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## Fractional Anisotropy Derived from the Diffusion Tensor Distribution Function Boosts Power to Detect Alzheimer's Disease Deficits

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### Abstract

**Purpose**—In diffusion MRI (dMRI), fractional anisotropy derived from the single-tensor model (FA<sup>DTI</sup>) is the most widely used metric to characterize white matter (WM) micro-architecture, despite known limitations in regions with crossing fibers. Due to time constraints when scanning patients in clinical settings, high angular resolution diffusion imaging (HARDI) acquisition protocols, often used to overcome these limitations, are still rare in clinical population studies. However, the tensor distribution function (TDF) may be used to model multiple underlying fibers by representing the diffusion profile as a probabilistic mixture of tensors.

**Methods**—We compared the ability of standard FA<sup>DTI</sup> and TDF-derived FA (FA<sup>TDF</sup>), calculated from a range of dMRI angular resolutions (41, 30, 15, and 7 gradient directions), to profile WM deficits in 251 individuals from the Alzheimer's Disease Neuroimaging Initiative (ADNI), and to detect associations with (1) AD diagnosis, (2) Clinical Dementia Rating scores, and (3) average hippocampal volume.

**Results**—Across angular resolutions and statistical tests, FA<sup>TDF</sup> showed larger effect sizes than FA<sup>DTI</sup>, particularly in regions preferentially affected by AD, and was less susceptible to crossing fiber anomalies.

**Conclusion**—The TDF “corrected” form of FA may be a more sensitive and accurate alternative to the commonly-used FA<sup>DTI</sup>, even in clinical quality dMRI data.

## Keywords

Alzheimer's disease; white matter; diffusion-weighted imaging; fractional anisotropy; tensor distribution function

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## INTRODUCTION

Diffusion-weighted MRI (dMRI) is a variant of standard MRI that can measure the diffusion of water molecules in biological tissues, such as the brain. By characterizing the diffusion process at the voxel level, we can make tentative inferences about the underlying white matter (WM) microstructure and factors that affect it (1). Since dMRI was developed, acquisition protocols have improved to increase angular, spatial, and spectral resolution. Multiple mathematical models have been developed to describe the diffusion process. One of the first – and still most popular – methods to summarize diffusion properties in a specific voxel is the single tensor model (2) commonly referred to as diffusion tensor imaging (DTI) (3). This model is limited as it assumes diffusion is purely Gaussian; it can only model a single fiber population, with a single dominant orientation, at every voxel. It cannot resolve complex WM architecture, such as dispersing, crossing or kissing fibers. Yet, at the current resolution of dMRI, between one-third and two-thirds of WM voxels contain fiber crossings (4, 5). dMRI can also be used to evaluate disease-related gray matter (GM) abnormalities, where the micro-architecture is even more complex (6). Nevertheless, the tensor-derived fractional anisotropy ( $FA^{DTI}$ ) metric is still the most widely used scalar measure to characterize tissue micro-architecture. It is widely used in research studies of schizophrenia, depression, autism, HIV/AIDS, and other developmental, psychiatric, and neurodegenerative disorders including Alzheimer's disease (7, 8).

In recent years, many new models have been proposed to overcome limitations of the tensor model, including  $q$ -ball imaging and diffusion orientation distribution functions (ODF) (9), constrained spherical deconvolution (CSD) (10), multicompartment models such as the “ball and stick” model (4), diffusion spectrum MRI (DSI) (11), and neurite orientation dispersion and density imaging (NODDI) (12). Due to the numerous types of biological, neuropsychiatric and imaging data often acquired for clinical populations, time constraints are often placed on imaging protocols to reduce patient attrition or motion, and ensure adequate sample sizes. This precludes state of the art models like DSI and NODDI, which require extremely dense or multi-shell acquisitions, and may prevent the reliable reconstruction of many other higher-order diffusion models. However, the tensor distribution function (TDF), as proposed by (13), may still be feasible. The TDF is a probabilistic extension of a multi-tensor model that describes crossing fibers mathematically as an ensemble of Gaussian tensors. However, unlike other multicompartment models (4, 12, 14–16) where one needs to specify in advance the total number of compartments in the tissue, the authors propose a continuous distribution of tensors, with a profile of “weights” or relative contributions estimated for tensors with a continuously varying range of shapes and sizes, in the tensor space.

The Alzheimer's Disease Neuroimaging Initiative (ADNI) is a large longitudinal multisite study of healthy elderly controls, individuals with mild cognitive impairment (MCI), and Alzheimer's disease (AD). The goal is to identify sensitive imaging biomarkers that can be used to track or predict changes in the brain, which is vital for drug trials to identify candidates for treatment and monitor effectiveness. In addition to the battery of cognitive tests, cerebrospinal fluid (CSF) and blood tests, ADNI collects several functional and structural MRI modalities including T1 and T2-weighted anatomical MRI, positron emission tomography (PET), arterial spin labeling (ASL), and resting state functional MRI, and dMRI. In such longitudinal studies, there is a real concern about patient attrition, especially in elderly individuals who may not be able to tolerate being confined to a scanner for long periods. In an effort to collect such a wide range of data-types and maintain patient enrollment, there are time constraints placed on possible dMRI protocols, including debates as to whether or not to continue acquiring dMRI in the next phase of ADNI. Clearly it is of great interest to maximize the power of the available scans and show that even clinical quality diffusion scans can be powerful tools for uncovering disease related abnormalities in tissue microstructure and WM neurocircuitry.

Here we aimed to find whether FA metrics derived using the TDF model ( $FA^{TDF}$ ) may be more sensitive to disease-related differences than the corresponding  $FA^{DTI}$  measure that is now widely used. Building upon preliminary findings in (17), our goal was to understand how the imaging protocol may influence the sensitivity of the metrics, and further compared performance for each metric computed from subsamples of the full ADNI dMRI 41 gradient direction angular resolution, including subsets of 30, 15, and 7 gradient directions. Voxel-wise association tests were used to compare  $FA^{TDF}$  and  $FA^{DTI}$  metrics computed from a range of angular resolutions, and their ability to detect microstructural differences between AD patients and healthy elderly controls (CN). We also evaluated associations between the two FA metrics and common AD biomarkers—hippocampal volume and Clinical Dementia Rating (CDR) scores. Finally, we evaluated the test-retest reliability of each model's fit and the resulting scalar FA maps. In comparing models, there is interest in detecting clinical associations with maximal sensitivity and power, ideally using improved metrics, which measure standard properties more accurately.

## METHODS

### Subjects and Image Acquisition

Standard MRI, dMRI, and clinical data were downloaded from the publicly available Alzheimer's Disease Neuroimaging Initiative (ADNI) database ([www.loni.usc.edu/ADNI](http://www.loni.usc.edu/ADNI)). We analyzed baseline data from 251 participants: 53 healthy controls (CN; mean age: 72.6+/-6.1 yrs; 24M/29F), 28 with significant memory concern (SMC; mean age: 72.4+/-4.5 yrs; 10M/18F), 121 with mild cognitive impairment (MCI; mean age: 72.6+/-7.3 yrs; 75M/46F) and 49 with AD (mean age: 75.0+/-8.6 yrs; 29M/20F). Of the 53 CN participants, 33 returned for follow-up evaluations after 3 months and their scans were used for test-retest analyses (mean age: 72.8+/-6.5 yrs; 16M/17F). All procedures were reviewed and approved by institutional review boards. All participants gave written informed consent.

All subjects underwent whole-brain MRI scanning on 3T GE Medical Systems scanners at 17 acquisition sites across North America. Anatomical T1-weighted SPGR (spoiled gradient echo) sequences (256×256 matrix; voxel size = 1.2×1.0×1.0 mm<sup>3</sup>; TI=400 ms; TR = 6.98 ms; TE = 2.85 ms; flip angle=11°), and dMRI (128×128 matrix; voxel size: 2.7×2.7×2.7 mm<sup>3</sup>; TR=9000 ms; dMRI scan time = 9 min) were collected. 46 separate images were acquired for each dMRI scan: 5 T2-weighted images with no diffusion sensitization ( $b_0$  images) and 41 diffusion-weighted images (DWI;  $b=1000$  s/mm<sup>2</sup>).

Baseline hippocampal volume summary metrics, processed using the FreeSurfer package (<http://surfer.nmr.mgh.harvard.edu/>), were downloaded from the ADNI database (n=243 available), as was the “sum-of-boxes” Clinical Dementia Rating (CDR-sob; n=238) (18).

### Image Preprocessing

Raw images were preprocessed as in (7). Extra-cerebral tissue was removed, raw DWI images were corrected for motion and eddy current distortions, and T1-weighted images underwent inhomogeneity normalization. Each T1-weighted image was linearly aligned to a standard brain template. The diffusion images were linearly and then elastically registered (19) to their respective T1-weighted scans to correct for echo-planar imaging (EPI) induced susceptibility artifacts. Gradient tables were corrected for DWI linear registrations.

