is free from the disease. The above relations 1, 2, and 3 still hold. If the number of virgins $\bar{x}$ is greater than the constant value $\frac{d+l+\rho}{\bar{k}}$ the number of sick will increase ; that is to say, there will be an epidemic rise. If it is less than the constant value, the number of sick will decline and the disease will fade out. Thus we see that the risk to which imported cases subject a community depends upon the number of susceptible persons in the area in which the community resides. If this number be greater than $\frac{d+l+\rho}{\bar{k}}$, there will be an epidemic ; if it be less, there can be no epidemic (apart from a few sporadic cases).

For this reason we may call the value of $\bar{x}$, which is equal to $\frac{d+l+\rho}{\bar{k}}$, a *threshold density*. Also we note, returning to the argument developed in the previous paragraph, that the

number of sick will reach its peak when the number of susceptibles is at its threshold density.

Two cases fall to be considered. The first is presented when the community may be thought of as isolated—that is to say, it receives no new recruits either by births or as the result of immigration. (Of course the exclusion of new individuals is never quite complete, but it often happens that the time required for an appreciable number of these new individuals to arrive is long in comparison with the average time taken by the disease to run its course.) The second case occurs when new recruits are flowing in at a relatively high rate, so that their effect on the course of the epidemic cannot be neglected.

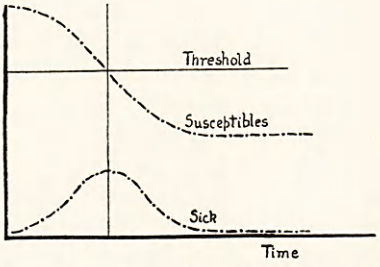

FIG. 11.—Diagram illustrating variation in numbers of susceptibles (upper curve) and numbers of sick (lower curve) during the course of an epidemic.

The first case then occurs when an epidemic of plague or influenza, for example, breaks out in a community, where the number of people infected is very large in comparison with the number of births and immigrations which are likely to occur whilst the epidemic is raging.

Let us enquire what course we would expect the epidemic to follow if it were controlled by the simple factors which we have abstracted in the schema.

As we have seen above, no epidemic, apart from a few sporadic cases, can occur unless the number of susceptibles exceeds the threshold value. If it exceeds this value, then the number of sick increases, and correspondingly the number of susceptibles is reduced. When the number of susceptibles is reduced to the threshold density, then the epidemic reaches its peak. Finally, the epidemic slowly exhausts itself as there are not sufficient susceptibles to keep it going; the number of susceptibles has in fact become less than the threshold. (See Fig. 11.)

A very simple deduction from the theory as shown by the model is that the number of people who become sick during an epidemic is approximately twice the amount by which the number of susceptibles at the commencement of the epidemic exceeded the threshold. In other words, at the end of an epidemic the number of susceptibles remaining will be just as much below the threshold as the original number was above it.

Let us now examine Fig. 12, which shows the number of

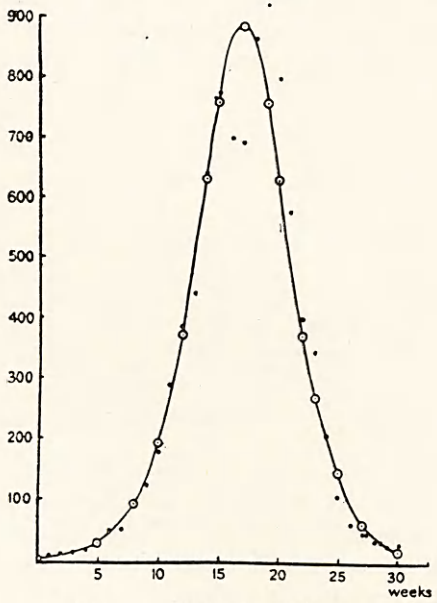

FIG. 12.—Epidemic of plague.
dots = observed ; lines with rings = calculated.

deaths which occurred during the course of an epidemic of plague in the city of Bombay. It should first be explained that we have to regard this human epidemic as a reflection of a parallel epizootic amongst the rats of the city. As will be seen, the number of deaths gradually increases to a maximum and then falls. The figure also shows the course which our model would lead us to expect that the epidemic would follow. Of course the constants occurring in the schema are adjusted so as to give the best fit. One of these constants is a simple multiplier, which makes it possible to adjust the human results to the theory which really applies to the primary rat epidemic.

It will be seen that the general course of the epidemic is well represented by the theoretical curve.

This result, that, according to the schema, the epidemic may exhaust itself long before all the susceptibles are affected, is one of considerable interest. You are aware of course that various theories have been put forward as to why epidemics die out. One theory is that those who remain are relatively immune, that all the susceptibles are in fact attacked. A second is that during the course of the epidemic the *materies morbi* becomes less virulent, that the strain of the organism becomes attenuated by repeated passage through successive hosts, and that the epidemic dies out because of this loss of virulence. From what I have shown you it will be realised that it is not necessary to introduce any such assumptions. This is not of course to assert that they are untrue, only that the characteristic features of the epidemic do not make it necessary to invoke them.

