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The Permeability of Vacuum and the Revised International System of Units

Ronald B. Goldfarb[*]
National Institute of Standards and Technology, Boulder, CO 80305, USA

Abstract

The International System of Units (SI) is expected to be revised such that all seven base units, including the kilogram, will be defined in terms of fixed numerical values of seven defining constants. The revised SI will include a redefinition of the ampere. One consequence is that the permeability of vacuum will not have a fixed numerical value but will become, in principle, a measurable quantity. The constitutive relation among magnetic flux density, magnetic field strength, and magnetization will not change. However, its expression in the centimeter-gram-second system of electromagnetic units (EMU), where the permeability of vacuum is unity, will no longer be ontologically equivalent, and quantities will not be exactly convertible to the SI. Already contrary to international convention, the still common EMU system will become obsolete.

Index Terms

Electromagnetism; permeability of vacuum; permeability of free space; magnetic constant; magnetic units; International System of Units; electromagnetic system of units

I. REVISION OF THE SI

At its 106th meeting in October 2017, the International Committee for Weights and Measures (**CIPM**) formally recommended a major redefinition of the International System of Units (**SI**). It is expected that it will be adopted at the 26th General Conference on Weights and Measures (**CGPM**) in November 2018 and that the revised SI would then come into practice on the following World Metrology Day, 20 May 2019. The revised SI will be the most significant change in units of measure since the meter-kilogram-second-ampere (**MKSA**) system was adopted by the CGPM in 1954.

In the revised SI, the kilogram, ampere, kelvin, and mole will be redefined in terms of newly fixed values of the Planck constant, the elementary charge, the Boltzmann constant, and the Avogadro constant, respectively. These four constants will join the cesium hyperfine transition frequency, the speed of light, and the luminous efficacy of radiation of frequency 540 THz as the seven defining constants of the SI [CGPM 2011]. Under the revised SI, the definition of the units will be distinct from their realization.

Corresponding author: Ronald B. Goldfarb (ron.goldfarb@nist.gov).
[*] Life Fellow, IEEE

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In 2014, the 25th CGPM believed that the uncertainties in the determinations of the four fundamental constants were not yet sufficiently small to revise the SI [CGPM 2014]. The new values of the constants will be based on determinations that have been published, or at least accepted for publication, by 1 July 2017, and which were used by the International Council for Science's Committee on Data for Science and Technology (**CODATA**) to recommend values and uncertainties of the four fundamental constants needed to revise the present SI [Mohr 2018]. The four values will later be fixed (i.e., will become exact), and the full set of seven fixed constants will define all units in the revised SI [Newell 2014].

In this letter, a distinction is made between fixed, defining constants under the revised SI (e.g., the Planck constant) and constants that are experimentally determined (e.g., the fine structure constant).

II. THE AMPERE

Part of the revised SI is constructed progressively: The hyperfine transition frequency of cesium-133 already defines the second. The second and the fixed value of the speed of light already define the meter. The second, the meter, and a newly fixed value of the Planck constant will redefine the kilogram. The elimination of the kilogram material artifact as a standard was one of the motivations for the revised SI; masses of official kilogram copies have, for unknown reasons, drifted on the order of 50 parts in 10^9 over 100 years with respect to the defining artifact [Mills 2011].

The second and a newly fixed value of the elementary charge will redefine the ampere: 1 ampere will be the electric current corresponding to the flow of $1/(1.602\ 176\ 634 \times 10^{-19})$ elementary charges per second [BIPM 2016]. (Here, the value of the elementary charge from the special 2017 CODATA adjustment [Mohr 2018] is used.) The definition of the ampere in the present SI is "that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length" [BIPM 2006]. That definition had the effect of fixing the value of the permeability of vacuum μ_0 to be exactly $4\pi \times 10^{-7}$ N/A$^2 \equiv 4\pi \times 10^{-7}$ H/m, as may be seen from Ampère's force law: $F/l = \mu_0 I_1 I_2 / 2\pi d$, with force per unit length $F/l = 2 \times 10^{-7}$ N/m, currents I_1 and $I_2 = 1$ A, and separation $d = 1$ m.

