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Developmental Change in the Influence of Domain-General Abilities and Domain-Specific Knowledge on Mathematics Achievement: An Eight-Year Longitudinal Study

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Abstract

The contributions of domain-general abilities and domain-specific knowledge to subsequent mathematics achievement were longitudinally assessed (n = 167) through 8th grade. First grade intelligence and working memory and prior grade reading achievement indexed domain-general effects and domain-specific effects were indexed by prior grade mathematics achievement and mathematical cognition measures of prior grade number knowledge, addition skills, and fraction knowledge. Use of functional data analysis enabled grade-by-grade estimation of overall domain-general and domain-specific effects on subsequent mathematics achievement, the relative importance of individual domain-general and domain-specific variables on this achievement, and linear and non-linear across-grade estimates of these effects. The overall importance of domain-general abilities for subsequent achievement was stable across grades, with working memory emerging as the most important domain-general ability in later grades. The importance of prior mathematical competencies on subsequent mathematics achievement increased across grades, with number knowledge and arithmetic skills critical in all grades and fraction knowledge in later grades. Overall, domain-general abilities were more important than domain-specific knowledge for mathematics learning in early grades but general abilities and domain-specific knowledge were equally important in later grades.

Keywords

domain-general abilities; domain-specific knowledge; mathematics achievement; mathematical development; mixed functional data analysis; longitudinal study

There is consensus that a combination of domain-general abilities, such as intelligence and working memory, and domain-specific knowledge contribute to academic and occupational learning, but their relative contribution to this learning is debated, including whether these contributions change over time or level of expertise (e.g., Ferrer & McArdle, 2004; Gustafsson & Undheim, 1992; Schmidt & Crano, 1974; Von Aster & Shalev, 2007). These

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are in fact longstanding issues in developmental, differential, and educational psychology (Ackerman, 2000; Ferrer & McArdle, 2004; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2016; Geary, 2011; Gustafsson & Undheim, 1992; Schmidt & Crano, 1974; Von Aster & Shalev, 2007). For instance, Cattell's (1987) influential investment theory focused on the importance of fluid intelligence (abstract problem solving), in combination with interests and personality, in the development of crystallized intelligence, including domain-specific knowledge (see also Ackerman & Beier, 2006); " … this year's crystallized ability level is a function of last year's fluid ability level–and last year's interest in school work" (Cattell, 1987, p. 139). Other differential and educational psychologists have noted that level of domain-specific knowledge can influence further gains in this knowledge (Ackerman, 2000; Sweller, 2012; Tricot & Sweller, 2013; Thorsen, Gustafsson, & Cliffordson, 2014).

The results from associated empirical studies have been mixed, however. Schmidt and Crano (1974) found that intelligence measured in 4th grade predicted gains on a composite measure of academic achievement through 6th grade, controlling prior achievement, consistent with Cattell's (1987) argument. However, the pattern was found only for students from middle socioeconomic status (SES) households, not for students from lower SES households. Ferrer and McArdle (2004) found that fluid intelligence predicted gains in mathematics achievement throughout childhood and adolescence but did not assess if the magnitude of this relation varied across grades. Moreover, once intelligence, general knowledge, and performance in nonmathematical academic areas were controlled, prior mathematics achievement was inversely related to subsequent achievement. Gustafsson and Undheim (1992), in contrast, found a strong positive relation between a composite of academic skills in 6th grade and academic skills in 9th grade, but intelligence in 6th grade did not predict gains in academic skills through 9th grade, above and beyond the influence of prior skills.

The mixed results are related in part to use of different domain-general and domain-specific measures across studies, as well as different age ranges and different analytic techniques. Assessments of these relations were initially based on cross-lagged correlations (e.g., Schmidt & Crano, 1974), whereby the relation between domain-general abilities and domain-specific knowledge at one age predicts abilities and knowledge at a later age, but critiques of these methods (Kenny, 1975; Rogosa, 1980) resulted in the development and use of more sophisticated autoregressive cross-lagged models (Hamaker, Kuiper, & Grasman, 2015; Preacher, 2015). These models provide greater flexibility in the estimation of time-related change in the influence of domain-general abilities and domain-specific knowledge on the outcome of interest. In one recent study in which these methods were used, Lee and Bull (2016) found a stable across-grade influence of working memory on subsequent-grade mathematics achievement and that the importance of prior mathematics achievement increased across grades.

Functional data analysis (FDA) may provide an even more flexible approach to the study of longitudinal relations because it is not constrained by the parametric assumptions of cross-lagged and related models; indeed, the latter may be considered subsets of FDA (Müller, 2009). The mixed FDA approach has been successfully applied to many areas including image analysis and signal analysis as well as estimation of growth curves. In particular, for the estimation of growth curves, it allows one to capture the underlying shape of the true curve much more accurately than other methods (Ramsay, Hooker, & Graves, 2009; Wang, Chiou, & Müller, 2016). In the context of this study, the FDA approach enabled the simultaneous assessment of domain-general and domain-specific effects on gains in children's mathematics achievement from 2nd to 8th grade, inclusive, and linear and non-linear changes in the relative contributions of these effects across grades.

As with other academic domains, the relative contributions of domain-general abilities and domain-specific knowledge to subsequent mathematics achievement are not fully understood and may vary across grade, level of student knowledge, and mathematical content (Bailey, Watts, Littlefield, & Geary, 2014; Frisovan den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2016; Geary, 2011; Lee & Bull, 2016; Von Aster & Shalev, 2007; Watts et al., 2015). Identifying the grade-to-grade contributions of domain-general and domain-specific effects and changes in the relative magnitude of these effects will contribute significantly to our understanding of the factors that drive children's mathematical development and will provide insights into when and where to target interventions to improve this development.

Domain-General Abilities, Domain-Specific Knowledge and Mathematics Achievement

Intelligence and one or more components of working memory are the most frequently studied domain-general predictors of mathematics achievement (e.g., Fuchs et al., 2016; Geary, 2011; Lee & Bull, 2016; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015), although reading- and language-related competencies are included in some studies (LeFevre et al., 2010; Watts, Duncan, Siegler, & Davis-Kean, 2014), as are non-cognitive constructs, such as mathematics self-concept (Watts et al., 2015). Among these measures, the most consistent effects are found for intelligence and the updating–holding information in mind while processing other information – component of working memory (Deary, Strand, Smith, & Fernandes, 2007; Friso-van den Bos et al., 2013; Geary, 2011; Lee & Bull, 2016; Östergren & Träff, 2013; Siegler et al., 2012), although the strength of these effects can vary across grades and with the novelty and complexity of the mathematics domain being assessed (e.g., Fuchs et al., 2010; Lee & Bull, 2016). On the basis of these findings, we included 1st grade measures of intelligence and the updating–component of working memory (or central executive) as domain-general abilities. Fluid intelligence and competence at updating improve over the timeframe assessed here (e.g., Fry & Hale, 1996; Gathercole, Pickering, Ambridge, & Wearing, 2004; Li & Geary, 2013), but individual differences in these domains are at least moderately stable (Mazzocco & Kover, 2007; Sameroff, Seifer, Baldwin, & Baldwin, 1993; Thorndike, 1933)

We also included prior grade word reading achievement as a domain-general effect. Word reading is obviously a domain-specific skill, but may also tap domain general abilities. In the traditional psychometric literature, all cognitive and academic tests share common variance, typically attributed to intelligence, as well as unique test-specific variance (Carroll, 1993). Fuchs et al. (2016), as an example, found that children's early fluency with identifying letter and high-frequency words predicted subsequence word reading and arithmetic achievement, but the magnitude of the effect of 3.5-fold higher for reading than arithmetic (see also Chu, vanMarle, & Geary, 2016). The effect for arithmetic may reflect the operation of domain-general mechanisms, whereas that for reading reflects these mechanisms and the influence of domain-specific skills. There is some evidence that this domain-general ability is not intelligence or working memory but rather the ease with which students form arbitrary visual-verbal associations (Koponen et al., 2013). This would explain the often-found relation between basic reading skills and memorization of arithmetic facts (e.g., Hecht, Torgesen, Wagner, & Rashotte, 2001), both of which may reflect the functional integrity of the domain-general hippocampal-dependent memory system (Qin et al., 2014).

