Multiscale computing

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Multiscale computing (MSC) involves the computation, manipulation, and analysis of information at different resolution levels. Widespread use of MSC algorithms and the discovery of important relationships between different approaches to implementation were catalyzed, in part, by the recent interest in wavelets. We present two examples that demonstrate how MSC can help scientists understand complex data. The first is from acoustical signal processing and the second is from computer graphics.

Multiscale computing (MSC) is concerned with methods for computing, manipulating, and analyzing information at different resolution levels. The field has undergone tremendous advances during the past decade because of the increase in inexpensive, powerful hardware. The concurrent development of advanced algorithms and data structures to reduce computational, data access, and communication costs has played an equally vital role. MSC is used in many disciplines, but its presence is often obscured, because it appears under several different names depending on the field of application. For instance, it appears as *multiresolution analysis* in wavelet theory, *compression* in signal analysis, *progressive meshing* in computer graphics, and *clustering* in the study of databases. The discovery of important relationships between different algorithms and implementation techniques for MSC was catalyzed, in part, by the development of wavelet theory. The primary aim of this paper is to describe how MSC can help scientists understand complex data through two examples: one from acoustical signal processing and a second from computer graphics. But first, we take a short digression to explain some basic concepts used in the examples.

Wavelets. Until recently scientists used approximation methods based on the ease of proving error bounds and computation. Polynomials and Fourier methods dominated the scientific landscape despite well known drawbacks. For instance, polynomials diverge on unbounded intervals and the *Gibbs phenomenon* plagues the Fourier expansion of discontinuous functions. The increase in powerful computers during the last decade freed scientists from methods that involve only simple computations. What are the consequences? Given a function f, our natural inclination is to use a basis with functions that converge quickly and have properties similar to f. Wavelet theory is based on this idea. The Fourier expansion usually works well for periodic functions that are smooth, because the associated basis functions (cosines and sines) are smooth and periodic. What if f is a piecewise constant function? We are more likely to accurately approximate f by using a basis that consists of *characteristic functions*[¶] with unit support[‖] such as $B_1 = \{\chi_{[m,m+1[}$ for $m \in \mathcal{Z}\}$ than by Fourier methods. Suppose additionally that the discontinuities of $f(x)$ lie at the points $x = 0.5$, 2.0, and 4.5. A better approximation could be made by using a basis with a finer scale, such as $B_{1/2} = \{\chi_{[n/2,(n+1)/2[}$ for $n \in \mathcal{Z}\}$. If we already have a coarse approximation by using B_1 and want one of a finer scale using $B_{1/2}$, we do not have to compute from scratch if we use wavelets.

In the context of wavelet theory, the basis elements χ in the example shown above are the *scaling functions*, and the corresponding wavelets are the *Haar* wavelets, which are smaller or larger versions of the *generating* function $\psi_{\text{Haar}}(x)$, which equals 1 for $0 \le x < \frac{1}{2}$, -1 for $\frac{1}{2} \le x < 1$, and 0 elsewhere. Addition and subtraction of Haar wavelets are efficient tools for refining the coarser approximation.

We are now prepared for the formal definition: *Wavelets are families of functions*,

$$\psi_{a,b} = |a|^{-1/2}\psi\left(\frac{x-b}{a}\right); \quad a,b \in \mathcal{R}, \quad a \neq 0,$$

generated from a single function ψ *by dilations and translations* (1). One of the applications of wavelet theory is to construct a basis for efficient and accurate approximation of functions and signals at different scales and to provide for a simple and fast means for moving between different scales according to user needs. To approximate functions, most engineers begin with a generating wavelet $\psi(x)$, which has *compact support*** and $\int \psi(x)dx = 0$, because a considerable number of theorems have been developed for this class of wavelets.[††] A second class of applications associated with wavelets involves time-frequency analysis of nonstationary signals. It has generated more interest than MSC. The exponential increase in wavelet-related patents issued in the United States during the last decade and the large proportion of those using time-frequency analysis are quantitative measures of this observation (2). Time-frequency analysis involves transforms, which are defined as follows: For wavelets with mother function ψ, the *continuous wavelet transform* for a function $f(x) \in L^2(\mathcal{R})$ is

$$\langle \psi_{a,b}, f \rangle = |a|^{-1/2} \int dx \cdot \psi\left(\frac{x-b}{a}\right) \cdot f(x)$$

for $a,b \in \mathcal{R}$, $a \neq 0$, and the *discrete wavelet transform* is

$$\langle \psi_{m,n}, f \rangle = |a_0|^{-m/2} \int dx \cdot \psi(a_0^{-m} x - n b_0) \cdot f(x)$$

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Abbreviation: MSC, multiscale computing.

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[¶]The *characteristic function* $\chi_{[a,b[}$ on the finite interval $[a,b[$ is one on $[a,b[$ and naught elsewhere.

[‖]The *support* of a function $f(x)$ is the set of x for which function is nonzero.

**A function has *compact support* if its support (the set of points on which the function is nonzero) is a compact (closed and bounded) set.

[††]Actually, most of the theorems apply to a wider class of wavelets ψ that satisfy the weaker *admissibility condition*: $2\pi \int d\xi\, \xi^{-1} \hat{\psi}(\xi)^2 < \infty$, where $\hat{\psi}$ denotes the Fourier transform of the wavelet and ξ the variable in the transform space. Because testing for convergence of the integral is cumbersome, the stricter requirement (given above) is normally used in practice because it is easier to verify.