The new definition of the ampere fixes the value of e instead of μ_0, and as a result, μ_0 must be determined experimentally [BIPM 2016]. Similarly, the permittivity of vacuum $\varepsilon_0 = 1/\mu_0 c^2$ must be determined experimentally (as it was before c was fixed in 1983). The product $\varepsilon_0 \mu_0 = 1/c^2$ remains exact.

III. THE FINE STRUCTURE CONSTANT

In fact, the experimental value of μ_0 will be based on that of the dimensionless fine structure constant α, the coupling constant of the electromagnetic force

$$\mu_0 = 2h\alpha/ce^2 \quad (1)$$

where h is the newly fixed Planck constant, c is the fixed speed of light in vacuum, and e is the newly fixed elementary charge (equal to the absolute value of the electron charge). The relative standard uncertainties in μ_0, ε_0, and α will be identical.

Another quantity of interest in magnetics, the magnetic flux quantum $\phi_0 = h/2e$, equal to the reciprocal of the Josephson constant K_J, will be fixed in the revised SI. However, the Bohr magneton $\mu_B = eh/4\pi m_e$ will depend on the value of the electron rest mass m_e, calculated from the experimentally determined Rydberg constant R_∞ and α : $m_e = 2hR_\infty/c\alpha^2$. Because e and h will have no uncertainty, the uncertainty of μ_B in the revised SI will be an order of magnitude smaller than at present [Mohr 2016, 2018].

The fine structure constant α, intriguingly almost equal to 1/137, has many physical interpretations in atomic physics, high-energy physics, quantum electrodynamics, and cosmology [Kragh 2003]. Because α is measurable by many different modalities, it is possible that its accepted value, and therefore that of μ_0, will evolve slightly over the years after the redefinition of the SI.

In the CODATA compilations under the present SI, h is a parameter that is adjusted according to an algorithm that forces agreement among fixed constants μ_0 and c and experimental constants α, h, and e in (1) [Mills 2006, Mohr 2016]. In the revised SI, the fixed constants will be h, c, and e and the experimental constant will be α. One way to represent that evolution is

$$(h/e^2)_{\mathrm{exp}} = (\mu_0 c/2)_{\mathrm{fixed}} \cdot (1/\alpha)_{\mathrm{exp}} \quad \text{[present SI]} \quad (2)$$

$$(\mu_0)_{\mathrm{exp}} = (2h/ce^2)_{\mathrm{fixed}} \cdot (\alpha)_{\mathrm{exp}} \quad \text{[revised SI]} \quad (3)$$

where the subscript “exp” denotes “experimental.”

If one uses the values of $(h, c, e)_{\mathrm{fixed}}$ and $(\alpha)_{\mathrm{exp}}$ from the special CODATA compilation [Mohr 2018] in (3), one obtains

$$\mu_0 = 1.256\,637\,0617 \times 10^{-6}\ \mathrm{H/m} \quad (4)$$

with some uncertainty in the last decimal digit owing to the uncertainty in α. This value is identical to $4\pi \times 10^{-7}$ H/m to 9 significant figures. The point is, μ_0 will be as close to $4\pi \times 10^{-7}$ H/m as allowed by the uncertainty in α and the CODATA truncation of h and e.

The practical consequences of μ_0 not being a fixed constant will be nil [Mills 2006]; however, the philosophical implication for researchers still using the centimeter-gram-second (**CGS**) system of electromagnetic units (**EMU**) should provoke introspection. (Gaussian units and EMU are the same for magnetic properties.) As noted by Davis [2017], "conversion factors to CGS systems, which presently make use of the exact relation $\{\mu_0/4\pi\} \equiv 10^{-7}$, will no longer be strictly correct after the revised SI takes effect." (The curly brackets mean that one removes the units associated with the quantity within.)