Studies of the influence of earlier domain-specific knowledge on the acquisition of later knowledge are either based on the relation between mathematics achievement scores in one grade and scores on the same or a similar measure in a later grade (Bailey et al., 2014; Duncan et al., 2007; Lee & Bull, 2016), or the relation between specific mathematical competencies in one grade and achievement in a later grade. The key mathematical knowledge identified in the latter studies includes an understanding of numbers and basic arithmetic in the early grades (Clements, Sarama, Wolfe, & Spitler, 2013; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Koponen, Salmi, Ekland, & Aro, 2013), and fractions in later grades (Bailey, Hoard, Nugent, & Geary, 2012; Siegler et al., 2012). Accordingly, in separate analyses domain-specific knowledge was indexed by prior grade mathematics achievement or by several measures of prior grade number, arithmetic, or fractions knowledge that have been shown to predict later mathematics achievement (Bailey et al., 2012; Geary, 2011; Geary et al., 2013; Booth & Siegler, 1996). For the latter, we assessed children's number knowledge and their basic arithmetic skills, beginning in 1st grade, and used these to predict mathematics achievement in the subsequent year; specifically, 2nd to 8th grade, inclusive. We also used a 6th grade measure of fractions knowledge to predict concurrent mathematics achievement (we did not have a 5th grade fractions assessment) and achievement in 6th, 7th and 8th grades. The inclusion of three grades allowed us to assess non-linear effects for this measure.

Current Study

As noted, we applied longitudinal mixed FDA techniques (Guo, 2002; Liu & Guo, 2011) to data from a unique longitudinal study to simultaneously estimate the contributions of domain-general abilities and domain-specific knowledge on mathematics achievement from 2nd to 8th grade, inclusive (below). The models also allowed us to estimate across-grade linear and nonlinear changes in these effects, across-grade changes in the contribution of individual domain-general and domain-specific variables, and across-grade change in the overall contribution of domain-general abilities and domain-specific knowledge on subsequent mathematics achievement. In all, the statistical approach and unique longitudinal data set that included assessment of key domain-general abilities and key domain-specific knowledge provided a more nuanced picture of stability and change in the influence of domain-general and domain-specific effects on children's mathematics achievement than previous studies and thus informs approaches to improve this achievement.

Method

Participants

All kindergarten children from 12 elementary schools that serve families from a wide range of socioeconomic backgrounds were invited to participate. Parental consent and child assent were received for 37% (n = 311) of these children and 288 of them completed the first year of testing (see Geary, Hoard, Nugent, & Bailey, 2012). The 167 children (78 boys) used in the current analyses completed all annual achievement assessments and the vast majority of mathematical cognition tasks (below) through the end of 8^{th} grade. 1 shows the measures used in the analyses or reported elsewhere in the text and the grades in which they were administered.

The intelligence of the sample used in the analyses was average (M = 102, SD = 15), as was their mathematics achievement at the end of kindergarten (M = 103, SD = 13) and 8^{th} grade (M = 94, SD = 18). Their reading achievement was high average at the end of kindergarten (M = 113, SD = 14) and average at the end of 8^{th} grade (M = 101, SD = 12). The children who did not complete all of these measures (n = 121) had lower intelligence (M = 95, SD = 14), $F(1,286) = 16.04$, $p<.0001$, and lower mathematics achievement at the end of kindergarten (M = 99, SD = 14), $F(1,286) = 6.24$, $p=.013$, than the sample used here; however, their kindergarten reading scores (M = 110, SD = 15) did not differ, $F(1,286) = 2.96$, $p=.0865$. Five percent of the sample identified as Hispanic. Seventy seven percent identified as White, 4% as Black, 6% as Asian, 8% as mixed race, and 5% as other or unknown. The sample averaged 6 years 2 months of age (SD = 4 months) at the kindergarten achievement assessment and 14 years 2 months (SD = 4 months) at the 8^{th} grade achievement assessment.

Participants' parents were asked to complete a survey that included items on their education level, income, and government assistance. Complete or partial information was available for the families of 149 participants. Of these parents, 4% had some schooling but no GED or high school degree; 24% had a high school diploma or GED; 4% had some college, technical school, or an associate's degree; 39% had a Bachelor's degree; and 29% had a postgraduate degree. The total household income was: \$0–\$25k (6%), \$25k –\$50k (22%), \$50k–\$75k (19%), \$75k–\$100k (14%), \$100k–\$150k (22%), \$150k or more (16%). Six percent of parents reported receiving food stamps; 1% reported receiving housing assistance. We used the education level of the students' primary care giver as an indictor of family SES; specifically, 4-year college degree (n = 101), high school diploma (n = 48), and no information (n = 18). The intelligence of children from each of these family types was in the average range. The IQ of children from high school households (M = 95, SD = 15) was lower than that of children from college (M = 105, SD = 14) and no-information (M = 103, SD = 17) households (ps < .05); the two latter groups did not differ (p > .50). The kindergarten mathematics achievement of children from college households (M = 106, SD = 13) was higher than that of children from high school (M = 98, SD = 9) and no-information (M = 98, SD = 12) households (ps < .005); the two latter groups did not differ (p > .50). At the end of 8^{th} grade, the mathematics achievement of children from high school households (M = 86, SD = 17) was lower than that of children from college households (M = 98, SD =

18, $p < .0002$); the children from no-information households ($M = 92$, $SD = 14$) did not differ from either group (ps > .15).

In preliminary analyses, the parental education and income categories were included as covariates ($n = 149$) but did not substantively change our results with inclusion of the domain-general variables. Thus, these were not included in the final analyses, which allowed us to use data from all 167 children. We include the information here to provide a more thorough description of our sample.

Mathematics Achievement

Mathematics achievement was assessed with the Numerical Operations subtest of the *Wechsler Individual Achievement Test-II: Abbreviated* (WIAT-II; Wechsler, 2001). The easier items include number discrimination, rote counting, number production, and basic arithmetic operations. More difficult items include rational numbers and simple algebra and geometry problems solved with pencil-and-paper. Spearman-Brown reliability estimates for the age ranges assessed here range from .87 to .96 (median = .91) (Wechsler, 2001).

Domain-General Measures

Intelligence—Full-scale IQ was estimated using the Vocabulary and Matrix Reasoning subtests of the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999).