### Diffusion Gradient Subsampling

To gain a better understanding of the dMRI parameters necessary to employ the TDF model, we used the framework presented in (20) to essentially downsample the angular resolution from 41 gradient directions to include only a subset of either 30, 15, or 7 gradient images. Gradient subsets were selected by optimizing the spherical angular distribution energy. Briefly, the angular distribution energy,  $E_{ij}$ , of a pair of points,  $i$  and  $j$ , on the unit sphere may be defined as the inverse of the sum of the squares of (1) the least spherical distance between point  $i$  and point  $j$ , and (2) the least spherical distance between point  $i$  and point  $j$ 's antipodal, symmetric point  $J$ . As in prior work, we acknowledge that protocols with fewer gradients would be independently optimized for angular distribution rather than subsample directions from an existing protocol, but the subsampling we use is designed to lead to the most equally distributed sampling on the sphere possible. It also helps us to assess effects of gradient count while keeping other factors of the patient's scan constant (e.g. motion, artifacts).

### dMRI Reconstruction Models and Scalar Maps

For each angular resolution (41, 30, 15, or 7 gradient directions), three different dMRI reconstruction models were used to generate scalar FA maps. First, a single diffusion tensor (3) – equivalent to a 3D ellipsoid capturing a single fiber orientation – was modeled at each voxel in the brain from the corrected DWI scans. This model assumes that the diffusion is a 3D Gaussian process, fitted using just six independent parameters of a tensor (3 eigenvalues describing its shape, and 3 Euler angles describing its orientation). Scalar fractional anisotropy (FA<sup>DTI</sup>) maps were obtained from the resulting diffusion tensor eigenvalues ( $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ):

$$FA = \sqrt{\frac{(\lambda_1 - \lambda_2)^2 + (\lambda_1 - \lambda_3)^2 + (\lambda_2 - \lambda_3)^2}{2 [\lambda_1^2 + \lambda_2^2 + \lambda_3^2]}}$$

In contrast to the single tensor model, the tensor distribution function (TDF) represents the diffusion profile as a probabilistic mixture of tensors that optimally explain the observed DWI data, allowing for the reconstruction of multiple underlying fibers per voxel, together with a distribution of weights. We applied the framework proposed in (13, 21) to the angular diffusion signal, to compute the voxel-wise optimal TDF  $P^*(D(\theta, \lambda))$ — the probability distribution function of all feasible Gaussian tensors  $D(\theta, \lambda)$  that best describes the observed signal. As in (13), to reduce the solution space, each tensor  $D(\theta, \lambda)$  at spherical angle  $\theta$  was assumed to be cylindrical such that  $\lambda = (\lambda_1, \lambda_2 = \lambda_3)$  and  $\lambda_1 \geq \lambda_2$ . However, unlike the gradient descent approach in (13) to solve for this optimal TDF, here we used a quadratic programming approach (see Appendix A for details). The tensor orientation distribution function (TOD), was then calculated by computing the marginal density function of the TDF with the eigenvalues  $\lambda = (\lambda_1, \lambda_2)$  integrated out.

$$TOD(\theta) = \int_{\lambda} P(D(\theta, \lambda)) d\lambda$$

For each  $\theta$ , the eigenvalues are calculated by computing the expected value of each eigenvalue along  $\theta$ , from which a corresponding scalar FA metric is calculated:

$$\lambda'_i(\theta) = \frac{\int P(D(\theta, \lambda)) \lambda_i d\lambda}{\int P(D(\theta, \lambda)) d\lambda}$$

$$FA(\theta) = \sqrt{\frac{(\lambda'_1(\theta) - \lambda'_2(\theta))^2 + (\lambda'_1(\theta) - \lambda'_3(\theta))^2 + (\lambda'_2(\theta) - \lambda'_3(\theta))^2}{2 [\lambda'_1(\theta)^2 + \lambda'_2(\theta)^2 + \lambda'_3(\theta)^2]}}$$

At each voxel, the final scalar  $FA^{TDF}$  metric across all  $\theta$  is then calculated as the sum of all  $FA(\theta)$  weighted by the probability that  $\theta$  is the principal fiber direction,  $TOD(\theta)$ .

$$FA^{TDF} = \int TOD(\theta) * FA(\theta) d\theta$$

A healthy control subject's  $FA^{DTI}$  and  $FA^{TDF}$  maps, calculated from various angular resolutions, are shown in Figure 1a and 1b for visual comparison. A voxel-wise two-tailed paired  $T$ -test was performed to quantitatively compare TDF and DTI FA values in the healthy control group. All resulting statistical maps were corrected for multiple comparisons

\*Many investigators within the ADNI contributed to the design and implementation of ADNI and/or provided data, but most of them did not participate in this analysis or writing this report. A complete list of ADNI investigators may be found at: [https://adni.loni.usc.edu/wp-content/uploads/how\\_to\\_apply/ADNI\\_Acknowledgement\\_List.pdf](https://adni.loni.usc.edu/wp-content/uploads/how_to_apply/ADNI_Acknowledgement_List.pdf)

using the standard false discovery rate (FDR) method at  $q=0.05$  (22), and thresholded at the FDR critical  $P$ -value.

For comparison to an established HARDI technique also designed to reconstruct multiple fiber orientations in a given voxel, we fitted orientation distribution functions (ODFs) at each voxel, with a non-parametric  $q$ -ball reconstruction technique, using the normalized and dimensionless constant solid angle (CSA) method (9, 23). The generalized FA (GFA<sup>ODF</sup>) was then calculated from the CSA-ODF. GFA<sup>ODF</sup> is analogous to FA<sup>DTI</sup>, but calculated at each diffusion direction of the ODF (9) and is defined as:

$$GFA = \frac{\text{std}(\Psi)}{\text{rms}(\Psi)} = \sqrt{\frac{n \sum_{i=1}^n (\Psi(u_i) - \langle \Psi \rangle)^2}{(n-1) \sum_{i=1}^n \Psi(u_i)^2}}$$

Here,  $\Psi(u)$  is the ODF,  $i$  is each diffusion direction and  $\langle \Psi \rangle = \frac{1}{n} \sum_{i=1}^n \Psi(u_i)$  is the mean of the ODF.

### Template Creation and Spatial Normalization

To avoid bias in the diffusion-based registrations, we created multi-channel study-specific minimal deformation templates (MDTs) with the ANTs registration software (24), equally weighting FA<sup>DTI</sup>, FA<sup>TDF</sup>, GFA<sup>ODF</sup> and T1-weighted maps. Similarly, to spatially normalize each subject's three FA maps we performed a 3-channel linear then non-linear registration to the MDT. In this way all FA maps were used to drive the registration and they were all normalized to the same space. To avoid differences in registration accuracy, the deformations from the full angular resolution registration were applied to the FA maps calculated from the various DWI gradient subsets for each individual.

Test-retest FA<sup>DTI</sup> and FA<sup>TDF</sup> maps generated from baseline and 3-month follow-up dMRI scans were each linearly aligned to an intermediate space half way between each subject's two time points (25). Baseline and follow-up maps were each spatially normalized to the baseline MDT with 2-channel linear then non-linear registrations. The deformations from the full angular resolution registration were applied to the various DWI gradient subsets as well as the respective FA maps

### Clinical Associations and Effect Sizes

To test for statistical effects of AD diagnosis on measures of white matter microstructure in FA<sup>DTI</sup>, FA<sup>TDF</sup>, and GFA<sup>ODF</sup> maps calculated at various angular resolutions, we ran voxel-wise, random-effects linear regressions, covarying for age and sex, and using the acquisition site as the random grouping variable. In an effort to try and tease apart microstructural associations from those driven by atrophy and registration, we also covaried for the log Jacobian determinant derived from the non-linear spatial normalization of each map to the template. In addition to AD diagnosis, we also tested for voxel-wise associations between FA and CDR-sob scores as well as average bilateral hippocampal volume (after covarying for intracranial volume) across the entire study sample. All statistical tests were limited to

voxels present in all subject scans, as some scans had a slightly cropped FOV. As such, we did not consider the inferior parts of the cerebellum and brain stem. All resulting statistical maps were corrected for multiple comparisons using the standard false discovery rate (FDR) method at  $q=0.05$  (22), and thresholded at the FDR critical  $P$ -value. We show regression coefficients ( $\beta$ -values) only in regions where the false discovery rate was controlled.

We computed cumulative distribution function (CDF) plots to visualize and rank effect sizes across voxel-wise tests. The sorted observed voxel  $P$ -values from each regression were plotted against the  $P$ -values from the expected null distribution. If the CDF curve initially rises at a rate steeper than 20 times the null CDF ( $y = 20x$ ), then the corresponding maps have supra-threshold or FDR significant voxels at  $q = 0.05$ . Curves that rise at a steeper rate than that line represent significant voxels and larger deviations represent larger effect sizes.

Effect sizes for detecting AD versus control group differences were also compared using Cohen's  $d$  calculated as  $(\mu_{AD} - \mu_{CN}) / s_{pooled}$ , where  $s_{pooled} = [(s_1^2 + s_2^2) / 2]$  (26). This metric has been widely used in studies of disease effects on imaging measures (27–29). For each FA metric and angular resolution, we used the average FA metric from both the respective statistical test's significant cluster. To avoid overfitting, a 10-fold cross validation approach was used. In each fold, 80% of the data were used for voxel-wise regressions to estimate the significant clusters (training data), and the remaining test data was used to compute the Cohen's  $d$  effect sizes.