Another interesting consequence of the above result, that the magnitude of the epidemic is approximately twice the excess of the susceptible population above the threshold, is one repeatedly referred to in the literature. In a virgin community which so far has been free from the disease, most of the population may be susceptible to it, and so a density of population far in excess of the threshold is built up. The result is that when the disease is actually introduced a widespread epidemic is initiated, which affects a large proportion of the community. The larger the original excess, the greater the catastrophe. Hence we have cases such as the epidemics of tuberculosis amongst North American Indian tribes, and the catastrophic epidemics which occurred in earlier years in the islands of the Pacific.

Let us now consider the second case where the population is being constantly supplied by an influx of fresh susceptibles whether by birth or by immigration. There is now the important difference, as compared with the first case, that a continuous process of disease is possible in the community, and that the disease does not necessarily die out. In fact, we have the case of *endemic* disease. When the schema is examined it is found that we again meet with the idea of the threshold. Here it turns up, for example, with the rate of immigration. It is found that in order to ensure the continuance of the endemic condition, this rate of immigration must exceed a certain minimum or threshold value, and that above this the

incidence of the disease is proportional to the excess. We call the state of affairs where we have a steady rate of immigration and a constant incidence of the disease a steady state condition, and we note that the steady state of the sick is the " endemic level." It is in fact a kind of balance or equilibrium, and if the system is disturbed we may expect to have oscillations about this steady state, just as a disturbed balance swings about its equilibrium point. These oscillations will have a period determined by the characteristics of the system, and the successive oscillations will appear as successive epidemics, alternating with periods during which the disease is relatively

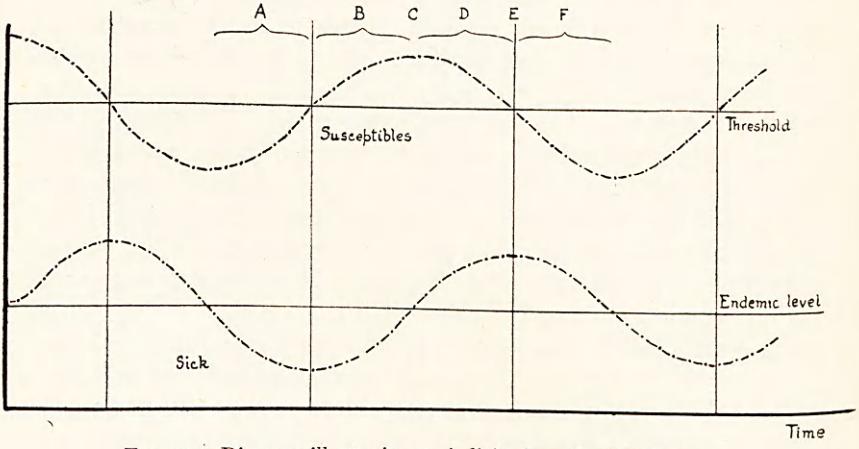

FIG. 13.—Diagram illustrating periodicity in endemic infection.
upper curve = susceptibles,
lower curve = sick.

quiescent. We are in fact led to the notion that under certain circumstances diseases may exhibit natural periodicities.

Let us look at this matter from a somewhat different point of view. We have seen that when the population of susceptibles is above the threshold density, and infection is introduced, an epidemic will occur which on the principles of the law of mass action will reduce the population as far below the threshold value as it was previously in excess of it (Fig. 11). If there is no immigration, of course, nothing more happens. If, however, fresh susceptibles are coming in (see Fig. 13), the population will gradually build itself up again. As long as the population of susceptibles is below the threshold level the disease will continue to die out (A), but once the threshold level

is exceeded (B), the number of ill will begin to increase. A point (C) will come when the disease is so intense that the number of individuals who turn ill per day is equal to the daily influx of susceptibles. At this point the population of susceptibles will have reached its maximum. The epidemic, however, will continue to grow (D) because the population of susceptibles still exceeds the threshold, and we will reach the maximum (E) of the epidemic as before when the population has returned to about its threshold density. After that point the chance of an individual who is ill recovering or dying

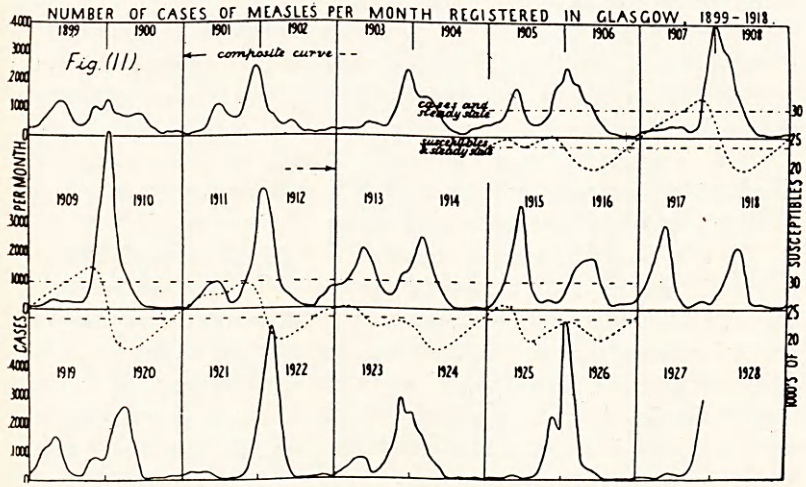

FIG. 14.—Periodicity of measles in Glasgow.
full line = cases,
dotted line = susceptibles.