Central executive—The central executive was assessed using three dual-task updating subtests of the *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001). Listening Recall requires the child to determine if a sentence is true or false and then to recall the last word in a series of sentences. Counting Recall requires the child to count a set of 4, 5, 6, or 7 dots on a card and then to recall, in order, the number of dots counted on each card at the end of that series of cards. Backward Digit Recall is a standard format backward digit span task. The subtests consist of span levels ranging from one to six or one to nine items to remember, and each span level has six trials. Failing three trials at one span level terminates the subtest, and passing four trials moves the child to the next level. The total number of trials answered correctly was used as the central executive measure, because these scores are more reliable than span scores ($\alpha = .77$). To keep the assessment time consistent with our IQ measure, we used 1st grade scores as the domain-general predictor. The students were administered the same measure in 5th grade and the across grade correlation, $r = .61$, $p < .0001$, indicated at least moderate stability in individual differences in the central executive.

Reading achievement—Reading achievement was assessed with the Word Reading subtest of the WIAT-II (Wechsler, 2001). The easier items include matching and identifying letters, rhyming, beginning and ending sounds, and more difficult items assess accuracy of reading increasingly difficult words. Spearman-Brown reliability estimates for the age ranges assessed here range from .95 to .98 (median = .97) (Wechsler, 2001).

Domain-Specific Mathematical Cognition Measures

As noted, we included specific measures of number skills, arithmetical competencies, and knowledge of fractions concepts that have been shown to predict concurrent or later mathematics achievement in previous studies (e.g., Bailey et al., 2012; Booth & Siegler, 2006; Clements et al., 2013; Jordan et al., 2009; Koponen et al., , 2013; Siegler et al., 2012). These included fluency in accessing and manipulating the quantities associated with Arabic numerals (number sets test), accuracy in placing numerals on the number line (number line estimation), knowledge of basic addition facts and ability to decompose numbers into smaller sets to solve more complex addition problems (addition strategy choices), and fluency in determining the larger of two fractions (fractions comparison test).

Number sets test—Two types of stimuli are used: objects (e.g., squares) in a 1/2” square and an Arabic numeral (18 pt font) in a 1/2” square. Stimuli are joined in domino-like rectangles with different combinations of objects and numerals. These dominos are presented in lines of 5 across a page. The last two lines of the page show three 3-square dominos. Target sums (5 or 9) are shown in large font at the top the page. On each page, 18 items match the target; 12 are larger than the target; 6 are smaller than the target; and 6 contain 0 or an empty square.

The tester begins by explaining two items matching a target sum of 4; then, uses the target sum of 3 for practice. The measure is then administered. The child is told to move across each line of the page from left to right without skipping any; to “circle any groups that can be put together to make the top number, 5 (9)”; and to “work as fast as you can without making many mistakes.” The child has 60 sec per page for the target 5; 90 sec per page for the target 9. Time limits were chosen to avoid ceiling effects and to assess fluent processing of quantities.

The variable used here was based on the d-prime measure; specifically, (z-hits z-false – alarms) x (maximum RT/actual RT). Thus, the scores of children who completed the test in less than the maximum time were adjusted upward. The adjustment was made because most of the children completed the test in less than the maximum time in later grades. The adjustment enabled us to maintain the sensitivity of the test, despite faster processing times across grades. The d-prime score appears to capture the speed and accuracy with which children can access the magnitudes associated with whole numbers and their implicit or explicit understanding of arithmetic (Moore, vanMarle, & Geary, 2016), and is highly reliable in all grades; the Spearman-Brown reliability estimates ranged from .89 to .92 (median =.90), based on the correlation between performance on the 5 and 9 target sums.

Number line estimation—Across grades, two different number line tasks were administered. The 1st to 5th grade, inclusive, task was a 0 to 100 number line. Here, a series of twenty-four 25cm number lines containing a blank line with the two endpoints (0 and 100) was presented, one at a time, to the child with a target number (e.g., 43) in a large font printed above the line. The child’s task was to mark the line where the target number should lie (Siegler & Booth, 2004); a pencil-and-paper version was used in first grade and a computerized version, where the child used the mouse to mark the line, thereafter. In later grades, the procedure was the same, but the task was a 0 to 1000 number line in 6th and 7th grade. The measure was the percent absolute error, because this correlates with later mathematics achievement (Booth & Siegler, 2006) and captures children's understanding of the line (Siegler et al., 2011) regardless of the strategies used to guide the placements (Rouder & Geary, 2014); αs = .61 to .84 (median = .72).

Addition strategy choices—Fourteen simple addition problems and six more complex problems were horizontally presented, one at a time, at the center of a computer monitor (using flash cards in 1st grade). The simple problems consisted of the integers 2 through 9, with the constraint that the same two integers (e.g., 2+2) were never used in the same problem; ½ of the problems summed to 10 or less and the smaller valued addend appeared in the first position for ½ of the problems. The complex items were 16+7, 3+18, 9+15, 17+4, 6+19, and 14+8.

The child was asked to solve each problem (without pencil-and-paper) as quickly as possible without making too many mistakes. It was emphasized that the child could use whatever strategy was easiest to get the answer, and was instructed to speak the answer into a microphone that was interfaced with the computer that in turn recorded reaction time (RT) from onset of problem presentation to microphone activation. After solving each problem the child was asked to describe how they got the answer. Using the child's description and the experimenter's observations, the trial was classified as counting fingers, verbal counting, retrieval, and decomposition. The combination of experimenter observation and child reports has proven to be a useful measure of children's strategy choices (Siegler, 1987). The validity of this information is supported by RT patterns; finger-counting trials have the longest RTs, followed respectively by verbal counting, decomposition, and direct retrieval (Geary, Hoard, & Bailey, 2012).

We used two variables from these tasks. The first was the frequency with which children correctly retrieved answers to the simple addition problems. This variable indexed their relative mastery of basic facts. The second was the frequency with which decomposition was correctly used to solve the more complex problems; for instance, to solve 7 + 16, first decomposing 7 into 3 and 4 and then adding 16 + 4 and then 20 + 3. Use of this strategy reflects children's conceptual understanding that numbers are composed of subsets of smaller numbers (Geary, Hoard, Byrd-Craven, & DeSoto, 2004).

Fractions comparison test—The 16 items require the child to circle the larger of two presented fractions in 120 seconds (Geary et al., 2013). This task consists of four types of comparisons. The first type presents two fractions with a constant numerator but different denominators (e.g., 1/5 vs. 1/9), which assesses the child's understanding of the inverse relation between the denominator and the fraction value. In the second type, both numerators and denominators differ and the fraction with the larger value always has the larger numerator and smaller denominator (e.g., 3/10 vs. 2/12). If the child focuses on the numerators and chooses the larger one, the child will always be correct. If the child focuses on the denominators, in contrast, and chooses the larger one, the child will always be incorrect. The numerators in each comparison pair have a ratio of 1.5 and denominators have ratios between 1.1 and 1.25; these ratios were chosen based on the Weber function for magnitude discrimination in adolescents (Halberda & Feigenson, 2008). In the third type of comparison, numerators and denominators are reversed (e.g., 3/2 vs. 2/3) to assess whether the child understands that the larger fraction should have the larger numerators and the smaller denominators. The comparisons in the fourth type involve a fraction with a 1/2 value as an anchor and the other fraction close to 1 (e.g., 20/40 vs. 8/9). A child who understands fraction magnitudes should be able to quickly identify the 1/2 fraction and choose the other one.