### Test-Retest Reliability and Model Fit

We used the framework defined in (30, 31) to evaluate the goodness of the fit of each dMRI model in healthy controls. We first compared voxel-wise root mean squared error (RMSE) between the observed signal (A) and expected signal (B) from each model in each voxel:

$$\text{RMSE}(A, B) = \sqrt{\frac{\sum_{i=1}^N (A_i - B_i)^2}{N}},$$

where  $N$  is the number of gradient directions or DWIs, and  $A_i, B_i$ —the observed and expected signal intensities in the given voxel in the  $i$ -th DWI. Additionally, baseline and 3-month follow-up test-retest data was used to cross-validate each model's fit—the model parameters were first estimated on baseline control subjects' DWI scans, and then used to predict the signal in the 3-month follow-up DWI scan. As proposed in (30), we defined the test-retest relative root mean squared error (rRMSE) in each voxel as:

$$\text{rRMSE} = \frac{(\text{RMSE}(M1, D2) + \text{RMSE}(M2, D1))}{2\text{RMSE}(D1, D2)}$$

Here,  $\text{RMSE}(M1, D2)$  is the RMSE between the data observed in the follow-up scan and predicted from the first scan,  $\text{RMSE}(M2, D1)$  is the RMSE between the observed data in the first scan and that predicted from the follow-up scan, and  $\text{RMSE}(D1, D2)$  is the RMSE between the observed data from both scans. A model that predicts the repeated measurement

more accurately than the original will result in an  $rRMSE < 1$  (30). To compute RMSE (M1,D2), we used parameters learned from the first scan, and the  $bvecs$  (scanner gradient directions) and  $b_0$  from the follow-up scan, and vice versa for RMSE(M2,D1). A voxel-wise two-tailed paired  $T$ -test was performed to compare TDF and DTI baseline RMSE and  $rRMSE$  values in the healthy control group. All resulting statistical maps were corrected for multiple comparisons using the standard false discovery rate (FDR) method at  $q=0.05$  (22); resulting maps were thresholded at the FDR critical  $P$ -value.

We also evaluated the test-retest reliability for  $FA^{DTI}$  and  $FA^{TDF}$  maps calculated from each angular resolution by computing the voxel-wise intra-class correlation (ICC) between baseline and 3-month follow-up healthy control FA maps, with the R PSYCH package ([personality-project.org/r/html/ICC.html](http://personality-project.org/r/html/ICC.html)). Again, the FDR method was used to correct for multiple comparisons.

## RESULTS

Both a visual comparison of  $FA^{DTI}$  and  $FA^{TDF}$  maps (Figure 1a and 1b) and  $T$ -test between maps (Figure 1c) reveal that  $FA^{TDF}$  maps have higher FA values not only in the core, coherent WM structures but throughout the tissue, including near gray/white matter boundaries. The standard  $FA^{DTI}$  measure tends to show loss of signal near cortical boundaries and in regions with known fiber crossings and complex gray matter architecture.

### Clinical Associations and Effect Sizes

As expected, across all of the FA metrics, AD diagnosis, greater cognitive impairment (higher CDR-sob score), and lower average hippocampal volume, were associated with significant WM deficits (lower FA) after correction for multiple comparisons (Figure 2a). However, across statistical tests, larger effect sizes, as denoted by greater  $\beta$ -value magnitude and more widespread differences, were detected with  $FA^{TDF}$  voxel maps, as compared to  $FA^{DTI}$  and  $GFA^{ODF}$ . Moreover,  $FA^{TDF}$  findings are highly localized to the temporal lobe and hippocampal regions most vulnerable to early changes in AD. CDF plots further reflect the increased sensitivity of  $FA^{TDF}$  for differentiating disease groups, and for detecting clinical associations (Figure 2b).

Across maps, some very small regions exhibited significant associations with FA in direction contrary to what would traditionally be accepted as showing impairment (i.e., higher FA with impairment; Figure 2a *boxed regions*). These regions were largely found at the junction of the corpus callosum commissural fibers and the *corona radiata*, a region notorious for fiber crossings that may reduce the FA, as computed from the tensor model (32). However, across analyses,  $FA^{TDF}$  showed fewer associations that were contrary to the hypothesized effects of the disease (Figure 2c). Relative to the total number of significant voxels,  $FA^{DTI}$  showed between ~13–15% of these voxels,  $GFA^{ODF}$  showed ~8–10%, while  $FA^{TDF}$  showed <0.5% across tests, suggesting that  $FA^{TDF}$  may be handling computations better for crossing fibers.

A comparison of the same three clinical associations on  $FA^{DTI}$  and  $FA^{TDF}$  computed from a subset of 30, 15 and 7 gradient directions reveals that even at 7 gradient directions,  $FA^{TDF}$  is



consistently the most sensitive metric across statistical tests (Table 1; Figure 3). In fact  $FA^{TDF}$  calculated from 7 gradient directions has larger effect sizes than  $FA^{DTI}$  calculated at the *full* angular resolution. Across statistical tests performed at each angular resolution,  $FA^{TDF}$  consistently showed less than 0.5% of significant voxels with a direction of association opposite to that hypothesized, compared to  $FA^{DTI}$  which showed ~5–15% (Table 1). Finally, the mean Cohen's *d* effect sizes for picking up AD versus CN group differences, calculated from test data from the 10-fold cross validation, once again confirm that across all angular resolutions,  $FA^{TDF}$  showed larger effect sizes (Table 2).

### Test-Retest Reliability and Model Fit

Mean maps of the RMSE calculated from the DTI and TDF model fit in the subset of 53 healthy control subjects are shown in Figure 4a and 4b. A voxel-wise *T*-test revealed significantly lower error in the TDF fit throughout the tissue regardless of angular resolution (Figure 5a; FDR critical *P*-value for 41 gradients = 0.041, 30 gradients = 0.041, 15 gradients = 0.042, and 7 gradients = 0.043). The mean rRMSE maps from DTI and TDF models from 33 healthy control individuals at two time points are shown in Figure 4c and 4d. While the rRMSE is high in both the TDF and DTI models in the superior cortical gray matter (mean rRMSE > 1), the fit is stable (< 1) in WM and overall temporal lobe regions where most of the AD-related effects were detected. A *T*-test between the DTI and TDF rRMSE maps (Figure 5b) reveals significantly lower rRMSE with the TDF fit, in not only the temporal lobes, but in the region of the superior WM where commissural fibers and the corona radiata intersect, often leading to depleted  $FA^{DTI}$ . The TDF model shows higher error only in CSF. TDF rRMSE is progressively more similar to DTI (i.e. less area of significant differences) with lower angular resolution (FDR critical *P*-value for 41 gradients = 0.024, 30 gradients = 0.021, 15 gradients = 0.017, and 7 gradients = 0.001).

In terms of test-retest reliability of the scalar FA maps, we found that across resolutions there was an overall stable and strong ICC between baseline and follow-up  $FA^{TDF}$  maps (mean ICC ~0.8; Figure 5c), while, as might be expected, there was a degradation in  $FA^{DTI}$  ICC at the lowest angular resolutions (Figure 5d).

## DISCUSSION AND CONCLUSIONS

Here we show that FA metrics derived from the tensor distribution function (TDF) may be more sensitive to disease related microstructural abnormalities than corresponding single tensor-derived FA metrics that are now widely used to assess clinical data. FA is highly affected by numerous factors including the number of dominant fiber directions and orientation coherence as well as partial volume effects from neighboring gray matter. By using the TDF approach, we can still employ an extension of the tensor model, adapted to identify contributions to FA from separate crossing fiber compartments in tissue with more complex micro-architecture and voxels on tissue boundaries that are susceptible to partial voluming.

Alzheimer's disease (AD) is characterized by cortical and hippocampal neuronal loss and widespread gray matter (GM) atrophy driven in part by cortical amyloid plaque and neurofibrillary tangle deposits, and vascular changes. Structural and diffusion MRI studies

show white matter (WM) injury – perhaps due to myelin degeneration, and neuronal loss leading to progressive disconnection of cortical and subcortical regions (7, 33–36). Standard anatomical MRI is still the imaging technique most often used in AD studies and clinical trials, but dMRI is sensitive to microscopic changes in WM integrity not always detectable with standard anatomical MRI (37, 38). In addition to WM, dMRI is an emerging tool for the evaluation of disease related gray matter (GM) abnormalities as well (6, 39–43). A growing number of studies assess cortical and subcortical GM diffusivity changes in AD that may reflect GM cellular microstructure breakdown (6). Several studies report microscopic changes in the hippocampi that may be detectable prior to volumetric changes (43, 44). As dMRI changes may be detectable prior to, and predict gross volume loss (45, 46), it is important to maximize the power to detect such changes.