becomes greater than his chance of infecting another individual, so that the number who are ill begins to fall (F) and continues to do so, and a point comes when it is so low that the daily number of those who turn ill becomes less than the daily inflow of new susceptibles. And so the process goes on, the number ill continuing to fall to a minimum which as before occurs when the number of susceptibles is at its threshold level.

You may ask whether this kind of rhythmic process, the direct consequence of the law of mass action proceeding under certain conditions, ever occurs in nature. As an answer, let us examine the curve (Fig. 14) showing the notified number of cases of measles in Glasgow during a prolonged period of

years (Soper, 1929). The periodic nature of the curve is evident. Actually the periodicity is not perfect. This is not unexpected in view of the manifold disturbing factors which must be present in any human community. Furthermore, even our schema when it is properly worked out does not lead to perfect oscillations—rather it is found they are oscillations which die out like a damped pendulum. Fresh disturbances are required to keep them going, and these may be expected to be supplied by the vagaries of climate and the accidents of communal life. Perhaps we get a fairly good analogy if we think of a pendulum subjected to odd knocks at random intervals; its period of swing would on the whole be that of its natural oscillation, but the knocks would introduce irregularities and at the same time compensate for the natural dying out of the oscillation.

The influence of the threshold density of the susceptible population has been emphasised in the foregoing, but a few further words on its nature and effects seem desirable.

In the first place we note that the attainment of a high threshold density, by whatever means this may be brought about, is much to be desired, because (1) any population having a density below that of the threshold is not at risk from imported cases of disease, and (2), if an epidemic starts (the initial density having been above the threshold), then the greater the threshold density, the sooner does the epidemic reach its peak, and the smaller is its magnitude.

Each disease has its own threshold density. The threshold rises with the rate of recovery, and the specific and non-specific death rates—that is to say, with all factors which deplete the number of sick. Thus any factor which reduces the period of illness, or more strictly the period of infectiveness, whether by death, recovery, or segregation, is advantageous to the community in the sense that it raises the threshold. Early death and early recovery act beneficially in the same direction, and so also does early segregation. The threshold may also be raised by cutting down the chances of infection. If this cutting down of the infectivity be done with sufficient energy, a very large population of susceptibles may be built up, and kept free of serious epidemics. But such a community would be very unstable in the sense that any relaxation of care would result in a sudden fall of the threshold level, and so a serious epidemic might eventuate. An example is to be found in the case of

foot-and-mouth disease, where very vigorous precautions against the spread of infection must be enforced the instant a case comes to light if a general epidemic of the disease is to be avoided. Calculation shows that if a population is near its threshold level a relatively small increase in infectivity may produce quite a serious epidemic. Conversely, a small decrease effected by the sanitarian may have a surprisingly beneficial effect upon the public health.

It is to be remembered that the factor $\bar{k}$ which we have called the " infectivity " is a composite one. It includes (*a*) the mobility of the members of the community whether sick or susceptible ; (*b*) the infectivity of the sick person ; and (*c*) the susceptibility of the persons at risk. Consequently any measures which reduce intercourse, infectivity, or susceptibility will raise the threshold and so reduce both the likelihood of occurrence of epidemics and their size if they do occur.

A further conclusion is that the immigration of healthy susceptible individuals may be of serious epidemiological importance. The population density will thereby be raised above the threshold, and a flare-up will be likely to occur which will affect not only the immigrants but also a proportion of the original inhabitants who would otherwise have escaped. In considering the effects of large immigrations of human or animal populations, this possibility must be borne in mind.

The foregoing is a mere sketch of a very large subject. I have tried to indicate some of the consequences which follow from a simple assumption, that a law of mass action can be applied to the interaction of human and animal populations just as it can be applied to populations of molecules or atoms. The principle is approximate rather than exact. Consequently it gives results which are only of a general character. The manifold complications of detail are neglected ; we concentrate our attention only on the wood and forget for the moment about the trees. It is, of course, important to deal with the individual trees, too, to study the details of particular epidemics, and the epidemiological peculiarities of particular diseases, but here we have looked from a wide perspective, and we have tried to perceive the common principle which lies behind a variety of phenomena exhibiting many superficial diversities. The common unifying principle is the same as that which plays so important a rôle in chemistry and in atomic physics, the principle of mass action.

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