The four types of comparisons were designed to examine whether the children conceptually understand the meanings of the numerator, the denominator, and the value of a fraction as a whole. The child received 1 point for circling the correct answer in each pair. Scores were summed across item type to create a single composite score (α = .83).

Procedure

Assessments—The achievement measures were administered in the spring semester of each academic year and the WASI was administered in the spring of first grade. The addition strategy and number sets tasks were administered in the fall of each academic year. The number line measure was administered in the fall of first grade and in the spring of subsequent academic years to accommodate time constraints in the fall assessment. The fractions comparison task was administered in the spring semester in 6^{th} grade. The majority of children were tested in a quiet location at their school site, and occasionally on the university campus or in a mobile testing van. Testing in the van occurred for children who had moved out of the school district or to a non-participating school and for administration of the WMTB-C (e.g., on the weekend or after school). The mathematical cognition and achievement assessments required about 40 min and the WMTB-C about 60 min per assessment.

Analyses—For the 167 students used in these analyses, the 1.3% of missing data were imputed using the Multiple Imputation procedure in SAS (SAS, 2014), using the EM algorithm with multivariate normality assumption. All variables were standardized (*M* = 0, *SD* = 1) at each grade level to provide a common metric for estimating across-grade change in the relative contribution of domain-general abilities and domain-specific knowledge on subsequent mathematics achievement, following the mixed FDA discussed in Guo (2002) and in Liu and Guo (2011). The method allows for both the estimation of regression effects that are linear functions of time (grade) from longitudinal data and the estimation of non-linearity in these functions of time. For n subjects assessed across multiple grades, subject i is observed at grades, $g_{i1} < \ldots < g_{im_i}$, $i = 1, ..., n$. The outcome of interest is mathematics achievement or y, such that y_{ij} denotes the observed value on subject i at grade g_{ij} , j = 1, ..., m_i, i = 1 , ... , n. Then the FDA model assumes that y_{ij} can be described by

$$y_{ij} = X_{ij}\beta(g_{ij}) + Z_{ij}\alpha_i(g_{ij}) + e_{ij}, \quad (1)$$

whereby, $\beta(g) = \{\beta_1(g), ..., \beta_p(g)\}$represents a set of fixed functions (i.e., domain-general and domain-specific regression effects), and represents a set of random $\alpha_i(g) = \{\alpha_{1i}(g)$, ...,

$\alpha_{qi}(g)\}$ functions (i.e., individual subject effects), x_{ij}, z_{ij} represent the two matrices of the study design, and e_{ij} is the error term. Liu and Guo (2011) suggest to represent both fixed and random functions by linear splines, which are piecewise linear polynomials joined end-to-end at the joints (grades). For each individual variable, the across-grade estimates are simultaneously and jointly calculated.

One major advantage is that the method allows estimation of the curve for each individual variable and results in smooth and more natural estimates for the overall regression functions, i.e., the grade-over-grade estimates of the relation between prior domain-general abilities and domain-specific knowledge on subsequent mathematics achievement. The estimates were obtained using the %fmixed SAS macro, following Liu and Guo (2011). All substantive models were variations of equation 1. The two domain-specific models, model 1 and model 2 were represented by,

$$\mathrm{NO}_i(g)=\beta_1(g)\mathrm{NO}_i(g-1)+\alpha_{1i}(g)+e_i(g), \quad (2)$$

$$\mathrm{NO}_i(g)=\beta_1(\mathrm{g})Ret_i(g-1)+\beta_2(g)Dec_i(g-1)+\beta_3(g)NS_i(g-1)+\beta_4(g)NL_i(g-1)+\alpha_{1i}(g)+e_i(g), \quad (3)$$

whereby NO is Numerical Operations scores, and $\beta_1(g)\mathrm{NO}_i(g-1)$ represents the prediction of these scores by Numerical Operations scores from the preceding grade. Model 1 (equation 2) thus estimates the effects of prior-grade mathematics achievement on subsequent mathematics achievement. For model 2 (equation 3), the effect of Numerical Operations scores are replaced by preceding grades' fact retrieval (*Ret*), use of decomposition (*Dec*), number sets fluency (*NS*), and number line accuracy (*NL*). Model 2 thus allows for the estimation of domain-specific effects for all of these variables unadjusted for domain-general effects and grade-to-grade changes in the magnitudes of each of these effects. Comparison of the fit of model 1 and model 2 assesses whether broad achievement measures or specific quantitative knowledge are the better indicators of domain-specific effects.

As noted, domain-general abilities included 1st grade intelligence (*IQ*) and 1st grade central executive (*CE*) scores, as well as reading scores (*Read*) from the prior grade, as represented by model 3 (equation 4). Model 3 thus provides an overall assessment of the importance of these domain-general abilities on subsequent mathematics achievement, unadjusted by domain-specific effects. The fourth model (equation not shown) combined all terms from models 2 and 3, and thus provides the critical adjusted simultaneous estimate of domain-specific and domain-general effects, and grade-to-grade changes in the magnitude of these effects, on subsequent mathematics achievement. The first through fourth models provide estimates for domain-specific and domain-general effects unadjusted or adjusted from 2nd to 8th grade, inclusive. The fifth model (equation not shown) added the fractions comparison measure at 6th grade to the model for combined effects, but only estimated effects from 6th to 8th grade, inclusive.

$$\mathrm{NO}_i(g)=\beta_1(g)IQ_i+\beta_2(g)Read_i(g-1)+\beta_3(g)CE_i+\alpha_{1i}(g)+\mathrm{e}_i(g) \quad (4)$$

Results

Independent or Unadjusted Domain-Specific and Domain-General Effects

Table 2 shows the variance components for the fixed and random effects from all of the models, as well as the mean R^2 over grades for each model. The spline variance components in particular indicate the extent to which across-grade estimates deviate from a straight line, with 0 estimates indicating linear trends and > 0 estimates indicating non-linear trends, or spline. The grade-to-grade estimates of fixed domain-specific effects for Numerical Operations (Model 1) and the four mathematical cognition measures (Model 2) are shown in Figure 1 (Panel A).

When no other variables are included in the model, the importance of prior grades' Numerical Operations scores on subsequent scores increases across grades, suggesting a gradual increase in the importance of domain-specific knowledge. It is also clear from Figure 1 that the importance of each individual mathematical cognition variable for predicting subsequent achievement is smaller than prior Numerical Operations scores, but in combination the mathematical cognition variables explain substantively more variation in subsequent achievement (mean R^2 = .822, Table 2) than does Numerical Operations (mean R^2 = .682). The latter indicates that domain-specific effects are better represented by the combination of mathematical cognition scores than by scores on prior mathematics achievement tests, and thus the mathematical cognition scores were included in combined models (below).

As shown in Table 2, estimation of the three domain-general effects (Model 3) explained a substantial amount of variance in subsequent Numerical Operations scores, mean R^2 = .814. The effects for intelligence and reading achievement were linear across grades. The estimates for the central executive increased and then decreased across grades (Figure 1, Panel B), but the deviation from linearity was not significant (p = .398).