IV. THE REJECTED ALTERNATIVE

Another possibility was under consideration, but rejected, by the Working Group on the SI (**WGSI**) of CIPM's Consultative Committee for Electricity and Magnetism (**CCEM**). In addition to fixing the Planck constant, WGSI could have recommended fixing the Planck charge $q_P = (2\varepsilon_0 hc)^{1/2} = (2h/\mu_0 c)^{1/2} = e/\alpha^{1/2}$ instead of fixing the elementary charge e [Stock 2006]. Fixing q_P would have kept μ_0 at its familiar value of $4\pi \times 10^{-7}$ H/m and made e dependent on measurements of α.

In its deliberations, the WGSI was partly influenced by the rationale for a fixed value of e laid out by Mills et al. [2006]: By the 1990s, the ampere was being realized by the Josephson effect for voltage and the quantum Hall effect for resistance (both functions of h and e) and Ohm's law, not by the force on currents in parallel wires. A definition of current in terms of a fixed value of e would bring the practical quantum electrical standards into exact agreement with the SI [BIPM 2016]. WGSI's decision informed CCEM's [2007] Recommendation E1 to the CIPM, which was reaffirmed in 2009.

More recently, the Consultative Committee for Thermometry [CCT 2017], the Consultative Committee for Mass and Related Quantities [CCM 2017], and the Consultative Committee for Units [CCU 2017] also recommended that the CIPM proceed with the planned redefinition of the kilogram, ampere, kelvin, and mole.

The change of μ_0 from a fixed constant to an experimentally determined value has implications for units of measure in magnetics.

V. A BRIEF HISTORY

Maxwell used the term "magnetic inductive capacity" for the ratio of flux density B and magnetic field strength H. The term "permeability" originated with Thomson [1872]. In Maxwell's treatment (in CGS units), B and H were fundamentally different quantities [Silsbee 1962]. This was codified in 1930 by the International Electrotechnical Commission (**IEC**), which assigned the unit "gauss" to B and the unit "oersted" to H, and specified that their ratio, the permeability of vacuum, had a numerical value of unity and physical dimensions yet to be determined [Kennelly 1931].[1] At meetings in 1931–1934, committees of the International Union of Pure and Applied Physics **(IUPAP)** endorsed the IEC

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[1] Despite the IEC 1930 compromise statement on dimensions, it was universally accepted by physicists that, in the EMU system, B and H have the same physical dimensions and the permeability of vacuum is, in fact, dimensionless [Birge 1934], just as described by Maxwell [Birge 1935].

resolutions and, while acknowledging the "practical" units, opined that "the CGS system of units is suitable for the physicist" [Kennelly 1933].

Although the IEC abandoned the EMU system in 1935 (a decision affirmed in 1938) in favor of the Giorgi [1901] MKSX system [Ascoli 1905, Giorgi 1905] (with the fourth fundamental unit "X" not assigned to the ampere "A" until 1950 by IEC and 1954 by CGPM) [Petley 1995], CGS has remained popular with generations of physicists. Indeed, when the MKSX system was adopted by CGPM [1948], it noted that "the International Union of [Pure and Applied] Physics … does not recommend that the CGS system be abandoned by physicists."

In 1960, the 11th CGPM established the name *Système International d'Unitès* (SI) for the system of MKSA units, the kelvin, and the candela [BIPM 2006]. In 1964, the U.S. National Bureau of Standards made it a policy to require SI units in its reports [Chisholm 1967]. In 1966, the Institute of Electrical and Electronics Engineers, via its Standards Coordinating Committee 14 on Quantities and Units, recommended SI units for all published work. Notably, a specific recommendation was that "the various CGS units of electrical and magnetic quantities are no longer to be used. This includes … the gilbert, oersted, gauss, and maxwell" [Page 1966]. IUPAP [1987] eventually recommended "that authors be encouraged to adapt [sic] the SI units for data in physics journals," and more recently IUPAP [2008] endorsed the projected revised SI.

VI. RECOMMENDATIONS

In the still popular EMU system, quantities are exactly convertible to the present SI by factors of 4π and powers of 10. The requirement that the permeability of vacuum have a value of unity precluded its actual experimental determination, just as it is a fixed constant in the present SI. However, the nature of electromagnetic reality will be very different in the revised SI. Compared to EMU, the permeability of vacuum not only will have dimensions (as it does in the present SI), but its value will also, in principle, be measurable. That is, the relationship between B and H will be ontologically different in the revised SI compared to the EMU system.