In this study, we found that compared to both  $GFA^{ODF}$  and  $FA^{DTI}$ ,  $FA^{TDF}$  showed increased power to detect subtle or diffuse disease effects, especially in hippocampal and temporal lobe regions. AD pathology targets GM regions especially in the temporal lobe and hippocampus. In these regions,  $FA^{DTI}$  might be sub-optimal, as it is best suited to detect differences in cohesive WM fiber bundles such as the corpus callosum. We also found more significant  $FA^{TDF}$  associations in voxels at GM or CSF boundaries that may otherwise be susceptible to partial volume effects with  $FA^{DTI}$ . Compared to  $FA^{DTI}$ , larger  $FA^{TDF}$  effect sizes were preserved even when the dMRI angular resolution was subsampled from 41 gradient directions to 30, 15, or even 7 gradient directions. Perhaps surprisingly,  $FA^{TDF}$  calculated from 7 gradient directions had larger effect sizes than  $FA^{DTI}$  calculated at the *full* angular resolution. While some higher order models require extremely dense or multi-shell acquisitions, TDF may better extract the information typically available in clinical settings, where time constraints limit scan times. It may also be helpful for studies of valuable but lower-resolution legacy data. The TDF as proposed by (13) makes no assumptions about the number of compartments per voxel and, unlike the tensor distribution function previously proposed by (47), does not impose the same exact anisotropy profile on all fiber compartments. This may lead to better estimates if there are higher levels of uncertainty in the data, such as may arise with low resolution data.

Furthermore,  $FA^{TDF}$  may also help to interpret apparent increases in  $FA^{DTI}$  found in disease. In many contexts, lower FA is hypothesized to reflect impairment. However, relative increases in FA have been reported in  $FA^{DTI}$  studies of AD, which may reflect a selective sparing or selective degeneration of one of the pathways in a region with crossing fibers (48). However, without histologic data, we cannot be certain whether selective degeneration or increased “integrity”, or some mix of both, is driving higher  $FA^{DTI}$  values in a neuroimaging study.  $FA^{TDF}$ , on the other hand, takes into account crossing fiber compartments. A relatively higher FA may more consistently reflect healthier tissue, while lower FA more consistently reflects deficits, making the direction of associations easier to interpret. Across analyses, we found that  $FA^{TDF}$  showed fewer ‘contrary to hypothesis’ regions (i.e., higher FA associated with greater deficits). Across all statistical tests and angular resolutions, compared to  $FA^{TDF}$ ,  $FA^{DTI}$  showed both a higher absolute number of these types of significant voxels, and a higher percentage relative to the total number of significant voxels (~5–15% of voxels versus <0.5% with  $FA^{TDF}$ ), suggesting that  $FA^{TDF}$  may in fact be resolving crossing fibers.

Analyses of test-retest reliability and model fit show that the TDF is quite reliable and robust in regions that show disease effects in this analysis. Some instability in test-retest reliability is to be expected when using the ADNI dataset – the scans are 3 months apart, and the healthy control participants are elderly (mean age: 72.8 +/- 6.5 yrs). These individuals may show some biological aging and white matter deterioration, even over the 3-month interscan interval. All registrations were visually evaluated, but age-related changes and minor geometric miscalibration of the scanner may also contribute to minor discrepancies in alignment between 2 scans from the same subject, contributing to both the final ICC and rRMSE measure in both the DTI and TDF models.

A prior study also showed that  $FA^{TDF}$  was a more stable metric with decreasing spatial resolution, where  $FA^{DTI}$  values decreased more rapidly due to more fiber incoherence and greater partial voluming in larger voxels (49). However, further analyses of  $FA^{TDF}$  limitations on a wider range of diffusion protocols and comparisons of performance to numerous other proposed scalar metrics are necessary. In addition to FA, there is also a growing interest in assessing complementary diffusivity metrics, including mean diffusivity (MD), axial diffusivity (AxD), and radial diffusivity (RD). As FA is an inherently normalized measure and diffusivity metrics are not, future work is necessary to define analogous measures within the TDF framework.

Multi-shell and other DSI or q-space techniques may ultimately outperform tensor model metrics, but they are often less feasible given the time constraints on dMRI protocols in clinical settings, as well as for recovering information from valuable legacy data. The TDF model may ultimately allow us to take advantage of such available clinical quality diffusion data with more sensitivity and fewer limitations than the classic DTI model.

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## Appendix A

The probability distribution we fit to the tensors,  $\mathcal{P}(D(\theta, \lambda))$ , is defined as a nonparametric distribution, we sample the tensor space to solve the optimization problem. As in (13), we set  $\lambda_2 = \lambda_3$  and  $\lambda_1 \neq \lambda_2$ , thus reducing the search of tensors to 2 eigenvalues and a principal eigenvector direction. The cost function could be rewritten (up to a constant multiplier  $S_0$ ) as:

$$I[P(D(\theta, \lambda))] = \sum_i (S_{obs}(q_i) - \sum_{k=1}^M P(D_k) \exp(-q_i^T D_k q_i))^2 = \frac{1}{2} x^T (2 * F F^T) x - x^T (2F s) + s^2$$

(eq1)

where  $x \in R^M$  – is the vector of probabilities:  $x_i = P(D_i)$ ,  $i=1, \overline{M}$ , where  $M$  is the number of elements in sampling;  $s \in R^N$  – is the vector of observed intensities of the DW measures, where  $N$  is the number of DW scans;  $F \in R^{M \times N}$  – is the matrix of elements,

$F_{ij} = \exp(-q_j^T D_i q_j)$ . Clearly, we search for a vector  $x$  that minimizes the cost function with two constraints:

$$x_i > 0, \quad i = \overline{1, M}$$

$$\sum_{i=1}^M x_i = 1$$

Which lends itself to a classic quadratic programming (QP) problem (50). By definition, the matrix  $H = 2F F^T$  is positive semi-definite, which means that the problem is convex.

However, by construction, the rank of the matrix  $H$  cannot exceed  $N$  (the number of observations). So if the tensor space sampling has more than  $N$  points, then the problem has multiple optimal solutions. The original gradient descent approach in (13) used the substitution  $P(D) = \exp(R(D))$  to meet the constraint  $P(D_i) > 0$ . It also used a feasible-direction gradient descent method, projecting the steepest descent direction onto the constraint  $\int P(D) dD = 1$ . Even so, as long as  $P$  is sought as a non-parametric distribution based on a sampling at pre-defined grid points, multiple optimal solutions still remain a problem. Instead, we use the primal-dual predictor-corrector interior-point method (51) to solve the optimization problem. The method that we use is one of the interior-point methods family, which converges to a unique solution from the feasible interior region to the optimum, following the central path (52). Generally, interior-point methods solve the problem:

$$\min c^T x + \frac{1}{2} x^T H x - \mu \sum_{j=1}^M \ln x_j$$

given (in our case)  $H = 2F F^T$ ,  $c = -2F s$ , and the constraint  $e^T x = 1$  and  $x_i = P(D_i) > 0$  as defined above. Starting with large  $\mu$ , this functional is being decreased on every step, enforcing the solution to be close to the line  $x_1 = x_2 = \dots = x_M$ , which is a minimizer for the

$\left(-\mu \sum_{j=1}^M \ln x_j\right)$  term. A QP solver implemented as compiled C++ code (<http://sigpromu.org/quadprog/>) was used for solving the minimization problem.

## Numerical implementation details

The following sampling scheme was chosen for the tensor space:  $\lambda_1 = [0.2 \ 0.4 \ \dots \ 2.0]$ ,  $\lambda_2 = [0.2 \ 0.4 \ \dots \ \lambda_1]$ , resulting in 55 sampling points for the *lambda* combinations. Principal eigenvectors were sampled at 2 resolution levels, as centers of faces of half an icosahedron (to represent the hemisphere), resulting in 10 directions at the low-resolution level and 40 directions at the higher-resolution level (by subdividing each icosahedral face into 4 new ones). A freely available set of functions (<https://www.mathworks.com/matlabcentral/fileexchange/37004-suite-of-functions-to-perform-uniform-sampling-of-a-sphere>) was used to build and subdivide an icosahedron.

The algorithm may be summarized as follows:

1. Pre-compute matrices F, H.
2. For each voxel:
  - a. Compute the vector  $c = -2Fs$  (given  $s$  – vector of DW observations in voxel)
  - b. Solve the QP problem (eq1)
  - c. Compute the TOD as:  $TOD(\theta) = \int_{\lambda} P(D(\theta, \lambda)) d\lambda$
  - d. For directions where the value of the TOD exceeds the threshold (as implemented in Leow et al., but set here to  $1/10 = 0.1$ ), upsample – replace each low-resolution direction with 4 corresponding higher-resolution directions to fine tune the tensor orientation probabilities.
  - e. Solve QP again only for the higher-resolution directions.
  - f. Compute higher-resolution output metrics (TDF-FA), as well as root mean squared error measure (RMSE).