Simultaneous or Adjusted Domain-Specific and Domain-General Effects

The core analyses are the simultaneous estimation of domain-specific and domain-general effects across grades in model 4, and the determination of whether the relative magnitudes of these effects change across grades. The associated estimates are shown in Table 3 and Figure 2. None of the across-grade effects differ significantly from a straight line. As shown in Table 2 and Figure 2, there were potential non-linear effects for the decomposition and central executive variables, but the overall deviation from a straight line was not significant for either variable (ps > .4432). There were nevertheless differences across the larger and smaller grade-level estimates for these two variables. The smallest effects for decomposition were for grades 4 and 5 and the largest for grades 2 and 8. Follow up contrasts of these grades confirmed larger effects for grades 2 and 8 than for grades 4 and 5 (*p*s < .039). Similar contrasts for the central executive revealed no differences between grade 2 (smallest effect) and grade 5 (largest effect) ($p = .107$), but there was a trend for the contrast of grades 5 and 8 ($p = .089$).

As can be seen in Table 3, the pointwise significance of the grade-level effects for the domain-specific variables is mixed through 5th grade, that is, some of the effects are significant (e.g., 1st grade number sets fluency predicting 2nd grade Numerical Operations scores, $p < .001$), but others are not (e.g., 1st grade fact retrieval predicting 2nd grade Numerical Operations scores, $p = .733$). After 5th grade, all of the individual domain-specific effects are significant (*p*s < .038), consistent with a gradual increase in the importance of domain-specific knowledge. With the exception of 7th grade Word Reading scores predicting 8th grade Numerical Operations scores (p=.084), all of the individual domain-general effects are significant in every grade.

To determine if there was an overall trend of increasing or decreasing effects of individual variables across 2nd to 8th grade for this combined model, we tested whether the linear slope of the across-grade fixed effect estimates was statistically different from 0; we excluded the central executive and decomposition because of the across-grade non-linearity noted above. The linear slope was non-significant for intelligence ($p = .7248$), reading ($p = .8882$), number sets ($p = .7309$), addition fact retrieval ($p = .1102$), and number line ($p = .3783$). With the possible exception of the central executive and decomposition, the pattern suggest that despite grade-to-grade differences in the significance of individual domain-specific and domain-general variables, the magnitude of the relation between these variables and subsequent mathematics achievement was not different from constants across grades.

A similar overall pattern is evident for 6th to 8th grade, with the inclusion of the fractions comparison measure (Figure 3, Table 4). With the inclusion of the latter, the effects of intelligence (p=.217) and Word Reading (p=.144) are no longer significant by 8th grade, but the central executive remains important (p=.007). For the same grade, four of the five mathematical cognition measures are significant (*p*s<.039), and the individual effect for fractions comparison ($\beta = .206$, p<.002) is at least as important as that of the central executive ($\beta = .18$, p<.007). Examination of 6th to 8th grade change in the relation between these variables and subsequent mathematics achievement revealed a trend for a decline in the importance of intelligence ($p = .096$) and a significant increase in the importance of decomposition ($p = .015$).

The overall importance of the domain-specific and domain-general variables on subsequent mathematics achievement is provided by the sum of the respective β estimates for each grade. As shown in Figure 4, the overall domain-general effects are relatively stable, ranging between .453 and .558. None of the adjacent grade comparisons differ significantly for these domain-general effects (*p*s > .126), nor does the contrast of the smallest (grade 8) and largest (grades 4 and 5) effects (*p*s > .106). The overall domain-specific effects, in contrast, are more variable, ranging from .315 to .551. Adjacent grade comparisons indicated a significant decrease in overall domain-specific effects from grade 2 to grade 3 ($p = .050$), but increases from grade 6 to grade 7 ($p = .034$), and grade 7 to grade 8 ($p = .024$). Moreover, the contrast of the smallest (grade 4) to largest (grade 8) overall domain-specific effect was highly significant ($p = .007$). There is a trend for larger overall domain-general than domain-specific effects in grade 3 ($p = .067$) and significant differences in grades 4 ($p =.014$) and 5 ($p = .023$). As shown in Figure 4, the overall domain-specific effect exceed the domain-general effect by 8th grade, but none of the overall differences are significant after 5th grade (*p*s > .117).

Discussion

The combination of longitudinal mixed functional data analysis and a unique data set enabled a more nuanced assessment of a long-standing question in psychology than afforded by previous studies and analytic approaches (e.g., Ackerman, 2000; Ferrer & McArdle, 2004; Fuchs et al., 2016; Geary, 2011; Gustafsson & Undheim, 1992; Schmidt & Crano, 1974; Thorsen et al., 2014; Von Aster & Shalev, 2007); specifically, the relative contributions of prior domain-specific knowledge and domain-general abilities on subsequent achievement and estimation of grade-over-grade change in the relative contribution of knowledge and abilities on this achievement. Moreover, the outcome itself, the development of mathematical competencies, is critically important for success in a wide range of jobs in the modern economy and for navigating the many now-routine activities of daily life (Bynner, 1997; Reyna, Nelson, Han, & Dieckmann, 2009), and thus the results are practically important.

Domain-General Abilities

Our finding that intelligence, the central executive, and reading achievement made significant contributions to subsequent mathematics achievement in most or all grades is consistent with many previous studies (Fuchs et al., 2016; Geary, 2011; Lee & Bull, 2016; LeFevre et al., 2010; Van de Weijer-Bergsma et al., 2015; Watts et al., 2015). There are nevertheless several aspects of our results that expand on this literature. The first is that the combination of these three measures, without inclusion of the domain-specific effects, explained substantial variance in mathematics achievement, suggesting these variables captured the bulk of broadly defined domain-general abilities. This does not necessarily mean that these are *the* core domain-general abilities that contribute to individual differences in mathematics achievement, but rather they provide a strong proxy for a broad set of abilities that are correlated with the measures assessed here and that also contribute to mathematics achievement (e.g., Fuchs et al., 2010; Fuchs et al., 2016).

Moreover, we did not have assessments of non-cognitive traits that could, in theory, also predict academic achievement (e.g., Cattell, 1987; Eccles, Vida, & Barber, 2004; Ma, 1999). Watts et al. (2015), for instance, found that mathematics self-concept in 6th grade was significantly correlated with later mathematics achievement and with earlier performance on working memory measures. Moreover, mathematics self-concept predicted later mathematics achievement, controlling working memory and domain-specific division and fractions knowledge, suggesting an important non-cognitive effect. As they note, however, mathematics self-concept could be a reflection of prior achievement and is related to later mathematics only because of the strong correlations between earlier and later achievement

(Duncan et al., 2007). In other words, it remains to be determined if the magnitude of our specific domain-general effects would change with inclusion of non-cognitive measures, such as mathematics self-concept or mathematics anxiety, that often predict later achievement.

Second, our overall results suggest stable across-grade domain-general effects on subsequent mathematics achievement. Some previous studies are also consistent with stable effects (Bailey et al., 2014), but more often than not the influence of domain-general abilities varies across grades when domain-specific variables are included as predictors (e.g., Fuchs et al., 2016; Lee & Bull, 2016; Van de Weijer-Bergsma et al., 2015). In the latter case, domain-general effects are often indirect, mediated by the relation between these abilities and prior domain-specific achievement or individual domain-specific competencies, such as arithmetic skills (e.g., Fuchs et al., 2016; Östergren & Träff, 2013). Although Cattell's (1987) ideas have not been incorporated into this literature, the associated results are consistent with his argument that measures of crystallized abilities or domain-specific knowledge reflect, in part, prior levels of fluid intelligence. Our results show this same pattern, whereby domain-general abilities influence subsequent mathematics achievement, and individual differences in this achievement or in specific mathematical competencies facilitate further mathematical gains.