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for $a_0 > 1$, $b_0 \neq 0$. Two other important topics associated with wavelets are filter banks (3) and operator theory (4).

The remarkable ability of wavelets to facilitate movement between families of bases that are constructed from a single function but of different scale lies at the heart of many algorithms associated with MSC, but it is not the only method for scaling data. The next section is a summary of auditory modeling work in which the Mellin transform is used to rescale human speech signals. References to scientifically important and beautiful examples of wavelet- and nonwavelet-based MSC in graphics are given in the section that follows.

Rescaling Human Speech Signals via the Mellin Transform. A fundamental question in the study of human auditory mechanisms is the following. Suppose that a man, woman, and child are asked to read aloud a sentence. *How does the human auditory system process the information and identify that the same sentence has been spoken by all three?* To answer this question, we first must tackle the simpler one of how the auditory system adjusts for the difference in body sizes and vocal tracts of the humans who are speaking. The existence of some kind of intelligent processing is conceivable, because we can appreciate and recognize the same, beautiful tune in *C* major whether it is played on a violin, viola, cello, or bass even when there is a shift in octaves.

Irino and Patterson (5) proposed a model to answer this question. They demonstrated that essential features in speech signals for the same vowel can be mapped consistently by the Mellin transform to the (nearly same) location in the time-scale plane regardless of the speaker's body size. The Mellin transform $F(p)$ of a function $f(x)$ is defined as

$$F(p) = \int_0^\infty f(x)x^{p-1}dx = \int_0^\infty f(x)e^{(p-1)\ln(x)}dx,$$

where x and s denote the respective variable parameters for the function and its transform. Irino and Patterson go on to note that the Mellin hypothesis enables us to interpret the peripheral auditory system as an optimal signal processor. The mathematical model operates as follows. First, the wavelet transform is used to simulate the auditory filterbank. The wavelet transform is suited for this role, because it is transparent to the Mellin transform, i.e., all of the wavelet kernels (even with dilations) are mapped into a uniform distribution by the Mellin transform, and thus it is easy to deconvolve the filter component from the components of sound in the outside world. Second, the kernel function for the auditory filter is derived as a solution to an eigenvalue problem that satisfies certain minimal uncertainty constraints in the time-scale plane; both psychophysical and physiological data are simulated well by this filter function (6).

The model in this section maps signals produced by humans of different scales to the same location in time-scale space. In the next section we consider the reverse mapping. Graphical models of objects are mapped to several different scales according to user needs or the virtual distance from the object to the user.

Multiscale Computer Graphics. One of the goals of modern computer graphics has been to develop increasingly better technologies for fast and accurate presentation of three-dimensional virtual objects at resolution levels that are appropriate for individual user needs. These technologies have been used in the development of flight simulators for training military and civilian pilots. The devices work as follows. When a simulated object is beyond a certain distance, a very crude model of the object is used. The model has relatively few meshpoints with virtually no rendering, shading, or texture. When a pilot approaches the object and reaches certain distance thresholds in virtual space, the model for the object is replaced with one of a finer scale with more detailed rendering, shading, and texture. Conversely, as one moves away from an object, successively coarser models are substituted. This technology has been extended for use in walk-throughs and fly-bys for electronic games.

Recently scientists have been exploring a new generation of MSC algorithms for a wider range of applications. Examples from this emerging area of research are the Digital Michelangelo Project at Stanford and the Multi-Res Group Modeling Group at Cal Tech. One goal of the Michelangelo project is to generate realistic three-dimensional images of some of the artist's sculptures from laser and range scanner data (7). Art historians will be able to "see" and study the chisel marks to learn about the shapes of the chisels and techniques used to carve the marble. The marks on the actual sculpture are so fine that they cannot be perceived in detail by the unaided eye. Another goal is to enhance the visits of museum goers and archives of virtual digital museums. For this second purpose, a coarser resolution image will suffice, because the objects will be viewed for their artistic beauty.

MSC of three-dimensional models has also been conducted by Schröder and coworkers (8, 9) by using wavelet- and nonwavelet-based approaches for industrial as well as academic applications. The works of various scientific groups in this discipline differ primarily in the choice of primitives used for meshing and how and what types of information are stored and compressed during MSC. Surveys on MSC of meshes are given on the home pages of Owen (www.andrew.cmu.edu/user/sowen/survey/) and Guibas (www-graphics.stanford.edu/guibas/GeomSem/99winter/schedule.html) and in the *Annual Proceedings of ACM SIGGRAPH* conference and *ACM Transactions on Graphics*. A good MSC algorithm for mesh generation will minimize the amount of storage space for descriptors (e.g., node locations and connectivities and normal and tangent vectors) as much as possible, facilitate movement between meshes of different scales, and facilitate efficient rendering and shading algorithms at all scales. The need for these requirements can be understood in light of an example from the Michelangelo project. A three-dimensional 0.25-mm resolution scan of the 2.7-mm statue of Saint Matthew generated 102,868,637 points, making a 644-MB file. Some approaches to multiscale meshing that have been proposed are mesh compression, wavelets, zerotree coders, and irregular subdivision. The plethora of topological primitives and their possible contortions make it impossible to conclude which algorithm will always be the *best*. The only foregone conclusion is that state-of-the-art algorithms for graphics will continue to be based on meshing, rendering, and shading algorithms that are designed in ways that complement each other's strengths to improve performance.

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