## References

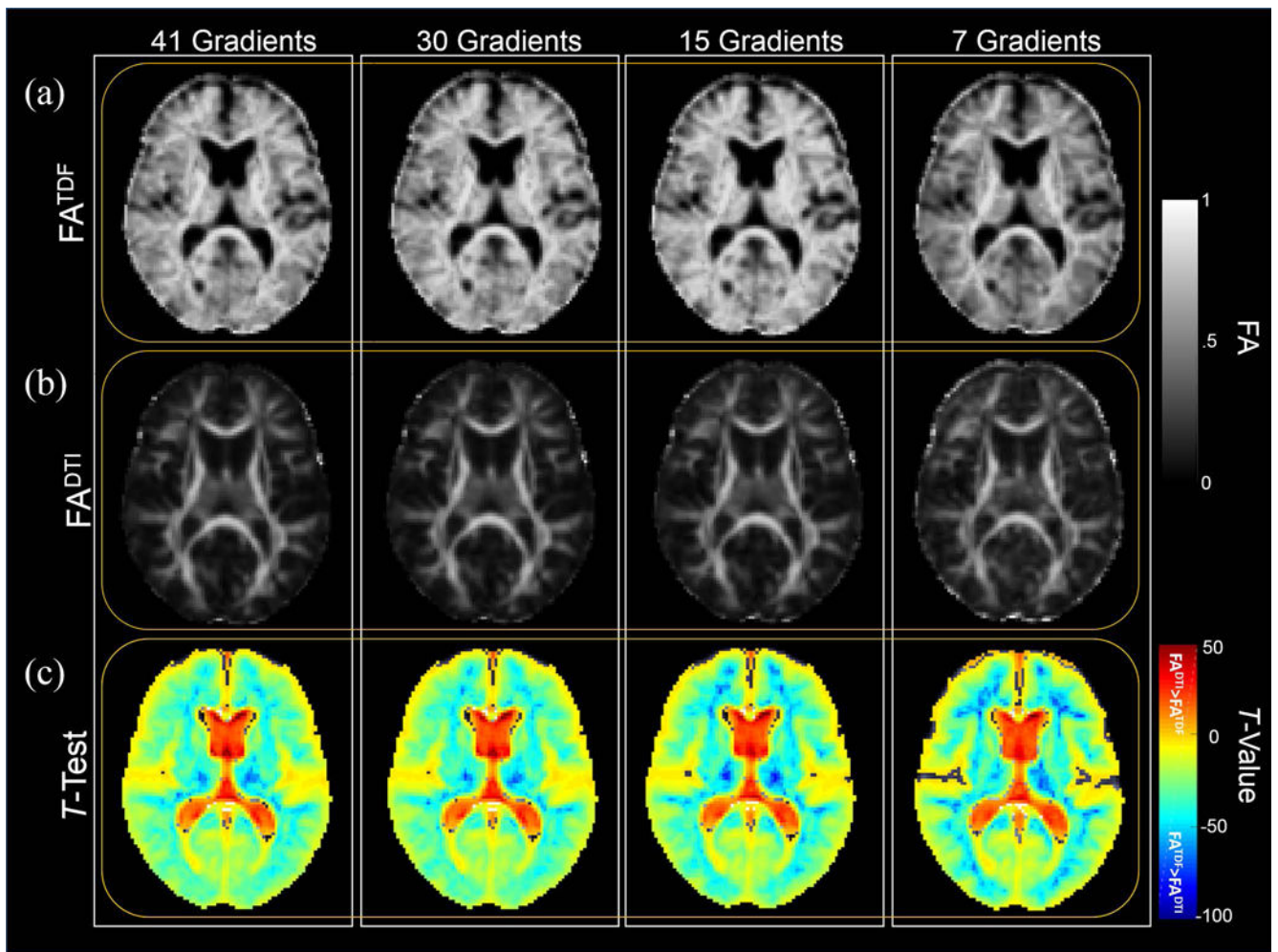
1. Descoteaux M, Poupon C. Diffusion-weighted MRI. Comprehensive Biomedical Physics Elsevier. 2012; 2:3.
2. Stejskal EO, Tanner JE. Spin Diffusion Measurements: Spin Echoes in the Presence of a Time-Dependent Field Gradient. The Journal of Chemical Physics. 1965; 42(1):288–92.
3. Basser PJ, Mattiello J, LeBihan D. MR diffusion tensor spectroscopy and imaging. Biophys J. 1994; 66(1):259–67. [PubMed: 8130344]
4. Behrens TE, Berg HJ, Jbabdi S, Rushworth MF, Woolrich MW. Probabilistic diffusion tractography with multiple fibre orientations: What can we gain? Neuroimage. 2007; 34(1):144–55. [PubMed: 17070705]
5. Descoteaux M, Deriche R, Knosche TR, Anwander A. Deterministic and probabilistic tractography based on complex fibre orientation distributions. IEEE Trans Med Imaging. 2009; 28(2):269–86. [PubMed: 19188114]

6. Weston PSJ, Simpson IJA, Ryan NS, Ourselin S, Fox NC. Diffusion imaging changes in grey matter in Alzheimer's disease: a potential marker of early neurodegeneration. *Alzheimer's Research & Therapy*. 2015; 7(1):47.
7. Nir TM, Jahanshad N, Villalon-Reina JE, Toga AW, Jack CR, Weiner MW, et al. Effectiveness of regional DTI measures in distinguishing Alzheimer's disease, MCI, and normal aging. *Neuroimage Clin*. 2013; 3:180–95. [PubMed: 24179862]
8. Nir TM, Villalon-Reina JE, Prasad G, Jahanshad N, Joshi SH, Toga AW, et al. Diffusion weighted imaging-based maximum density path analysis and classification of Alzheimer's disease. *Neurobiol Aging*. 2015; 36(Suppl 1):S132–40. [PubMed: 25444597]
9. Tuch DS. Q-ball imaging. *Magn Reson Med*. 2004; 52(6):1358–72. [PubMed: 15562495]
10. Tournier JD, Calamante F, Gadian DG, Connelly A. Direct estimation of the fiber orientation density function from diffusion-weighted MRI data using spherical deconvolution. *Neuroimage*. 2004; 23(3):1176–85. [PubMed: 15528117]
11. Wedeen VJ, Hagmann P, Tseng WY, Reese TG, Weisskoff RM. Mapping complex tissue architecture with diffusion spectrum magnetic resonance imaging. *Magn Reson Med*. 2005; 54(6):1377–86. [PubMed: 16247738]
12. Zhang H, Schneider T, Wheeler-Kingshott CA, Alexander DC. NODDI: practical in vivo neurite orientation dispersion and density imaging of the human brain. *Neuroimage*. 2012; 61(4):1000–16. [PubMed: 22484410]
13. Leow AD, Zhu S, Zhan L, McMahon K, de Zubicaray GI, Meredith M, et al. The tensor distribution function. *Magn Reson Med*. 2009; 61(1):205–14. [PubMed: 19097208]
14. Alexander DC. A general framework for experiment design in diffusion MRI and its application in measuring direct tissue-microstructure features. *Magn Reson Med*. 2008; 60(2):439–48. [PubMed: 18666109]
15. Barazany D, Basser PJ, Assaf Y. In vivo measurement of axon diameter distribution in the corpus callosum of rat brain. *Brain*. 2009; 132(Pt 5):1210–20. [PubMed: 19403788]
16. Alexander DC, Hubbard PL, Hall MG, Moore EA, Ptito M, Parker GJ, et al. Orientationally invariant indices of axon diameter and density from diffusion MRI. *Neuroimage*. 2010; 52(4):1374–89. [PubMed: 20580932]
17. Nir, TM, Zavaliangos-Petropulu, A, Jahanshad, N, Villalon-Reina, JE, Zhan, L, Leow, AD., et al., editors. 2016 IEEE 13th International Symposium on Biomedical Imaging (ISBI). 2016. Diffusion Tensor Distribution Function Metrics Boost Power to Detect Defecits in Alzhemier's Disease. April 13 2016–April 16 2016
18. Berg L. Clinical Dementia Rating (CDR). *Psychopharmacol Bull*. 1988; 24(4):637–9. [PubMed: 3249765]
19. Leow AD, Yanovsky I, Chiang MC, Lee AD, Klunder AD, Lu A, et al. Statistical properties of Jacobian maps and the realization of unbiased large-deformation nonlinear image registration. *IEEE Trans Med Imaging*. 2007; 26(6):822–32. [PubMed: 17679333]
20. Zhan L, Leow AD, Jahanshad N, Chiang MC, Barysheva M, Lee AD, et al. How does angular resolution affect diffusion imaging measures? *Neuroimage*. 2010; 49(2):1357–71. [PubMed: 19819339]
21. Zhan L, Leow AD, Zhu S, Baryshev M, Toga AW, McMahon KL, et al. A novel measure of fractional anisotropy based on the tensor distribution function. *Med Image Comput Comput Assist Interv*. 2009; 12(Pt 1):845–52. [PubMed: 20426067]
22. Benjamini Y, Hochberg Y. Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society Series B (Methodological)*. 1995:289–300.
23. Aganj I, Lenglet C, Sapiro G, Yacoub E, Ugurbil K, Harel N. Reconstruction of the orientation distribution function in single- and multiple-shell q-ball imaging within constant solid angle. *Magn Reson Med*. 2010; 64(2):554–66. [PubMed: 20535807]
24. Avants BB, Tustison NJ, Song G, Cook PA, Klein A, Gee JC. A reproducible evaluation of ANTs similarity metric performance in brain image registration. *Neuroimage*. 2011; 54(3):2033–44. [PubMed: 20851191]