Magnetics has been one of the scientific disciplines most resistant to adoption of the SI. With the revised SI, the "peaceful coexistence" of two systems of units [Silsbee 1962] is no longer feasible. The following recommendations warrant consideration.

1. Scholarly journals that publish articles in magnetics should require use of the SI and disallow EMU such as oersted, gauss, and "emu per cubic centimeter." Authors who find the expression of magnetic field strength H in units of ampere per meter to be inconvenient could instead refer to $\mu_0 H$ in units of tesla (or milli-, micro-, nano-, or picotesla). Similarly, magnetization M could be expressed as $\mu_0 M$ or as magnetic polarization J in units of tesla or millitesla.
2. For the benefit of future generations of magneticians, professors should use SI in classroom instruction. Commercial instruments and magnetometers should be programmed to report measurement results in SI.

3. In writing equations, it is adequate to use phrases such as “where μ_0 is the permeability of vacuum” (or “the vacuum magnetic permeability” or “the permeability of free space” or “the magnetic constant”) without giving a numerical value. This follows typical usage when referring to the speed of light c, the Boltzmann constant k, or the Bohr magneton μ_B.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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References

Ascoli M. On the systems of electric units. J Inst Elect Eng. 1905; 34:176–180. DOI: 10.1049/jiee-1.1905.0012

BIPM. The International System of Units. 8th. Sèvres, France: Bureau Int. des Poids et Mesures; 2006. [Online]. Available: http://www.bipm.org/utils/common/pdf/si_brochure_8.pdf

BIPM. The International System of Units. Draft 9th. Sèvres, France: Bureau Int. des Poids et Mesures; 2016. [Online]. Available: http://www.bipm.org/utils/common/pdf/sibrochure-draft-2016b.pdf

Birge RT. On electric and magnetic units and dimensions. Amer Phys Teacher. 1934; 2:41–48. DOI: 10.1119/1.1992863

Birge RT. On the establishment of fundamental and derived units, with special reference to electric units. Part II. Amer Phys Teacher. 1935; 3:171–179. DOI: 10.1119/1.1992965

CCEM. Recommendation E1: Proposed changes to the International System of Units (SI). Bureau International des Poids et Mesures; Sèvres, France: 2007. [Online]. Available: http://www.bipm.org/cc/CCEM/Allowed/25/CCEM2007-44.pdf

CCM. Recommendation G1: For a new definition of the kilogram in 2018. Bureau International des Poids et Mesures; Sèvres, France: 2017. [Online]. Available: http://www.bipm.org/cc/CCM/Allowed/16/06E_Final_CCM-Recommendation_G1-2017.pdf

CCT. Recommendation T1: For a new definition of the kelvin in 2018. Bureau Int. des Poids et Mesures; Sèvres, France: 2017. [Online]. Available: https://www.bipm.org/cc/CCT/Allowed/28/Recommendation-CCT-T1-2017-EN.pdf

CCU. Recommendation U1: On the possible redefinition of the kilogram, ampere, kelvin and mole in 2018. Bureau Int. des Poids et Mesures; Sèvres, France: 2017. [Online]. Available: http://www.bipm.org/cc/CCU/Allowed/23/CCU_Final_Recommendation_U1_2017.pdf

CGPM. Resolution 6 of the 9th CGPM. 1948. [Online]. Available: http://www.bipm.org/en/CGPM/db/9/6/

CGPM. Resolution 1 of the 24th CGPM on the Possible Future Revision of the International System of Units, the SI. 2011. [Online]. Available: http://www.bipm.org/en/CGPM/db/24/1/

CGPM. Resolution 1 of the 25th CGPM on the Future Revision of the International System of Units, the SI. 2014. [Online]. Available: http://www.bipm.org/en/CGPM/db/25/1/

Chisholm, LJ. Units of Weight and Measure. Washington, DC, USA: GPO; 1967. National Bureau of Standards, Misc. Pub. 286[Online]. Available: https://www.govinfo.gov/app/details/GOVPUB-C13-a8ff2e478908a0307a1b097ecfe6a1cc

Davis RS. Determining the value of the fine-structure constant from a current balance: Getting acquainted with some upcoming changes to the SI. Amer J Phys. 2017; 85:364–368. DOI: 10.1119/1.4976701

Giorgi G. Unità razionali di elettromagnetismo. Atti dell’ Associazione Elettrotecnica Italiana. 1901; 5:402–418.