Our domain-general measures, however, do not provide pure assessments of fluid intelligence and thus are not a direct test of Cattell's (1987) investment hypothesis. Nevertheless, the combination of standard IQ scores and working memory used here will be highly correlated with fluid abilities (e.g., Conway, Cowan, Bunting, Therriault, & Minkoff, 2002; Engle, Kane, & Tuholski, 1999; Deary, 2000; Geary, 2005), and therefore our results, though not definitive, are consistent with Cattell's hypothesis. With the inclusion of prior reading achievement in the set of domain-general variables, our results could also be interpreted as being consistent with Carroll's (1993) general intelligence that subsumes fluid intelligence and other processes that influence learning; including ease of associative learning that may underlie the relation between word reading and mathematics achievement (Chu et al., 2016; Koponen et al., 2013). However the results are framed, the key finding is that domain-general abilities, including those assessed 7 years earlier, influence mathematics achievement throughout much of formal schooling, even with control of domain-specific competencies.

Of these abilities, the central executive component of working memory (updating) emerged as particularly important. Our results for this measure were consistent with a recent cross-sequential preschool to 9th grade study in which Numerical Operations scores were predicted by prior achievement, an updating measure of working memory, and intelligence (Lee & Bull, 2016). The relation between working memory and subsequent mathematics achievement was stronger in earlier than later grades, but statistically constraining the relation to be identical across grades fitted the data nearly as well as allowing these relations to vary: The most parsimonious explanation is that the relation between working memory and subsequent achievement is stable across grades.

At the same time, Lee and Bull (2016) found that intelligence did not predict achievement in early grades, once working memory and prior achievement were controlled, but was important in 6th to 9th grade. Our results suggest a more consistent relation between intelligence and achievement across grades, but perhaps a declining influence in later grades, once fractions knowledge is included as a domain-specific effect. The differences might be related to different analytic approaches, use of different intelligence tests (they used a block design test), estimation of domain-specific effects using prior achievement versus our mathematical cognition variables, or some combination. Either way, both studies are consistent with intelligence as an important domain-general ability (see also Deary et al., 2007), although perhaps less important than working memory in some grades and more important in others, depending on mathematical content and students' prior knowledge.

The inclusion of prior reading achievement is not as straightforward as a domain-general ability as working memory and intelligence, although use of reading- and-language related measures in similar studies is common (e.g., Fuchs et al., 2016; LeFevre et al., 2010; Watts et al., 2015). Still, word reading should have little if any direct effect on solving problems on the Numerical Operations test, but as noted may index the ease of associative learning (Koponen et al., 2013) and functional integrity of the hippocampal-dependent memory system (Qin et al., 2014). The latter would be consistent with Cattell's (1987) 'rote learning' contributions to the development of domain-specific knowledge and is a component of Carroll's (1993) model of general intelligence. Nevertheless, further work is needed on the basic cognitive and neural mechanisms, above and beyond intelligence and working memory, that contribute to ease of learning some aspects of reading competencies and some aspects of mathematical competencies (Geary, 1993).

Domain-Specific Mathematical Knowledge

As we found here, Lee and Bull (2016) reported that the relative importance of prior mathematics achievement on subsequent achievement increased across grades. The across-grade increase in the importance of mathematic knowledge for the learning of new mathematical knowledge, with relatively stable domain-general effects, supports instructional approaches that focus on learning domain-specific knowledge rather than teaching domain-general problem-solving competencies (Tricot & Sweller, 2013), and is consistent with individual differences in adult levels of expertise in mathematics and other domains (Ackerman, 2000; Ackerman & Beier, 2006). Our results and those of Lee and Bull (2016) also suggest that individual differences in domain-specific mathematical competencies may be a critical factor driving the greater variability among students in mathematics achievement in later than earlier grades. The implication is that addressing prerequisite domain-specific skill deficits has the potential to substantially reduce individual differences in mathematical competencies at school completion. Our results suggest these prerequisite skills include number knowledge and basic arithmetic in the early grades and fractions knowledge in later grades, consistent with other studies (e.g., Siegler et al., 2012). However, the same caveat for our domain-general measures applies here; although our measures were carefully chosen based on prior studies (Bailey et al., 2012; Booth & Siegler, 2006; Clements et al., 2013; Jordan et al., 2009; Koponen et al., 2013; Siegler et al., 2012), we cannot conclude that these measures capture all of *the* key domain-specific knowledge needed to progress in mathematics. It is likely that our measures are important, but other measures are likely to be just as important, and indeed the set of critical prior skills may vary to some extent across grades and with the mathematical outcome of interest. For instance, the Numerical Operations test does not include many geometry items and thus the importance of prior geometrical knowledge on future mathematics achievement could not be assessed.

Finally, a few words for our decomposition variable: Although the overall U-shaped relation between use of decomposition and subsequent-grade mathematics achievement was not significant, there was evidence that students' who frequently used decomposition had higher achievement than their peers in earlier and later grades. In early grades, only children with the most sophisticated understanding of numbers and the relations among them use decomposition with any frequency and thus its importance early in elementary school makes sense (Geary et al., 2012). The reason for the re-emergence of decomposition for predicting achievement in later grades is less clear, however. Given that mean use of decomposition did not change after 4th grade (not reported here), the effect for later grades is likely due to changes in the content of the items on the Numerical Operations test; specifically, complex whole number arithmetic problems where decomposition might be a useful problem solving strategy.

Summary and Limitations

As noted, we did not include all potential domain-general and domain-specific measures in our study and thus it is possible that alternative variables might emerge in future studies. Although the data set is unique in many ways, the sample size is relatively small which may have reduced the statistical power of some of our analyses, and it is unclear how sample recruitment, attrition, and diversity (77% white) influenced our findings. The measurement of intelligence and the central executive in 1st grade and the domain-specific competencies in the grade prior to the mathematical outcome may have biased the results in favor of domain-specific competencies. We do not believe this is a strong bias, however, because Lee and Bull (2016) assessed working memory in each grade and, as noted, found the same across-grade pattern as emerged here. Moreover, if assessment timing was critical, the importance of domain-general abilities would have declined across grades, not remained stable. Of course the data itself is correlational and does not support strong causal inferences. Follow up studies will be needed to fully assess the validity of our conclusions; such as, improvements in domain-specific skills will reduce individual differences in subsequent mathematics achievement.

Despite these limitations, the study yielded three key findings. First, the overall magnitude of domain-general effects on mathematics achievement remained constant across grades, whereas the overall magnitude of domain-specific effects increased across grades. Second and in keeping with previous studies, domain-general effects were larger than domain-specific effects in the early grades; however, the overall contributions of domain-general abilities and domain-specific knowledge did not differ in later grades. Third, the combination of several specific measures of mathematical knowledge provided a substantively larger estimate of domain-specific effects than the more commonly used prior mathematics achievement. The later suggests that use of prior achievement scores may underestimate the importance of domain-specific knowledge for further learning in the domain, mathematics in this case.