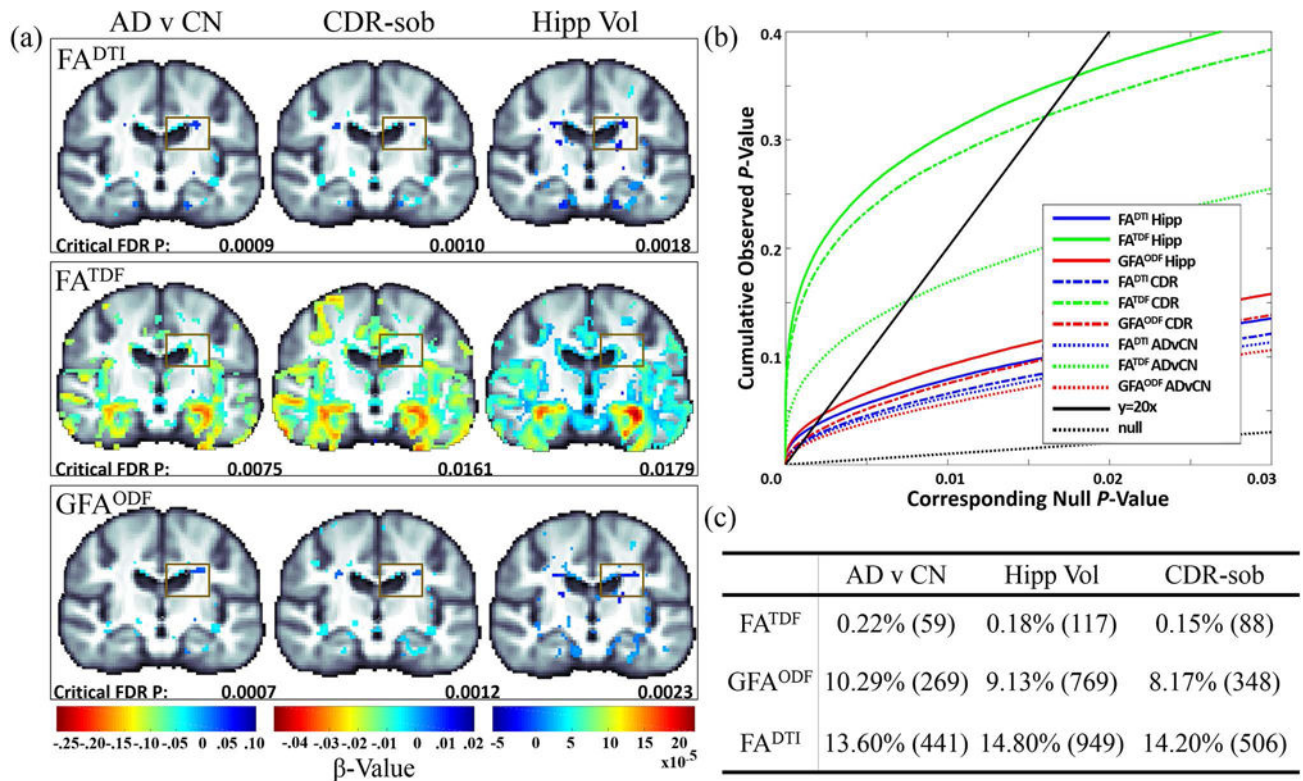
25. Smith SM, Zhang Y, Jenkinson M, Chen J, Matthews PM, Federico A, et al. Accurate, robust, and automated longitudinal and cross-sectional brain change analysis. *Neuroimage*. 2002; 17(1):479–89. [PubMed: 12482100]
26. Cohen, J. *Statistical power analysis for the behavioral sciences*. Hillsdale, N.J: L. Erlbaum Associates; 1988.
27. Schmaal L, Hibar DP, Samann PG, Hall GB, Baune BT, Jahanshad N, et al. Cortical abnormalities in adults and adolescents with major depression based on brain scans from 20 cohorts worldwide in the ENIGMA Major Depressive Disorder Working Group. *Molecular psychiatry*. 2016
28. van Erp TG, Hibar DP, Rasmussen JM, Glahn DC, Pearlson GD, Andreassen OA, et al. Subcortical brain volume abnormalities in 2028 individuals with schizophrenia and 2540 healthy controls via the ENIGMA consortium. *Molecular psychiatry*. 2016; 21(4):585. [PubMed: 26283641]
29. Hibar DP, Westlye LT, van Erp TG, Rasmussen J, Leonardo CD, Faskowitz J, et al. Subcortical volumetric abnormalities in bipolar disorder. *Molecular psychiatry*. 2016
30. Rokem A, Yeatman JD, Pestilli F, Kay KN, Mezer A, van der Walt S, et al. Evaluating the accuracy of diffusion MRI models in white matter. *PLoS One*. 2015; 10(4):e0123272. [PubMed: 25879933]
31. Pestilli F, Yeatman JD, Rokem A, Kay KN, Wandell BA. Evaluation and statistical inference for human connectomes. *Nat Methods*. 2014; 11(10):1058–63. [PubMed: 25194848]
32. Oishi, K. *MRI atlas of human white matter*. Amsterdam: Elsevier Academic Press; 2011.
33. Delbeuck X, Van der Linden M, Collette F. Alzheimer's disease as a disconnection syndrome? *Neuropsychol Rev*. 2003; 13(2):79–92. [PubMed: 12887040]
34. Oishi K, Mielke MM, Albert M, Lyketsos CG, Mori S. DTI analyses and clinical applications in Alzheimer's disease. *Journal of Alzheimer's disease: JAD*. 2011; 26(Suppl 3):287–96. [PubMed: 21971468]
35. Toga AW, Thompson PM. Connectomics sheds new light on Alzheimer's disease. *Biol Psychiatry*. 2013; 73(5):390–2. [PubMed: 23399468]
36. Jack CR Jr, Barnes J, Bernstein MA, Borowski BJ, Brewer J, Clegg S, et al. Magnetic resonance imaging in Alzheimer's Disease Neuroimaging Initiative 2. *Alzheimers Dement*. 2015; 11(7):740–56. [PubMed: 26194310]
37. Xie S, Xiao JX, Gong GL, Zang YF, Wang YH, Wu HK, et al. Voxel-based detection of white matter abnormalities in mild Alzheimer disease. *Neurology*. 2006; 66(12):1845–9. [PubMed: 16801648]
38. Canu E, McLaren DG, Fitzgerald ME, Bendlin BB, Zoccatelli G, Alessandrini F, et al. Microstructural diffusion changes are independent of macrostructural volume loss in moderate to severe Alzheimer's disease. *Journal of Alzheimer's disease: JAD*. 2010; 19(3):963–76. [PubMed: 20157252]
39. Bozzali M, Cercignani M, Sormani MP, Comi G, Filippi M. Quantification of brain gray matter damage in different MS phenotypes by use of diffusion tensor MR imaging. *AJNR American journal of neuroradiology*. 2002; 23(6):985–8. [PubMed: 12063230]
40. Cercignani M, Bozzali M, Iannucci G, Comi G, Filippi M. Magnetisation transfer ratio and mean diffusivity of normal appearing white and grey matter from patients with multiple sclerosis. *Journal of Neurology, Neurosurgery, and Psychiatry*. 2001; 70(3):311–7.
41. Chiapponi C, Piras F, Piras F, Fagioli S, Caltagirone C, Spalletta G. Cortical grey matter and subcortical white matter brain microstructural changes in schizophrenia are localised and age independent: a case-control diffusion tensor imaging study. *PLoS One*. 2013; 8(10):e75115. [PubMed: 24124469]
42. Whitwell JL, Avula R, Senjem ML, Kantarci K, Weigand SD, Samikoglu A, et al. Gray and white matter water diffusion in the syndromic variants of frontotemporal dementia. *Neurology*. 2010; 74(16):1279–87. [PubMed: 20404309]
43. Kantarci K, Petersen RC, Boeve BF, Knopman DS, Weigand SD, O'Brien PC, et al. DWI predicts future progression to Alzheimer disease in amnesic mild cognitive impairment. *Neurology*. 2005; 64(5):902–4. [PubMed: 15753434]
44. Muller MJ, Greverus D, Dellani PR, Weibrich C, Wille PR, Scheurich A, et al. Functional implications of hippocampal volume and diffusivity in mild cognitive impairment. *Neuroimage*. 2005; 28(4):1033–42. [PubMed: 16084115]