Giorgi G. Proposals concerning electrical and physical units. J Inst Elect Eng. 1905; 34:181–185. DOI: 10.1049/jiee-1.1905.0013

IUPAP. Resolutions, IUPAP 19th General Assembly, Washington, USA. 1987. [Online]. Available: http://iupap.org/general-assembly/19th-general-assembly

IUPAP. Resolutions, IUPAP 26th General Assembly, Tsukuba, Japan. 2008. Resolution proposal submitted to the IUPAP Assembly by Commission C2 (SUNAMCO). [Online]. Available: http://iupap.org/general-assembly/26th-general-assembly/; http://archive.iupap.org/ga/ga26/file_50095.pdf

Kennelly AE. Magnetic-circuit units as adopted by the I.E.C. Trans Amer Inst Elect Eng. 1931; 50:737–743. DOI: 10.1109/T-AIEE.1931.5055863 DOI: 10.1109/T-AIEE.1931.5055864

Kennelly A E. Recent actions taken by the symbols, units, and nomenclature (S.U.N.) committee of the International Union of Pure and Applied Physics (I.P.U.) in reference to C.G.S. magnetic units. Trans Amer Inst Elect Eng. 1933; 52:647–658. DOI: 10.1109/T-AIEE.1933.5056365 DOI: 10.1109/T-AIEE.1933.5056366

Kragh H. Magic number: A partial history of the fine-structure constant. Arch Hist Exact Sci. 2003; 57:395–431. DOI: 10.1007/s00407-002-0065-7

Mills IM, Mohr PJ, Quinn TJ, Taylor BN, Williams ER. Redefinition of the kilogram, ampere, kelvin and mole: A proposed approach to implementing CIPM recommendation 1 (CI-2005). Metrologia. 2006; 43:227–246. DOI: 10.1088/0026-1394/43/3/006

Mills IM, Mohr PJ, Quinn TJ, Taylor BN, Williams ER. Adapting the International System of Units to the twenty-first century. Philosoph Trans Roy Soc A. 2011; 369:3907–3924. DOI: 10.1098/rsta.2011.0180

Mohr PJ, Newell DB, Taylor BN. CODATA recommended values of the fundamental physical constants: 2014. Rev Mod Phys. 2016; 88:035009.doi: 10.1103/RevModPhys.88.035009

Mohr PJ, Newell DB, Taylor BN, Tiesinga E. Data and analysis for the CODATA 2017 special fundamental constants adjustment. Metrologia. 2018; to be published. doi: 10.1088/1681-7575/aa99bc

Newell DB. A more fundamental International System of Units. Phys Today. 2014; 67:35–41. DOI: 10.1063/PT.3.2448

Page CH, Barrow BB, Kreer JG, Mason W, Wintringham WT. IEEE recommended practice for units in published scientific and technical work. IEEE Spectr. 1966; 3:169–173. DOI: 10.1109/MSPEC.1966.5216747

Petley BW. A brief history of the electrical units to 1964. Metrologia. 1995; 31:481–494. DOI: 10.1088/0026-1394/31/6/007

Silsbee FB. Systems of electrical units. J Res Nat Bur Stand. 1962; 66C:137–183. DOI: 10.6028/jres.066C.014

Stock M, Witt TJ. CPEM 2006 round table discussion ‘Proposed changes to the SI,’. Metrologia. 2006; 43:583–587. DOI: 10.1088/0026-1394/43/6/014

Thomson, W. Reprint of Papers on Electrostatics and Magnetism. London, U.K: Macmillan; 1872. p. 482-493.[Online]. Available: https://books.google.com/books?id=xGZDAAAAIAAJ&pg=PA482