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Educational Impact And Implications Statement

The current study identifies the factors that influence students' grade-to-grade gains in mathematical competencies from school entry through middle school. The key educational finding is that the importance of students' prior mathematical knowledge for further gains in this knowledge increases across grades. The implication is that interventions focused on improving domain-specific skills, such as fractions knowledge, will likely yield longer-term gains in mathematics achievement than will interventions focused on domain-general abilities, such as working memory.

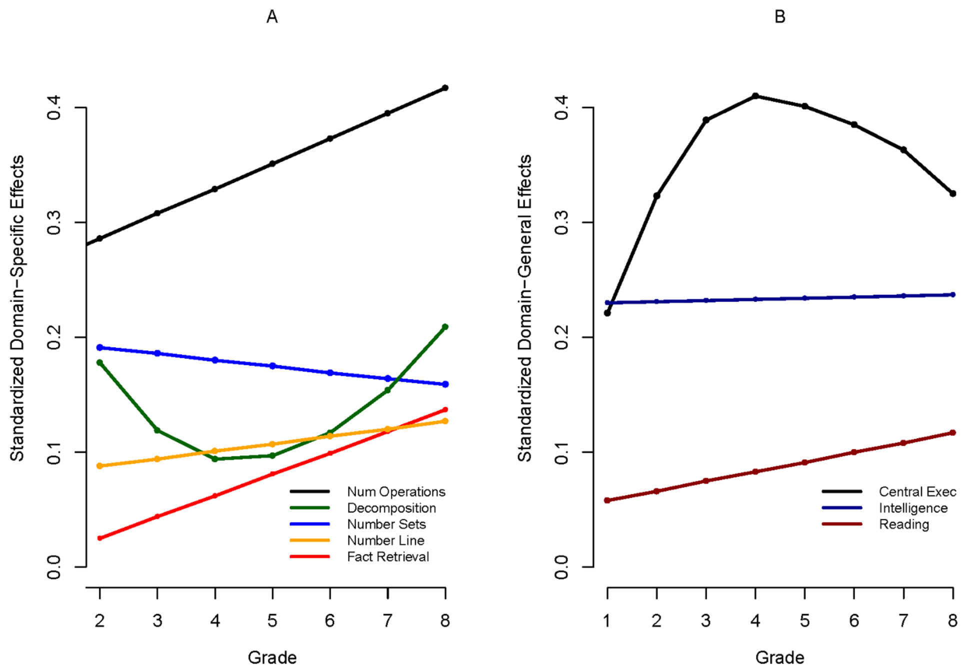

Figure 1.

Standardized domain-specific effects (Panel A) from model 1 (Numerical Operations) and model 2 (mathematical cognition), and standardized domain-general effects (Panel B) from model 3.

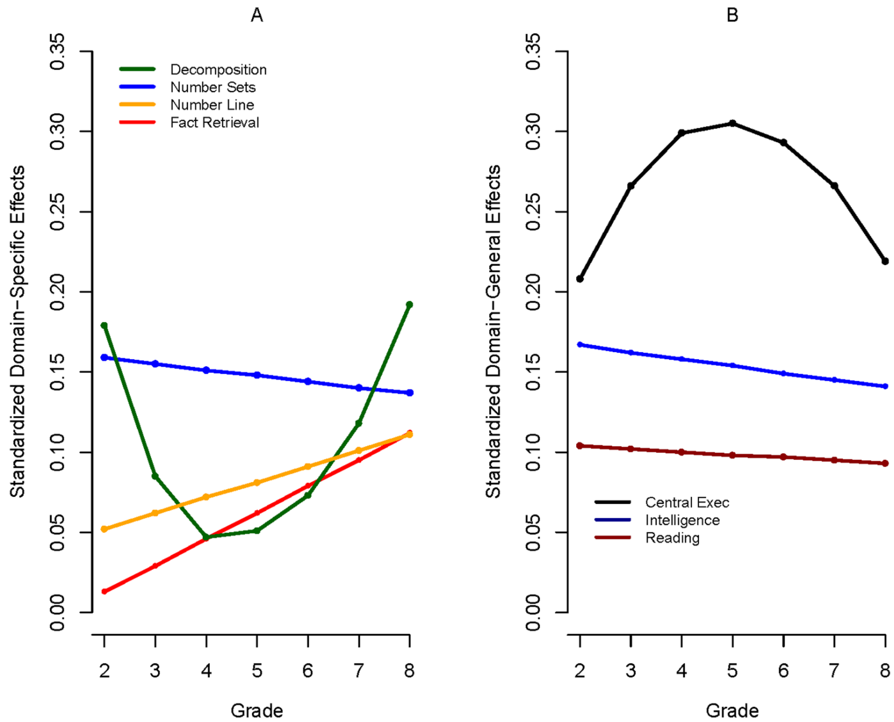

Figure 2.
Standardized domain-specific (Panel A) and domain-general (Panel B) effects from 2nd to 8th grade from model 4.

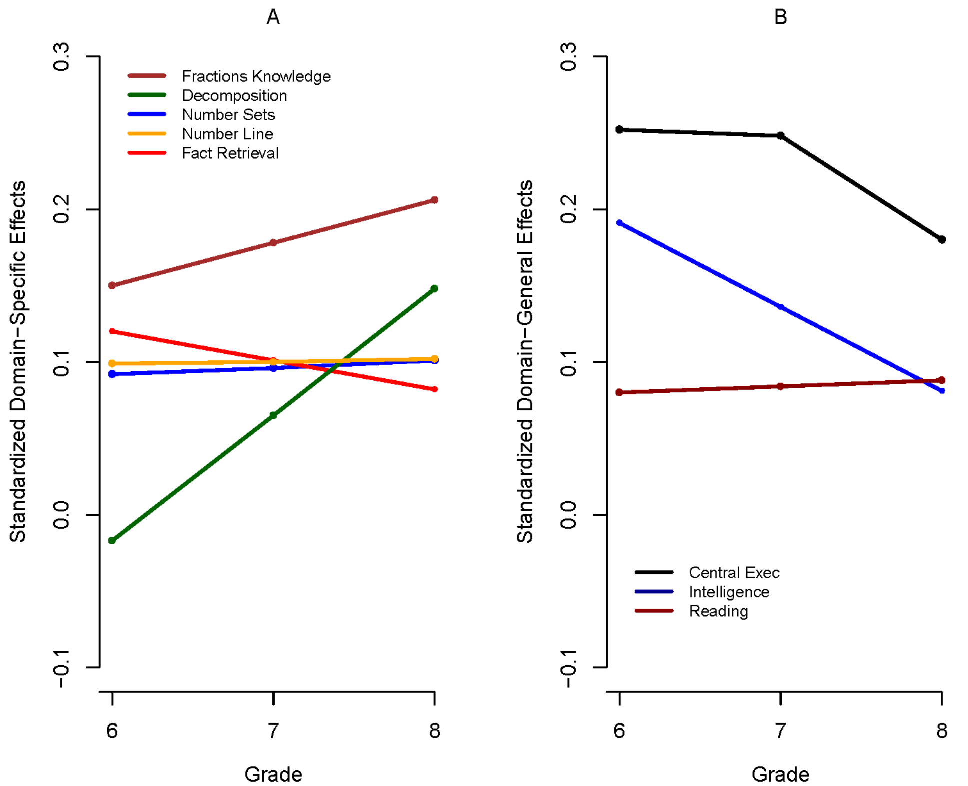

Figure 3.