45. Hugenschmidt CE, Peiffer AM, Kraft RA, Casanova R, Deibler AR, Burdette JH, et al. Relating imaging indices of white matter integrity and volume in healthy older adults. *Cereb Cortex*. 2008; 18(2):433–42. [PubMed: 17575289]
46. Nir TM, Jahanshad N, Toga AW, Bernstein MA, Jack CR Jr, Weiner MW, et al. Connectivity network measures predict volumetric atrophy in mild cognitive impairment. *Neurobiol Aging*. 2015; 36(Suppl 1):S113–20. [PubMed: 25444606]
47. Jian B, Vemuri BC, Ozarslan E, Carney PR, Mareci TH. A novel tensor distribution model for the diffusion-weighted MR signal. *Neuroimage*. 2007; 37(1):164–76. [PubMed: 17570683]
48. Douaud G, Jbabdi S, Behrens TE, Menke RA, Gass A, Monsch AU, et al. DTI measures in crossing-fibre areas: increased diffusion anisotropy reveals early white matter alteration in MCI and mild Alzheimer's disease. *Neuroimage*. 2011; 55(3):880–90. [PubMed: 21182970]
49. GadElkarim, JJ.Zhan, L.Yang, SL.Zhang, AF.Altshuler, L.Lamar, M., et al., editors. 2011 IEEE International Symposium on Biomedical Imaging: From Nano to Macro. 2011. TDF-TRACT: Probabilistic tractography using the tensor distribution function. March 30 2011–April 2 2011
50. Nocedal, J., Wright, S. Numerical optimization. Springer Science & Business Media; 2006.
51. Mehrotra S. On the Implementation of a Primal-Dual Interior Point Method. *SIAM Journal on Optimization*. 1992; 2(4):575–601.
52. Gondzio J. Interior point methods 25 years later. *European Journal of Operational Research*. 2012; 218(3):587–601.



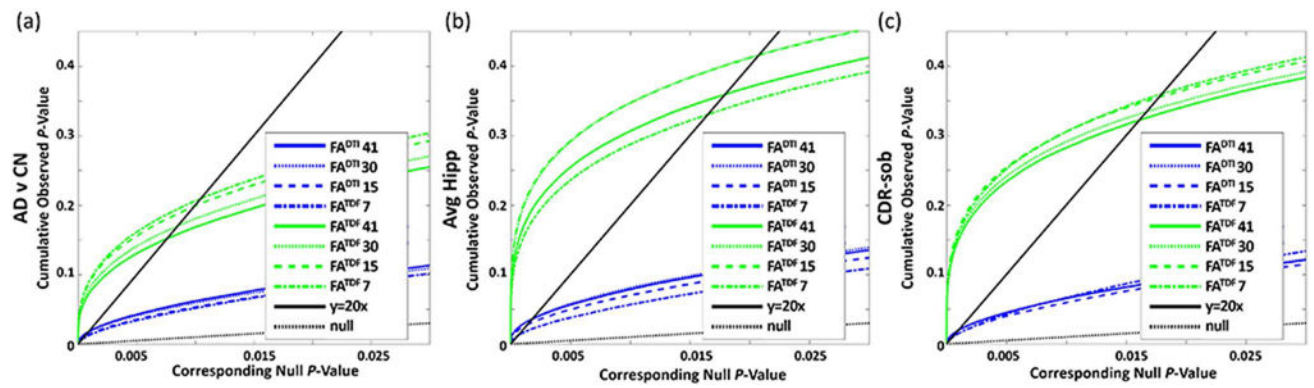


**Figure 1.** Diffusion fractional anisotropy (FA) maps, (a) FA<sup>TDF</sup> and (b) FA<sup>DTI</sup>, are shown for a single subject calculated from 41, 30, 15 and 7 gradient direction sets. The FA<sup>TDF</sup> maps show more sharply defined white matter (WM) boundaries, with much less signal drop-out in regions near the cortex that tend to have less coherent WM, compared to FA<sup>DTI</sup> maps. (c) *T*-maps in regions where FA<sup>DTI</sup> and FA<sup>TDF</sup> maps are significantly different reveal lower FA<sup>DTI</sup> values (negative association) throughout the tissue regardless of angular resolution (FDR critical *P*-value for 41 gradients = 0.047, 30 gradients = 0.047, 15 gradients = 0.047, and 7 gradients = 0.046).



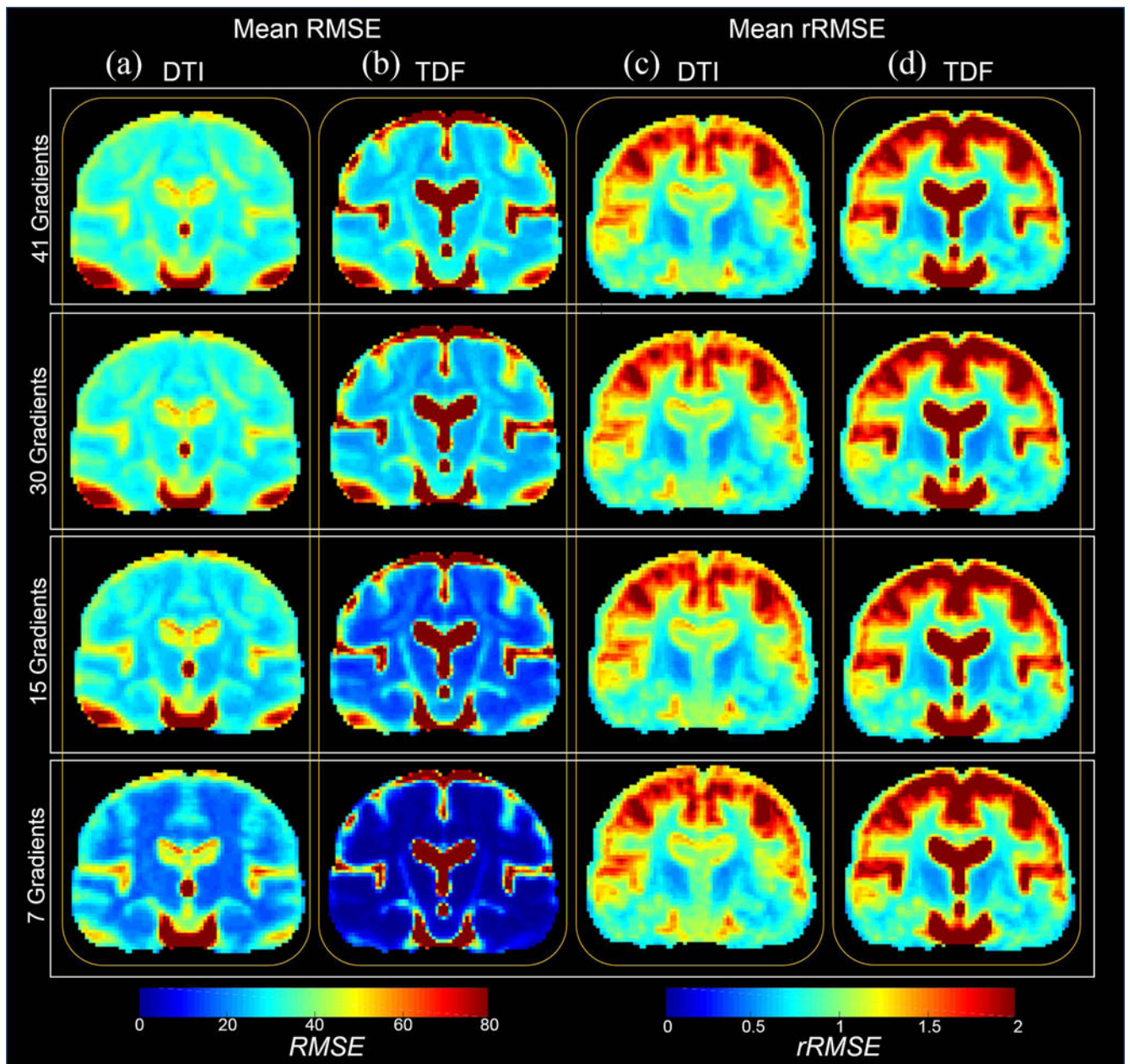
**Figure 2.**

(a) *Beta*-maps show regions where lower FA<sup>DTI</sup>, FA<sup>TDF</sup>, and GFA<sup>ODF</sup> is significantly associated with AD diagnosis, higher CDR-sob cognitive deficits, and lower average bilateral hippocampal volume. Across tests, FA<sup>TDF</sup> maps (*left, middle row,*) consistently show larger effect sizes in temporal lobe and hippocampal regions. This is denoted by greater  $\beta$ -value magnitudes and more pervasive significant associations. The patterns are also more in line with the expected topography of the disease effects. (b) Cumulative distribution function (CDF) plots show effect sizes for FA<sup>DTI</sup>, FA<sup>TDF</sup>, and GFA<sup>ODF</sup> statistical associations. FA<sup>TDF</sup> maps (*green curves*) are consistently the most sensitive metric (denoted by the higher critical *P*-values controlling the FDR, i.e., the highest non-zero *x*-coordinate where the CDF crosses the  $y=20x$  line). (c) The absolute number and percentage of total significant voxels surviving FDR correction, showing an association direction opposite to that traditionally accepted as showing impairment—*dark blue* voxels highlighted by boxes in (a). FA<sup>DTI</sup> and GFA<sup>ODF</sup> associations show ~8–15%, while FA<sup>TDF</sup> tests show <0.5% suggesting that FA<sup>TDF</sup> may be handling computations better in areas with crossing fibers.