Standardized domain-specific (Panel A) and domain-general (Panel B) effects from 6th to 8th grade from model 5.

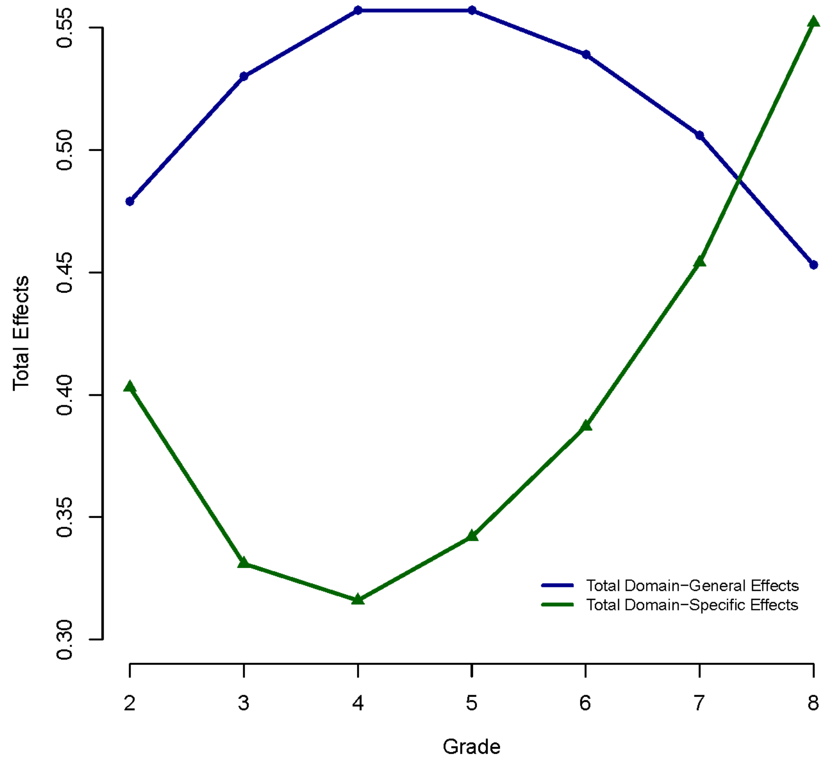

Figure 4.
Total domain-specific and domain-general effects from 2nd to 8th grade.

Table 1

Measures Used in Analyses or Reported in Text and Grade of Administration

	Grade								
Variable	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Mathematics Achievement	x	x	x	x	x	x	x	x	x
Domain-General Predictors									
Intelligence		x							
Central Executive		x				x			
Reading Achievement	x	x	x	x	x	x	x	x	x
Domain-Specific Predictors									
Number Sets Test		x	x	x	x	x	x	x	
Number Line		x	x	x	x	x	x	x	
Addition Retrieval		x	x	x	x	x	x	x	
Addition Decomposition		x	x	x	x	x	x	x	
Fractions Comparison Test							x		

Table 2

Variance Components Associated with Fixed Effects

	Model				
	1	**2**	**3**	**4**	**5**
	Fixed Effect Spline Variance Component				
IQ	-	-	0.0000	0.0000	0.0000
Reading (g-1)	-	-	0.0000	0.0000	0.0000
Central Executive	-	-	0.8860	0.3981	0.0919
Numerical Operations (g-1)	0.0000	-	-	-	-
Number Sets (g-1)	-	0.0000	-	0.0000	0.0000
Retrieval(g-1)	-	0.0000	-	0.0000	0.0000
Decomposition(g-1)	-	0.4709	-	0.8808	0.0000
Number line (g-1)	-	0.0000	-	0.0000	0.0000
Fractions comparison	-	-	-	-	0.0000
Random Effect Variance Component					
Intercept	0.2581	0.3661	0.3309	0.2317	0.3144
Slope	0.0353	0.2325	0.5232	0.2569	0.0973
Spline	0.0000	6.6547	10.1853	6.6063	0.0000
Residual VC	0.3587	0.2405	0.2544	0.2341	0.1731
Mean R^2 over grade	.682	.822	.814	.828	.784

Table 3

Domain-Specific and Domain-General Effect Estimates from Model 4

	Domain-Specific Effect Estimates							
Grade	**Number Sets (g-1)**	***p***	**Retrieval (g-1)**	***p***	**Decomposition (g-1)**	***p***	**Number Line (g-1)**	***p***
2	.159 (.040)	0.000	.013 (.037)	0.733	.179 (.051)	0.000	.052 (.041)	0.209
3	.155 (.032)	0.000	.029 (.030)	0.332	.085 (.036)	0.020	.062 (.033)	0.058
4	.151 (.026)	0.000	.046 (.025)	0.068	.047 (.034)	0.175	.072 (.026)	0.005
5	.148 (.024)	0.000	.062 (.023)	0.008	.051 (.034)	0.141	.081 (.022)	0.000
6	.144 (.026)	0.000	.079 (.026)	0.003	.073 (.035)	0.038	.091 (.024)	0.000
7	.140 (.032)	0.000	.095 (.032)	0.003	.118 (.037)	0.002	.101 (.030)	0.001
8	.137 (.040)	0.001	.112 (.039)	0.005	.192 (.051)	0.000	.111 (.039)	0.004
	Domain-General Effect Estimates							
Grade	IQ	*p*	Central Executive	*p*	Reading (g-1)	*p*		
2	.167 (.059)	.005	.208 (.066)	.002	.104 (.051)	.041		
3	.162 (.002)	.052	.266 (.055)	.000	.102 (.043)	.017		
4	.158 (.048)	.001	.299 (.053)	.000	.100 (.038)	.008		
5	.154 (.046)	.001	.305 (.052)	.000	.098 (.036)	.006		
6	.149 (.048)	.002	.293 (.052)	.000	.097 (.039)	.013		
7	.145 (.052)	.006	.266 (.055)	.000	.095 (.045)	.036		
8	.141 (.059)	.017	.219 (.065)	.001	.093 (.054)	.084		

Note. g = grade. Parenthetical values are standard errors.

Table 4

Domain-Specific and Domain-General Effect Estimates from Model 5

	Domain-Specific Effect Estimates									
Grade	**Number Sets (g-1)**	***p***	**Retrieval (g-1)**	**p**	**Decomposition (g-1)**	***p***	**Number Line (g-1)**	***p***	**Fractions**	***p***
6	.092 (.048)	.056	.120 (.048)	.014	−.017 (.050)	.727	.099 (.047)	.034	.150 (.065)	.021
7	.096 (.036)	.007	.101 (.035)	.004	.065 (.038)	.085	.100 (.031)	.001	.178 (.057)	.002
8	.101 (.047)	.031	.082 (.045)	.071	.148 (.050)	.003	.102 (.049)	.039	.206 (.068)	.002
	Domain-General Effect Estimates									
Grade	IQ	*p*	Central Executive	*p*	Reading (g-1)	*p*				
6	.191 (.065)	.003	.252 (.067)	.000	.080 (.063)	.203				
7	.136 (.057)	.016	.248 (.061)	.000	.084 (.050)	.095				
8	.081 (.066)	.217	.180 (.067)	.007	.088 (.060)	.144				

Note. g = grade. Parenthetical values are standard errors.