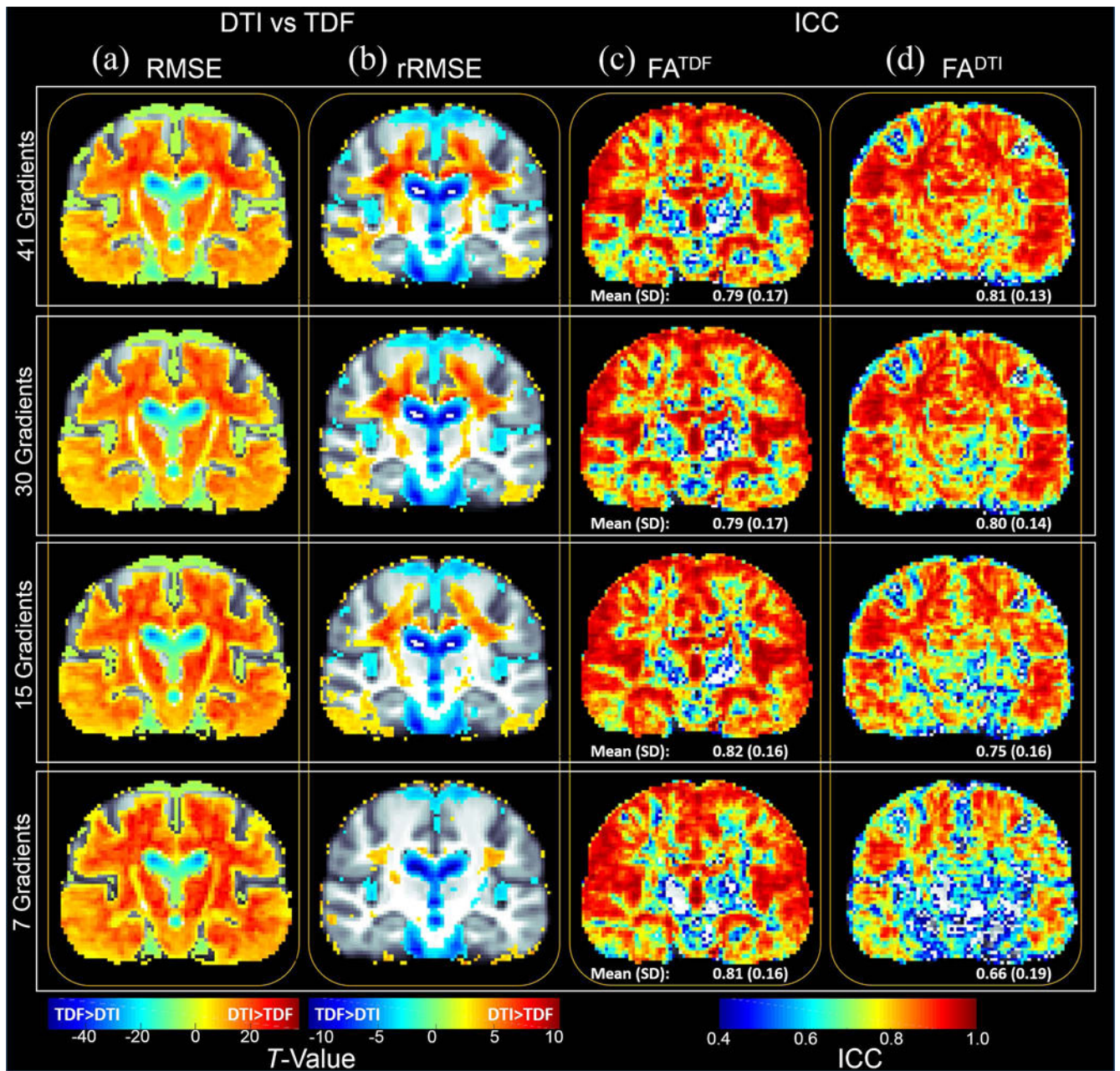
**Figure 3.**

Cumulative distribution function (CDF) plots of statistical associations between (a) AD diagnosis, (b) average bilateral hippocampal volume, and (c) CDR-sob and FA<sup>DTI</sup> or FA<sup>TDF</sup> maps computed from 41, 30, 15 and 7 gradient direction sets. FA<sup>TDF</sup> maps (*green curves*) are consistently the most sensitive metric (denoted by the higher critical *P*-values controlling the FDR, i.e., the highest non-zero *x*-coordinate where the CDF crosses the  $y=20x$  line) across all gradient subsets. Curves correspond to values listed in Table 1.



**Figure 4.**

The left two columns show the root mean squared error (RMSE) maps from the (a) DTI and (b) TDF model fit, averaged across 53 healthy control subjects. The right two columns show the relative RMSE (rRMSE) maps from the (c) DTI and (d) DTI model fit, trained on baseline scans and tested on 3-month follow-up scans in each of 33 control subjects individually, and averaged across the group.



**Figure 5. Statistical differences in reliability between TDF and DTI models**

(a)  $T$ -maps in regions where the root mean squared error (RMSE) maps of the TDF and DTI model fit in 53 healthy controls are significantly different reveal higher error for DTI (positive association) throughout the tissue (*red*) regardless of angular resolution (FDR critical  $P$ -value for 41 gradients = 0.041, 30 gradients = 0.041, 15 gradients = 0.042, and 7 gradients = 0.043). (b)  $T$ -maps in regions where the relative RMSE (rRMSE) maps of the TDF and DTI model fit, trained on 33 healthy controls baseline scans and tested on 3-month follow-up scans, are significantly different reveal higher error for DTI (positive association) in the tissue (*red*), particularly in regions of known crossing fibers (FDR critical  $P$ -value for 41 gradients = 0.024, 30 gradients = 0.021, 15 gradients = 0.017, and 7 gradients = 0.001);

the TDF model shows higher error only in CSF. **(c)** Intra-class correlation (ICC) maps in regions with a significant ICC between baseline and 3-month follow-up FA<sup>TDF</sup> maps (FDR critical  $P$ -value for 41 gradients = 0.049, 30 gradients = 0.049, 15 gradients = 0.049, and 7 gradients = 0.049) and **(d)** FA<sup>DTI</sup> maps (FDR critical  $P$ -value for 41 gradients = 0.050, 30 gradients = 0.050, 15 gradients = 0.049, and 7 gradients = 0.047). The mean ICC and standard deviation (SD) of the ICC across all voxels are reported below each mapped coronal slice.

**Table 1**

Comparison of FA<sup>DTI</sup> and FA<sup>TDF</sup> statistical effect sizes, based on FDR critical *P*-value and percent of voxels surviving FDR (22), for the full 41 gradient angular resolution, and each of the 30, 15, and 7 subsampled gradient subsets, followed by the absolute number and percent of total significant voxels surviving FDR correction, showing an association direction opposite to that traditionally accepted as showing impairment (i.e. higher FA with increased deficits).

Gradients	FDR Critical P-Value				Surviving Voxels				Surviving Voxels w/Opposite Association				
	41	30	15	7	41	30	15	7	41	30	15	7	
ADVCN	FA <sup>TDF</sup>	0.0076	0.0085	0.0099	0.0105	15.17%	16.99%	19.83%	20.91%	0.22% (59)	0.17% (53)	0.10% (34)	0.16% (59)
	FA <sup>DTI</sup>	0.0009	0.0008	0.0005	0.0004	1.80%	1.63%	1.07%	0.71%	13.60% (441)	13.75% (404)	13.43% (259)	5.92% (76)
Avg Hipp	FA <sup>TDF</sup>	0.0179	0.0208	0.0208	0.0165	35.88%	41.57%	41.53%	32.99%	0.18% (117)	0.14% (101)	0.10% (76)	0.17% (98)
	FA <sup>DTI</sup>	0.0018	0.0018	0.0014	0.0006	3.56%	3.65%	2.80%	1.21%	14.80% (949)	13.81% (909)	13.91% (701)	10.11% (220)
CDR-sob	FA <sup>TDF</sup>	0.0161	0.0167	0.0177	0.0180	32.18%	33.42%	35.48%	36.00%	0.15% (88)	0.16% (97)	0.13% (80)	0.18% (117)
	FA <sup>DTI</sup>	0.0010	0.0009	0.0005	0.0004	1.98%	1.82%	1.05%	0.88%	14.20% (506)	14.27% (468)	13.70% (260)	4.52% (72)

**Table 2**

Mean and standard deviation (SD) Cohen's  $d$  effect sizes across 10 folds, for picking up  $FA^{DTI}$  and  $FA^{TDF}$  group differences between AD patients and healthy CN, across angular resolutions.

Gradients	$FA^{DTI}$ Mean (SD)	$FA^{TDF}$ Mean (SD)
41	1.64 (0.11)	1.90 (0.06)
30	1.63 (0.14)	1.88 (0.06)
15	1.64 (0.15)	1.87 (0.07)
7	1.77 (0.39)	1.95 (0.08)

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